ALGEBRA 1 LECTURE

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LECTURER

ALGEBRA 1

ALGEBRA 1 SUBTOPICS

- Number System
- Significant Figures
- LCM and GCF
- Algebraic Equations
- Logarithms
- Remainder and Factor Theorem
- Quadratic Equations
- Depressed Equations
- Pythagorean Mean
- Factoring/Roots of Equation
- Descartes' Rule of Signs

ALGEBRA 1

WHAT IS ALGEBRA?

Algebra is a type of mathematics that uses letters to represent numbers.

Algebra comes from the magic word, "al-jabr" which means restoration.

Algebra is the science of restoration, reduction, completion and balancing.

ALGEBRA 1

ARITHMETIC

5 + 7 = ?

Reema have 5 coins. Her friend had 7 coins. How much did they have altogether?

ALGEBRA

5 + x = 12

Reema have 5 coins. She wants to buy a CD for 12 coins. How much more money does she need? Let x be the amount of money that she needs.

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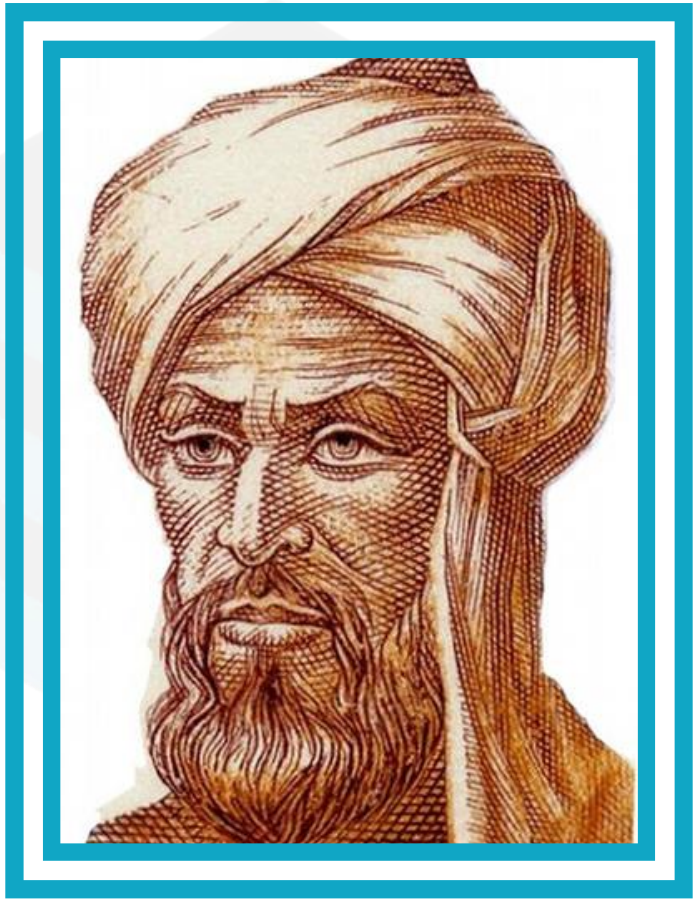
ALGEBRA 1

FATHER OF ALGEBRA



He is credited with developing the foundational concepts and methods of algebra, including the use of algebraic symbols and equations to represent unknown quantities and solve mathematical problems. Al-Khwarizmi's book, "Kitab al-Jabr wa-l-Muqabala" (The Compendious Book on Calculation by Completion and Balancing), introduced the algebraic method and served as a major influence on the development of mathematics in the Islamic world and Europe for centuries to come.

NUMBER SYSTEM

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Number

- Is an item that describes **magnitude** and **position.**

Cardinal

- Number that describes **magnitude** (size) or quantity of the collection of objects.
  Example: 1,2,3,.....

Ordinal

- Number that describes **position** relative to an ordering.
  Example: 1st, 2nd, 3rd,...

Roman Numerals

➡ Consist of seven symbols written in Latin Alphabet.

