# The LibTMCG Reference Manual

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## An Implementation of the Toolbox for Mental Card Games

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This is the reference manual of LibTMCG.

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# Table of Contents

# 1 Introduction

'LibTMCG' is a C++ library for creating secure and fair online card games. The library contains a sort of useful classes, algorithms, and high-level protocols to support an application programmer in writing such software. The most remarkable feature is the absence of a trusted third party (TTP), i.e., neither a central game server nor trusted hardware components are necessary. Thus, with the present library there is no need for an independent referee, because the applied protocols provide a basic level of confidentiality and fairness by itself. Consequently, the library is well-suited for peer-to-peer (P2P) environments where no TTP is available. Of course, we cannot avoid that malicious players share information about their private cards, but the protocols ensure that the shuffle of the deck is performed randomly (presumed that at least one player is honest) and thus the cards will be distributed uniformly among the players. Further, no coalition can learn the private cards of a player against his will (except for trivial conclusions). The corresponding cryptographic problem, actually called "Mental Poker", has been studied since 1979 (Shamir, Rivest, and Adleman) by many authors. LibTMCG provides the first practical implementation of such protocols.

The security and the fairness rely on advanced cryptographic techniques—the so-called zero-knowledge proofs. Using these 'building blocks' the high-level protocols minimize the effect of coalitions and preserve the confidentiality of the players' strategy, i.e., the players are not required to reveal their cards at the end of the game to show that they did not cheat. This important property is often required in card games like Poker.

LibTMCG is *Free Software* according to the definition of the Free Software Foundation. The source code is released under the GNU *General Public License* Version 2.

## 1.1 Further Reading

The cryptographic background and a detailed discussion of the implementation issues are beyond the scope of this manual. The interested reader is referred to the following papers:

**[Sc98]:** Christian Schindelhauer. *Toolbox for Mental Card Games.*
Technical Report A-98-14, University of Lübeck, 1998.
`http://citeseer.ist.psu.edu/schindelhauer98toolbox.html`

**[BS03]:** Adam Barnett and Nigel P. Smart. *Mental Poker Revisited.*
In K.G. Paterson (Ed.): Cryptography and Coding 2003, Lecture Notes in Computer Science 2898, pp. 370–383, 2003.

**[Gr05]:** Jens Groth. *A Verifiable Secret Shuffle of Homomorphic Encryptions.*
Cryptology ePrint Archive, Report 2005/246, 2005.
`http://eprint.iacr.org/2005/246`

**[Gr10]:** Jens Groth. *A Verifiable Secret Shuffle of Homomorphic Encryptions.*
Journal of Cryptology, Volume 23 Issue 4, pp. 546–579, 2010.
`http://dx.doi.org/10.1007/s00145-010-9067-9`

**[HSSV09]:** Sebastiaan de Hoogh, Berry Schoenmakers, Boris Skoric, and Jose Villegas. *Verifiable Rotation of Homomorphic Encryptions*
Public Key Cryptography 2009, Lecture Notes in Computer Science 5443, pp. 393–410, 2009.
`http://dx.doi.org/10.1007/978-3-642-00468-1_22`

**[St04]:** Heiko Stamer. *Kryptographische Skatrunde.* (in German)
Offene Systeme (ISSN 1619-0114), 4:10–30, 2004.
http://www.nongnu.org/libtmcg/OS-4-2004-openskat_rev2005.pdf

**[St05]:** Heiko Stamer. *Efficient Electronic Gambling: An Extended Implementation of the Toolbox for Mental Card Games.*
Proceedings of the Western European Workshop on Research in Cryptology (WEWoRC 2005), Lecture Notes in Informatics P-74, pp. 1–12, 2005.
http://www.nongnu.org/libtmcg/WEWoRC2005_proc.pdf

## 1.2 Getting Started

This manual describes the application programming interface of LibTMCG. All relevant data types, public classes and security parameters are explained. The reader should have an advanced knowledge in applied cryptography and C++ programming. Reference is made at this point to the famous *Handbook of Applied Cryptography* for a brief introduction of this topic.

This document follows, in style and rarely in phrasing, the *Reference Manual of the GNU Crypto Library*. Thus don't be surprised, if you recognize some obvious analogies.

## 1.3 Preliminaries

The most card games are played with a regular card deck, i.e., cards where the pattern on the front side (face) determines the card type (e.g. the King of Spades ♠, the Seven of Hearts ♡, the Ace of Club ♣, or the Jack of Diamonds ♢) and where the back sides of all cards are indistinguishable. Only such 'regular' card decks are supported by LibTMCG.

### 1.3.1 Terminology

The following list defines some common terms that are subsequently used in the manual.

**Player:** A *player* is an active participant in an electronic card game.

**Observer:** An *observer* is an passive party who watches the game.

**Card:** The term *card* means the electronic representation of a playing card.

**Card Type:** The *card type* is a nonnegative integer which corresponds to the pattern on the picture side of a real playing card. We assume here that such a natural encoding always exists.

**Masking:** *Masking* is a process which aim is to transform the card representation such that the input card and the result cannot be linked (except for trivial conclusions). Roughly speaking, masking is the (re-)encryption of a card representation such that the original card type is preserved.

**Card Secret:** The *card secret* contains all random values used in a masking operation. These values must be kept secret until the card is publicly revealed. Otherwise the corresponding output of the masking transformation is linkable and other players may learn the card type.

**Open Card:** An *open card* is a card whose type can be easily determined by all players and usually by observers as well.

**Masked Card:** A *masked card* (also known as face-down card) is a card whose type is unknown to a subset of players. It can be only revealed, if all players cooperate in a common computation of the type.

**Private Card:** A *private card* is a card whose type is only known to its owner. As long as the owner does not corporate the type of the private card stays hidden to all other players (except for trivial conclusions).

**Stack:** A *stack* is a not necessarily disjoint subset of the whole card deck.

**Prover and Verifier:** The *prover* is a player who shows some property to another party called *verifier*. For example, he wants to show that a masking operation was performed correctly, i.e., the card type is preserved by the transformation.

### 1.3.2 Security

"Mental Poker" solutions cannot prevent that malicious players exchange private information, for example, by telephone or Internet chat. Cryptographic protocols can only minimize the effect of such colluding parties and should try to protect the confidentiality for honest players. But even this small protection often relies on number-theoretical assumptions which are only believed to be true, i.e., problems like factoring products of large primes or computing discrete logarithms are only believed to be hard. That means, strict mathematical proofs[1] for the hardness of these problems are not known, and it is not very likely that such proofs will ever be found. However, almost all public key cryptosystems rely on such assumptions and therefore you should not care about this issue, as long as reasonable security parameters are chosen.

LibTMCG was designed to provide security in the "honest-but-curious" (aka semi-honest) adversary model. That means, all participants follow the protocol instructions properly but they may gather information and share them within a coalition to obtain a game advantage. Thus we are not concerned with robustness and availability issues which are hard to solve in asynchronous environments like the Internet. However, the most operations are verifiable such that cheating can be detected. To obtain this verifiability, the protocols deploy so-called zero-knowledge proofs which yield no further knowledge but the validity of a statement. The soundness error of these proofs is bounded by a security parameter $t$. Depending on your application scenario this parameter should be chosen such that there is a reasonable tradeoff between the cheating probability (which is less or equal than $2^{-t}$) and the produced computational and communication complexity.

Unfortunately, in practice there is a substantial problem with the detection of cheaters. Reliable cheater detection requires that an authenticated broadcast channel has been established, where all players have read/write access. LibTMCG does not yet contain the necessary protocols (reliable broadcast or even atomic broadcast) for creating such a channel. Thus you should take into account that not necessarily the player acting as prover is the source of evil, if a verification procedure fails. This level of uncertainty is also a reason for our restricted adversary model.

Note that it is not known, whether the used protocols retain their zero-knowledge property, if they are composed and executed in a concurrent setting. Thus the application programmer should be careful and avoid parallel protocol sessions. It is an open research project to create a protocol suite whose security can be proven in the UC-framework of Canetti (see

[1] For instance, a "tight reduction" to a known hard problem in the sense of complexity theory.

*Universally Composable Security: A New Paradigm for Cryptographic Protocols*, Cryptology ePrint Archive: Report 2000/067). Furthermore, the protocols should employ concurrent zero-knowledge proofs (see Dwork, Naor, Sahai: *Concurrent Zero-Knowledge*, Journal of the ACM 51(6):851–898, 2004).

LibTMCG was carefully implemented with respect to timing attacks (see Kocher: *Cryptanalysis of Diffie-Hellman, RSA, DSS, and other cryptosystems using timing attacks*, CRYPTO '95, LNCS 963, 1995). Therefore we loose some efficiency, e.g., during modular exponentiations. However, it is strongly recommended to leave the timing attack protection turned on, unless you know exactly where it is really not needed.

> **Security Advice:** We have implemented all cryptographic primitives according to the cited research papers and to the best of our knowledge. However, we can not eliminate any possibility of contained flaws or bugs, because the implementation of such complex protocols is always an error-prone process. Thus we encourage readers with advanced cryptographic background to review the source code of LibTMCG. Please report any complaint or correction proposal.

## 1.4 Preparation

LibTMCG depends on three other basic libraries. Therefore you will need the corresponding development files to build LibTMCG and your application properly. The following list gives a short exposition of the used features and specifies the required versions:

- GNU Multiple Precision Arithmetic Library (`libgmp`), Version $\geq$ 4.1.0

  The library provides a powerful framework for performing arbitrary precision arithmetic on integers. Further reasons for choosing this dependency are the license compatibility, the portability, the vital maintenance, and of course, the reasonable performance.
- GNU Crypto Library (`libgcrypt`), Version $\geq$ 1.2.0

  The library provides some basic cryptographic algorithms (e.g. RIPEMD-160) and an easily accessible interface for cryptographically strong pseudo random numbers.
- GNU Privacy Guard Error Code Library (`libgpg-error`), Version $\geq$ 0.5

  This library defines common error values, e.g., returned by the GNU Crypto Library.

We suppose that the reader is familiar with these libraries because their correct installation, configuration, and usage is crucial to the security of the entire application.

## 1.5 Header Files and Name Spaces

The interface definitions of classes, data types, and security parameters[2] are provided by the central header file `libTMCG.hh`. You have to include this file in all of your sources, either directly or through some other included file. Thus often you will simply write:

```
#include <libTMCG.hh>
```

There are no uniform C++ name spaces for the most parts of the library. Some classes and data types have the common prefix `TMCG_*` resp. `VTMF_*` while others are composed of the author names and an abbreviation of the title from the related research paper. Further

[2] The security parameters are fixed at compile time of LibTMCG. Please don't change anything unless you know exactly what you are doing! Beside the apparent security concerns you will probably break the compatibility with other LibTMCG applications.

there are internally used C functions which might produce conflicting names. These function names are prepended by `mpz_*` because they are extensions for the large integer support of the GNU Multiple Precision Arithmetic Library.

## 1.6 Building Sources

If you want to compile a source file including the `libTMCG.hh` header, you must make sure that the compiler can find it in the directory hierarchy. This is achieved by adding the path of the corresponding directory to the compilers include file search path.

However, the path to the include file has been determined at the time the source is configured. To solve this problem, LibTMCG ships with a small helper program `libTMCG-config` that knows the path to the include file and a few other configuration options. The options that need to be added to the compiler invocation are output by the `--cflags` option to `libTMCG-config`. The following example shows how it can be used at the command line:

```
g++ -c foo.cc `libTMCG-config --cflags`
```

Adding the output of '`libTMCG-config --cflags`' to the compilers command line will ensure that the compiler can find the LibTMCG header file.

A similar problem occurs when linking your program with LibTMCG. Again, the compiler has to find the library files. Therefore the correct installation path has to be added to the library search path. To achieve this, the option `--libs` of `libTMCG-config` can be used. For convenience, this option also outputs all other stuff that is required to link your program with LibTMCG (in particular, the '`-lTMCG`' option).

