Experimentally Determining the Accuracy of an 80 Sappho Model

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Introduction

The Database of Asteroid Models from Inversion Techniques (DAMIT) was developed by researchers in Finland and the Czech Republic to create 3-dimensional models of asteroids using light curve data.

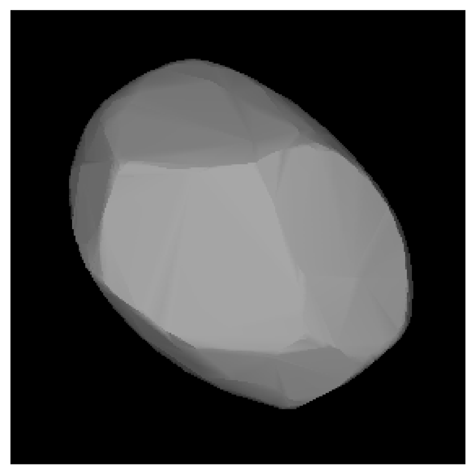


Figure 1: 3D Model of 80 Sappho

Figure 1 is a computer rendering of 80 Sappho, created using the model from DAMIT. The purpose of this experiment is to determine the accuracy of DAMIT models, in this case 80 Sappho, in comparison to light curve data collected by the Minor Planet Center.

Materials and Methods

The 80 Sappho Model The model was printed using a MakerBot Replicator 2 3D printer. It was printed in white PLA plastic.

The Experimental Environment The experiment was conducted within a dark box. The dark box was constructed from insulation foam and matte black paint, resulting in a box with a volume of roughly 8 cubic feet (or 0.23 cubic meters). The asteroid was rotated using a stepper motor controlled by an Arduino. The same Arduino controlled the data collection system. The light was provided by a small LED flashlight. Figure 2 contains images of the experimental setup.

Data Collection Using a photometric array connected to an Arduino, light intensities were collected to create four different light curves. The four light curves were created for:

- Asteroid rotating along its x axis.
- Asteroid rotating along its y axis.
- Asteroid rotating along its z axis.
- Control light curve without asteroid.

A measurement was taken at each one of the stepper motors steps (384 steps in one rotation). 10 full rotations were measured for each light curve resulting in a total of 15,360 measurements.

Materials and Methods (continued)

Data Analysis Using the principle of superposition these four light curves were combined together to create a composite light curve. Adding the x, y, and z light curves would create a light curve representing the asteroid as if it were rotating on all three axes at one time. Subtracting the control light curve would remove data noise caused by the stepper motor and the support structure holding the asteroid in place.

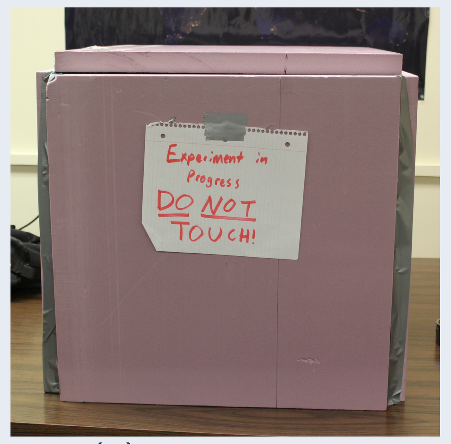
The data was collected in light intensities. To compare this dataset with the one obtained from the Minor Planet Center it had to be converted to the magnitude scale, which is done with the use of equation 1.

$$m = -2.5 \log_{10}\left(\frac{I_1}{I_{ref}}\right) \tag{1}$$

To determine the accuracy of the measured light curve the consistency between the experimental and actual curves was calculated at each point. This was done using equation 2. If is equation 2 is true and $z \leq 2$ then the values are consistent. The accuracy can then be given as the percentage of the experimental light curve that is consistent with the actual light curve.