I = 1

V= 5

X = 10

L = 50

C = 100

D = 500

M = 1000

Example:

MCMXCIV

M + CM + XC + IV

1000 + 900 + 90 +4

= 1994

Another Example:

The roman numeral MMDXCV has a value of:

1000 + 1000 + 500 + 90 + 5

= 2595

Number System

Complex Numbers

- Composed of Real and Imaginary Numbers
- a.k.a. "Gaussian Integer"
- Plot in "Argand Diagram"
- a + bi (Where a & b are both real numbers)

If a = 0, 0 + bi ⟶ "Pure Imaginary Real Number"

If b = 0, a + 0i ⟶ "Real Number"

PROBLEM 1

Which of the following best describe 5+3i?

a. Irrational number
b. Real Number
c. Imaginary Number
d. Surd

Which of the following best describe 5+0i?

a. Irrational number
b. Real Number
c. Imaginary Number
d. Surd

ALGEBRA 1

Number System

Complex Number

- Real Numbers
  - Rational Numbers
  - Irrational Numbers
- Imaginary Numbers

→ $(i) = \sqrt{(-1)}$

(i) Leonhard Euler

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ALGEBRA 1

Imaginary Numbers (i)

Leonhard Euler

$(i) = \sqrt{(-1)}$

mode 2: (complex)

Factorial Sign "!" ⟶ Christian Kramp

Equal Sign "=" ⟶ Robert Recorde

PROBLEM 2

Evaluate $\sqrt{(-7)} \cdot \sqrt{(-10)}$

a. $\sqrt{(-70)}$
b. $-\sqrt{70}$
c. $-70i$
d. 70

MODE 2 (COMPLEX)

$\therefore -\sqrt{70}$

PROBLEM 3

What is the value of (1+i)^3?

MODE 2

a. -2+2i
b. -4-4i
c. 3+4i
d. 2i

What is the value of (1+i)^9?

$[(1+i)^3]^3$

a. 4+4i
b. -16-4i
c. 20+3i
d. 16+16i

WHAT IF:

$(1+i)^{10}$

$((1+i)^3)^3(1+i)$

ALGEBRA 1

Real Numbers

Rational Numbers

- Can be expressed in ratio of two integer.

  Example: 2/3, -5, 0.25, 0.11111...

Irrational Numbers

- Cannot be expressed in ratio of two integer.

  Example: π, e, √2 (first known irrational)

  A.k.a. :Transcendental Numbers"

PROBLEM 4

The number 0.099099099099... is:

a. Rational number
b. Irrational number
c. Imaginary
d. composite

$= \frac{11}{111}$

PROBLEM 5

The number $4+\sqrt{2}$ is:

a. Irrational number
b. Integer
c. Rational number
d. Complex

The number $4+\sqrt{4}$ is:

$4+2=6$

a. Irrational number
b. Imaginary number
c. Rational number
d. Complex

PROBLEM 6

Which of the following number is a surd?

a. $\sqrt{3}$

b. $\sqrt{4}+5$ $= 2+5 = 7$ ✗

c. $3 + 5i$ ✗

d. $5 + \sqrt{0}$ ✗

RADICAND · EXPRESSING IRRATIONAL NUMBER

Number System

Rational Numbers

- Integers
  - Negative
  - Zero
  - Positive
- Non-Integers
  - Terminating Decimals Ex: 0.5, 0.125, etc...
  - Non-Terminating but Repeating Decimals Ex: 0.636363...

Integer

- Consist of zero, positive natural numbers (Whole Numbers/Counting Numbers) and their opposites.

  Example: (...,-3,-2,-1,0,1,2,3...,)

Counting Numbers

- (1,2,3...,)

Prime Numbers

- is a natural number greater than 1 that has no positive divisors other than 1 and itself.

  Examples: 2,3,5,7, etc.

Composite Numbers

- it is a positive integer that has at least one divisor other than 1 and itself.

  Examples: 4,6,8,9,10,etc.

4

$1 \times 4 = 4$

$2 \times 2 = 4$

Twin Primes

- is a prime number that is either 2 less or 2 more than another prime number.
  Examples: (41,43) or (11,13)

EMIRP

- is a prime number that results in a different prime when its decimal digits are reversed.
  Examples: (13,31) or (17,71)

PROBLEM 7

Which of the following number is a PRIME NUMBER?

a. 377
b. 313
c. 357
d. 333

377

1 x 377
13 x 29

313

| x | y |

MODE 7 (TABLE)

$f(x) = \frac{313}{x}$

START : 2
END : 20
STEP: 1

ALGEBRA 1

SPECIAL (TRIVIAL NUMBERS)

- Happy Numbers
- Sad Numbers
- Perfect Numbers
- Abundant Numbers (excessive number
- Deficient Numbers
- Polite Numbers
- Friendly Number (Amicable numbers)

(220,84)

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Happy Numbers

- is a number defined by the following process:
  Starting with any positive integer, replace the number by the sum of the squares of its digits, and repeat the process until the number equals 1.
  Examples: 1,7,13,19,23,31,49,70,79,91,97,100, etc.