The example shows how to link `foo.o` with LibTMCG to a program called `foo`:

```
g++ -o foo foo.o `libTMCG-config --libs`
```

Of course, you can also combine both examples to a single command by calling the shell script `libTMCG-config` with both options:

```
g++ -o foo foo.c `libTMCG-config --cflags --libs`
```

### 1.6.1 Building Sources Using GNU Automake

You can use GNU Automake to obtain automatically generated Makefiles. If you do so then you do not have to care about finding and invoking the `libTMCG-config` script at all. LibTMCG provides an Automake extension that does all the stupid work for you.

`AM_PATH_LIBTMCG` ([*minimum-version*], [*action-if-found*], [*action-if-not-found*]) [Macro]

> Check whether LibTMCG (at least version *minimum-version*, if given) exists on the host system. If it is found, execute *action-if-found*, otherwise do *action-if-not-found*.
>
> Additionally, the macro defines `LIBTMCG_CFLAGS` to the flags needed for compilation in order to find the necessary header files, and `LIBTMCG_LIBS` to the corresponding linker flags.

You can use the defined variables in your `Makefile.am` as follows:

```
AM_CPPFLAGS = $(LIBTMCG_CFLAGS)
LDADD = $(LIBTMCG_LIBS)
```

## 1.7 Initializing the Library

The first step is the initialization of LibTMCG. The following function must be invoked early in your program, i.e., before you make use of any other capability of LibTMCG.

`bool init_libTMCG ()` [Function]

> The function checks whether the installed third-party libraries match their required versions. Further it initializes them and returns `true`, if everything was sound. Otherwise `false` is returned and an appropriate error message is sent to `std::cerr`.

Additionally, the function `version_libTMCG` returns a string containing the version number of the library in a common format. It is strongly recommended to check, whether the installed version matches your requirements.

`std::string version_libTMCG ()` [Function]

> This function returns the version of the library in the format *major.minor.revision*.

# 2 Application Programming Interface

## 2.1 Preprocessor Defined Global Symbols

Please note that the following macros are fixed at compile time of LibTMCG and cannot be changed by your application. They are only provided here for informational purposes.

`TMCG_MR_ITERATIONS` [Macro]

Defines the number of iterations for the Miller-Rabin primality test. The default value is `64` which implies a soundness error probability $\leq 4^{-64}$.

`TMCG_GROTH_L_E` [Macro]

Defines the security parameter $\ell_e$ of Groth's interactive shuffle argument [Gr05]. The default value is `80` which implies a soundness error probability $\leq 2^{-80}$. For the intended purposes of LibTMCG this bound seems to be reasonable.

`TMCG_DDH_SIZE` [Macro]

Defines the security parameter (field size in bit) of the group $G$ which is used by the card encoding scheme of Barnett and Smart [BS03]. The underlying assumptions are DDH, CDH, and DLOG. The default value is `1024`.

`TMCG_DLSE_SIZE` [Macro]

Defines the security parameter (subgroup size in bit) of the group $G$ which is used by the card encoding scheme of Barnett and Smart [BS03]. The underlying assumptions are DLSE (related to DDH) and DLOG. The default value is `160`.

`TMCG_GCRY_MD_ALGO` [Macro]

Defines the message digest algorithm for digital signatures and the Fiat-Shamir heuristic (see Fiat, Shamir: *How to prove yourself: Practical Solutions to Identification and Signature Problems*, 1986). The security of the most non-interactive zero-knowledge proofs (NIZK) is related to the so-called random oracle model, i.e., we suppose that the instantiated hash function behaves like an ideal random function (which cannot hold in a real world scenario). However, this assumption seems to be reasonable, if the hash function is collision-resistant and carefully implemented. The default value `GCRY_MD_RMD160`[1] chooses the hash algorithm RIPEMD-160 (see Dobbertin, Bosselaers, Preneel: *RIPEMD-160, a strengthened version of RIPEMD*, 1996) which has an output length of 160 bit. Thus we gain a security level of approximately $2^{80}$, assuming that a birthday-attack is the best known attack against this hash function.

`TMCG_KEYID_SIZE` [Macro]

Defines the length (in characters w.r.t. `TMCG_MPZ_IO_BASE`) for the distinctive suffix of the unique TMCG key identifier. The default value is `8` which spans a reasonable name space for at least $2^{20}$ different TMCG keys (see `TMCG_PublicKey`). However, sometimes it is required to use even smaller sizes.

Each key identifier starts with the string `"ID"` followed by the decimal encoded value of `TMCG_KEYID_SIZE` and the appended carret symbol `"^"`. The final suffix contains `TMCG_KEYID_SIZE` alphanumerical characters from the self signature of TMCG key. The signature has enough entropy included to be used as unique key identifier.

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[1] This is a constant defined by the GNU Crypto Library.

`TMCG_KEY_NIZK_STAGE1` [Macro]
: Defines the security parameter (number of iterations) of the NIZK proof (stage 1) which convince all verifiers that the TMCG key was correctly generated. The default value is `16` which implies a soundness error probability $\leq d^{-16}$, where $d = \gcd(m, \phi(m))$. This parameter is only relevant for the card encoding scheme of Schindelhauer.

`TMCG_KEY_NIZK_STAGE2` [Macro]
: Defines the security parameter (number of iterations) of the NIZK proof (stage 2) which convince all verifiers that the TMCG key was correctly generated. The default value is `128` which implies a soundness error probability $\leq 2^{-128}$. This parameter is only relevant for the card encoding scheme of Schindelhauer.

`TMCG_KEY_NIZK_STAGE3` [Macro]
: Defines the security parameter (number of iterations) of the NIZK proof (stage 3) which convince all verifiers that the TMCG key was correctly generated. The default value is `128` which implies a soundness error probability $\leq 2^{-128}$. This parameter is only relevant for the card encoding scheme of Schindelhauer.

`TMCG_LIBGCRYPT_VERSION` [Macro]
: Defines the required minimum version number of the GNU Crypto Library. The default value is `"1.2.0"`. During the initialization of LibTMCG (see `init_libTMCG`) it is checked, whether the version number of the linked shared object fulfills this condition.

`TMCG_LIBGMP_VERSION` [Macro]
: Defines the required minimum version number of the GNU Multiple Precision Arithmetic Library. The default value is `"4.1.0"`. During the initialization of LibTMCG (see `init_libTMCG`) it is checked, whether the version number provided by the header file `gmp.h` and used at compile time of LibTMCG fulfills this condition.

`TMCG_MAX_CARDS` [Macro]
: Defines the maximum number of stackable cards. The default value is `128`.

`TMCG_MAX_PLAYERS` [Macro]
: Defines the maximum number of players. The default value is `32`. This parameter is only relevant for the card encoding scheme of Schindelhauer.

`TMCG_MAX_TYPEBITS` [Macro]
: Defines the maximum number of bits to represent the card type in the scheme of Schindelhauer. On the other hand, this value determines the maximum size of the message space in the scheme of Barnett and Smart. The default value is `8` which implies that 256 different card types are possible.

`TMCG_MPZ_IO_BASE` [Macro]
: Defines the input and output base of the `std::iostream` operators `<<` and `>>` which are used to encode large integers (`mpz_t`). The default value is `36` which is currently the largest base supported by the GNU Multiple Precision Arithmetic Library.

`TMCG_PRAB_K0` [Macro]

Defines the security parameter $k_0$ (in characters) of the PRab scheme (see Bellare, Rogaway: *The Exact Security of Digital Signatures – How to Sign with RSA and Rabin*, 1996). The default value is `20` which implies a security level around $2^{80}$.

`TMCG_QRA_SIZE` [Macro]

Defines the security parameter (size of the modulus $m = p \cdot q$ in bit) of the TMCG key. The underlying assumptions are QRA and FACTOR. The default value is `1024`. This parameter is only relevant for TMCG keys and Schindelhauer's encoding scheme.

`TMCG_SAEP_S0` [Macro]

Defines the security parameter $s_0$ (in characters) of the Rabin-SAEP scheme (see Boneh: *Simplified OAEP for the RSA and Rabin Functions*, 2002). The default value is `20` which implies a security around $2^{80}$ against CCA (Chosen Ciphertext Attacks).

`TMCG_HASH_COMMITMENT` [Macro]

Defines whether shortened commitments are used in the shuffle verification procedure of Schindelhauer. The default value is `true`, because it will decrease the communication complexity significantly. However, as an immediate consequence the soundness property is violated, if the used hash function `TMCG_GCRY_MD_ALGO` is broken.

`TMCG_MAX_FPOWM_T` [Macro]

Defines the maximum size of admissible exponents (in bit) used by fast exponentiation procedures. The default value is `2048`. Note that this parameter has a strong influence on the amount of memory allocated by LibTMCG since it determines the size of the precomputed tables. However, it should be at least greater than `TMCG_DDH_SIZE` and `TMCG_QRA_SIZE`.

## 2.2 Data Types and Classes

This section describes all public data types and classes that are necessary to create a secure card game. Private methods and only internally used members are not explained.

### 2.2.1 Data Types

LibTMCG provides several data structures for cards, stacks, and cryptographic keys.

#### 2.2.1.1 Encoding Schemes for Cards

There exist two different encoding schemes that can be used for the digital representation of playing cards. In the scheme of Schindelhauer [Sc98] the type of a card is shared among the players through bit-wise representation by quadratic (non-)residues. Thus the security relies on the well-known QRA (Quadratic Residuosity Assumption). Unfortunately, the size of a card grows linearly in the number of players and logarithmically in the number of card types. Recently the much more efficient solution of Barnett and Smart [BS03] has been implemented. This encoding works on a cyclic group of prime order and requires that the DDH (Decisional Diffie-Hellman Assumption) holds there.

For both schemes LibTMCG provides a structure whose name contains the suffix `Card`. This data type is used to represent an open or even a masked card. Further, there is a corresponding structure whose name contains the suffix `CardSecret`. This data type is used to represent the secret values involved in a card masking operation.

Because of the reduced computational and communication complexity (see [St05] for details) the usage of the second card encoding scheme, i.e. `VTMF_Card` and `VTMF_CardSecret`, is highly recommended.

`TMCG_Card` [Data type]

This `struct` represents a card in the encoding scheme of Schindelhauer [Sc98]. The type of the card is shared among the players by quadratic residues and non-residues, respectively. Thus the security relies on the Quadratic Residuosity Assumption.

`std::vector< std::vector<MP_INT> > z` [Member of `TMCG_Card`]

This $k \times w$-matrix encodes the type of the corresponding card in a shared way. For each of the $k$ players there is a separate row and for each of the $w$ bits in the binary representation of the type there is a column. The elements are numbers from the group $\mathbf{Z}^{\circ}_{m_i}$ where $m_i$ is the public modulus of the $i$th player.

`TMCG_Card ()` [Constructor on `TMCG_Card`]

This default constructor initializes the card with an empty $1 \times 1$-matrix. Later the method `TMCG_Card::resize` can be used to enlarge the card representation.

`TMCG_Card (size_t` *k*, `size_t` *w*`)` [Constructor on `TMCG_Card`]

This constructor initializes the card with an empty $k \times w$-matrix. The parameter $k$ is the number of players and $w$ is the maximum number of bits used by the binary representation of the card type.

`TMCG_Card (const TMCG_Card&` *that*`)` [Constructor on `TMCG_Card`]

This is a simple copy-constructor and *that* is the card to be copied.

`TMCG_Card& = (const TMCG_Card&` *that*`)` [Operator on `TMCG_Card`]

This is a simple assignment-operator and *that* is the card to be assigned.

`bool == (const TMCG_Card&` *that*`)` [Operator on `TMCG_Card`]

This operator tests two card representations for equality.

