

Familias Paramétricas Especiales de Distribuciones Univariadas

Distribución	$f(x)$	f.g.m= $\Psi_X(t)$	$E(X)$	$\text{Var}(X)$	$F(x)$
Uniforme Discreta	$\frac{1}{N} I_{\{1,2,\dots,N\}}(x)$	$\frac{e^t(1-e^{N+1})}{N(1-e^t)} \quad t \neq 0$ $1 \quad t = 0$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$	$\frac{x}{N}$
Uniforme Discreta1	$\frac{1}{N} I_{\{x_1, x_2, \dots, x_N\}}(x)$	No es útil	$\frac{1}{N} \sum_{i=1}^N x_i$	$\frac{1}{N} \sum_{i=1}^N [x_i - E(X)]^2$	
Bernoulli	$\pi^x (1-\pi)^{1-x} I_{\{0,1\}}(x)$	$(1-\pi) + \pi e^t$	π	$\pi(1-\pi)$	
Binomial	$\binom{n}{x} \pi^x (1-\pi)^{n-x} I_{\{0,1,2,\dots,n\}}(x)$	$[(1-\pi) + \pi e^t]^n$	$n\pi$	$n\pi(1-\pi)$	
Hipergeométrica	$\frac{\binom{A}{x} \binom{N-A}{n-x}}{\binom{N}{n}} I_{\{\max(0, n-N+A), \dots, \min(n, A)\}}(x)$	No es útil	$n \frac{A}{N}$	$n \frac{A}{N} \left(1 - \frac{A}{N}\right) \left(\frac{N-n}{N-1}\right)$	
Hipergeométrica Negativa	$\frac{\binom{x-1}{k-1} \binom{N-x}{A-k}}{\binom{N}{A}} I_{\{k, \dots, N-A+k\}}(x)$		$\frac{k(N+1)}{A+1}$	$\frac{k(N+1)(N-A)}{(A+1)(A+2)} \left[1 - \frac{k}{A+1}\right]$	

V	$\frac{\binom{A+x-1}{x} \binom{(N-A)+(n-x)-1}{n-x}}{\binom{N+n-1}{n}} I_{\{0,1,2,\dots,n\}}(x)$	No es útil	$n \frac{A}{N}$	$\frac{n A (N+n)(N-A)}{N^2 (N+1)}$	
Poisson	$\frac{e^{-\mu} \mu^x}{x!} I_{\{0,1,2,\dots\}}(x)$	$\exp[\mu(e^t - 1)]$	μ	μ	
Geométrica	$\pi (1-\pi)^x I_{\{0,1,2,\dots\}}(x)$	$\frac{\pi}{1-(1-\pi)e^t}$	$\frac{1-\pi}{\pi}$	$\frac{1-\pi}{\pi^2}$	$1-(1-\pi)^{x+1}$
Geométrica1	$\pi (1-\pi)^{x-1} I_{\{1,2,\dots\}}(x)$	$\frac{\pi e^t}{1-(1-\pi)e^t}$	$\frac{1}{\pi}$	$\frac{1-\pi}{\pi^2}$	$1-(1-\pi)^x$
Pascal	$\binom{k+x-1}{x} \pi^k (1-\pi)^x I_{\{0,1,2,\dots\}}(x)$	$\left[\frac{\pi}{1-(1-\pi)e^t} \right]^k$	$\frac{k(1-\pi)}{\pi}$	$\frac{k(1-\pi)}{\pi^2}$	
Pascal1	$\binom{x-1}{k-1} \pi^k (1-\pi)^{x-k} I_{\{k,k+1,k+2,\dots\}}(x)$	$\left[\frac{\pi e^t}{1-(1-\pi)e^t} \right]^k$	$\frac{k}{\pi}$	$\frac{k(1-\pi)}{\pi^2}$	
Zeta o Zipf	$\frac{C(\alpha+1)}{x^{\alpha+1}} I_{\{1,2,3,\dots\}}(x)$ donde $C(\alpha+1) = \left[\sum_{x=1}^{\infty} \left(\frac{1}{x} \right)^{\alpha+1} \right]^{-1}$	No es útil	$\frac{C(\alpha+1)}{C(\alpha)}$ para $\alpha > 0$	$\frac{C(\alpha+1)}{C(\alpha-1)} - \left[\frac{C(\alpha+1)}{C(\alpha)} \right]^2$ para $\alpha > 1$	
Weibull Discreta Tipo 1-a	$\left[q^{x^\beta} - q^{(x+1)^\beta} \right] I_{\{0,1,2,\dots\}}(x),$ $0 < q < 1, \beta > 0, [q = \text{cuantil}]$				$1 - q^{(x+1)^\beta}$