  PS. Numbers do not end in 1 are unhappy numbers (or sad numbers)

PROBLEM 8

Which of the following number is a happy number?

a. 15
b. 91
c. 50
d. 34

A. 15

$1^2 + 5^2 = 26$

$2^2 + 6^2 = 40$

$4^2 + 0^2 = 16$

$1^2 + 6^2 = 37$

$3^2 + 7^2 = 58$

B. 91

$9^2 + 1^2 = 82$

$8^2 + 2^2 = 68$

$6^2 + 8^2 = 100$

$1^2 + 0^2 + 0^2 = 1$

Perfect Numbers

- an integer that is equal to the sum of all its possible divisors except the number itself.

  Examples: 6,28

6

$1 \times 6 = 6$

$2 \times 3 = 6$

$1 + 2 + 3 = 6$

$6 = 6$ ✓

Abundant Numbers

- an integer in which the sum of all possible divisors except the number itself is greater than the integer.

  Examples: 12,18,20 and 24

12

$1 \times 12 = 12$

$3 \times 4 = 12$

$2 \times 6 = 12$

$6 + 2 + 1 + 3 + 4 = 18 > 12$

Deficient Numbers

- an integer in which the sum of all possible divisors except the number itself is less than the integer.

  Examples: 8 and 15

8

$1 \times 8 = 8$

$2 \times 4 = 8$

$1 + 2 + 4 = 7 < 8$

Polite Numbers

- is a positive integer that can be written as the sum of two ore more consecutive positive integers.

Examples:

6 = 1+2+3

18 = 3+4+5+6

41 = 20+21

13+14+15 = 42

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Impolite Numbers

- Numbers that are not polite

  Examples are 1 and powers of 2

$2^1, 2^2, 2^3, 2^4 \ldots\ldots$

Amicable/Friendly Numbers

➡ Two integer numbers are said to be amicable numbers if each is the sum of all possible divisors of the other.

220 and 284

Divisor of 220		Divisor of 284	Sum of the Divisor
220 = 1 x 220	220 = 20 x 11	284 = 1 x 284	1 + 2 + 4 + 71 + 142 = 220
220 = 2 x 110	220 = 22 x 10	284 = 2 x 142	
220 = 4 x 55	220 = 44 x 5	284 = 4 x 71	Sum of the Divisor
220 = 5 x 44	220 = 55 x 4	284 = 71 x 4	1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 +
220 = 10 x 22	220 = 110 x 2	284 = 142 x 2	44 + 55 + 110 = 284
220 = 11 x 20			

PROBLEM 9

The number 25 is what type of special number?

A. Perfect Number
B. Deficient number
C. Abundant number
D. Happy Number

25

$1 \times 25 = 25$

$5 \times 5 = 25$

$1 + 5 + 5 = 11 < 25$

SIGNIFICANT FIGURES

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Significant Figures (Digits)

The significant figures of a number are those digits that carry meaning contributing to its precision.

Without Decimal

➡ Trailing zero's are insignificant.

~~1000~~	1 ✓
100~~100~~	4 ✓

With Decimal

➡ Count from right to left and only stop at a NON-ZERO digit.

100.00	5
~~0.000~~1	1
~~0~~.100	3
~~0.00~~1002	4

Significant Figures (Digits)

➡ EXAMPLES:

1234 → 4
1000 → 1
100100 → 4
100.00 → 5
0.00001 → 1
0.100 → 3
0.001002 → 4

5000. → 4
5000 → 1

PROBLEM 10

Which of the following statements is correct?

a. 5000 has four significant figures → 1

b. 0.0005 has five significant figures → 1

c. 100050 has six significant figures → 5

d. 0.00300 has three significant figures → 3

LCM AND GCF

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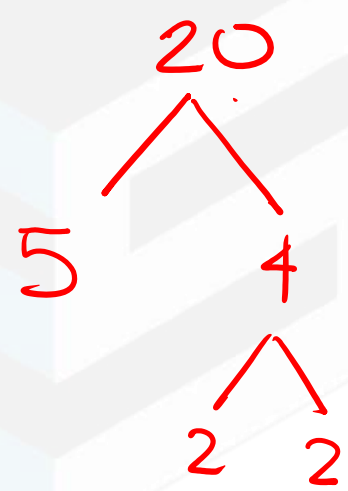
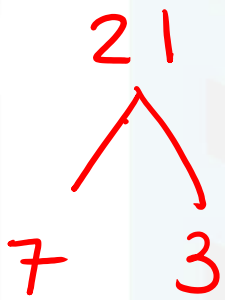
Least Common Multiple (LCM)

- Is the smallest counting numbers which is a multiple of each the given numbers.
- The LCM is the product of the prime factors with the highest power in the factorization.