`bool != (const TMCG_Card&` *that*`)` [Operator on `TMCG_Card`]

This operator tests two card representations for inequality.

`void resize (size_t` *k*, `size_t` *w*`)` [Method on `TMCG_Card`]

This method resizes the representation of the card. The current content of the member `z` will be released and a new $k \times w$-matrix is created. The parameter $k$ is the number of players and $w$ is the maximum number of bits used by the binary representation of the card type.

`bool import (std::string` *s*`)` [Method on `TMCG_Card`]

This method imports the content of the member `z` from the correctly formatted input string *s*. It returns `true`, if the import was successful.

`~TMCG_Card ()` [Destructor on `TMCG_Card`]

This destructor releases all occupied resources.

`std::ostream& << (std::ostream& out, const TMCG_Card& card)` [Operator on `TMCG_Card`]

This operator exports the content of the member `z` (of the given `TMCG_Card` *card*) to the output stream *out*.

`std::istream& >> (std::istream& in, TMCG_Card& card)` [Operator on `TMCG_Card`]

This operator imports the content of the member `z` (of the given `TMCG_Card` *card*) from the input stream *in*. The data has to be delimited by a newline character. The `failbit` of the stream is set, if any parse error occurred.

`TMCG_CardSecret` [Data type]

This `struct` represents the secret used for a card masking operation in the original encoding scheme of Schindelhauer [Sc98].

`std::vector< std::vector<MP_INT> > r` [Member of `TMCG_CardSecret`]

This $k \times w$-matrix encodes the first part of the secret. For each of the $k$ players there is a separate row and for each of the $w$ bits in the binary representation of the corresponding card type there is a column. The elements are numbers from the group $\mathbf{Z}^{\circ}_{m_i}$ where $m_i$ is the public modulus of the $i$th player.

`std::vector< std::vector<MP_INT> > b` [Member of `TMCG_CardSecret`]

This $k \times w$-matrix encodes the second part of the secret. For each of the $k$ players there is a separate row and for each of the $w$ bits in the binary representation of the corresponding card type there is a column. The elements are simply numbers from $\{0, 1\}$.

`TMCG_CardSecret ()` [Constructor on `TMCG_CardSecret`]

This default constructor initializes both members with an empty $1 \times 1$-matrix. Later the method `TMCG_CardSecret::resize` can be used to enlarge the card representation.

`TMCG_CardSecret (size_t k, size_t w)` [Constructor on `TMCG_CardSecret`]

This constructor initializes both members with an empty $k \times w$-matrix. The parameter $k$ is the number of players and $w$ is the maximum number of bits used by the binary representation of the corresponding card type.

`TMCG_CardSecret (const TMCG_CardSecret& that)` [Constructor on `TMCG_CardSecret`]

This is a simple copy-constructor and *that* is the secret to be copied.

`TMCG_CardSecret& = (const TMCG_CardSecret& that)` [Operator on `TMCG_CardSecret`]

This is a simple assignment-operator and *that* is the secret to be assigned.

`void resize (size_t k, size_t w)` [Method on `TMCG_CardSecret`]

This method resizes the representation of the secret. The current content of the members `r` and `b` will be released and new $k \times w$-matrices are created. The parameter $k$ is the number of players and $w$ is the maximum number of bits used by the binary representation of the corresponding card type.

`bool import (std::string s)` [Method on `TMCG_CardSecret`]
: This method imports the content of the members `r` and `b` from the correctly formatted input string *s*. It returns `true`, if the import was successful.

`~TMCG_CardSecret ()` [Destructor on `TMCG_CardSecret`]
: This destructor releases all occupied resources.

`std::ostream& << (std::ostream& out, const TMCG_CardSecret& cardsecret)` [Operator on `TMCG_CardSecret`]
: This operator exports the content of the members `r` and `b` (of the given `TMCG_CardSecret` *cardsecret*) to the output stream *out*.

`std::istream& >> (std::istream& in, TMCG_CardSecret& cardsecret)` [Operator on `TMCG_CardSecret`]
: This operator imports the content of the members `r` and `b` (of the given `TMCG_CardSecret` *cardsecret*) from the input stream *in*. The data has to be delimited by a newline character. The `failbit` of the stream is set, if any parse error occurred.

`VTMF_Card` [Data type]
: This `struct` represents a card in the encoding scheme of Barnett and Smart [BS03]. Here we use the discrete logarithm based instantiation of their general cryptographic primitive VTMF (Verifiable $k$-out-of-$k$ Threshold Masking Function). The security relies on the DDH assumption in the underlying abelian group $G$.

`mpz_t c_1` [Member of `VTMF_Card`]
: This is the first part of the encrypted card type. It is an element from the underlying group $G$.

`mpz_t c_2` [Member of `VTMF_Card`]
: This is the second part of the encrypted card type. It is also an element from the underlying group $G$.

`VTMF_Card ()` [Constructor on `VTMF_Card`]
: This default constructor initializes an empty card where the members `c_1` and `c_2` are set to zero.

`VTMF_Card (const VTMF_Card& that)` [Constructor on `VTMF_Card`]
: This is a simple copy-constructor and *that* is the card to be copied.

`VTMF_Card& = (const VTMF_Card& that)` [Operator on `VTMF_Card`]
: This is a simple assignment-operator and *that* is the card to be assigned.

`bool == (const VTMF_Card& that)` [Operator on `VTMF_Card`]
: This operator tests two card representations for equality.

`bool != (const VTMF_Card& that)` [Operator on `VTMF_Card`]
: This operator tests two card representations for inequality.

`bool import (std::string s)` [Method on `VTMF_Card`]
: This method imports the content of the members `c_1` and `c_2` from a correctly formatted input string *s*. It returns `true`, if the import was successful.

`~VTMF_Card ()` [Destructor on `VTMF_Card`]

This destructor releases all occupied resources.

`std::ostream& << (std::ostream& out, const VTMF_Card& card)` [Operator on `VTMF_Card`]

This operator exports the content of the members `c_1` and `c_2` (of the given `VTMF_Card` *card*) to the output stream *out*.

`std::istream& >> (std::istream& in, VTMF_Card& card)` [Operator on `VTMF_Card`]

This operator imports the content of the members `c_1` and `c_2` (of the given `VTMF_Card` *card*) from the input stream *in*. The data has to be delimited by a newline character. The `failbit` of the stream is set, if any parse error occurred.

`VTMF_CardSecret` [Data type]

This `struct` represents the secrets used in the card masking operation by the encoding scheme of Barnett and Smart [BS03].

`mpz_t r` [Member of `VTMF_CardSecret`]

This member is the exponent (randomizer) used in the masking operation. It should be chosen uniformly and randomly from $\mathbf{Z}_q$ where $q$ is the order of the finite abelian group $G$ for which the DDH assumption holds.

According to the results of Koshiba and Kurosawa (see *Short Exponent Diffie-Hellman Problems*, PKC 2004, LNCS 2947) the length of this exponent can be shorten to a more efficient size (e.g. 160 bit), if the corresponding generator of $G$ is adjusted as well. Under the additional DLSE (Discrete Logarithm with Short Exponents) assumption the DDH problem in $G$ seems to be still hard. By such an optimization trick we gain a great performance advantage for almost all modular exponentiations that are computed during the masking operation, if the VTMF primitive was instantiated by the later explained class `BarnettSmartVTMF_dlog_GroupQR`. Furthermore, the size of the card secret is substantially reduced which results in an improved communication complexity.

`VTMF_CardSecret ()` [Constructor on `VTMF_CardSecret`]

This default constructor initializes the secret with an empty member `r`.

`VTMF_CardSecret (const VTMF_CardSecret& that)` [Constructor on `VTMF_CardSecret`]

This is a simple copy-constructor and *that* is the secret to be copied.

`VTMF_CardSecret& = (const VTMF_CardSecret& that)` [Operator on `VTMF_CardSecret`]

This is a simple assignment-operator and *that* is the secret to be assigned.

`bool import (std::string s)` [Method on `VTMF_CardSecret`]

This method imports the content of the member `r` from the correctly formatted input string *s*. It returns `true`, if the import was successful.

`~VTMF_CardSecret ()` [Destructor on `VTMF_CardSecret`]

This destructor releases all occupied resources.

`std::ostream& << (std::ostream& out, const VTMF_CardSecret& cardsecret)` [Operator on `VTMF_CardSecret`]

This operator exports the content of the member `r` (of the given `VTMF_CardSecret` *cardsecret*) to the output stream *out*.

`std::istream& >> (std::istream& in, VTMF_CardSecret& cardsecret)` [Operator on `VTMF_CardSecret`]

This operator imports the content of the member `r` (of the given `VTMF_CardSecret` *cardsecret*) from the input stream *in*. The data has to be delimited by a newline character. The `failbit` of the stream is set, if any parse error occurred.

#### 2.2.1.2 Stacks

All of the following data types are generic containers that can be instantiated as C++ templates with the former explained `Card` and `CardSecret` data types, respectively. Note the maximum number of stackable data is upper-bounded by `TMCG_MAX_CARDS`. There is no error reported, if this limit is exceeded.

`TMCG_Stack<CardType>` [Data type]

This `struct` is a simple container for cards of the specified *`CardType`*. Currently, the elements can be either of type `TMCG_Card` or `VTMF_Card` depending on which kind of encoding scheme is used. The `TMCG_Stack` structure is mainly used to represent a stack of masked cards, i.e., playing cards that are stacked in a face-down manner. It can be either a public stack where all participants have access to or even a private stack, e.g. the players' hand. If the corresponding card types are known it can also serve as an "open stack", although `TMCG_OpenStack` is more suitable in that case.

`std::vector<CardType> stack` [Member of `TMCG_Stack`]

This is the container that is used internally for storing the cards.

`TMCG_Stack ()` [Constructor on `TMCG_Stack`]

This default constructor initializes an empty stack.

`TMCG_Stack& = (const TMCG_Stack<CardType>& that)` [Operator on `TMCG_Stack`]

This is a simple assignment-operator and *that* is the stack to be assigned.

`bool == (const TMCG_Stack<CardType>& that)` [Operator on `TMCG_Stack`]

This operator tests two stacks for equality. It checks whether the sizes of the stacks and the contained cards are equal with respect to the implied order.

`bool != (const TMCG_Stack<CardType>& that)` [Operator on `TMCG_Stack`]

This operator tests two stacks for inequality. It returns `true`, if either the sizes does not match or at least two corresponding cards are not equal.

`const CardType& [] (size_t n)` [Operator on `TMCG_Stack`]

This operator provides read-only random access to the contained cards. It returns a const-reference to the *n*th card from the top of the stack.

`CardType& [] (size_t n)` [Operator on `TMCG_Stack`]

This operator provides random access to the contained cards. It returns a reference to the *n*th card from the top of the stack.

`size_t size ()` [Method on `TMCG_Stack`]
This method returns the size of the stack.

`void push (const CardType& c)` [Method on `TMCG_Stack`]
This method pushes the card *c* to the back of the stack.

`void push (const TMCG_Stack<CardType>& s)` [Method on `TMCG_Stack`]
This method pushes the stack *s* to the back of the stack.

`void push (const TMCG_OpenStack<CardType>& s)` [Method on `TMCG_Stack`]
This method pushes the cards of the open stack *s* to the back of the stack.

`bool empty ()` [Method on `TMCG_Stack`]
This method returns `true`, if the stack is empty.

`bool pop (CardType& c)` [Method on `TMCG_Stack`]
This method removes a card from the back and stores the data in *c*. It returns `true`, if the stack was not empty and thus *c* contains useful data.

`void clear ()` [Method on `TMCG_Stack`]
This method clears the stack, i.e., it removes all cards.

`bool find (const CardType& c)` [Method on `TMCG_Stack`]
This method returns `true`, if the card *c* was found in the stack.

