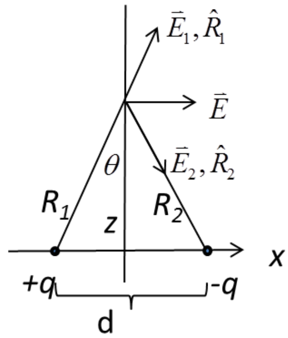
EM(1) Quiz # 2

Name: ____________________________ ID: _________________________________

1. (10%) (a) Find the electric field $\vec{E}$ =? (magnitude and direction) a distance z above the midpoint between two charges (+q, -q) with a distance d apart.

   (b) From(a), find $\vec{E}$ =? at distance far away (z>>d)



(Hint: Coulomb's' law $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1^2}\hat{R}_1 + \frac{q_2}{R_2^2}\hat{R}_2 + ...\right)$)

$$R_1 = R_2 = \sqrt{z^2 + \left(\tfrac{d}{2}\right)^2} \qquad \hat{R}_1 + \hat{R}_2 = 2\sin\theta = 2\frac{d/2}{\sqrt{z^2 + \left(\frac{d}{2}\right)^2}}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1^2}\hat{R}_1 + \frac{q_2}{R_2^2}\hat{R}_2\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}2\sin\theta\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2 + \left(\frac{d}{2}\right)^2}2\frac{d/2}{\sqrt{z^2 + \left(\frac{d}{2}\right)^2}}\hat{x} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}\hat{x}$$

$$\text{z>>d} \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}\hat{x} \underset{z>>d}{\approx} \frac{1}{4\pi\varepsilon_0}\frac{qd}{z^3} \propto \frac{p}{z^3} \quad p = qd$$

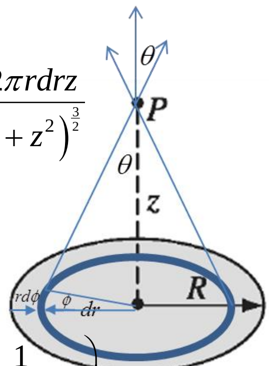
2. (10%) (a) Find the electric field a distance z above the center of a circular disk with radius R and a uniform surface charge density σ

   (b) From a, find electric field E in the limit R>>z

   (hint: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\vec{r}\,')}{\mathfrak{R}^2}\hat{\mathfrak{R}}da' \quad \vec{\mathfrak{R}} = \vec{r} - \vec{r}\,'$

   First for a ring using symmetry $d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{\phi=0}^{\phi=2\pi}\frac{(\sigma dr)}{\mathfrak{R}^2}\cos\theta\hat{z}\, rd\phi \quad \mathfrak{R} = \sqrt{r^2 + z^2}, \cos\theta = \frac{z}{\sqrt{r^2 + z^2}}$

   Then integrate over r $\vec{E} = \int_{r=0}^{r=R}(dE)$



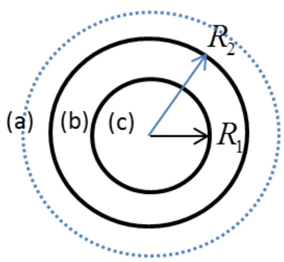
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{\phi=0}^{\phi=2\pi}\frac{(\sigma dr)}{r^2 + z^2}\frac{z}{\sqrt{r^2 + z^2}}\hat{z}\, rd\phi = \hat{z}\frac{1}{4\pi\varepsilon_0}\frac{\sigma rdrz}{\left(r^2 + z^2\right)^{\frac{3}{2}}}\int_{\phi=0}^{\phi=2\pi}d\phi = \hat{z}\frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi rdrz}{\left(r^2 + z^2\right)^{\frac{3}{2}}}$$

$$\vec{E} = \int_{r=0}^{r=R}\hat{z}\frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi rz}{\left(r^2 + z^2\right)^{\frac{3}{2}}}dr = \hat{z}\frac{2\pi z\sigma}{4\pi\varepsilon_0}\int_{r=0}^{r=R}\frac{r}{\left(r^2 + z^2\right)^{\frac{3}{2}}}dr$$

$$= \hat{z}\frac{2\pi z\sigma}{4\pi\varepsilon_0}\int_{r=0}^{r=R}\frac{\frac{1}{2}}{\left(r^2 + z^2\right)^{\frac{3}{2}}}d\left(r^2 + z^2\right) = \hat{z}\frac{2\pi z\sigma}{4\pi\varepsilon_0}\frac{1}{2}\left.\frac{\left(r^2 + z^2\right)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right|_{r=0}^{r=R} = \hat{z}\frac{2\pi z\sigma}{4\pi\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}}\right)$$

$$\text{R>>z} \quad \vec{E} = \hat{z}\frac{2\pi z\sigma}{4\pi\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}}\right) \approx \hat{z}\frac{2\pi z\sigma}{4\pi\varepsilon_0}\frac{1}{z} = \frac{\sigma}{2\varepsilon_0}\hat{z} \quad \text{same as flat surface}$$

3. (10%) Use Gauss's law $\oint \vec{E}\cdot da = \int \nabla\cdot\vec{E}d\tau = \int\frac{\rho}{\varepsilon}d\tau = \frac{1}{\varepsilon}Q_{enclosed}$ to find the electric field between two spherical ***shell*** with radius R_1 and R_2 and surface charge density σ_1 , σ_2 such that the total charge Q on each shell are equal but with opposite sign. (a) E ($r>R_2$) (b) E ($R_1<r<R_2$) (c) E ($r<R_1$)



$$(a) \;\; r > R_2 \;\; \int\frac{\rho}{\varepsilon}d\tau = \frac{1}{\varepsilon}Q_{enclosed} = 0$$

$$0 = \oint \vec{E}\cdot da = 4\pi r^2 E \to E\left(r > R_2\right) = 0$$

$(b)\;\; R_1 < r < R_2 \;\; \int \frac{\rho}{\varepsilon} d\tau = \frac{1}{\varepsilon} Q_{enclosed} = \frac{1}{\varepsilon} \sigma_1 4\pi R_1^2$

$\oint \vec{E} \cdot da = 4\pi r^2 E = \frac{1}{\varepsilon} \sigma_1 4\pi R_1^2 \rightarrow E\left(R_1 < r < R_2\right) = \frac{\sigma_1 4\pi R_1^2}{4\pi\varepsilon r^2} = \frac{Q}{4\pi\varepsilon r^2}$

$(c)\;\; r < R_1 \;\; \int \frac{\rho}{\varepsilon} d\tau = \frac{1}{\varepsilon} Q_{enclosed} = 0$

$0 = \oint \vec{E} \cdot da = 4\pi r^2 E \rightarrow E\left(r < R_1 \;\right) = 0$