What is the LCM of 20, 21 and 22?

a. 7650
b. 5530
c. 9240 ✓
d. 4620 ✓





$20: 5 \times 2^2$

$21: 7 \times 3$

$22: 11 \times 2$

$LCM = 5 \times 7 \times 3 \times 11 \times 2^2$

$= 4620$

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PROBLEM 11

The red bulbs of a Christmas light flash every 10 seconds; the blue bulbs flash every 15 seconds and the green bulbs flash every 20 seconds. If they all start flashing together, in every how many seconds will they flash simultaneously again?

a. 1000 ×
b. 2000 ×
c. 3000 ✓
d. 4000 ×

LCM

20 IS 10

Greatest Common Factor (GCF)

- Is the largest counting numbers which is a factor of each the given numbers.
- The GCF is the product of the smallest prime factors common to both.

What is the GCF of 150 and 225?

a. 55 ✗
b. 35 ✗
c. 25 ✓
d. 75 ✓

ALGEBRAIC EQUATIONS

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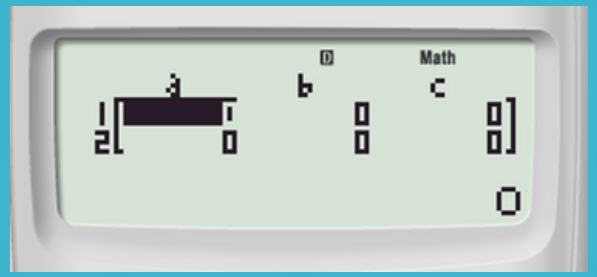
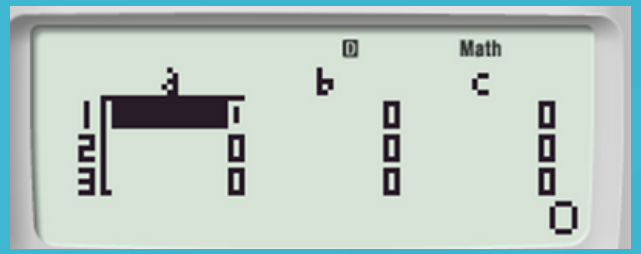
2 equations, 2 unknowns

MODE 5:1



3 equations , 3 unknowns

MODE 5:2



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SAMPLE PROBLEMS

```
1:anX+bnY=cn
2:anX+bnY+cnZ=dn
```

Solve for x and y in the equations

$$3x + 5y = 16$$
$$7x + 10y - 14 = 0$$

$3x + 5y = 16$
$7x + 10y = 14$

MODE 5:1

$x = -18$
$y = 14$

Solve for m, k and z in the equations

$$5m + 2k - 3z = 5$$
$$7m + 4z - 3k - 7 = 0$$
$$2z + 6m - 4k = 15$$

$5m + 2k - 3z = 5$
$7m - 3k + 4z = 7$
$6m - 4k + 2z = 15$

MODE 5:2

$m = 0.91$
$k = -3.69$
$z = -2.61$

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ALGEBRA 1

Repeated Algebraic Equations

PROBLEM 12

Solve for the value of x:

$$2x = \sqrt{2 - \sqrt{2 - \sqrt{2 - \cdots}}}$$

a. 0.618

b. 0.500

c. 0.554

d. 1.212

$$2x = \sqrt{2 - 2x}$$

SHIFT SOLVE:

$$x = 0.5$$

Repeated Algebraic Equations

PROBLEM 13

Solve the value of x from the following equation

$$x^{x^{x^{x.....}}} = 5$$

a. 1.745619
b. 2.128419
c. 1.258925
d. 1.379729

$x^{x^{x^{x\cdots}}} = 5$

$x^5 = 5$

$x = 5^{1/5} = 1.379729$ //

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PROBLEM 14

Let $x = \sqrt{2}$

$y = \sqrt{3}$

Expand $(x - y)^3$

$= -0.032$ //

$a. x^3 + 2x^2y - 3xy^2 + y^3 = 2.215$

$b. x^3 - 3x^2y + 3xy^2 - y^3 = -0.032$ //

$c. x^3 + 3x^2y + 3xy^2 - y^3 = 20.75$

$d. x^3 - 3x^2y + 3xy^2 - 4y^3 = -15.62$

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PROBLEM 15

Let $x = \sqrt{2}$

Resolve $\frac{x+2}{x^2-7x+12}$ into partial fraction:

a. $\frac{4}{x-4} - \frac{2}{x-3}$ $= -0.285$

b. $\frac{3}{x-2} - \frac{5}{x-3}$ $= X$

c. $\frac{6}{x-4} - \frac{5}{x-3}$ $= 0.832$

d. $\frac{6}{x-4} - \frac{8}{x-6}$ $= X$

$= 0.832$

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PROBLEM 16

Find the value of B in the partial fraction equation:

$$\frac{x+10}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

a. -1
b. 3
c. -2
d. 5

$x^2-4 = (x+2)(x-2)$

$(x+2)(x-2)\left(\frac{x+10}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}\right)(x+2)(x-2)$

$x+10 = A(x+2) + B(x-2)$

Let $x = -2$

$-2+10 = A(-2+2) + B(-2-2)$

$-2+10 = 0 + B(-2-2)$

$B = -2$

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PROBLEM 17

Simplify the given expression:

$$\frac{5x}{2x^2+7x+3}-\frac{x+3}{2x^2-3x-2}+\frac{2x+1}{x^2+x-6}$$

a. $\frac{4}{x+3}$

b. $\frac{3}{x+6}$

c. $\frac{2}{x+5}$

d. $\frac{1}{x+3}$

CALC $x=\sqrt{2}$

Answer: A

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PROBLEM 18

Simplify the given expression:

$$3\sqrt{3x^3} + x\sqrt{27x} - 2x\sqrt{48x}$$

Let $x = \sqrt{2}$

$= -5.825$

a. $-5x\sqrt{3x}$
b. $-2x\sqrt{3x}$ $= -5.825$
c. $-3x\sqrt{6x}$
d. $-5x\sqrt{6x}$

IT'S DEYN'S TIME!
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$\log_3 3^x = \log_3 243$

$x = \log_3 243$

LOGARITHMS

$3^x = 243$

$x \log 3 = \log 243$

$x = \frac{\log 243}{\log 3}$

ALGEBRA 1

LOGARITHM

Logus – Ratio
Arithmus - Number

JOHN NAPIER

- Naperian Logarithm
- Natural Logarithm
- Base "e"
- $log_e = ln$

What is the natural logarithm of e^{xy}?

$\ln e^{xy}$

$\log_e(e^{xy})$

xy

HENRY BRIGGS

- Brigssian Logarithm
- Common Logarithm
- Base "10"
- $log_{10} = log$

What is the common logarithm of 10^{xy}?

$\log 10^{xy}$

$\log_{10} 10^{xy}$

xy

LOGARITHM

➡ The logarithm of number “N” to the base “b” is the exponent or power to which the base must be raised to obtain “N”.

$$N = b^x \rightarrow x = log_b N$$

$b^x = N$

$N = b^x$

$\log_b b^x = \log_b N$

$x = \log_b N$

ALGEBRA 1

PROPERTIES OF LOGARITHM

- $\log\ (xy) = \log\ (x) + \log\ (y)$
- $\log \left(\frac{x}{y}\right) = \log\ (x) - \log\ (y)$
- $\log\ x^n = \mathrm{n} \log\ x$
- $\log_y x = \frac{\log x}{\log y}$

$\log\ 0 = -\infty$

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PROPERTIES OF LOGARITHM

- $\log\ x = 0.4343 \ln x$
- $\ln\ x\ = 2.3026 \ln x$

Modulus of logarithm

$$\frac{\log x^{s}}{\ln x^{s}} = 0.4343$$

$$\frac{\ln s}{\log s} = 2.3026$$

PROPERTIES OF LOGARITHM

Characteristics

➡ The integral part of common logarithm.

Mantissa

➡ The non-negative decimal part.

Note: Characteristics + Mantissa = logarithm of number

Cologarithm

➡ $colog\,(x) = \log\left(\frac{1}{x}\right) = -\log(x)$

$\log(1) - \log(x)$ (0)

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PROBLEM 19

What is the characteristic and mantissa of the natural logarithm of 35?

$\ln(35) = 3.555$

Note: Characteristics + Mantissa = logarithm of number

$C = 3$

$M = 0.555$

$C + M = 3.555$

PROBLEM 20

What is the characteristic and mantissa of the common logarithm of 0.0071?

