Index Tracking

With Autoencoders

Introduction

- Index Tracking is an investment strategy that aims to replicate the performance of a specific market index. Instead of actively selecting stocks or other securities, index trackers invest in the same components and proportions as the index they are tracking.

Types of Index Tracking

- 1. Full Replication: Involves buying all the securities in the index in the same proportions.

- 2. Partial Replication (Sampling): Involves buying a representative sample of the securities in the index.

- 3. Synthetic Replication: Uses financial derivatives such as swaps to replicate the index's performance without actually owning the securities.

Examples of Index Tracking

- S&P 500 Index: Many mutual funds and ETFs, like the Vanguard 500 Index Fund (VFIAX) and SPDR S&P 500 ETF (SPY), track the performance of the S&P 500.

- NIFTY 50 Index (India): Funds like the UTI Nifty Index Fund and HDFC Index Fund track the performance of the NIFTY 50.

Index Tracking in the Indian Scenario

- In India, index tracking is commonly applied to major indices such as:

- NIFTY 50: Tracks the performance of the 50 largest and most liquid stocks listed on the National Stock Exchange (NSE).

- BSE Sensex: Tracks the performance of 30 financially sound and established companies listed on the Bombay Stock Exchange (BSE).

Mathematical Details of Index Tracking

- The objective of an index tracker is to minimize the tracking error, which is the difference between the returns of the index and the returns of the fund.

- Tracking Error Formula

$$\text{Tracking Error} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (R_i - I_i)^2}$$

- Where:

- R_i is the return of the fund on day i,

- I_i is the return of the index on day i,

- n is the number of days in the period.

Available Traditional Methods for Index Tracking

- 1. Buy and Hold Strategy: Investors purchase and hold all the securities in the index for the long term.

- 2. Rebalancing: Regularly adjusting the holdings to match the index's composition.

- 3. Optimization Techniques: Mathematical models are used to select a subset of index components that will minimize tracking error.

- 4. Stratified Sampling: Dividing the index into different strata and selecting a representative sample from each stratum.

- 5. Equity Derivatives: Using futures, options, and swaps to gain exposure to the index without owning the underlying securities.

Autoencoders

Introduction

- Autoencoders are a type of artificial neural network used to learn efficient codings of unlabeled data. They are an unsupervised learning technique that aims to discover patterns in data.

- All students are aware of Autoencoders.
  - Lets here from you.

Components of Autoencoders

- 1. Encoder: Compresses the input data into a lower-dimensional representation.

- 2. Latent Space (Bottleneck): The compressed representation of the input data.

- 3. Decoder: Reconstructs the input data from the latent representation.

Training Autoencoders

- The goal of training an autoencoder is to minimize the difference between the input data and its reconstruction. This difference is often measured using a loss function such as mean squared error (MSE).

Types of Autoencoders

- 1. Undercomplete Autoencoders: Latent space dimension is smaller than the input dimension.

- 2. Overcomplete Autoencoders: Latent space dimension is larger than the input dimension.

- 3. Denoising Autoencoders: Reconstruct the original input from a corrupted version.

- 4. Sparse Autoencoders: Impose a sparsity constraint on the latent representation.

- 5. Variational Autoencoders (VAEs): Probabilistic models with a prior distribution on the latent space.

Applications of Autoencoders

- 1. Dimensionality Reduction: Similar to PCA, preserving important features.

- 2. Data Denoising: Remove noise from data, useful in image and audio processing.

- 3. Anomaly Detection: Detect anomalies that deviate from normal patterns.

- 4. Generative Modeling: Generate new data samples similar to the training data.