

Machine Tools and Machining

Question Bank

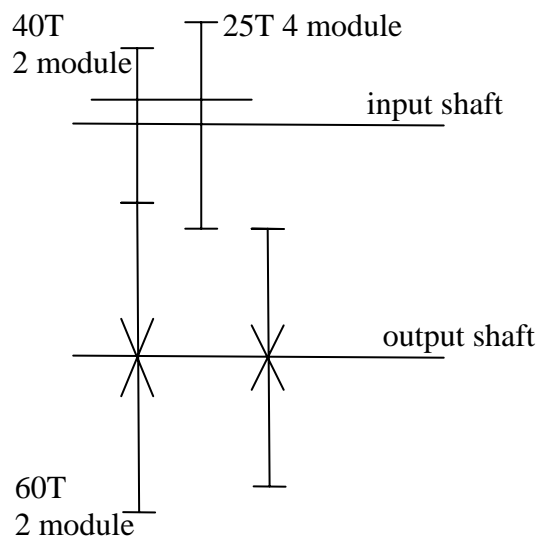
1. The apron constant is 50. The spindle speed is 200 rpm. The longitudinal feed in a centre lathe is 40 mm/min. The number of teeth on the pinion responsible for longitudinal motion is 12 with module of 2 mm. Calculate the rpm of the feed rod.
2. For wire EDM the generatrix statement is: 'G-X-T-F'. What is the directrix statement?
3. The pitch of the leadscrew in a centre lathe is 10 mm. The single start thread that is being machined is having pitch of 1.5 mm. Find out the transmission ratio of the change gears, if the same for the Norton cone and the Meander drive are 1 and 0.5 respectively. The output of the Meander drive is directly connected to the leadscrew.
4. The motor is connected to the speed gear box of a centre lathe using belt pulley mechanism. The diameter of the pulley mounted on the motor is 300 mm and the speed of the motor is 1500 rpm. If the input speed to the speed gear box is 2000 rpm then find out diameter of the pulley mounted on the speed gear box.
5. The cross feed in a centre lathe is 0.1 mm/rev. The spindle speed is changed from 100 rpm to 200 rpm. Find out the new cross feed in mm/rev and mm/min.
6. Name any two machining operations in a centre lathe that would have the following generatrix and directrix statements:

G-X-T-F
D-CM-W-Tr
7. Write the generatrix statement of thread cutting with a single point tool in a centre lathe
8. Write the directrix statement of facing in a centre lathe.

9. The Norton cone has 12 gears and the Meander drive can be set to 4 different transmission ratios. How many different combinations of longitudinal feed can be obtained?
10. Can one rotate the spindle of a centre lathe by applying torque on the pinion responsible for longitudinal feed? Give reasons.
11. If principal cutting edge angle is 90° , and the inclination angle is 0° of a single point turning tool. The orthogonal rake is always -5° . The angle between the cutting planes is 80° . Draw the top view of the cutting tool showing the master line.
12. For a single point turning tool, the principal cutting edge and master line of principal flank are parallel. What information can be inferred about the tool geometry?
13. For a single point turning tool, if the principal cutting edge angle is 90° and the back rake is 15° , then what would be the inclination angle? The orthogonal rake is 0° and the inclination of the auxiliary cutting edge is 5° .
14. A single point thread cutting tool is used to cut metric thread. Its back and side rake angles are both zero. What is its orthogonal rake? Would the orthogonal rake change if it is thread cutting tool for British thread?
15. In a CNC turning centre, straight turning is undertaken on a steel bar of 200 mm diameter with a coated carbide insert at 350 m/min. If the longitudinal feed is 111 mm/min, calculate the feed in mm/rev, when the cutting velocity and diameter of the bar are changed to 200 m/min and 100 mm respectively.

16. Two shafts are connected (as shown below) by the one set of sliding cluster gears mounted on the input shaft and two fixed gears on the output shaft. The cluster gears have 40 and 25 number of teeth. The cluster gear with 40 numbers of teeth meshes with a fixed gear (mounted on the output shaft) of 60 teeth and they have a module of 2. The cluster gear with 25 numbers of teeth has a module of 4.

If the input shaft is rotating at 1000 rpm, find out all possible rpm of the output shaft.



17. Draw the schematic of a cylindrical grinding operation in a lathe using a grinding attachment and state the generatrix and directrix statements.
18. The apron constant is 50. The longitudinal feed is 30 mm/min. The pinion in mesh with rack under the lathe bed is having a 16 teeth and module of the rack is 2. What is the rotational speed of the feed rod?
19. The motor is connected to the speed gear box of a centre lathe through a belt pulley system. The diameter of the pulley on the motor is 200 mm and that on the speed gear box is 300 mm. The gear positions in the speed gear box are such that the transmission ratio of the speed gear box is $1/16$. The speed gear box is connected to the Norton Cone via a set of change gears. The change gears are having a transmission ratio of $1/4$. The shaft of the Norton Cone is having a rotational speed of 15 rpm. What is the speed of the motor?