$C = -3$

$M = 0.8513$

$C = -2$

$M = 0.1487$

Mantissa

➡ The non-negative decimal part.

$\log(0.0071)$

$= -2.1487 + 3 = 0.8513$

$\hookrightarrow M$

$C = -2$

$M = -0.1487$

PROBLEM 21

What is the characteristic and mantissa of the natural logarithm of 0.00055?

Mantissa

➡ The non-negative decimal part.

$\ln(0.00055)$

$= -7.5056 + 8$

$= 0.4944 \rightarrow M$

$C = -8$

PROBLEM 22

If $log_x 10 = \frac{1}{4}$, *what is* $log_{10} X$?

a. 4
b. -1
c. 3
d. 2

SHIFT SOLVE

$x = 10000$

$\log_{10}(10000) = 4$

PROBLEM 23

Solve the value of x in the equation: $x^{3logx} = 100x$

a. 7
b. 5
c. 10
d. 9

SHIFT SOLVE

x = 10

PROBLEM 24

Find the value of x:

$$\log(x^2 + 3x) + log5x = 1 + log2x$$

a. 2
b. 1
c. 4
d. 3

SHIFT SOLVE

$x = 1$

ANOTHER SOLUTION:

USE CALC!

$$\log(x^2+3x) + \log 5x - 1 - \log 2x = 0$$

CALC CHOICES

PROBLEM 25

If log 2 = x and log 3 = y, what is the value of log 1.2?

a. $3x - y + 2$ = 2.426
b. $x + y + 1$ = 1.778
c. $2x + y - 1$ = 0.079
d. $X + 2y + 1$ =

$\log(1.2) = 0.079$

$X \rightarrow \log 2$

$y \rightarrow \log 3$

REMAINDER AND FACTOR THEOREM

PROBLEM 26

What is the remainder if $3x^4 + 2x^3 - 4x^2 + x + 4$ is divided by (x-2).

a. 50
b. 54
c. 52
d. 56

$3x^4 + 2x^3 - 4x^2 + x + 4$

$x - 2 \rightarrow x = 2$

	3	2	-4	1	4
2		6	16	24	50
	3	8	12	25	54

ALGEBRA 1

REMAINDER THEOREM

$$\frac{f(x)}{x - r} \quad Solve\ for\ x \quad x = r$$

PROBLEM 27

What is the remainder if $3x^4 + 2x^3 - 4x^2 + x + 4$ is divided by (x-2).

a. 50
b. 54
c. 52
d. 56

$3x^4 + 2x^3 - 4x^2 + x + 4$

CALC $x = 2$

$= 54$

ALGEBRA 1

FACTOR THEOREM

$$\frac{f\ (x)}{x\ -r}$$

$Equate\ the\ divisor\ to\ zero, Solve\ for\ x$:

x = r

$f\ (r) \rightarrow 0\ \ \therefore (x - r)\ is\ a\ factor$

PROBLEM 28

Which of the following is a factor of $3x^5 + 6x^4 + 25x - 1290$

a. x + 3 → x = -3
b. x + 5 → x = -5
c. x - 7 → x = 7
d. x - 3 → x = 3

$3x^5 + 6x^4 + 25x - 1290$

CALC x = 3

= 0 //

PROBLEM 29

Find the value of k for which x+4 is a factor of $x^3 + 2x^2 - 7x + k$.

a. 4
b. 1
c. 6
d. 9

$x + 4 \rightarrow x = -4$

$x^3 + 2x^2 - 7x + k$

$(-4)^3 + 2(-4)^2 - 7(-4) + k = 0$

$k = -4$

QUADRATIC EQUATIONS

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QUADRATIC EQUATIONS

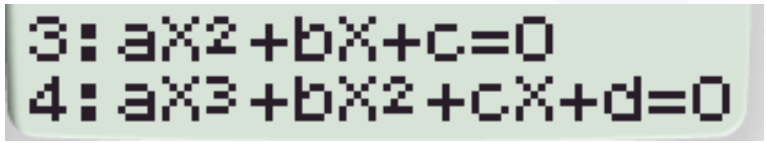
General formula $= ax^2 + bx + c = 0$

Quadratic Formula (Roots) $= x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Sum of Roots $= x_1 + x_2 = \frac{-B}{A}$

Product of Roots $= x_1 x_2 = \frac{C}{A}$

QUADRATIC EQUATIONS→→ $MODE$ 5: 3
CUBIC EQUATIONS→→ $MODE$ 5: 4



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ES MECHANICAL ENGINEERING REVIEW

PROBLEM 30

Given the equation $2x^2 + x - 10 = 0$.
Find the sum and product of the roots.

