The Makam Metalanguage
Reducing the cost of PL experimentation

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- Dependent types: capture program invariants in types
- VeriML: programs to prove the invariants
- Ur/Web: avoid SQL injections etc. in webapps statically

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- VeriML: programs to prove the invariants
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… but running experiments takes huge up-front cost

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- Implement a language from scratch?
- Extend an existing language?
- Practical aspects are important but tricky
    - efficiency? error-messages?
- Extensibility/malleability of new design is key but runs counter to doing full-fledged implementation

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$\rightarrow$ Many PL ideas stay at prototype level

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a metalanguage for quick PL prototyping

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- can model type systems, transformations to existing languages, etc.

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- can model type systems, transformations to existing languages, etc.

- reduce time for prototype from months to days
- fast changes to key design decisions
- specifications are easy to extend
- metalanguage takes care of tricky parts

Let's use Makam to model the simply-typed lambda calculus.

Abstract syntax.

$$\tau ::= Int \mid Bool \mid \tau_1 \to \tau_2$$

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typ : sort.
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tint : typ. tbool : typ.
tarrow : typ -> typ -> typ.
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$$\begin{aligned} e ::= {} & e_1 + e_2 \mid e_1 < e_2 \mid n \mid \text{if } e \text{ then } e_1 \text{ else } e_2 \\ & \mid e_1\ e_2 \mid \lambda x.e \end{aligned}$$

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```
expr : sort.
plus : expr -> expr -> expr.
lt : expr -> expr -> expr.
intconst : int -> expr.
```

Typing and evaluation relations.

Typing

$$\Gamma \vdash e : \tau$$

```
typeof : expr -> typ -> prop.
```

Big-step semantics

$$e \Downarrow e'$$

```
eval : expr -> expr -> prop.
```

Sample rule.

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```
typeof (lt E1 E2) tbool <-
  typeof E1 tint,
  typeof E2 tint.
```

Sample rule.

Let's do an easy rule first.

$$\boxed{\frac{\Gamma \vdash e_1 : \mathit{Int} \qquad \Gamma \vdash e_2 : \mathit{Int}}{\Gamma \vdash e_1 < e_2 : \mathit{Bool}}}$$

```
typeof (lt E1 E2) tbool <-
  typeof E1 tint,
  typeof E2 tint.
```

Easy: Just as in Prolog.

If-then-else.

$$\boxed{\dfrac{e \Downarrow \mathit{True} \qquad e_1 \Downarrow v}{\textsf{if } e \textsf{ then } e_1 \textsf{ else } e_2 \Downarrow v}}$$

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```
eval (ifthenelse E E1 E2) V <-
  eval E btrue, eval E1 V.
```

Application is easy too.

$$\boxed{\frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}}$$

```
app : expr -> expr -> expr.
tarrow : typ -> typ -> typ.

typeof (app E1 E2) T' <-
  typeof E1 (tarrow T T'),
  typeof E2 T.
```

Lambda-case.

$$\boxed{\frac{\Gamma,\ x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda\, x.e : \tau \to \tau'}}$$

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```
var : string -> expr. lam : string -> expr -> expr.
typeof (lam X E) (tarrow T T’) <-
  (typeof (var X) T ->
  typeof E T’).
```

Lambda-case.

$$\boxed{\frac{\Gamma,\ x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda\, x.e : \tau \to \tau'}}$$

```
var : string -> expr. lam : string -> expr -> expr.
typeof (lam X E) (tarrow T T’) <-
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  typeof E T’).
```

Now let's do the evaluation rules.

$$\boxed{\dfrac{e_1 \Downarrow \lambda x.e' \qquad e_2 \Downarrow v \qquad e'[v/x] \Downarrow v'}{e_1\ e_2 \Downarrow v'}}$$

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$$\begin{array}{l}
x[e/x] = e \\
y[e/x] = y \\
(\lambda x.e)[e'/x] = \lambda x.e \\
(\lambda y.e)[e'/x] = \lambda y.(e[e'/x])\ \ if\ \ y \notin fv(e') \\
(e_1\ e_2)[e/x] = e_1[e/x]\ e_2[e/x] \\
\ldots
\end{array}$$

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&(e_1\ e_2)[e/x] = e_1[e/x]\ e_2[e/x]\\
&\ldots
\end{aligned}$$

This seems like a lot of work. Let's backtrack...