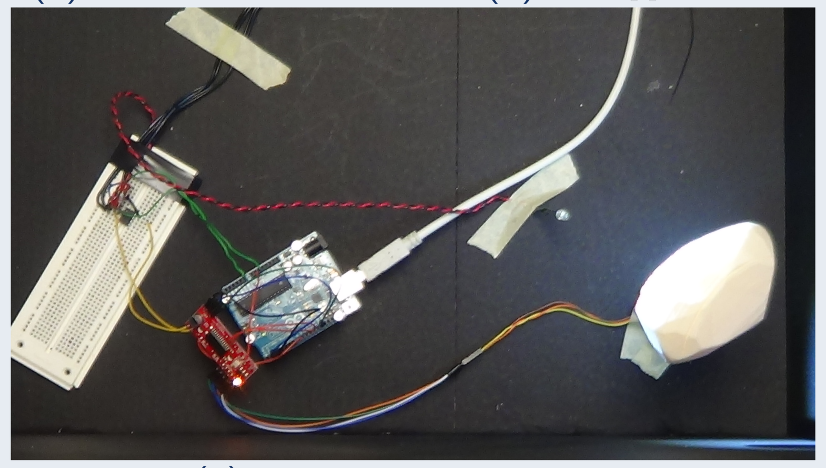
$$z = \frac{|m_{experimental} - m_{actual}|}{\sqrt{(\Delta m_{experimental})^2 + (\Delta m_{actual})^2}} \leq 2 \tag{2}$$

The Experimental Environment



(a) The Dark Box

(b) 80 Sappho Model



(c) Complete Arduino Setup

Figure 2: Portions of the experimental setup.

Materials and Methods (continued)

Error Determination Measurement error was taken into account at every step of data analysis. For each of the four component light curves, 10 full rotations were collected. The 10 rotations were averaged together and the statistical error was calculated using equation 3. The average curves, plotted with the calculated error, are shown in Fig. 3(a-d).

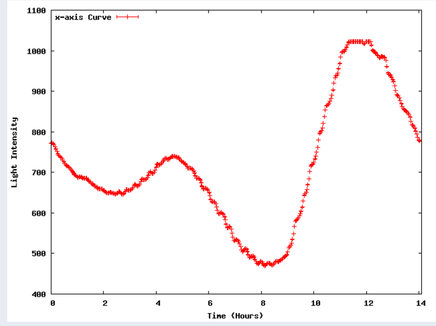
$$\Delta I = \frac{\sigma}{\sqrt{10}} \tag{3}$$

In equation 2 σ represents the standard deviation. The statistical errors were then propagated through the calculations that produced the composite curve. Equation 4 demonstrates error propagation.

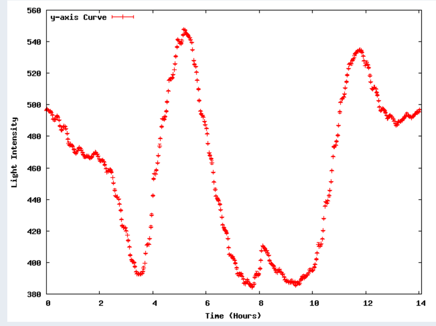
$$\begin{aligned} \bar{x} &= f(\bar{a}, \bar{b}) \\ \Delta x_a &= f(\bar{a} \pm \Delta a, \bar{b}) - \bar{x} \\ \Delta x_b &= f(\bar{a}, \bar{b} \pm \Delta b) - \bar{x} \\ \Delta x &= \sqrt{(\Delta x_a)^2 + (\Delta x_b)^2} \end{aligned} \tag{4}$$