Weibull Discreta Tipo 1-b	$\left[q^{(x-1)^\beta} - q^{x^\beta} \right] I_{\{1,2,3,\dots\}}(x),$ $0 < q < 1, \beta > 0, [q = \text{cuantil}]$				$1 - q^{x^\beta}$
Uniforme Continua	$\frac{1}{\beta - \alpha} I_{[\alpha, \beta]}(x)$	$\frac{e^{\beta t} - e^{\alpha t}}{t(\beta - \alpha)}$	$\frac{\alpha + \beta}{2}$	$\frac{(\beta - \alpha)^2}{12}$	$\frac{x - \alpha}{\beta - \alpha} I_{[\alpha, \beta]}(x)$ $+ I_{[\beta, \infty]}(x)$
Triangular	$\frac{2(x-a)}{(b-a)(c-a)} I_{[a, c]}(x)$ $+ \left[\frac{2(b-x)}{(b-a)(b-c)} \right] I_{[c, b]}(x)$	No es útil	$\frac{a+b+c}{3}$	$\frac{a^2 + b^2 + c^2 - ab - ac - bc}{18}$	
Normal	$\frac{\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]}{\sqrt{2\pi}\sigma} I_{[-\infty, \infty]}(x)$	$\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$	μ	σ^2	
LogNormal	$\frac{\exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]}{x\sqrt{2\pi}\sigma} I_{[0, \infty)}(x)$	No es útil	$\exp\left(\mu + \frac{\sigma^2}{2}\right)$	$[\exp(\sigma^2) - 1] \exp(2\mu + \sigma^2)$	
Gamma	$\frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)} I_{[0, \infty)}(x)$	$\left(\frac{\lambda}{\lambda - t}\right)^r$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	
Exponencial	$\lambda e^{-\lambda x} I_{[0, \infty)}(x)$	$\left(\frac{\lambda}{\lambda - t}\right)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$1 - e^{-\lambda x}$

Hipoexponencial	$\sum_{i=1}^n \lambda_i e^{-\lambda_i x} \left(\prod_{j=1, j \neq i}^n \frac{\lambda_j}{\lambda_j - \lambda_i} \right)$		$E(X) = \sum_{i=1}^n \frac{1}{\lambda_i}$ $E(X^2) = \sum_{i=1}^n \frac{1}{\lambda_i^2}$		
Ji-Cuadrado	$\frac{\left(\frac{1}{2}\right)^{\frac{k}{2}} x^{\frac{k}{2}-1} e^{-\frac{1}{2}x}}{\Gamma\left(\frac{k}{2}\right)} I_{[0,\infty)}(x)$	$\left(\frac{1}{1-2t}\right)^{\frac{k}{2}}$	k	$2k$	
Beta	$\frac{(x-\alpha)^{a-1} (\beta-x)^{b-1}}{B(a,b)(\beta-\alpha)^{a+b-1}} I_{[\alpha,\beta]}(x)$	No es útil	$\alpha + (\beta - \alpha) \frac{a}{a+b}$	$\frac{(\beta - \alpha)^2 ab}{(a+b+1)(a+b)^2}$	
Beta Estándar	$\frac{x^{a-1} (1-x)^{b-1}}{B(a,b)} I_{[0,1]}(x)$	No es útil	$\frac{a}{a+b}$	$\frac{ab}{(a+b+1)(a+b)^2}$	
Beta Tipo II	$\frac{x^{m-1}}{B(m,n)(1+x)^{m+n}} I_{[0,\infty)}(x)$		$\frac{m}{n-1}$	$\frac{m(m+n-1)}{(n-1)^2(n-2)}$	
Distribución	$f(x)$	f.g.m= $\Psi_X(t)$	$E(X)$	$\text{Var}(X)$	$F(x)$
Weibull	$\frac{\mu}{\theta} \left(\frac{x-r}{\theta}\right)^{\mu-1} \exp\left[-\left(\frac{x-r}{\theta}\right)^{\mu}\right] I_{[r,\infty)}(x)$	No es útil	$r + \theta \Gamma\left(1 + \frac{1}{\mu}\right)$	$\theta^2 \left\{ \Gamma\left(1 + \frac{2}{\mu}\right) - \left[\Gamma\left(1 + \frac{1}{\mu}\right) \right]^2 \right\} 1 - \exp\left[-\left(\frac{x-r}{\theta}\right)^{\mu}\right]$	