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lam : ?
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```
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```

Idea: use meta-level function type to represent binding. Meta-level application is object-level substitution.
(Higher-order abstract syntax.)

Lambda-case.

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```
lam : (expr -> expr) -> expr.
```

Lambda-case.

$$\boxed{\frac{\Gamma,\ x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda\, x.e : \tau \to \tau'}}$$

```
lam : (expr -> expr) -> expr.
```

```
typeof (lam EF) (tarrow T T’) <-
  (x:expr ->
  typeof x T -> typeof (EF x) T’).
```

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lam : (expr -> expr) -> expr.
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```
typeof (lam EF) (tarrow T T’) <-
  (x:expr ->
  typeof x T -> typeof (EF x) T’).

eval (app E1 E2) V’ <-
  eval E1 (lam EF), eval E2 V,
  eval (EF V) V’.
```

Querying.

$$\boxed{\Gamma \vdash e : ?}$$

```
typeof (lam (fun x => lam (fun y =>
        ifthenelse x y (plus y y))))
      T ?
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```
typeof (lam (fun x => lam (fun y =>
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        T ?

» T := tarrow tbool (tarrow tint tint)
```

What about more complicated typing features?

Polymorphism.

$$\boxed{\dfrac{\Delta,\ \alpha; \Gamma \vdash e : \tau}{\Delta; \Gamma \vdash \Lambda\alpha.e : \Pi\alpha.\tau}}$$

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```
pi : (typ -> typ) -> typ.
lamt : (typ -> expr) -> expr.

typeof (lamt EF) (pi TF) <-
  (a:typ -> typeof (EF a) (TF a)).
```

Polymorphism.

$$\boxed{\frac{\Delta; \Gamma \vdash e : \Pi\alpha.\tau}{\Delta; \Gamma \vdash e\,\tau' : \tau[\tau'/\alpha]}}$$

```
appt : expr -> typ -> expr.
typeof (appt E T) (TF T) <-
  typeof E (pi TF).
```

HM-style generalization.

```
typeof (lam (fun x => x)) ?
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typeof (lam (fun x => x)) ?
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typeof (lam (fun x => x)) ?
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HM-style generalization.

```
typeof (lam (fun x => x)) ?
» T := tarrow T1 T1

gen (tarrow T1 T1) T ?
» T := tpi (fun a => tarrow a a)

gen : typ -> typ -> prop.
gen T T <- not(getunif T (X : typ) _).
gen T (pi TF) <-
  getunif T (X : typ) T’,
  (a:typ -> gen (T’ a) (TF a)).
```

HM-style generalization.

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typeof (lam (fun x => x)) ?
» T := tarrow T1 T1

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```

Given T, get the first unification variable of type typ and abstract over it.

Mutually recursive definitions.

$$\boxed{\dfrac{\Gamma, \overrightarrow{xs} : \overrightarrow{\tau} \vdash es_i : \tau_i \qquad \Gamma, \overrightarrow{xs} : \overrightarrow{\tau} \vdash e : \tau'}{\Gamma \vdash \textsf{letrec}\ \overrightarrow{xs} = \overrightarrow{es}\ \textsf{in}\ e : \tau'}}$$

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```
letrec : bindmany expr (list expr * expr)
         -> expr.

typeof (letrec F) T' <-
  open F as (Defs, Body) binding xs in
  assumemany typeof xs TS in
  (map typeof Defs TS,
  typeof Body T').
```

Scalable in terms of expressivity.

- Big part of the OCaml type system in ~500 lines of code.
- Mutually recursive definitions of types and expressions.
- Algebraic datatypes.
- Pattern matching.
- Modules and module signatures.
- HM-style generalization.
- Type synonyms with expansion.
- Extensions are possible: e.g. type classes.
- No code modified, new code: ~100 lines of code.

Existential types.

```
typeof (pack T’ E) (sigma TF) <-
  typeof E (TF T’).

typeof (unpack E EF) T’ <-
  typeof E (sigma TF),
  (a:typ -> x:term ->
    typeof x (TF a) -> typeof (EF a x) T’).
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```

These two rules are comparably effective to serious type inferencing for existential types. How is this possible?