Results

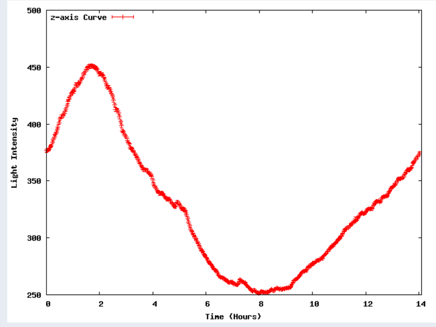
Light Curves



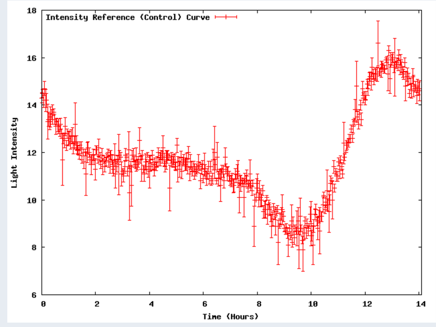
(a) x axis



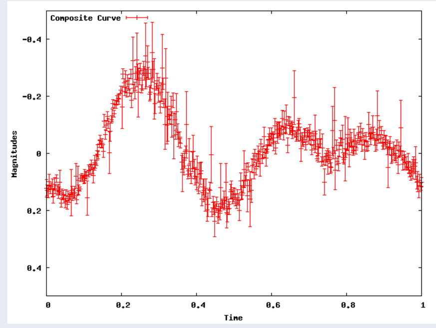
(b) y axis



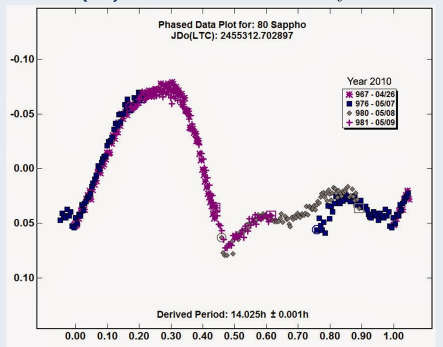
(c) z axis



(d) Control & I_{ref}



(e) Composite



(f) Actual

Figure 3:

Results (continued)

Figure 3 contains the experimentally created (a-e) and the actual (f) light curves for 80 Sappho. The first four are the component curves plotted as time, as a fraction of the total period of 14.025 hours, versus light intensity. The composite curve is the result of applying the superposition principle and equation 1 to the four component curves. The composite curve has also been normalized for better comparison to the actual light curve taken from the Minor Planet Center.

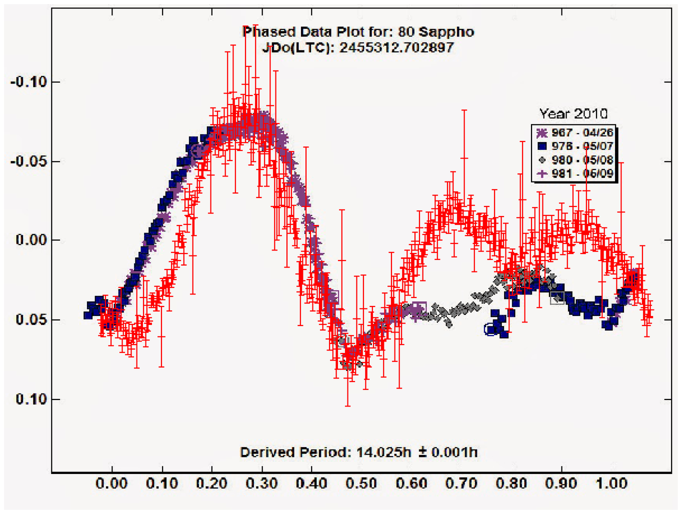


Figure 4: Overlay of the Experimental and Actual Curves

Figure 4 displays the experimental curve overlaid with the actual curve. During the first half of the rotation period the two curve are very similar; however, in the second half of the rotation there is a large variance. Pending data from the Astronomical Society of Las Cruces, the group that produced Figure 3(f), the exact accuracy of the experimental curve cannot be determined.

Conclusions

This experiment used a simplified model of how asteroids spin through space. Accounting for the asteroid's nutation, precession, and rotation around the Sun could explain the variance in the second half of the rotation in Figure 4. Correcting for printing and procedural error would also increase the overall accuracy. Future experiments should account for these errors and test a range of the 381 asteroids represented in DAMIT.

Acknowledgements

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