Cauchy	$\frac{\lambda}{\pi \left[\lambda^2 + (x - \mu)^2 \right]}$	No tiene, pero si F. Característica $\Phi(t) =$ $\exp[i\mu t - \lambda t]$	Indefinida	Indefinida	$\frac{1}{2} +$ $+ \frac{1}{\pi} \arctg \left(\frac{x - \mu}{\lambda} \right)$
Laplace	$\frac{\exp \left[-\frac{ x - \mu }{\lambda} \right]}{2\lambda}$	F. Característica: $\Phi(t) = \frac{e^{i\mu t}}{1 + \lambda^2 t^2}$	μ	$2\lambda^2$	$\frac{e^{\left(\frac{\mu - x}{\lambda} \right)}}{2} I_{(-\infty, \mu)}(x) +$ $\left[1 - \frac{e^{\left(\frac{x - \mu}{\lambda} \right)}}{2} \right] I_{[\mu, \infty)}(x)$
Logística	$\frac{\exp \left[-\left(\frac{x - \alpha}{\beta} \right) \right]}{\beta \left\{ 1 + \exp \left[-\left(\frac{x - \alpha}{\beta} \right) \right] \right\}^2}$	No es útil	α	$\frac{\pi^2 \beta^2}{3}$	$\frac{1}{1 + \exp \left[-\left(\frac{x - \alpha}{\beta} \right) \right]}$
Pareto	$\frac{\beta \alpha^\beta}{x^{\beta+1}} I_{[\alpha, \infty)}(x)$	No es útil	$\frac{\alpha \beta}{\beta - 1}$	$\left(\frac{\alpha}{\beta - 1} \right)^2 \left(\frac{\beta}{\beta - 2} \right)$	$1 - \left(\frac{\alpha}{x} \right)^\beta$
Pareto Tipo II	$\frac{\beta \alpha^\beta}{(\alpha + x)^{\beta+1}} I_{[0, \infty)}(x)$	No es útil	$\frac{\alpha}{\beta - 1}, \beta > 1$	$\frac{\alpha^2 \beta}{(\beta - 1)^2 (\beta - 2)}, \beta > 2$	$1 - \left(\frac{\alpha}{\alpha + x} \right)^\beta$