`bool remove (const CardType& c)` [Method on `TMCG_Stack`]
This method removes the top-most card from the stack which is equal to *c*. It returns `true`, if the card was found and successfully removed.

`size_t removeAll (const CardType& c)` [Method on `TMCG_Stack`]
This method removes every card from the stack which is equal to *c*. It returns the number of removed cards.

`bool import (std::string s)` [Method on `TMCG_Stack`]
This method imports the stack from the correctly formatted input string *s*. It returns `true`, if the import was successful.

`~TMCG_Stack ()` [Destructor on `TMCG_Stack`]
This destructor releases all occupied resources.

`std::ostream& << (std::ostream& out, const TMCG_Stack<CardType>& stack)` [Operator on `TMCG_Stack`]
This operator exports the given *stack* to the output stream *out*.

`std::istream& >> (std::istream& in, TMCG_Stack<CardType>& stack)` [Operator on `TMCG_Stack`]
This operator imports the given *stack* from the input stream *in*. The data has to be delimited by a newline character. The `failbit` of the stream is set, if any parse error occurred.

`TMCG_OpenStack<`*CardType*`>` [Data type]

This `struct` is a simple container for cards of the specified *CardType* whose types are known. The elements are pairs where the first component is the type and the second component is the corresponding card. The card type is represented by a `size_t` integer. Currently, the cards can be either of type `TMCG_Card` or `VTMF_Card` depending on which kind of encoding scheme is used.

`std::vector<std::pair<size_t,` *CardType*`> > stack` [Member of `TMCG_OpenStack`]

This is the container that is used internally for storing the pairs.

`TMCG_OpenStack ()` [Constructor on `TMCG_OpenStack`]

This default constructor initializes an empty stack.

`TMCG_OpenStack& = (const TMCG_OpenStack<`*CardType*`>&` *that*) [Operator on `TMCG_OpenStack`]

This is a simple assignment-operator and *that* is the stack to be assigned.

`bool == (const TMCG_OpenStack<`*CardType*`>&` *that*) [Operator on `TMCG_OpenStack`]

This operator tests two stacks for equality. It checks whether the types, the sizes, and the contained cards are equal with respect to the stack order.

`bool != (const TMCG_OpenStack<`*CardType*`>&` *that*) [Operator on `TMCG_OpenStack`]

This operator tests two stacks for inequality. It returns `true`, if either the sizes resp. types does not match or at least two corresponding cards are not equal.

`const std::pair<size_t,` *CardType*`>& [] (size_t` *n*) [Operator on `TMCG_OpenStack`]

This operator provides read-only random access to the contained pairs. It returns a const-reference to the *n*th pair from the top of the stack.

`std::pair<size_t,` *CardType*`>& [] (size_t` *n*) [Operator on `TMCG_OpenStack`]

This operator provides random access to the contained pairs. It returns a reference to the *n*th pair from the top of the stack.

`size_t size ()` [Method on `TMCG_OpenStack`]

This method returns the size of the stack.

`void push (const std::pair<size_t,` *CardType*`>&` *p*) [Method on `TMCG_OpenStack`]

This method pushes the pair *p* to the back of the stack. The first component is the type and the second component is the corresponding card representation.

`void push (size_t` *type*, `const` *CardType*`&` *c*) [Method on `TMCG_OpenStack`]

This method pushes a pair to the back of the stack. The parameter *type* is the card type and *c* is the corresponding card representation.

`void push (const TMCG_OpenStack<CardType>& s)` [Method on `TMCG_OpenStack`]
: This method pushes the pairs of the stack *s* to the back of this stack.

`bool empty ()` [Method on `TMCG_OpenStack`]
: This method returns `true`, if the stack is empty.

`bool pop (size_t& type, CardType& c)` [Method on `TMCG_OpenStack`]
: This method removes a pair from the back of the stack. It stores the card type in *type* and the representation in *c*. It returns `true`, if the stack was not empty and thus *type* and *c* contain useful data.

`void clear ()` [Method on `TMCG_OpenStack`]
: This method clears the stack, i.e., it removes all pairs.

`bool find (size_t type)` [Method on `TMCG_OpenStack`]
: This method returns `true`, if a pair with the first component *type* was found in the stack.

`bool remove (size_t type)` [Method on `TMCG_OpenStack`]
: This method removes the top-most pair with the first component *type* from the stack. It returns `true`, if such a pair was found and successfully removed.

`size_t removeAll (size_t type)` [Method on `TMCG_OpenStack`]
: This method removes every pair from the stack whose first component is equal to *type*. Further it returns the number of removed pairs.

`bool move (size_t type, TMCG_Stack<CardType>& s)` [Method on `TMCG_OpenStack`]
: This method moves the top-most card representation of the given *type* to another stack *s*. It returns `true`, if such a pair was found and successfully moved.

`~TMCG_OpenStack ()` [Destructor on `TMCG_OpenStack`]
: This destructor releases all occupied resources.

`TMCG_StackSecret<CardSecretType>` [Data type]
: This `struct` is a simple container for the secrets involved in the masking operation of cards. Additionally, the permutation of a corresponding shuffle of the stack is stored. The elements are pairs where the first component is a permutation index of type `size_t` and the second component is a card secret of the specified *CardSecretType*. Currently, such secrets can be either of type `TMCG_CardSecret` or `VTMF_CardSecret` depending on which kind of encoding scheme is used.

`std::vector<std::pair<size_t, CardSecretType> > stack` [Member of `TMCG_StackSecret`]
: This is the container that is used internally for storing the pairs.

`TMCG_StackSecret ()` [Constructor on `TMCG_StackSecret`]
: This default constructor initializes an empty stack secret.

TMCG_StackSecret& = (const TMCG_StackSecret<*CardSecretType*>& *that*) [Operator on TMCG_StackSecret]

This is a simple assignment-operator and *that* is the stack secret to be assigned.

const std::pair<size_t, *CardSecretType*>& [] (size_t *n*) [Operator on TMCG_StackSecret]

This operator provides read-only random access to the contained pairs. It returns a const-reference to the *n*th pair from the top of the stack secret.

std::pair<size_t, *CardSecretType*>& [] (size_t *n*) [Operator on TMCG_StackSecret]

This operator provides random access to the contained pairs. It returns a reference to the *n*th pair from the top of the stack secret.

size_t size () [Method on TMCG_StackSecret]

This method returns the size of the stack secret.

void push (size_t *index*, const *CardSecretType*& *cs*) [Method on TMCG_StackSecret]

This method pushes a pair to the back of the stack secret. The parameter *index* is the permutation index and *cs* is the corresponding card secret.

void clear () [Method on TMCG_StackSecret]

This method clears the stack secret, i.e., it removes all pairs.

size_t find_position (size_t *index*) [Method on TMCG_StackSecret]

This method searches for a given permutation index in the stack secret. It returns the corresponding position[2] in the stack secret, if the *index* was found. Otherwise, the size of the stack secret is returned. Please note that in this case the returned value is not a valid position for an access to the stack secret.

bool find (size_t *index*) [Method on TMCG_StackSecret]

This method searches for a given permutation index in the stack secret. It returns true, if such an *index* was found.

bool import (std::string *s*) [Method on TMCG_StackSecret]

This method imports the stack secret from a correctly formatted input string *s*. It returns true, if the import was successful.

~TMCG_StackSecret () [Destructor on TMCG_StackSecret]

This destructor releases all occupied resources.

std::ostream& << (std::ostream& *out*, const TMCG_StackSecret<*CardSecretType*>& *stacksecret*) [Operator on TMCG_StackSecret]

This operator exports the given *stacksecret* to the output stream *out*.

std::istream& >> (std::istream& *in*, TMCG_StackSecret<*CardSecretType*>& *stacksecret*) [Operator on TMCG_StackSecret]

This operator imports the given *stacksecret* from the input stream *in*. The data has to be delimited by a newline character. The failbit of the stream is set, if any parse error occurred.

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[2] According to the behavior of the []-operator, the zero denotes always the top-most position.

#### 2.2.1.3 Cryptographic Keys

LibTMCG only provides corresponding data types for keys used by the encoding scheme of Schindelhauer [Sc98], because it is not efficient to perform the key generation in every new game session. Furthermore, in general you can encrypt and sign messages to ensure confidentiality and integrity, even if the scheme of Barnett and Smart [BS03] has been applied for the card encoding. Therefore these structures may be of independent interest, for example to establish authenticated communication channels between players. However, like for all public key cryptosystems a trusted PKI (Public Key Infrastructure) is needed. This might not be a serious requirement in distributed game environments, because the players can compare their key fingerprints by telephone or a service provider can issue public key certificates.

`TMCG_SecretKey` [Data type]

This `struct` represents the secret part of the TMCG key. The underlying public key cryptosystem is due to Rabin with minor modifications for encryption padding (SAEP scheme of Boneh) and digital signatures (PRab scheme of Bellare and Rogaway).

`std::string name` [Member of `TMCG_SecretKey`]

This string contains the name or a pseudonym of the key owner.

`std::string email` [Member of `TMCG_SecretKey`]

This string contains the email address of the key owner.

`std::string type` [Member of `TMCG_SecretKey`]

This string contains information about the key type. The common prefix is `TMCG/RABIN`. It is followed by the decimal encoded bit size of the modulus $m$. The suffix `NIZK` signals that the correctness of the key is shown by an appended non-interactive zero-knowledge proof. The single parts are separated by underscore characters `_`, e.g., `TMCG/RABIN_1024_NIZK` has the correct form. However, the suffix can be left empty, if the key is only used for encryption and signing (non-NIZK key).

`std::string nizk` [Member of `TMCG_SecretKey`]

This string contains two stages of the non-interactive zero-knowledge proof of Gennaro, Micciancio, and Rabin (*An Efficient Non-Interactive Statistical Zero-Knowledge Proof System for Quasi-Safe Prime Products*, ACM CCS 1998). The proof shows that $m$ was correctly generated as product of two primes both congruent to 3 (modulo 4). Further there is another non-interactive zero-knowledge proof appended which shows that the condition $y \in \mathbf{NQR}^\circ_m$ holds.

`std::string sig` [Member of `TMCG_SecretKey`]

This string contains the self signature of the public key.

`mpz_t m` [Member of `TMCG_SecretKey`]

This is the public modulus $m = p \cdot q$ which is the product of two secret primes $p$ and $q$. The size of $m$ is determined by the security parameter `TMCG_QRA_SIZE`.

`mpz_t y` [Member of `TMCG_SecretKey`]

This is the public quadratic non-residue $y \in \mathbf{NQR}^\circ_m$ which is used in several zero-knowledge proofs of Schindelhauer's encoding scheme [Sc98].

`mpz_t p` [Member of `TMCG_SecretKey`]
: This is the secret prime number $p$ which is a factor of the modulus $m$.

`mpz_t q` [Member of `TMCG_SecretKey`]
: This is the secret prime number $q$ which is a factor of the modulus $m$.

`TMCG_SecretKey ()` [Constructor on `TMCG_SecretKey`]
: This default constructor initializes an empty secret key.

`TMCG_SecretKey (const std::string&` *n*`, const std::string&` *e*`, unsigned long int` *keysize* `=TMCG_QRA_SIZE, bool` *nizk_key* `=true)` [Constructor on `TMCG_SecretKey`]
: This constructor generates a new secret key where *n* is the name or a pseudonym of the owner, *e* is a corresponding email address, *keysize* is the desired bit length of the modulus $m$, and *nizk_key* indicates whether or not a NIZK proof will be appended. The default value of the third argument is set to `TMCG_QRA_SIZE`, if *keysize* is omitted in the call. The default value of the fourth argument is set to `true`, whenever it is omitted in the call. Depending on *keysize* and *nizk_key* the generation is a very time-consuming task and dots are sent to `std::cerr` as a progress indicator.