Behind the hood.

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- Meta-level functions for HOAS $\rightarrow$ *terms of the simply-typed lambda calculus*.

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- Prolog uses *s-expressions* as atomic terms.
- Typed abstract syntax $\rightarrow$ *typed s-expressions*.
- Meta-level functions for HOAS $\rightarrow$ *terms of the simply-typed lambda calculus*.
- Polymorphic types (e.g. lists) $\rightarrow$ *terms of the polymorphic lambda calculus*.

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- Undecidable in the general case; restrict to decidable subset.
- Even the restriction subsumes most common type inferencing problems.
- Also useful elsewhere: e.g. semantics of pattern matching.

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- … e.g. generic binding structures

```
bnil : B -> bindmany A B.
bcons : (A -> bindmany A B) -> bindmany A B.
```

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- Useful for all sorts of things:
- implementing DSLs (e.g. LM: invertible ML-like language)
- doing parsing (PEG combinators + LM semantic actions) with pretty-printing for free
- (eventually) implementing intermediate languages for Makam and bootstrapping

Parsing.

Invertible functional-style code.

```
forward : lm A B -> (A -> B -> prop) -> prop.
backward : lm A B -> (B -> A -> prop) -> prop.
```

PEG combinators compiled to invertible code.

```
pegcompile : peg A -> lm string A -> prop.
pegparse = forward ○ pegcompile.
pegprint = backward ○ pegcompile.
```

Parsing.

```
pterm ->
    ”λ” id:ident ”.” body:pterm
    { return lam (bind id body) }
  / f:pbaseterm args:rep(pbaseterm)
    { foldl (return app) f args }

pbaseterm ->
    id:ident { lookup id }
  / e:parenthesized(pterm) { e }
```

Guaranteed to produce well-typed, well-bound abstract syntax.

Conclusion.

Summary.

- Makam: a tool to simplify prototype PL implementation.
- Declarative, Prolog-style rules for specifying different aspects of languages.
- Re-use tricky stuff as implemented in the meta-language.
- Higher-order features allow powerful abstractions.
- Surprisingly expressive formalism – still figuring things out!

Current & future work.

- Base language features are fairly stable.
- Doing a profiling and optimization phase.
- Plan to experiment with further type systems using Makam (VeriML, Ur/Web, etc.)

Backup slides.

Existential types.

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```
sigma : (typ -> typ) -> typ.
pack : typ -> expr -> expr.

typeof (pack T’ E) (sigma TF) <-
  typeof E (TF T’).
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$$\boxed{\dfrac{\Delta; \Gamma \vdash e : \Sigma\alpha.\tau \qquad \Delta,\ \alpha'; \Gamma,\ x : \tau[\alpha'/\alpha] \vdash e' : \tau' \qquad \alpha' \notin fv(\tau')}{\Delta; \Gamma \vdash \textsf{let}\ \langle\ \alpha,\ x\ \rangle = e\ \textsf{in}\ e' : \tau'}}$$

```
unpack : expr ->
         (typ -> expr -> expr) ->
         expr.

typeof (unpack E EF) T’ <-
  typeof E (sigma TF),
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- ?!?

?!?

Let’s revisit the case with multiple bindings.

```
typeof (letrec F) T’ <-
  open F as (Defs, Body) binding xs in
  assumemany typeof xs TS in
  (map typeof Defs TS,
  typeof Body T’).
```

?!?

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assumemany : (A -> B -> prop) -> list A -> list B
      -> prop -> prop.
assumemany P [] [] Q <- Q.
assumemany P (X :: XS) (T :: TS) Q <-
  (P X T -> assumemany P XS TS Q).
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```

Staging in Makam.

Connectives, clauses, etc. are normal terms.

```
and : prop -> prop -> prop.
or : prop -> prop -> prop.
newvar : (A -> prop) -> prop.
newmeta : (A -> prop) -> prop.
assume : prop -> prop -> prop.
...
```

Predicates can compute propositions; we can then use the result normally.

```
invert : prop -> prop -> prop.
invert (and P Q) (and Q’ P’) <-
  invert P P’, invert Q Q’.
```