Sum of Roots $= x_1 + x_2 = \frac{-B}{A}$

Product of Roots $= x_1 x_2 = \frac{C}{A}$

$Ax^2 + Bx + C = 0$

$A = 2$

$B = 1$

$C = -10$

$S = \frac{-B}{A} = \boxed{\frac{-1}{2}}$

$P = \frac{C}{A} = \frac{-10}{2} = \boxed{-5}$

ANOTHER SOLUTION:

MODE 5 : 3

$x_1 = 2$

$x_2 = -5/2$

$S = 2 + \frac{-5}{2} = \boxed{\frac{-1}{2}}$

$P = 2 \times \left(\frac{-5}{2}\right) = \boxed{-5}$

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QUADRATIC EQUATIONS

Nature of Roots: $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Discriminant: $B^2 - 4AC$

$B^2 - 4AC = 0$ One root (real and equal)

$B^2 - 4AC > 0$ Roots are real and unequal

$B^2 - 4AC < 0$ Roots are imaginary/complex

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PROBLEM 31

Find the value of k so that the two roots of $2x^2 + 3kx + 9 = 0$ are equal.

a. $2\sqrt{2}$
b. $\sqrt{5}$
c. $3\sqrt{2}$
d. $\sqrt{7}$

$Ax^2 + Bx + C = 0$

$A = 2$

$B = 3k$

$C = 9$

$B^2 - 4AC = 0$

$(3k)^2 - 4(2)(9) = 0$

$k = 2.828$

$= 2\sqrt{2}$

PROBLEM 32

Renz and Maynard attempted to solve a quadratic equation. Renz made a mistake in writing down the constant term and ended up in roots (4,3). Maynard made a mistake in writing down coefficient of x to get the roots (3,2). The correct roots of equations are:

a. (4,2)
b. (4,-3)
c. (6,1)
d. (3,-3)

RENZ

$(x-4)(x-3)$

$x^2-4x-3x+12$

$x^2-7x+12$

MAYNARD

$(x-3)(x-2)$

$x^2-3x-2x+6$

x^2-5x+6

x^2-7x+6

MODE 5: 3

$x_1 = 6$

$x_2 = 1$

DEPRESSED EQUATION

ALGEBRA 1

DEPRESSED EQUATION

➡ When you divide an equation by one of its binomial factors, the resulting equation is called depressed equation.

PROBLEM 33

What is the depressed equation of

$$4x^3 - 2x^2 + x - 3 = 0$$

a. $4x^3 - 2x^2 + x$
b. $5x^2 + 2x + 3$
c. $5x^2 + 2x + 7$
d. $4x^2 + 2x + 3$

MODE 5: 4

x = 1

x = -0.25 + 0.829i

x = -0.25 - 0.829i

$4x^2 + 2x + 3 = 0$

	4	-2	1	-3
1		4	2	3
	4	2	3	0

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PYTHAGOREAN MEAN

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ALGEBRA 1

PYTHAGOREAN MEAN

$$AM\ (x_1, \ldots, x_n) = \frac{1}{n}(x_1 + \cdots + x_n)$$

$$\mathrm{G}M\ (x_1, \ldots, x_n) = \sqrt[n]{|x_1 \ldots \times x_n|}$$

$$\mathrm{H}M\ (x_1, \ldots, x_n) = \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

$$GM = \sqrt{AM\ x\ HM}$$

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PROBLEM 34

A quadratic equation in x such that the arithmetic mean of its roots is 5 and geometric mean of its roots is 4. The equation is:

a. $x^2 + 4x - 12 = 0$
b. $x^2 - 8x - 10 = 0$
c. $x^2 - 10x + 16 = 0$
d. $x^2 + 12x - 20 = 0$

$x_1 \quad x_2$

$AM = 5$

$\frac{x_1 + x_2}{2} = 5 \rightarrow (1)$

$GM = 4$

$(x_1 \cdot x_2)^{1/2} = 4$

$(x_1)(x_2) = 4^2$

$x_2 = \frac{16}{x_1} \rightarrow (2)$

$\frac{x_1 + \frac{16}{x_1}}{2} = 5$

$x_1 = 8$

$x_2 = \frac{16}{x_1} = \frac{16}{8}$

$x_2 = 2$

$(x-8)(x-2)$

$x^2 - 8x - 2x + 16$

$x^2 - 10x + 16 = 0$

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ALGEBRA 1

ROOT MEAN SQUARE

$$\text{RMS} = \sqrt{\frac{A^2 + B^2 + C^2 + \cdots \ldots}{n}}$$

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PROBLEM 35

WHAT IS THE ROOT MEAN SQUARE OF 11, 23 and 35?