`TMCG_SecretKey (const std::string&` *s*`)` [Constructor on `TMCG_SecretKey`]
: This constructor initializes the key from a correctly formatted input string *s*.

`TMCG_SecretKey (const TMCG_SecretKey&` *that*`)` [Constructor on `TMCG_SecretKey`]
: This is a simple copy-constructor and *that* is the key to be copied.

`TMCG_SecretKey& = (const TMCG_SecretKey&` *that*`)` [Operator on `TMCG_SecretKey`]
: This is a simple assignment-operator and *that* is the key to be assigned.

`bool check ()` [Method on `TMCG_SecretKey`]
: This method tests whether the self signature is valid and whether the non-interactive zero-knowledge proofs are sound. It returns `true`, if all checks have been successfully passed. Due to the computational complexity of the verification procedure these checks are extremely time-consuming.

`std::string fingerprint ()` [Method on `TMCG_SecretKey`]
: This method returns the fingerprint of the key. The fingerprint is the hexadecimal notation of the hash value (algorithm `TMCG_GCRY_MD_ALGO`) on the members `name`, `email`, `type`, `m`, `y`, `nizk`, and `sig`.

`std::string selfid ()` [Method on `TMCG_SecretKey`]
: This method returns the real value of the self signature. The string `ERROR` is returned, if any parse error occurred. The string `SELFSIG-SELFSIG-SELFSIG-SELFSIG-SELFSIG-SELFSIG` is returned, if the self signature `sig` was empty.

`std::string keyid (size_t` *size* `=TMCG_KEYID_SIZE)` [Method on `TMCG_SecretKey`]
: This method returns the unique key identifier of length *size*. The default value of the first argument is set to `TMCG_KEYID_SIZE`, if *size* is omitted in the call.

`size_t keyid_size (const std::string& s)` [Method on `TMCG_SecretKey`]
: This method returns the length of the unique key identifier *s*. Zero is returned, if any parse error occurred.

`std::string sigid (std::string s)` [Method on `TMCG_SecretKey`]
: This method returns the unique key identifier which is included in the signature *s*. The string `ERROR` is returned, if any parse error occurred.

`bool import (std::string s)` [Method on `TMCG_SecretKey`]
: This method imports the key from a correctly formatted input string *s*. It returns `true`, if the import was successful.

`bool decrypt (char* value, std::string s)` [Method on `TMCG_SecretKey`]
: This method decrypts the given encryption packet *s* and stores the content in *value* which is a pointer to a character array of size `TMCG_SAEP_S0`. The method returns `true`, if the decryption was successful.

`std::string sign (const std::string& data)` [Method on `TMCG_SecretKey`]
: This method returns a digital signature on *data*.

`std::string encrypt (const char* value)` [Method on `TMCG_SecretKey`]
: This method encrypts the content of *value* which is a pointer to a character array of size `TMCG_SAEP_S0`. The method returns a corresponding encryption packet that can be decrypted by the owner of the secret key.

`bool verify (const std::string& data, std::string s)` [Method on `TMCG_SecretKey`]
: This method verifies whether the signature *s* on *data* is valid or not. It returns `true`, if everything was sound.

`~TMCG_SecretKey ()` [Destructor on `TMCG_SecretKey`]
: This destructor releases all occupied resources.

`std::ostream& << (std::ostream& out, const TMCG_SecretKey& key)` [Operator on `TMCG_SecretKey`]
: This operator exports the given *key* to the output stream *out*.

`std::istream& >> (std::istream& in, TMCG_SecretKey& key)` [Operator on `TMCG_SecretKey`]
: This operator imports the given *key* from the input stream *in*. The data has to be delimited by a newline character. The `failbit` is set, if any parse error occurred.

`TMCG_PublicKey` [Data type]
: This `struct` represents the public part of the TMCG key.

`std::string name` [Member of `TMCG_PublicKey`]
: This string contains the name or a pseudonym of the key owner.

`std::string email` [Member of `TMCG_PublicKey`]
: This string contains the email address of the key owner.

`std::string type` [Member of `TMCG_PublicKey`]
This string contains information about the key type. The common prefix is `TMCG/RABIN`. It is followed by the decimal encoded bit size of the modulus $m$. The suffix `NIZK` signals that the correctness of the key is shown by an appended non-interactive zero-knowledge proof. The single parts are separated by underscore characters `_`, e.g., `TMCG/RABIN_1024_NIZK` has the correct form. However, the suffix can be left empty, if the key is only used for encryption and signing.

`std::string nizk` [Member of `TMCG_PublicKey`]
This string contains two stages of non-interactive zero-knowledge proof of Gennaro, Micciancio and Rabin (ACM CCS, 1998). They show that the modulus $m$ was correctly generated. Further there is another non-interactive zero-knowledge proof appended which shows that the condition $y \in \mathbf{NQR}^{\circ}_{m}$ holds.

`std::string sig` [Member of `TMCG_PublicKey`]
This string contains the self signature of the public key.

`mpz_t m` [Member of `TMCG_PublicKey`]
This is the public modulus $m = p \cdot q$ which is the product of two secret primes $p$ and $q$. The size of $m$ is determined by the security parameter `TMCG_QRA_SIZE`.

`mpz_t y` [Member of `TMCG_PublicKey`]
This is the public quadratic non-residue $y \in \mathbf{NQR}^{\circ}_{m}$ which is used by several zero-knowledge proofs of the toolbox.

`TMCG_PublicKey ()` [Constructor on `TMCG_PublicKey`]
This default constructor initializes an empty public key.

`TMCG_PublicKey (const TMCG_SecretKey&` *skey*`)` [Constructor on `TMCG_PublicKey`]
This constructor initializes the key using public values of the secret key *skey*.

`TMCG_PublicKey (const TMCG_PublicKey&` *pkey*`)` [Constructor on `TMCG_PublicKey`]
This is a simple copy-constructor and *pkey* is the key to be copied.

`TMCG_PublicKey& = (const TMCG_PublicKey&` *that*`)` [Operator on `TMCG_PublicKey`]
This is a simple assignment-operator and *that* is the key to be assigned.

`bool check ()` [Method on `TMCG_PublicKey`]
This method tests whether the self signature is valid and whether the non-interactive zero-knowledge proofs are sound. It returns `true`, if all checks have been successfully passed. Due to the computational complexity of the verification procedure these checks are extremely time-consuming.

`std::string fingerprint ()` [Method on `TMCG_PublicKey`]
This method returns the fingerprint of the key. The fingerprint is the hexadecimal notation of the hash value (algorithm `TMCG_GCRY_MD_ALGO`) on the members `name`, `email`, `type`, `m`, `y`, `nizk`, and `sig`.

`std::string selfid ()` [Method on `TMCG_PublicKey`]

This method returns the real value of the self signature. The string `ERROR` is returned, if any parse error occurred. The string `SELFSIG-SELFSIG-SELFSIG-SELFSIG-SELFSIG-SELFSIG` is returned, if the self signature `sig` was empty.

`std::string keyid (size_t` *size* `=TMCG_KEYID_SIZE)` [Method on `TMCG_PublicKey`]

This method returns the unique key identifier of length *size*. The default value of the first argument is set to `TMCG_KEYID_SIZE`, if *size* is omitted in the call.

`size_t keyid_size (const std::string&` *s*`)` [Method on `TMCG_PublicKey`]

This method returns the length of the unique key identifier *s*. Zero is returned, if any parse error occurred.

`std::string sigid (std::string` *s*`)` [Method on `TMCG_PublicKey`]

This method returns the unique key identifier which is included in the signature *s*. The string `ERROR` is returned, if any parse error occurred.

`bool import (std::string` *s*`)` [Method on `TMCG_PublicKey`]

This method imports the key from a correctly formatted input string *s*. It returns `true`, if the import was successful.

`std::string encrypt (const char*` *value*`)` [Method on `TMCG_PublicKey`]

This method encrypts the content of *value* which is a pointer to a character array of size `TMCG_SAEP_S0`. The method returns a corresponding encryption packet that can be decrypted by the owner of the secret key.

`bool verify (const std::string&` *data*`, std::string` *s*`)` [Method on `TMCG_PublicKey`]

This method verifies whether the signature *s* on *data* is valid or not. It returns `true`, if everything was sound.

`~TMCG_PublicKey ()` [Destructor on `TMCG_PublicKey`]

This destructor releases all occupied resources.

`std::ostream& << (std::ostream&` *out*`, const TMCG_PublicKey&` *key*`)` [Operator on `TMCG_PublicKey`]

This operator exports the given *key* to the output stream *out*.

`std::istream& >> (std::istream&` *in*`, TMCG_PublicKey&` *key*`)` [Operator on `TMCG_PublicKey`]

This operator imports the given *key* from the input stream *in*. The data has to be delimited by a newline character. The `failbit` is set, if any parse error occurred.

`TMCG_PublicKeyRing` [Data type]

This `struct` is just a simple container for TMCG public keys. There are no particular methods provided by `TMCG_PublicKeyRing`. You have to use the regular interface of the STL container `std::vector` to access the single keys of the ring.

`std::vector<TMCG_PublicKey> keys` [Member of `TMCG_PublicKeyRing`]

This is the real container that is used to store the keys.

`TMCG_PublicKeyRing ()` [Constructor on `TMCG_PublicKeyRing`]
This default constructor initializes an empty public key ring.

`TMCG_PublicKeyRing (size_t` *n*`)` [Constructor on `TMCG_PublicKeyRing`]
This constructor initializes the container for storing exactly $n$ keys.

`~TMCG_PublicKeyRing ()` [Destructor on `TMCG_PublicKeyRing`]
This destructor releases all occupied resources.

### 2.2.2 Classes

LibTMCG consists of several C++ classes. Some of them are only extensions or optimizations, but other provide necessary interfaces to perform the basic operations in secure card games, e.g., the creation of open cards, the masking of cards, the opening of masked cards, the verifiable secret shuffle of a stack, and more general tasks like distributed key generation procedures. Each class implements the main functionality of the corresponding research paper [Sc98,BS03,Gr05,HSSV09]. The author names are a prefix of the class name and the then following part is an abbreviation of the title.

#### 2.2.2.1 Verifiable *k*-out-of-*k* Threshold Masking Function

The two classes of this subsection are concrete instantiations of Barnett and Smart's VTMF primitive [BS03]. More formally, the authors specify four different protocols:

- Key Generation Protocol
- Verifiable Masking Protocol
- Verifiable Re-masking Protocol
- Verifiable Decryption Protocol

Each protocol uses low-level operations on an appropriately chosen algebraic group $G$. The choice of this group is crucial to the security of the card encoding scheme and thus to the high-level operations on cards resp. stacks.

There are just a few methods and members of these classes that might be of general interest for an application programmer, e.g. the methods of the key generation protocol. The other stuff is only used internally by high-level operations of `SchindelhauerTMCG`. Therefore this manual omits the description of such internal functions and members.

`BarnettSmartVTMF_dlog` [Class]
This class implements the discrete logarithm instantiation of the VTMF primitive in the field $\mathbf{Z}/p\mathbf{Z}$, where $p$ is a large prime number. The mathematical computations are performed in the finite cyclic subgroup $G$ of prime order $q$ such that $p = kq + 1$ holds for some $k \in \mathbf{Z}$. The security relies on the DDH assumption in $G$, i.e., the distribution $\{g^a, g^b, g^{ab}\}$ is computationally indistinguishable from $\{g^a, g^b, g^c\}$, where $g$ is a generator of $G$ and $a, b, c$ are chosen at random from $\mathbf{Z}_q$. Currently, this well-established assumption is believed to hold, if $p$ and $q$ are chosen according to the predefined security parameters of LibTMCG.

`mpz_t p` [Member of `BarnettSmartVTMF_dlog`]
This is the public prime number $p$ which defines the underlying field $\mathbf{Z}/p\mathbf{Z}$.