20. The cutting speed during turning a low carbon steel bar has been adjusted to be 120 m/min when the diameter of the bar is 200 mm. The transmission ratios of the change gear quadrant (connecting the speed gear box to the feed gear box), Norton Cone cum Tumbler and Meander drive are $\frac{1}{4}$, 1 and $\frac{1}{2}$ respectively. If the cross feed is 0.01 mm/rev when the pitch of the screw (responsible for cross or transverse feed) is 4 mm, find the transmission ratio between the feed rod and the screw (responsible for cross or transverse feed)
21. Thread cutting is being done in a centre lathe. The pitch of the lead screw (that engages with the half nut) is 5 mm. The transmission ratios of the change gear quadrant (connecting the speed gear box to the feed gear box), Norton Cone cum Tumbler and Meander drive are $\frac{1}{4}$, 2 and 2 respectively. What is the pitch of the thread that is being cut?
22. Taper turning is being undertaken in a CNC turning centre (similar to CNC lathe). Draw the schematic and write down the generatrix and directrix statements
23. The feed gear box is not connected to speed gear box via the shaft carrying the chuck or the spindle. It is connected directly to the input shaft of the speed gear box. The spindle speed is 200 rpm and the longitudinal feed is 0.2 mm/rev. Now by changing the gears within the speed gear box the spindle is doubled (i.e. made 400 rpm). What would be the new longitudinal feed in mm/rev and mm/min?
24. A centre lathe is having a two speed motor. It rotates at 1500 or 3000 rpm. The spindle speed and cross feed for particular combinations of gear positions are 500 rpm and 0.1 mm/rev when the motor speed is 1500 rpm. When the motor is doubled (i.e. made 3000 rpm) then what would be the spindle speed in rpm and longitudinal feed in mm/rev and mm/min?
25. The feed rod is rotating at 15 rpm. The pinion in mesh with the rack is having a module of 2 and 12 number of teeth. If the longitudinal feed is 0.1 mm/rev when the spindle is rotating at 200 rpm. Find out the apron constant.
26. What would happen to the longitudinal feed in mm/rev and in mm/min in a centre lathe if the motor speed is doubled?

27. What would happen to cross feed in mm/rev and in mm/min if the belt pulleys are interchanged in a centre lathe?
28. What is apron constant?
29. Why is the spindle of a centre lathe hollow?
30. The plane A (π_A) of single point turning tool (being used in a centre lathe) is at an angle θ from the auxiliary cutting plane towards the operator as measured on the reference plane. The same plane (π_A) is orthogonal to the reference plane and it is in between the orthogonal plane and auxiliary cutting plane. Find out the expression of the rake angle as measured on π_A as a function of orthogonal rake, inclination angle of the principal cutting edge and other relevant tool angles **using graphical method.**
31. A single point turning tool is designated as $0^\circ 6^\circ 10^\circ 8^\circ 15^\circ 90^\circ 1$ mm in orthogonal rake system (ORS). Draw the top view of the cutting tool and show the relevant angles. Determine the back rake and side rake. Also determine the angle between the cutting plane and auxiliary cutting plane as measured on reference plane.
32. The master line of the rake surface of a single point turning tool is obtained by intersection of the rake surface and back plane (or any plane parallel to the reference plane) by extending the rake surface. The master line (of the rake surface) of a single point turning tool is parallel to the auxiliary cutting edge. The principal cutting edge angle and auxiliary cutting edge angle are 75° and 15° respectively. The inclination angle of the principal or main cutting edge is -10° . Draw the top view of the cutting tool showing the master line. Find out the orthogonal rake associate with the principal cutting edge (γ_o) and with the auxiliary cutting edge (γ'_o).
33. The master line of the rake surface of a single point turning tool parallel to the main or principal cutting edge. The auxiliary cutting edge angle is 15° . It also happens to be perpendicular to the machine longitudinal plane (II_X). The orthogonal rake is -10° . Determine back rake, side rake, inclination angle and principal cutting edge angle and draw the top view of the cutting tool showing the master line and relevant angles.

34. The Norton gear cone has 13 gears. The first gear has 24 teeth and the numbers of teeth on the successive gears are in AP with a common difference of 2. The transmission ratios of the Meander Drive are 4, 2, 1 and 0.5. The numbers of teeth on the change gear quadrant are 20, 60, 15 and 30 where the gears with 60 and 15 teeth are on same shaft. The numbers of teeth on the Tumbler gears are both 24. The power flows from speed gear box through the change gear quadrant, Norton cone along with tumbler, Meander Drive to the leadscrew. The transmission ratio between the Meander Drive and Leadscrew is 0.5.

