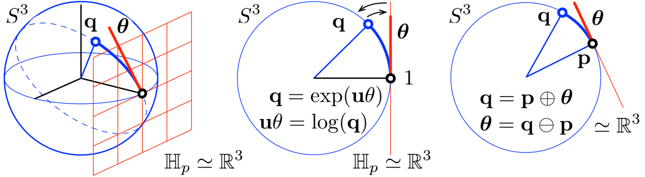

Figure 4. The S^3 manifold is a unit 3-sphere (blue) in the 4-space of quaternions $\mathbb{H}$, where the unit quaternions $\mathbf{q}^* \, \mathbf{q} = 1$ live. The Lie algebra is the space of pure imaginary quaternions $ix + jy + kz \in \mathbb{H}_p$, isomorphic to the hyperplane $\mathbb{R}^3$ (red grid), and any other tangent space $\mathcal{T}S^3$ is also isomorphic to $\mathbb{R}^3$. Tangent vectors (red segment) wrap the manifold over the great arc or *geodesic* (dashed). The centre and right figures show a side-cut through this geodesic (notice how it ressembles S^1 in Fig. 3). Mappings exp and log (arrows) map (wrap and unwrap) elements of $\mathbb{H}_p$ to/from elements of S^3 (blue arc). Increments between quaternions are expressed in the tangent space via the operators $\oplus, \ominus$ (see text).