`mpz_t q` [Member of `BarnettSmartVTMF_dlog`]

This is the public prime number $q$ which defines the underlying cyclic group $G$. $G$ is a subgroup of $\mathbf{Z}/p\mathbf{Z}$ and is exactly of order $q$.

`mpz_t g` [Member of `BarnettSmartVTMF_dlog`]

This is the fixed public generator $g$ of the underlying group $G$.

`mpz_t k` [Member of `BarnettSmartVTMF_dlog`]

This is a public integer $k$ such that $p = kq + 1$ holds.

`mpz_t h` [Member of `BarnettSmartVTMF_dlog`]

This is the common public key $h = \prod_{i=1}^{k} h_i$ which contains the public keys $h_i$ of each player $P_i$. Note that in the above formula $k$ denotes the number of players.

`BarnettSmartVTMF_dlog` [Constructor on `BarnettSmartVTMF_dlog`]
(`unsigned long int` *fieldsize* `=TMCG_DDH_SIZE`, `unsigned long int` *subgroupsize* `=TMCG_DLSE_SIZE`)

This constructor creates a new VTMF instance. That means, the primes $p$ and $q$ are randomly and uniformly chosen such that they have length *fieldsize* bit and *subgroupsize* bit, respectively. Further, a generator $g$ for the unique subgroup of order $q$ is chosen at random. If the arguments are omitted, then *fieldsize* and *subgroupsize* are set to their default values `TMCG_DDH_SIZE` and `TMCG_DLSE_SIZE`, respectively. Depending on *fieldsize* and *subgroupsize* the group generation is a very time-consuming task and some dots are sent to `std::cerr` as a progress indicator.

`BarnettSmartVTMF_dlog` [Constructor on `BarnettSmartVTMF_dlog`]
(`std::istream&` *in*, `unsigned long int` *fieldsize* `=TMCG_DDH_SIZE`, `unsigned long int` *subgroupsize* `=TMCG_DLSE_SIZE`)

This constructor initializes the VTMF instance from a correctly formatted input stream *in*. For example, such a stream can be generated by calling the method `PublishGroup` of an already created instance. The arguments *fieldsize* and *subgroupsize* are stored for later following usage, e.g. by the method `CheckGroup` as explained below. If these arguments are omitted, then they are set to the default values `TMCG_DDH_SIZE` and `TMCG_DLSE_SIZE`, respectively.

`bool CheckGroup ()` [Method on `BarnettSmartVTMF_dlog`]

This method checks whether $p$ and $q$ have appropriate sizes with respect to the bit lengths given during the initialization of the corresponding instance. Further, it checks whether $p$ has the correct form (i.e. $p = kq + 1$), whether $p$ and $q$ are probable prime, and whether $g$ is a generator of the subgroup $G$. It returns `true`, if all of these checks have been passed successfully.

`void PublishGroup (std::ostream&` *out*`)` [Method on `BarnettSmartVTMF_dlog`]

This method exports all necessary group parameters of $G$ to the given output stream *out*, so other VTMF instances of $G$ can be initialized, e.g. with the second constructor of `BarnettSmartVTMF_dlog`.

`void` [Method on `BarnettSmartVTMF_dlog`]
`KeyGenerationProtocol_GenerateKey ()`

This method generates a VTMF key pair and stores the pair internally for a later following usage. It must be called before any other part of the key generation protocol is executed. Otherwise, the produced results are wrong.

`void` [Method on `BarnettSmartVTMF_dlog`]
`KeyGenerationProtocol_PublishKey (std::ostream& out)`

This method exports the public part of the generated VTMF key pair to the given output stream *out*. Further, it appends a non-interactive zero-knowledge proof of knowledge which shows that the instance knows the secret part. Due to the non-interactive nature of this proof the method has to be called only once while the computed output can be reused multiple times if necessary.

`bool` [Method on `BarnettSmartVTMF_dlog`]
`KeyGenerationProtocol_UpdateKey (std::istream& in)`

This method reads the public part of a VTMF key and the proof of knowledge from the input stream *in*. It appends the key to the common public key and returns `true`, if the given proof was sound. Otherwise, `false` is returned.

`bool` [Method on `BarnettSmartVTMF_dlog`]
`KeyGenerationProtocol_RemoveKey (std::istream& in)`

This method reads the public part of a VTMF key and the corresponding proof of knowledge from the input stream *in*. It removes the key from the common public key and returns `true`, if the key was previously appended by `KeyGenerationProtocol_UpdateKey` as explained above.

`void` [Method on `BarnettSmartVTMF_dlog`]
`KeyGenerationProtocol_Finalize ()`

This method must be called after any update (`KeyGenerationProtocol_UpdateKey`) or removal (`KeyGenerationProtocol_RemoveKey`) has been performed on the common public key.

`~BarnettSmartVTMF_dlog ()` [Destructor on `BarnettSmartVTMF_dlog`]

This destructor releases all occupied resources.

`BarnettSmartVTMF_dlog_GroupQR` [Subclass of `BarnettSmartVTMF_dlog`]

This subclass implements the discrete logarithm instantiation of the VTMF primitive in the field $\mathbf{Z}/p\mathbf{Z}$, where $p$ is a large prime number. The mathematical computations are performed in the finite cyclic subgroup $G$ (quadratic residues modulo $p$) of prime order $q$, where $p = 2q + 1$ holds. The security relies on the DDH assumption in $G$, i.e., the distribution $\{g^a, g^b, g^{ab}\}$ is computationally indistinguishable from $\{g^a, g^b, g^c\}$, where $g$ is a generator of $G$ and $a, b, c$ are chosen at random from $\mathbf{Z}_q$. Currently, this well-established assumption is believed to hold, if $p$ and $q$ are chosen according to the predefined security parameters of LibTMCG.

`mpz_t p` [Member of `BarnettSmartVTMF_dlog_GroupQR`]

This is the public prime number $p$ which defines the underlying field $\mathbf{Z}/p\mathbf{Z}$.

`mpz_t q` [Member of `BarnettSmartVTMF_dlog_GroupQR`]

This is the public prime number $q$ which defines the underlying cyclic group $G$. $G$ denotes the unique subgroup of quadratic residues modulo $p$ which is exactly of order $q$, if $p = 2q + 1$ holds.

`mpz_t g` [Member of `BarnettSmartVTMF_dlog_GroupQR`]

This is the fixed public generator $g$ of the underlying group $G$.

`mpz_t k` [Member of `BarnettSmartVTMF_dlog_GroupQR`]

This integer is fixed here by $k = 2$.

`mpz_t h` [Member of `BarnettSmartVTMF_dlog_GroupQR`]

This is the common public key $h = \prod_{i=1}^{k} h_i$ which contains the public keys $h_i$ of each player $P_i$. Note that in the above formula $k$ denotes the number of players.

`BarnettSmartVTMF_dlog_GroupQR` [ on `BarnettSmartVTMF_dlog_GroupQR`]
(`unsigned long int` *fieldsize* =`TMCG_DDH_SIZE`, `unsigned long int` *exponentsize* =`TMCG_DLSE_SIZE`)

This constructor creates a new VTMF instance. That means, the safe prime $p$ is randomly and uniformly chosen such that it has a length of *fieldsize* bit. Further, the generator $g$ is initially set up by 2 and then shifted by *fieldsize* − *exponentsize* bit positions, according to the procedure described by Koshiba and Kurosawa (see *Short Exponent Diffie-Hellman Problems*, PKC 2004, LNCS 2947). If the arguments of the constructor are omitted, then *fieldsize* and *exponentsize* are set to their default values `TMCG_DDH_SIZE` and `TMCG_DLSE_SIZE`, respectively. Depending on *fieldsize* and *exponentsize* the group generation is a very time-consuming task and some dots are sent to `std::cerr` as a progress indicator.

`BarnettSmartVTMF_dlog_GroupQR` [ on `BarnettSmartVTMF_dlog_GroupQR`]
(`std::istream&` *in*, `unsigned long int` *fieldsize* =`TMCG_DDH_SIZE`, `unsigned long int` *exponentsize* =`TMCG_DLSE_SIZE`)

This constructor initializes the VTMF instance from a correctly formatted input stream *in*. For example, such a stream can be generated by calling the method `PublishGroup` of an already created instance. The arguments *fieldsize* and *exponentsize* are stored for later following usage, e.g. by the method `CheckGroup` as explained below. If these arguments are omitted, then they are set to the default values `TMCG_DDH_SIZE` and `TMCG_DLSE_SIZE`, respectively.

`bool CheckGroup ()` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method checks whether $p$ and $q$ have appropriate sizes with respect to the bit lengths given during the initialization of the corresponding instance. Further, it checks whether $p$ has the correct form (i.e. $p = 2q + 1$), whether $p$ and $q$ are probable prime, and whether $g$ is a generator of the subgroup $G$. It returns `true`, if all of these checks have been passed successfully.

`void PublishGroup (std::ostream& out)` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method exports all necessary group parameters of $G$ to the given output stream *out*, so other VTMF instances of $G$ can be initialized, e.g. with the second constructor of `BarnettSmartVTMF_dlog_GroupQR`.

`void KeyGenerationProtocol_GenerateKey ()` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method generates a VTMF key pair and stores the pair internally for a later following usage. It must be called before any other part of the key generation protocol is executed. Otherwise, the produced results are wrong.

`void KeyGenerationProtocol_PublishKey (std::ostream& out)` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method exports the public part of the generated VTMF key pair to the given output stream *out*. Further, it appends a non-interactive zero-knowledge proof of knowledge which shows that the instance knows the secret part. Due to the non-interactive nature of this proof the method has to be called only once while the computed output can be reused multiple times if necessary.

`bool KeyGenerationProtocol_UpdateKey (std::istream& in)` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method reads the public part of a VTMF key and the proof of knowledge from the input stream *in*. It appends the key to the common public key and returns `true`, if the given proof was sound. Otherwise, `false` is returned.

`bool KeyGenerationProtocol_RemoveKey (std::istream& in)` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method reads the public part of a VTMF key and the corresponding proof of knowledge from the input stream *in*. It removes the key from the common public key and returns `true`, if the key was previously appended by `KeyGenerationProtocol_UpdateKey` as explained above.

`void KeyGenerationProtocol_Finalize ()` [Method on `BarnettSmartVTMF_dlog_GroupQR`]

This method must be called after any update (`KeyGenerationProtocol_UpdateKey`) or removal (`KeyGenerationProtocol_RemoveKey`) has been performed on the common public key.

`~BarnettSmartVTMF_dlog_GroupQR ()` [ on `BarnettSmartVTMF_dlog_GroupQR`]

This destructor releases all occupied resources.

#### 2.2.2.2 Verifiable Secret Shuffle of Homomorphic Encryptions

Recently, Groth [Gr05] has proposed a very efficient solution to perform a verifiable shuffle of homomorphically encrypted values. He describes an honest verifier zero-knowledge argument which shows the correctness of a shuffle. Beside other applications (e.g. verifiable mix networks, electronic voting) his protocol can be used to show (with overwhelming probability) that the secret shuffle of a deck of cards was performed correctly. The computational complexity and the produced communication traffic are superior to previously deployed techniques (e.g. Schindelhauer's cut-and-choose method). LibTMCG provides the first known implementation of Groth's famous protocol. However, it can only be used along with the VTMF card encoding scheme of Barnett and Smart [BS03].

Our implementation uses the statistically hiding and computationally binding homomorphic commitment scheme due to Pedersen (see *Non-interactive and Information-theoretic Secure Verifiable Secret Sharing*, CRYPTO '91, LNCS 576). The binding property relies on the hardness of computing discrete logarithms in $G$, and thus a commitment is only binding for computationally bounded provers.[3] But this choice seems to be reasonable for the intention of LibTMCG, because all players are supposed to be computationally bounded. The security parameters of the commitment scheme (in particular the group $G$) are determined by the corresponding VTMF instance.