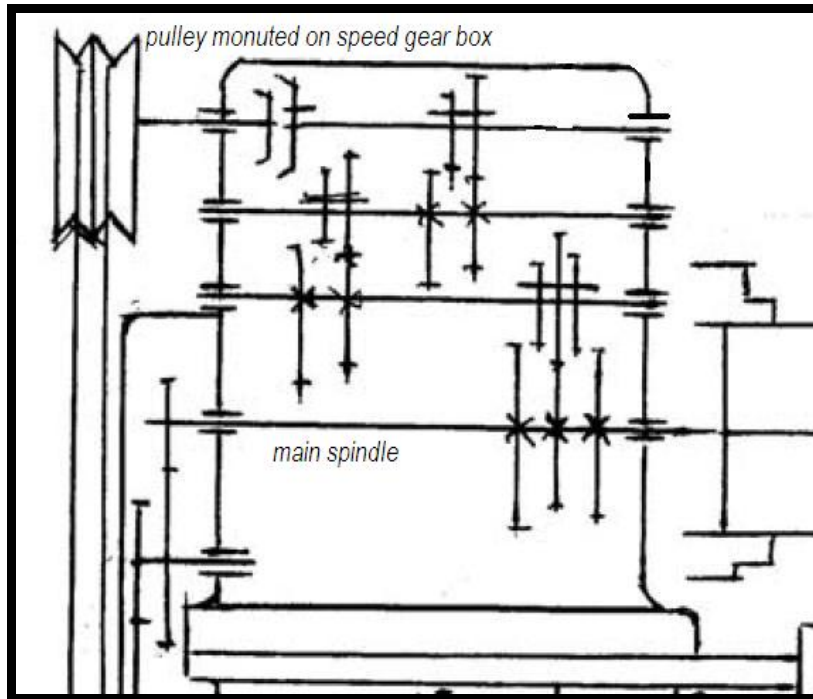
If the minimum pitch of a single start thread that can be cut in this lathe is 0.5 mm, find out the pitch of the leadscrew.

Also determine the maximum pitch of a single start thread that can be machined in this lathe.

The apron constant is 120. The number of teeth on the pinion matching with the rack under the lathe bed is 12 with a module of 1.5. Determine the minimum and maximum longitudinal feed. The transmission ratio between the output of the Meander Drive and feed rod is 1.

35. A metric thread is being machined on a circular medium carbon steel bar of diameter 100 mm with a pitch of 10 mm at cutting speed of 20 m/min. The orthogonal rake (γ_o) and the inclination angle of the principal cutting edge (λ) are both zero. Determine the side rake in work reference system (WRS) i.e. taking into account the effect of feed.
36. Draw the schematic diagram of the following machining operation. Identify the generatrix and directrix on the diagram and write the generatrix and directrix statements
- Taper turning in a CNC turning centre
 - Slab milling
 - Gear cutting in a milling machine
 - Gear hobbing
 - Cylindrical grinding with longitudinal feed using grinding attachment in a centre lathe
 - Thread cutting in a centre lathe

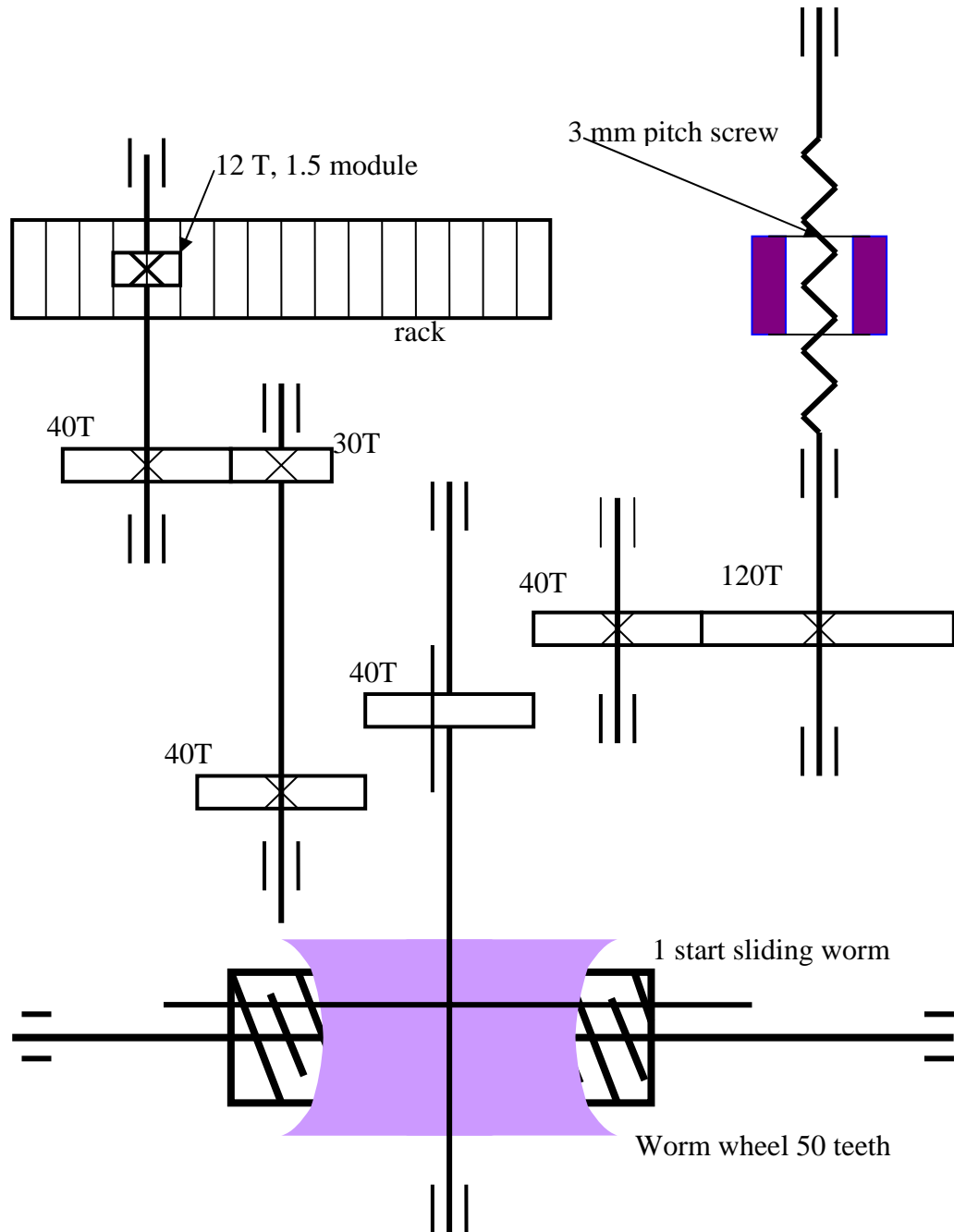
37.