r	$\frac{(1-x^2)^{\frac{n-4}{2}}}{B\left(\frac{1}{2}, \frac{n-2}{2}\right)} I_{[-1,1]}(x)$	No es útil	0	$\frac{1}{n-1}$	
Burr	$\frac{ckx^{c-1}}{(1+x^c)^{k+1}} I_{[0,\infty)}(x), \quad c, k > 0$		$E(X^r) = k \cdot \text{Beta}\left(1-\frac{r}{c}, k+\frac{r}{c}\right)$	$E(X^2) - [E(X)]^2$	$1 - (1+x^c)^{-k}$
Gumbel para máximos asintóticos	$\alpha e^{-\alpha(x-\mu)} \exp\left\{-e^{-\alpha(x-\mu)}\right\} I_{(-\infty,\infty)}(x)$		$\mu + \frac{\gamma}{\alpha} \approx \mu + \frac{0.577}{\alpha}$	$\frac{\pi^2}{6\alpha^2}$	$\exp\left\{-e^{-\alpha(x-\mu)}\right\}$
Gumbel para mínimos asintóticos	$\alpha e^{\alpha(x-\mu)} \exp\left\{-e^{\alpha(x-\mu)}\right\} I_{(-\infty,\infty)}(x)$		$\mu - \frac{\gamma}{\alpha} \approx \mu - \frac{0.577}{\alpha}$	$\frac{\pi^2}{6\alpha^2}$	$1 - \exp\left\{-e^{\alpha(x-\mu)}\right\}$
Fréchet para máximos asintóticos	$\frac{k}{\mu} \left(\frac{\mu}{x-\alpha}\right)^{k+1} e^{-\left(\frac{\mu}{x-\alpha}\right)^k} I_{[\alpha,\infty)}(x)$		$E(X) = \alpha + \mu \Gamma\left(1 - \frac{1}{k}\right)$ con $k > 1$ $E(X^p) = \alpha^p + \mu^p \Gamma\left(1 - \frac{p}{k}\right)$ con $k > p$	$\mu^2 \left[\Gamma\left(1 - \frac{2}{k}\right) - \Gamma^2\left(1 - \frac{1}{k}\right) \right]$ con $k > 2$	$\exp\left\{-\left(\frac{\mu}{x-\alpha}\right)^k\right\}$
Fréchet para mínimos asintóticos	$\frac{\alpha}{\mu} \left(\frac{x-\theta}{\mu}\right)^{\alpha-1} \exp\left[-\left(\frac{x-\theta}{\mu}\right)^\alpha\right] \cdot I_{(\theta,\infty)}(x)$		$E(X) = \theta + \mu \Gamma\left(1 + \frac{1}{\alpha}\right)$ con $\alpha > 1$ $E(X^p) = \theta^p + \mu^p \Gamma\left(1 + \frac{p}{\alpha}\right)$ con $p > 0$ y $\alpha > p$	$\mu^2 \left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right) \right]$ con $\alpha > 2$	$1 - \exp\left\{-\left(\frac{x-\theta}{\mu}\right)^\alpha\right\}$

Weibull para máximos asintóticos	$\left(\frac{k}{w-\mu}\right)\left(\frac{w-x}{w-\mu}\right)^{k-1} \exp\left[-\left(\frac{w-x}{w-\mu}\right)^k\right]$ $I_{(-\infty, w)}(x)$		$E(X) = w - (w-\mu)\Gamma\left(1+\frac{1}{k}\right)$ $E(w-X)^p = (w-\mu)^p \cdot \Gamma\left(1+\frac{p}{k}\right)$ con $p > 0$	$(w-\mu)^2 \cdot \left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]$	$\exp\left\{-\left(\frac{w-x}{w-\mu}\right)^k\right\}$
Distribución	$f(x)$	f.g.m= $\Psi_X(t)$	$E(X)$	$\text{Var}(X)$	$F(x)$
Weibull para mínimos asintóticos	$\left(\frac{k}{\mu-\varepsilon}\right)\left(\frac{x-\varepsilon}{\mu-\varepsilon}\right)^{k-1} \exp\left\{-\left(\frac{x-\varepsilon}{\mu-\varepsilon}\right)^k\right\}$ $I_{[\varepsilon, \infty)}(x)$		$E(X) = \varepsilon + (\mu-\varepsilon)\Gamma\left(1+\frac{1}{k}\right)$ $E(X-\varepsilon)^p = (\mu-\varepsilon)^p \Gamma\left(1+\frac{p}{k}\right)$	$(\mu-\varepsilon)^2 \left[\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right]$	$1 - \exp\left\{-\left(\frac{x-\varepsilon}{\mu-\varepsilon}\right)^k\right\}$