Further, to the best of our knowledge it is not known, whether Groth's protocol retains the zero-knowledge property when it is executed in a concurrent setting. Thus the application programmer should be careful and avoid parallel invocations of the same instance.

`GrothVSSHE` [Class]

This class provides the low-level interface for Groth's protocol. There are just a few methods that might be of general interest. All other components are only used internally by high-level operations and thus their description is omitted here.

`GrothVSSHE (size_t` *n*, `mpz_srcptr` *p_ENC*, `mpz_srcptr` *q_ENC*, `mpz_srcptr` *k_ENC*, `mpz_srcptr` *g_ENC*, `mpz_srcptr` *h_ENC*, `unsigned long int` *ell_e* `=TMCG_GROTH_L_E`, `unsigned long int` *fieldsize* `=TMCG_DDH_SIZE`, `unsigned long int` *subgroupsize* `=TMCG_DLSE_SIZE)` [Constructor on `GrothVSSHE`]

This constructor creates a new instance. The low-level operations are later used to show the correctness of a shuffle of at most $n$ cards. The protocol and some parameters of the commitment scheme are initialized by the members of the corresponding VTMF instance. Consequently, *p_ENC* is the prime number $p$ which determines the field $\mathbf{Z}/p\mathbf{Z}$, *q_ENC* is the order of the underlying subgroup $G$, i.e. the prime number $q$, and *k_ENC* is the integer such that $p = qk + 1$ holds. Further, *g_ENC* is the generator $g$, and finally *h_ENC* is the common public key $h$. The positive integer *ell_e* is the security parameter which controls the soundness error probability ($2^{-\ell_e}$) of the protocol. The default value is defined by `TMCG_GROTH_L_E`, if this argument is omitted. The *fieldsize* and the *subgroupsize* are supplied to internal classes and are only of interest, if *p_ENC* or *q_ENC* have lengths different from the default. If these arguments are omitted, they are set to `TMCG_DDH_SIZE` and `TMCG_DLSE_SIZE`, respectively.

Note that the generators $g'_1, \ldots, g'_n$ of the Pedersen commitment scheme are randomly and uniformly chosen from $\mathbf{Z}_q$. Therefore this constructor should be instantiated only once by the session leader. All other instances must be created by the second constructor. Further, it is very important that the VTMF key

[3] Strictly speaking, due to this reason Groth's protocol is a zero-knowledge *argument* instead of a zero-knowledge *proof*. However, for convenience we will not distinguish between these terms here.

generation protocol has been finished before the value of *h* is passed to the constructor. Otherwise, the correctness verification will definitely fail.

`GrothVSSHE (size_t` *n*, `std::istream&` *in*, `unsigned long int` *ell_e* `=TMCG_GROTH_L_E`, `unsigned long int` *fieldsize* `=TMCG_DDH_SIZE`, `unsigned long int` *subgroupsize* `=TMCG_DLSE_SIZE)` [Constructor on `GrothVSSHE`]

This constructor initializes the instance from a correctly formatted input stream *in*. For example, such a stream can be generated by calling the method `PublishGroup` of an already created instance. Later the instance can be used to show the correctness of a shuffle of at most *n* cards. The positive integer *ell_e* controls the soundness error probability of the protocol. The default value is defined by `TMCG_GROTH_L_E`, if this argument is omitted.

`bool CheckGroup ()` [Method on `GrothVSSHE`]

This method checks whether the initialized commitment scheme is sound. It returns `true`, if all tests have been passed successfully.

`void PublishGroup (std::ostream&` *out*`)` [Method on `GrothVSSHE`]

This method exports the instance configuration to the output stream *out* such that other instances can be initialized, e.g. with the second constructor.

`~GrothVSSHE ()` [Destructor on `GrothVSSHE`]

This destructor releases all occupied resources.

#### 2.2.2.3 Verifiable Rotation of Homomorphic Encryptions

Hoogh, Schoenmakers, Skoric, and Villegas [HSSV09] has proposed an efficient solution to perform a verifiable rotation (also known as cyclic shift) of homomorphically encrypted values. Other solutions (e.g. Reiter and Wang, *Fragile Mixing*, ACM CCS, 2004) does not provide that level of efficency. LibTMCG provides the first known implementation of their protocol. It can only be used with the VTMF card encoding scheme of Barnett and Smart [BS03].

Further, to the best of our knowledge it is not known, whether their protocol retains the zero-knowledge property when it is executed in a concurrent setting. Thus the application programmer should be careful and avoid parallel invocations of the same instance.

`HooghSchoenmakersSkoricVillegasVRHE` [Class]

This class provides the low-level interface for their protocol. There are just a few methods that might be of general interest. All other components are only used internally by high-level operations and thus their description is omitted here.

`HooghSchoenmakersSkoricVillegasVRHE` [Constructor on `HooghSchoenmakersSkoricVillegasVRHE`]
`(mpz_srcptr` *p_ENC*, `mpz_srcptr` *q_ENC*, `mpz_srcptr` *k_ENC*, `mpz_srcptr` *g_ENC*, `mpz_srcptr` *h_ENC*, `unsigned long int` *fieldsize* `=TMCG_DDH_SIZE`, `unsigned long int` *subgroupsize* `=TMCG_DLSE_SIZE)`

This constructor creates a new instance. The low-level operations are later used to show the correctness of a rotation of the cards. The protocol and some of its parameters are initialized by the members of the corresponding VTMF instance. Consequently, *p_ENC* is the prime number $p$ which determines the field $\mathbf{Z}/p\mathbf{Z}$, *q_ENC* is the order of the underlying subgroup $G$, i.e. the prime number $q$, and *k_ENC* is the integer such that $p = qk + 1$ holds. Further, *g_ENC* is the generator $g$, and finally *h_ENC* is the common public key $h$. The *fieldsize* and the *subgroupsize* are supplied to internal classes and are only of interest, if *p_ENC* or *q_ENC* have lengths different from the default. If these arguments are omitted, they are set to `TMCG_DDH_SIZE` and `TMCG_DLSE_SIZE`, respectively.

This constructor should be instantiated only once by the session leader. All other instances must be created by the second constructor. Further, it is very important that the VTMF key generation protocol has been finished before the value of $h$ is passed to the constructor. Otherwise, the correctness verification will definitely fail.

`HooghSchoenmakersSkoricVillegasVRHE` [Constructor on `HooghSchoenmakersSkoricVillegasVRHE`]
`(std::istream&` *in*, `unsigned long int` *fieldsize* `=TMCG_DDH_SIZE`, `unsigned long int` *subgroupsize* `=TMCG_DLSE_SIZE)`

This constructor initializes the instance from a correctly formatted input stream *in*. For example, such a stream can be generated by calling the method `PublishGroup` of an already created instance. Later the instance can be used to show the correctness of a rotation.

`bool CheckGroup ()` [Method on `HooghSchoenmakersSkoricVillegasVRHE`]

This method checks whether the initialized commitment scheme is sound. It returns `true`, if all tests have been passed successfully.

`void PublishGroup` [Method on `HooghSchoenmakersSkoricVillegasVRHE`]
`(std::ostream&` *out*`)`

This method exports the instance configuration to the output stream *out* such that other instances can be initialized, e.g. with the second constructor.

`~HooghSchoenmakersSkoricVillegasVRHE` [Destructor on `HooghSchoenmakersSkoricVillegasVRHE`]
`()`

This destructor releases all occupied resources.

#### 2.2.2.4 Toolbox for Mental Card Games

This section explains the main class of LibTMCG which provides all "high-level operations" from Schindelhauer's toolbox [Sc98]. Even if the more efficient card encoding scheme of Barnett and Smart [BS03] is deployed, at least one instance of the following class must be created to perform any card or stack operations.

`SchindelhauerTMCG` [Class]

This class implements the main core of Schindelhauer's toolbox, i.e. important functions like masking, opening, and shuffling of cards and stacks, respectively. Some exotic operations are still missing, e.g., the possibility to insert a masked card secretly into a stack or the verifiable subset properties of stacks. All implemented operations are available for the original encoding scheme of Schindelhauer (see `TMCG_Card`) and, of course, for the more efficient encoding scheme of Barnett and Smart (see `VTMF_Card` and `BarnettSmartVTMF_dlog`) as well.

`unsigned long int TMCG_SecurityLevel` [Member of `SchindelhauerTMCG`]
This read-only nonnegative integer represents the security parameter $t$ which was given to the constructor of this class. It defines the number of protocol iterations and hence the soundness error probability ($2^{-t}$) of the zero-knowledge proofs in the encoding scheme of Schindelhauer. Further it defines the soundness error probability (also $2^{-t}$) of the shuffle argument in the encoding scheme of Barnett and Smart, if the efficient protocol of Groth [Gr05] is not used.

`size_t TMCG_Players` [Member of `SchindelhauerTMCG`]
This read-only nonnegative integer represents the number of players as given to the constructor of this class.

`size_t TMCG_TypeBits` [Member of `SchindelhauerTMCG`]
This read-only nonnegative integer contains the number of bits that are necessary to encode the card types in the binary representation. It was given as an argument to the constructor of this class.

`SchindelhauerTMCG (unsigned long int` *security*, `size_t` *k*, `size_t` *w*`)` [Constructor on `SchindelhauerTMCG`]
This constructor creates an instance, where *security* is a nonnegative integer that represents the security parameter $t$. The parameter $k$ is the number of players and $w$ is the number of bits which are necessary to represent all possible card types in a binary representation.

The integer $t$ controls the maximum soundness error probability ($2^{-t}$) of the zero-knowledge proofs in the encoding scheme of Schindelhauer. Specifically, *security* defines the number of sequential iterations of the involved protocols and thus has a major impact on the computational and communication complexity. If the encoding scheme of Barnett and Smart [BS03] is used, then it only defines the soundness error probability (also $2^{-t}$) of the shuffle proof. However, if only the efficient shuffle verification protocol of Groth [Gr05] is used, then the parameter *security* is dispensable, because the parameter *ell_e* given during instantiation of `GrothVSSHE` (e.g. the LibTMCG default security parameter `TMCG_GROTH_L_E`) determines this soundness error probability ($2^{-\ell_e}$).

Unfortunately, the parameters $k$ and $w$ have a major impact on the complexity in the encoding scheme of Schindelhauer, too. Therefore you should always use reasonable values. For example, to create a deck with $M$ different card types simply set $w$ to $\lceil \log_2 M \rceil$ which is an tight upper-bound for the binary representation. Furthermore, set $k$ to the number of players which are really involved and not to a possible maximum value. Note that $k$ and $w$ are limited by the global constants `TMCG_MAX_PLAYERS` and `TMCG_MAX_TYPEBITS`, respectively.

`void TMCG_CreateOpenCard (TMCG_Card&` *c*, `const TMCG_PublicKeyRing&` *ring*, `size_t` *type*`)` [Method on `SchindelhauerTMCG`]
This method initializes the open card $c$ with the given *type* using the encoding scheme of Schindelhauer. The *type* MUST be an integer from the interval

$[0, 2^w - 1]$, where $w$ is the number given to the constructor of this class. The $w$ MUST be the same number as used at creation of $c$ (see `TMCG_Card`). The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. The $k$ MUST be the same number as used at the creation of $c$.

`void TMCG_CreateOpenCard (VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf, size_t type)` [Method on `SchindelhauerTMCG`]

This method initializes the open card *c* with the given *type* using the encoding scheme of Barnett and Smart. The *type* MUST be an integer from the interval $[0, 2^w - 1]$, where $w$ is the number given to the constructor of this class. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol was successfully finished (see `BarnettSmartVTMF_dlog` and `BarnettSmartVTMF_dlog_GroupQR`, respectively).