The line diagram of a speed gear box showing the kinematic structure of a centre lathe is shown:

The numbers of teeth of the sliding cluster gears are 40 and 60. The numbers of teeth on the last cluster gear (meshing with 3 keyed in gears on the main spindle) are 40, 60 and 30. Currently all the cluster gears are meshing with fixed gears (3 in number) having 60 number of teeth. The motor (not shown in the diagram) speed is 1440 rpm and the motor is connected to the speed gear box through belt pulley arrangement. The diameter of the pulley on the motor is 200 mm and that of the pulley on the speed gear box is 300 mm. Determine all the possible spindle speeds.

38.



The line diagram of a sliding apron mechanism is shown above

Calculate the apron constant

39. Sketch (line diagram) a 12 speed gear box used in a centre lathe

40. The Norton gear cone has 13 gears. The first gear has 24 teeth and the numbers of teeth on the successive gears are in AP with a common difference of 2. The transmission ratios of the Meander Drive are 4, 2, 1 and 0.5. The numbers of teeth on the change gear quadrant are 20, 40, 15 and 30 where the gears with 40 and 15 teeth are on same shaft. The numbers of teeth on the Tumbler gears are both 24. The power flows from speed gear box through the change gear quadrant, Norton cone along with tumbler, Meander Drive to the leadscrew. The transmission ratio between the Meander Drive and Leadscrew is 0.5.

If the maximum pitch of a single start thread that can be cut in this lathe is 16 mm, find out the pitch of the leadscrew.

Also determine the minimum pitch of a single thread that can be machined in this lathe.

The apron constant is 50. The number of teeth on the pinion matching with the rack under the lathe bed is 12 with a module of 1.5. Determine the minimum and maximum longitudinal feed. The transmission ratio between the output of the Meander Drive and feed rod is 0.25.

41. Explain with schematic diagram, how does the clearance angle get affected
- (i) during thread cutting and
 - (ii) due to tool setting error while grooving.
42. Why is a rack provided under the lathe bed?
43. What is/are the purpose(s) of a tailstock?
44. Sketch a line diagram of a feed gear box of a centre lathe
45. The speed of a main spindle motor is 1500 rpm and the input rpm to the speed gearbox is 1000 rpm – how it is achieved? Why do you propose such a connection?
46. Derive the interrelation between shear angle, rake angle and chip reduction coefficient
47. Derive the interrelation between chip reduction coefficient, friction coefficient and rake angle

48. Derive the interrelation between average cutting strain, shear angle and rake angle
49. The plane A (π_A) of single point turning tool (being used in a centre lathe) is at an angle θ from the orthogonal plane towards the operator as measured on the reference plane. The same plane (π_A) is orthogonal to the reference plane. Find out the expression of the rake angle as measured on π_A as a function of orthogonal rake and inclination angle of the principal cutting edge **using graphical method.**
50. The plane A (π_A) of single point turning tool (being used in a centre lathe) is at an angle θ from the orthogonal plane towards the operator as measured on the reference plane. The same plane (π_A) is orthogonal to the reference plane. Find out the expression of the rake angle as measured on π_A as a function of orthogonal rake and inclination angle of the principal cutting edge **using vector method.**
51. Draw a neat schematic diagram of a speed gear box of a centre lathe having 12 spindle speeds employing four shafts (the last shaft is the spindle itself) and combination of two sets of clutches and one sliding cluster having three gears.
52. The pitch of the lead screw of a centre lathe is 6 mm. A metric thread of 2 mm pitch is required to be machined. The transmission ratios of the change gear quadrant and Meander drive are both 0.5. The output of the meander is directly connected to the lead screw. If the numbers of teeth on the gears in Tumbler (connected to the Norton cone) are both 36, determine the number of teeth on the matching gear on the Norton Cone.
53. The observed tool lives while machining medium carbon steel with uncoated carbide insert are 24 and 12 minutes respectively for cutting velocities of 60 m/min and 90 m/min. Determine the cutting velocity, when a tool life of 15 min. is expected.
54. The transmission ratios of a Meander drive are 1, 0.5, 0.25 and 0.125. Draw the schematic of the same with suitable gears.