a. 21
b. 22
c. 24
d. 25

$$RMS = \sqrt{\frac{11^2 + 23^2 + 35^2}{3}}$$

$$= 25$$

FACTORING/ROOTS OF EQUATIONS

PROBLEM 36

Factor completely the given $x^3 - 5x^2 - 48x + 108 = 0$

a. (x+8)(x+4)(x-5)=0
b. (x+5)(x-3)(x-6)=0
c. (x+7)(x-8)(x+9)=0
d. (x+6)(x-9)(x-2)=0

MODE 5:4

$x = -6$

$x = 9$

$x = 2$

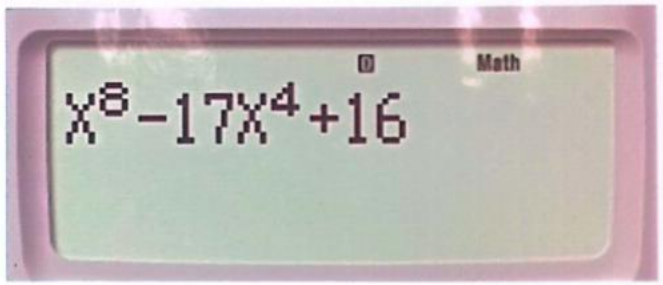
$(x+6)(x-9)(x-2) = 0$ //

PROBLEM 37

Solve the real roots of
$x^8 - 17x^4 + 16 = 0$

a. $\pm 2, \pm 1$
b. $\pm 4, \pm 3$
c. $\pm 6 \ \pm 8$
d. $\pm 5 \ \pm 7$

USE CALC: input data



X? +2 −2 +1 −1

DESCARTES' RULE OF SIGNS

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DESCARTES' RULE OF SIGNS

- Is a technique for determining an upper bound on the number of positive or negative real roots if a polynomial.
- It is not a complete criterion, because it does not provide the exact number of positive or negative roots.
- The rule is applied by counting the number of sign changes in the sequence formed by the polynomial's coefficients.

DESCARTES' RULE OF SIGNS

No. of possible positive roots

$f\ (+x)$ = equal to the number of sign differences
or less than it by an even number

No. of possible negative roots

$f\ (-x)$ = equal to the number of sign differences
or less than it by an even number

PROBLEM 38

The equation $3x^6 + 4x^5 + 3x^3 - x - 3 = 0$ has how many maximum possible positive real roots?

a. Two
b. Three
c. Four
d. one

No. of possible positive roots :

$f(+x) = 3x^6 + 4x^5 + 3x^3 - x - 3$

1 change in sign

$\therefore$ ONE MAX. POSSIBLE POSITVE!!!

PROBLEM 39

The equation $3x^6 + 4x^5 + 3x^3 - x - 3 = 0$ has how many maximum possible negative real roots?

a. Two
b. Three
c. Four
d. one

No. of possible negative roots :

$f(-x) = 3(-x)^6 + 4(-x)^5 + 3(-x)^3 - (-x) - 3$

$f(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$ 3 changes in sign

1 1 1 = 3

PROBLEM 40

The equation $3x^6 + 4x^5 + 3x^3 - x - 3 = 0$ has how many maximum possible real roots?

a. Two
b. Three
c. Four
d. one

No. of possible positive roots :

$f(+x) = 3x^6 + 4x^5 + 3x^3 - x - 3$

1 change in sign

No. of possible negative roots :

$f(-x) = 3(-x)^6 + 4(-x)^5 + 3(-x)^3 - (-x) - 3$

$f(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$

3 changes in sign

$\therefore$ 4 max. possible real !!!

$1 + 3 = 4$

PROBLEM 41

The equation $3x^6 + 4x^5 + 3x^3 - x - 3 = 0$ has how many minimum possible real roots?

a. Two
b. Three
c. Four
d. one

$total\ roots = 6$

No. of roots = ... or less than it by an even number

$\therefore$ 4 max. possible real

$\therefore$ 2 min. possible real !!!

$6 - 4 = \boxed{2}$