55. Specific cutting force is the ratio of the main cutting force in turning to the chip load. Derive the relationship between the specific cutting force and dynamic yield shear strength of the work material for a ductile work material. Orthogonal rake is zero.
56. During orthogonal machining of an engineering alloy, the angle between the friction force and resultant force has been observed to be 75° . The machining constant $C = 1.36$. $S = \text{feed} = 0.2 \text{ mm/rev}$, $t = \text{depth of cut} = 3 \text{ mm}$, $V_C = \text{cutting velocity} = 90 \text{ m/min}$, $\phi = \text{principal cutting edge angle} = 60^\circ$, $\gamma_o = \text{orthogonal rake} = 0^\circ$. Estimate chip thickness.
57. The amount of heat taken away by the chip from the primary shear zone is 70% while machining low carbon steel. The thermal conductivity, specific heat and density of low carbon steel are 50 W/m-K , 500 J/kg-K and 7800 kg/m^3 . The feed, depth of cut and cutting velocity are 0.4 mm/rev , 4 mm and 90 m/min . The chip reduction coefficient is 2 and the principal cutting edge angle is 90° . The orthogonal rake is 0° and dynamic yield shear strength is 500 MPa . Determine rise in temperature in the chip due to primary shear zone.
58. In orthogonal turning of an engineering alloy, the following information are available:
 $S = \text{feed} = 0.2 \text{ mm/rev}$, $t = \text{depth of cut} = 3 \text{ mm}$, $V_C = \text{cutting velocity} = 90 \text{ m/min}$, $\phi = \text{principal cutting edge angle} = 60^\circ$, $a_2 = \text{chip thickness} = 0.37 \text{ mm}$, $\gamma_o = \text{orthogonal rake} = 0^\circ$.
 The alloy exhibit 50% elongation during tensile testing. $P_Y = \text{axial or longitudinal feed force} = 200 \text{ N}$. Determine dynamic yield shear strength.
59. A low carbon steel bar is being turned by a single point turning tool having a nose radius of r , when the depth of cut is t . $r > t$. The inclination angle is zero. Neglecting the effect of auxiliary cutting edge, determine the angle and sense (towards work material or tool) of chip deviation. $\phi = \text{principal cutting edge angle} = 75^\circ$ and $\phi_1 = \text{auxiliary cutting edge angle} = 15^\circ$.

60. A steel bar of diameter 100 mm and length 400 mm is being straight turned with uncoated carbide insert at a cutting velocity of 90 m/min and feed of 0.2 mm/rev. The tool life is 20 minutes. The idle and tool setting time is 1 min. The tool change time is 1 min. The approach and over travel are both 5 mm. Determine the time to produce one item.
61. Under orthogonal turning, it has been observed that $P_{XY} = \text{resultant thrust force} = F = \text{friction force}$ and $P_Z = \text{main cutting force} = N = \text{normal force}$. The chip thickness and uncut chip thickness are 0.4 mm and 0.24 mm respectively. Calculate shear angle.
62. Find an expression for the rake angle of single point turning tool as measured on the auxiliary cutting plane as a function of orthogonal rake angle, inclination angle and other relevant angles.
63. Average normal stress at the chip tool interface is the ratio of normal force to the contact area. Contact length is equal to the plastic contact length during high speed machining. Determine the same for a ductile work material under high speed machining when orthogonal rake is zero.
64. For a single point turning tool, the following information is available: $\gamma_X = \text{back rake} = \text{negative}$, $\gamma_Y = \text{side rake} = \text{positive}$, $\phi < 90^\circ$, $\phi_l < 45^\circ$. Draw the master line for the rake surface. Derive the interrelation between inclination angle with γ_X , γ_Y and ϕ .

65. Assume orthogonal machining. The friction force acting at the chip tool interface while machining an engineering alloy is 500 N. The **orthogonal rake is zero**. The resultant cutting force is 1300 N. The feed, depth of cut and cutting velocity are 0.2 mm/rev, 2 mm and 90 m/min. The principal cutting edge angle is 90° . The chip thickness is 0.5 mm. Determine the following:
- I. material removal rate
 - II. shear angle
 - III. main cutting force
 - IV. normal force acting at chip tool interface
 - V. force acting along the intersection of orthogonal plane and reference plane
 - VI. shear force
 - VII. normal force acting at the shear plane
 - VIII. dynamic yield shear strength
 - IX. cutting power
 - X. specific cutting energy
66. Assume orthogonal machining. The friction force acting at the chip tool interface while machining a ductile engineering alloy is 500 N. The orthogonal rake is zero. The feed, depth of cut and cutting velocity are 0.2 mm/rev, 2 mm and 90 m/min. The principal cutting edge angle is 90° . The chip thickness is 0.5 mm. Determine the following:
- I. material removal rate
 - II. shear angle
 - III. main cutting force
 - IV. normal force acting at chip tool interface
 - V. force acting along the intersection of orthogonal plane and reference plane
 - VI. shear force
 - VII. normal force acting at the shear plane
 - VIII. dynamic yield shear strength
 - IX. cutting power
 - X. specific cutting energy

67. Assume orthogonal machining. The friction force acting at the chip tool interface while machining a semi ductile engineering alloy is 500 N. The material constant (C) is 0.8. The orthogonal rake is zero. The feed, depth of cut and cutting velocity are 0.2 mm/rev, 2 mm and 90 m/min. The principal cutting edge angle is 90° . The chip thickness is 0.5 mm. Determine the following:

- I. material removal rate
- II. shear angle
- III. main cutting force
- IV. normal force acting at chip tool interface
- V. force acting along the orthogonal plane
- VI. shear force
- VII. normal force acting at the shear plane
- VIII. dynamic yield shear strength
- IX. cutting power
- X. specific cutting energy

68. Assume orthogonal machining. $S = \text{feed} = 0.24 \text{ mm/rev}$, $t = \text{depth of cut} = 4 \text{ mm}$, $V_C = \text{cutting velocity} = 120 \text{ m/min}$, orthogonal rake = $+10^\circ$, principal cutting edge angle = 75° , $P_Y = \text{radial thrust force} = 100 \text{ N}$, $F/N = 0.4$, work material = engineering alloy

Determine the following

- I. friction angle
- II. resultant cutting force
- III. main cutting force and resultant thrust force
- IV. shear force and normal force acting at the primary shear plane
- V. dynamic yield shear strength

69. Assume orthogonal machining. $S = \text{feed} = 0.24 \text{ mm/rev}$, $t = \text{depth of cut} = 4 \text{ mm}$, $V_C = \text{cutting velocity} = 120 \text{ m/min}$, orthogonal rake = $+10^\circ$, principal cutting edge angle = 75° , $P_Y = \text{radial thrust force} = 100 \text{ N}$, $a_2 = \text{chip thickness} = 0.4 \text{ mm}$, % elongation during tensile testing = 50%

Determine the following

- I. friction angle
- II. resultant cutting force
- III. main cutting force and resultant thrust force
- IV. shear force and normal force acting at the primary shear plane
- V. dynamic yield shear strength

70. Assume orthogonal machining. $S = \text{feed} = 0.24 \text{ mm/rev}$, $t = \text{depth of cut} = 4 \text{ mm}$, $V_C = \text{cutting velocity} = 120 \text{ m/min}$, orthogonal rake = $+10^\circ$, principal cutting edge angle = 75° , specific cutting energy = 0.8 J/mm^3 , $a_2 = \text{chip thickness} = 0.4 \text{ mm}$, work material = semi-ductile, $C = 1.0$
- Determine the following
- I. friction angle
 - II. resultant cutting force
 - III. main cutting force and resultant thrust force
 - IV. shear force and normal force acting at the primary shear plane
 - V. dynamic yield shear strength
71. Assume orthogonal machining. Find out the ratio of main cutting force and resultant thrust force (P_{XY}) for general engineering alloy
72. Assume orthogonal machining. Find out the ratio of main cutting force and resultant thrust force (P_{XY}) for ductile engineering alloy and explain how it would vary with feed if rake angle is zero. The machining zone does not exhibit built up edge formation.
73. While orthogonal turning with a coated carbide, it has been observed that the ratio of main cutting force and resultant thrust force (P_{XY}) is one. Under what condition(s) is it feasible?
74. Assume orthogonal machining. A low carbon steel pin is being grooved at a feed of 0.05 mm/rev in a lathe. The width of the groove is 5 mm and the cutting edge is parallel to the longitudinal feed direction. The inclination angle and orthogonal rake are both zero. The chip thickness is 0.13 mm and the angle between the friction force and the resultant cutting force is 30° . Determine main cutting force.
75. Merchant's 2nd assumes that the dynamic yield shear strength varies as following: $\tau_s = \tau_o + k\sigma_n$. Determine the ratio $\frac{\tau_s}{\tau_o}$. $C = \text{machining constant in degrees} = 55^\circ$. $\gamma_o = \text{orthogonal rake} = 10^\circ$. $\zeta = \text{chip reduction coefficient} = 2.6$.
76. Draw a HSS twist drill and show the cutting forces acting on the drill.
77. Draw a slab milling cutter and show the cutting forces acting on the same.

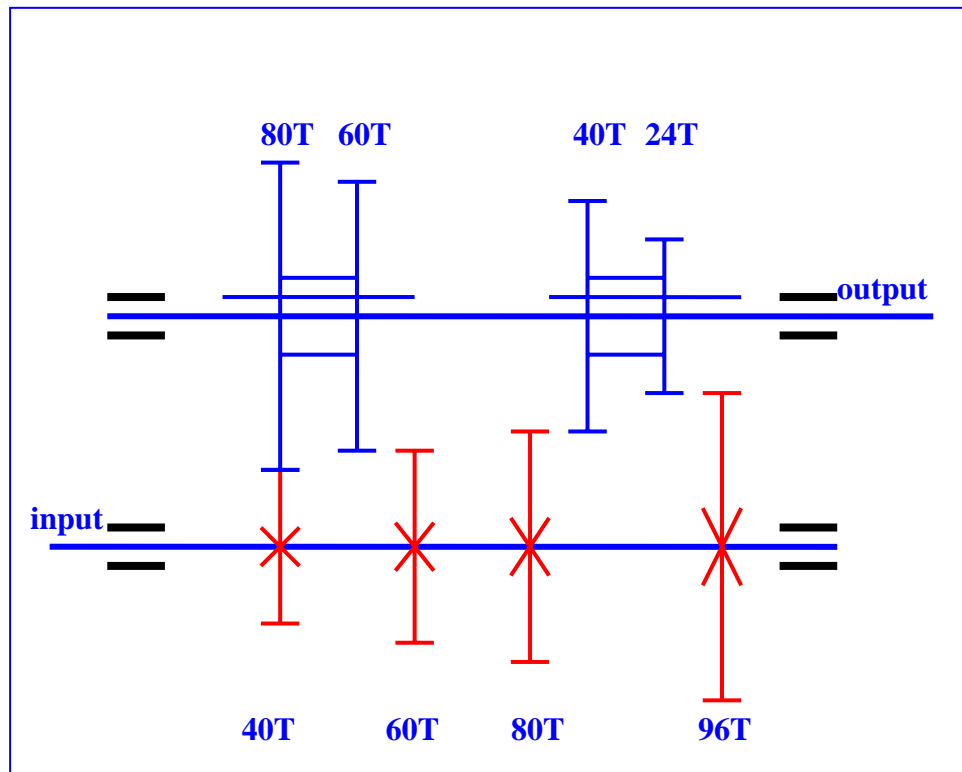
78. Draw a single point turning tool and show the cutting forces acting on it for the following operations:

straight turning
facing
grooving
parting off
taper turning

79. A facing operation of a flange is being undertaken from the periphery towards the centre in a CNC lathe with constant cutting velocity of V_C m/min and feed of S mm/rev. The outer and inner diameters in mm are d_O and d_B . Determine machining time assuming no approach and overtravel.
80. A facing operation of a flange is being undertaken from the periphery towards the centre in a CNC lathe with constant cutting velocity of 300 m/min and feed of 0.2 mm/rev. The outer and inner diameters in mm are 300 and 50. Determine machining time assuming no approach and overtravel.
81. A facing operation of a flange is being undertaken from the periphery towards the centre in a CNC lathe with constant cutting velocity of 300 m/min and feed of 0.2 mm/rev. The outer and inner diameters in mm are 300 and 10. Determine machining time assuming no approach and overtravel. Note that the maximum spindle speed is 3000 rpm.
82. Typically it is observed that the effect of cutting velocity (V_C) on cutting temperature (θ_C) is more than that of feed. If it is further observed that $\theta_C \propto \frac{\text{cutting power}}{\text{chip tool contact area}}$, then explain.
83. It is typically observed that increase in feed leads to reduction in chip reduction coefficient. Explain.

Hint: Use Kronenberg's equation

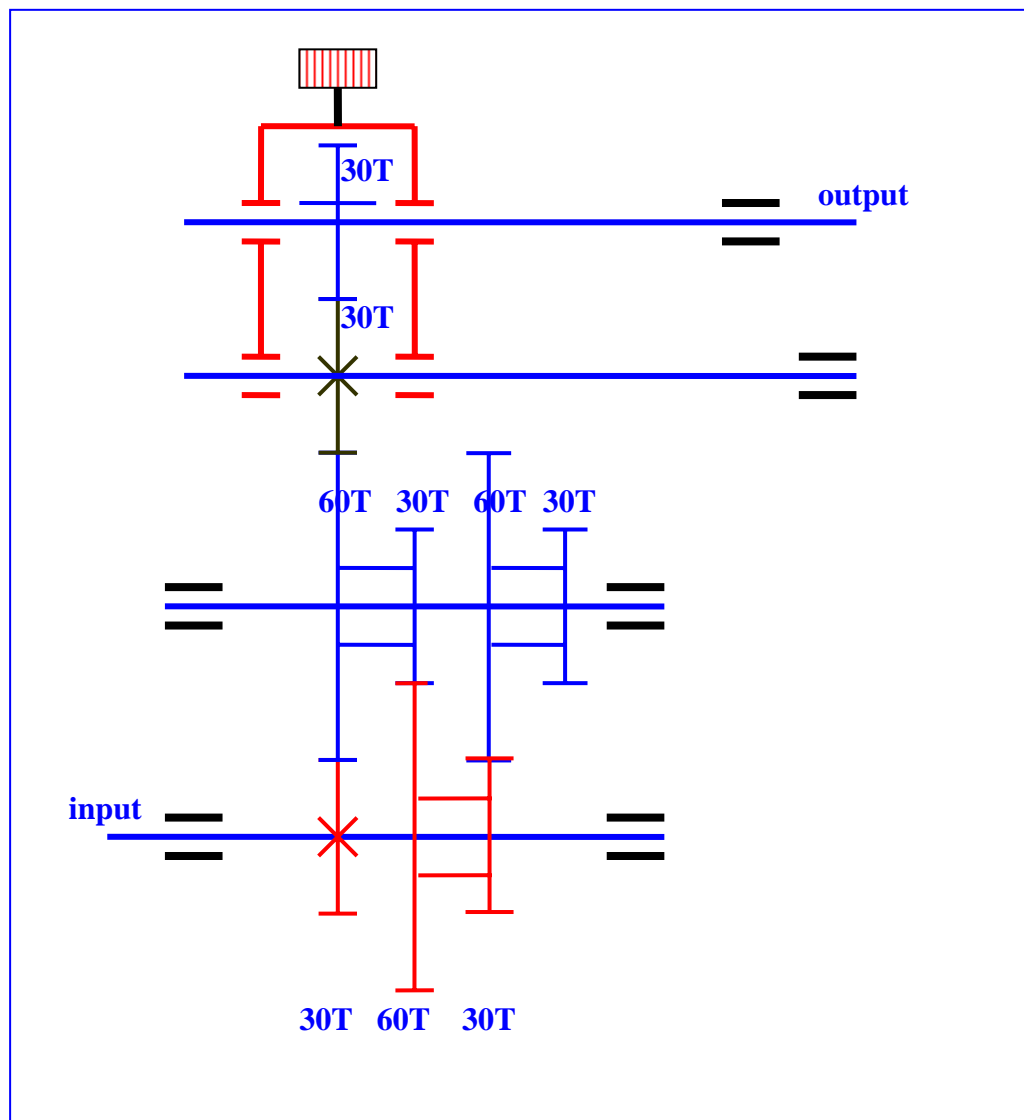
84.



What is the name of the above mechanism? Where is it used? What are the different transmission ratios? Find out the different output speed if the input shaft is rotating at 40 rpm.

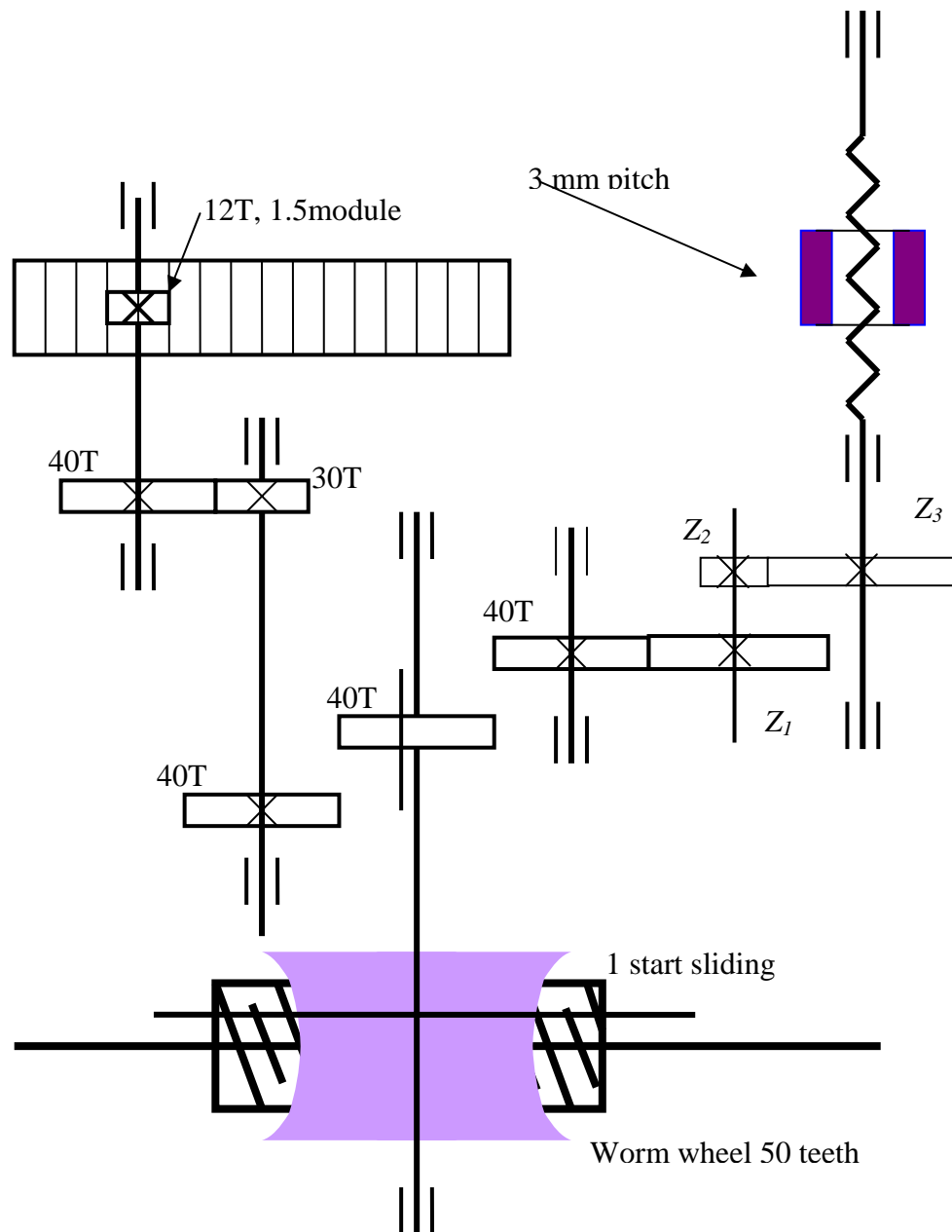
85. Draw the Merchant's circle diagram. Scale 1 mm = 200 N. Orthogonal rake is -10° . The chip reduction coefficient is 2.6. The work material is ductile. Main cutting force is 1000 N. Solve graphically and list all the forces visible on Merchant's circle diagram. *Do not use Kronenberg's relation.*
86. Orthogonal rake is -8° . The angle between the friction force vector and shear force vector 118° . The work material is ductile. Main cutting force is 1000 N. Solve and list all the forces visible on Merchant's circle diagram. *Do not use Kronenberg's relation.* Also determine chip reduction coefficient.

87.



What is the name of the above mechanism? Where is it used? What are the different transmission ratios? Find out the different output speed if the input shaft is rotating at 40 rpm. What would be the transmission ratios if the input and outputs are interchanged?

88.



The ratio of cross feed to longitudinal feed needs to be 0.4. Determine Z_1 , Z_2 and Z_3 .