`void TMCG_CreateCardSecret (TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, size_t index)` [Method on `SchindelhauerTMCG`]

This method initializes the card secret *cs* with random values which is necessary to perform later a masking operation on a card. The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. It MUST be the same number as used at the creation of *cs* (see `TMCG_CardSecret`). The parameter *index* is from the interval $[0, k - 1]$ and determines the position of the players public key in the container *ring*.

`void TMCG_CreateCardSecret (VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf)` [Method on `SchindelhauerTMCG`]

This method initializes the card secret *cs* with a random value which is necessary to perform later a masking operation on a card. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished (see `BarnettSmartVTMF_dlog` and `BarnettSmartVTMF_dlog_GroupQR`, respectively).

`void TMCG_CreatePrivateCard (TMCG_Card& c, TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, size_t index, size_t type)` [Method on `SchindelhauerTMCG`]

This method initializes a masked card *c* with the given *type* and a corresponding card secret *cs* using the encoding scheme of Schindelhauer. The *type* MUST be an integer from the interval $[0, 2^w - 1]$, where $w$ is the number given to the constructor of this class. The $w$ MUST be the same number as used at creation of $c$ (see `TMCG_Card`) and *cs* (see `TMCG_CardSecret`). The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. The $k$ MUST be the same number as used at the creation of $c$ and *cs*. The parameter *index* is from the interval $[0, k - 1]$ and determines the position of the players public key in the container *ring*. Internally, `TMCG_CreatePrivateCard` calls

1. `TMCG_CreateOpenCard` to initialize *c* with *type*,
2. `TMCG_CreateCardSecret` to initialize *cs* with random values, and

3. `TMCG_MaskCard` to mask *c* with the secret *cs*.

`void TMCG_CreatePrivateCard (VTMF_Card& c, VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf, size_t type)` [Method on `SchindelhauerTMCG`]

This method initializes a masked card *c* with the given *type* and a corresponding card secret *cs* using the encoding scheme of Barnett and Smart. The *type* MUST be an integer from the interval $[0, 2^w - 1]$, where $w$ is the number given to the constructor of this class. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished (see `BarnettSmartVTMF_dlog` and `BarnettSmartVTMF_dlog_GroupQR`, respectively). Specifically, `TMCG_CreatePrivateCard` directly executes the masking operation of the verifiable masking protocol.

`void TMCG_MaskCard (const TMCG_Card& c, TMCG_Card& cc, const TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, bool TimingAttackProtection =true)` [Method on `SchindelhauerTMCG`]

This method performs a masking operation on the open or already masked card *c* using the encoding scheme of Schindelhauer. Finally it returns the result in *cc*. The parameter *cs* MUST be an initialized fresh card secret which has NEVER been involved in a masking operation before. The parameters *c*, *cc*, and *cs* MUST be created such that their $k$ and $w$ corresponds to the numbers given to the constructor of this class, respectively. The parameter *ring* is a container with exactly $k$ public keys. The protection against timing attacks is turned on, if *TimingAttackProtection* is set to `true`.

`void TMCG_MaskCard (const VTMF_Card& c, VTMF_Card& cc, const VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf, bool TimingAttackProtection =true)` [Method on `SchindelhauerTMCG`]

This method performs a masking operation on the open or already masked card *c* using the encoding scheme of Barnett and Smart. Finally it returns the result in *cc*. Specifically, `TMCG_MaskCard` directly executes the masking operation of the verifiable re-masking protocol. The parameter *cs* MUST be an initialized fresh card secret which has NEVER been involved in a masking operation before. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished (see `BarnettSmartVTMF_dlog` and `BarnettSmartVTMF_dlog_GroupQR`, respectively). The protection against timing attacks is turned on, if *TimingAttackProtection* is set to `true`.

`void TMCG_ProveMaskCard (const TMCG_Card& c, const TMCG_Card& cc, const TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method should be called by the prover after `TMCG_MaskCard` to show that he performed the masking operation correctly. The parameters *c*, *cc*, and *cs* are the input, the result, and the used card secret of `TMCG_MaskCard`, respectively.

They MUST be created such that their *k* resp. *w* corresponds to the numbers given to the constructor of this class. The parameter *ring* is a container with exactly *k* public keys. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`void TMCG_ProveMaskCard (const VTMF_Card& c, const VTMF_Card& cc, const VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method should be executed by the prover after calling `TMCG_MaskCard` to show that he performed the masking operation correctly. Specifically, `TMCG_ProveMaskCard` directly calls the prove operation of the verifiable re-masking protocol. The parameters *c*, *cc*, and *cs* are the input, the result, and the used card secret of `TMCG_MaskCard`, respectively. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`bool TMCG_VerifyMaskCard (const TMCG_Card& c, const TMCG_Card& cc, const TMCG_PublicKeyRing& ring, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method should be executed by the verifier to check whether or not a masking operation was performed correctly. The parameters *c* and *cc* are the input and the result of `TMCG_MaskCard`, respectively. They MUST be created such that their *k* resp. *w* corresponds to the numbers given to the constructor of this class. The parameter *ring* is a container with exactly *k* public keys. The input/output protocol messages from and to the prover are transmitted on the streams *in* and *out*, respectively. The method returns `true`, if everything was sound.

`bool TMCG_VerifyMaskCard (const VTMF_Card& c, const VTMF_Card& cc, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method should be executed by the verifier to check whether or not a masking operation was performed correctly. Specifically, `TMCG_VerifyMaskCard` directly calls the verify operation of the verifiable re-masking protocol. The parameters *c* and *cc* are the input and the result of `TMCG_MaskCard`, respectively. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the prover are transmitted on the streams *in* and *out*, respectively. The method returns `true`, if everything was sound.

`void TMCG_ProveCardSecret (const TMCG_Card& c, const TMCG_SecretKey& key, size_t index, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method is used to reveal the card type of *c* to a verifier. Every player must execute this method as prover. The card *c* MUST be created such that its *k* resp. *w* corresponds to the numbers given to the constructor of this class.

The parameter *key* is the corresponding secret key (see `TMCG_SecretKey`) of the prover. The parameter *index* is from the interval $[0, k-1]$ and contains the position of the provers public key in the container *ring* (same as in `TMCG_CreateCardSecret`). The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`void TMCG_ProveCardSecret (const VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method is used to reveal the card type of *c* to a verifier. Every player must execute this method as prover. Specifically, `TMCG_ProveCardSecret` directly calls the prove operation of the verifiable decryption protocol. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`bool TMCG_VerifyCardSecret (const TMCG_Card& c, TMCG_CardSecret& cs, const TMCG_PublicKey& key, size_t index, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method is used to verify and accumulate card type information regarding *c* that are supplied by a prover. It is the opposite method of `TMCG_ProveCardSecret` and must be executed by the player who wants to know the type. The secrets provided by the single provers are accumulated in the parameter *cs*. Thus *c* and *cs* MUST be created such that their $k$ resp. $w$ corresponds to the numbers given to the constructor of this class. The parameter *key* is the corresponding public key (see `TMCG_PublicKey`) of the prover. The parameter *index* is from the interval $[0, k-1]$ and contains the position of the provers public key in the container *ring* (same as in `TMCG_CreateCardSecret`). The input/output protocol messages from and to the prover are transmitted on the streams *in* and *out*, respectively.

`bool TMCG_VerifyCardSecret (const VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)` [Method on `SchindelhauerTMCG`]

This method is used to verify and accumulate card type information regarding *c* that are supplied by a prover. It is the opposite method of `TMCG_ProveCardSecret` and must be executed by the player who wants to know the type. The secrets provided by the single provers are accumulated internally, thus this method cannot be interleaved with the opening of other cards. Specifically, `TMCG_VerifyCardSecret` directly calls the verify and update operation of the verifiable decryption protocol. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`void TMCG_SelfCardSecret (const TMCG_Card& c, TMCG_CardSecret& cs, const TMCG_SecretKey& key, size_t index)` [Method on `SchindelhauerTMCG`]

This method is used to compute and accumulate card type information regarding *c*. Analogously to `TMCG_VerifyCardSecret` it must be executed by the player who wants to know the type of *c*. The information is accumulated in the parameter *cs*. Thus *c* and *cs* MUST be created such that their $k$ resp. $w$ corresponds to the numbers given to the constructor of this class. The parameter *key* is the corresponding secret key (see `TMCG_SecretKey`) of the player. The parameter *index* is from the interval $[0, k-1]$ and contains the position of the players public key in the container *ring* (same as in `TMCG_CreateCardSecret`).

`void TMCG_SelfCardSecret (const VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf)` [Method on `SchindelhauerTMCG`]

This method is used to compute and accumulate card type information regarding *c*. It MUST be called by the player who wants to know the type of *c* BEFORE `TMCG_VerifyCardSecret` and `TMCG_TypeOfCard` are executed. The secrets provided by the player are accumulated internally, thus this method cannot be interleaved with the opening of other cards. Specifically, `TMCG_SelfCardSecret` directly calls the initialize operation of the verifiable decryption protocol. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished.

`size_t TMCG_TypeOfCard (const TMCG_CardSecret& cs)` [Method on `SchindelhauerTMCG`]

This method returns the type of a masked card provided that the type information were properly accumulated in *cs* before (by calling `TMCG_SelfCardSecret` and `TMCG_VerifyCardSecret`, respectively).

`size_t TMCG_TypeOfCard (const VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf)` [Method on `SchindelhauerTMCG`]

This method returns the type of a masked card *c* provided that the type information regarding *c* were properly accumulated internally before (by calling `TMCG_SelfCardSecret` and `TMCG_VerifyCardSecret`, respectively). It returns the value `TMCG_MaxCardType`, if the opening operation failed or if the card type was not among the set of valid types. This method MUST be performed by the player who wants to know the type AFTER `TMCG_SelfCardSecret` and `TMCG_VerifyCardSecret` are executed. Specifically, `TMCG_TypeOfCard` directly calls the finalize operation of the verifiable decryption protocol. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished.

`size_t TMCG_CreateStackSecret (TMCG_StackSecret<TMCG_CardSecret>& ss, bool cyclic, const TMCG_PublicKeyRing& ring, size_t index, size_t size)` [Method on `SchindelhauerTMCG`]

This method initializes the stack secret *ss* with a randomly and uniformly chosen permutation (using the algorithm of Knuth) and fresh card secrets. Later this stack secret can be used to perform a secret shuffle operation on a stack. If the parameter *cyclic* is set to true, then the permutation is only a cyclic shift which might be of interest for particular operations, e.g. cutting the deck. The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. The parameter *index* is from the interval $[0, k-1]$ and contains the position of the players public key in the container *ring*. The parameter *size* determines the size of the created stack secret, i.e. the number of cards in the corresponding stack. The *size* is upper-bounded by TMCG_MAX_CARDS. The method returns the offset of the cyclic shift, if *cyclic* was set to true. Otherwise, the value 0 is returned.

size_t TMCG_CreateStackSecret (TMCG_StackSecret<VTMF_CardSecret>& *ss*, bool *cyclic*, size_t *size*, BarnettSmartVTMF_dlog* *vtmf*) [Method on SchindelhauerTMCG]

This method initializes the stack secret *ss* with a randomly and uniformly chosen permutation (using the algorithm of Knuth) and fresh card secrets. Later this stack secret can be used to perform a secret shuffle operation on a stack. If the parameter *cyclic* is set to true, then the permutation is only a cyclic shift which might be of interest for particular operations, e.g. cutting the deck. The parameter *size* determines the size of the created stack secret, i.e. the number of cards in the corresponding stack. The *size* is upper-bounded by TMCG_MAX_CARDS. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The method returns the offset of the cyclic shift, if *cyclic* was set to true. Otherwise, the value 0 is returned.

void TMCG_MixStack (const TMCG_Stack<TMCG_Card>& *s*, TMCG_Stack<TMCG_Card>& *s2*, const TMCG_StackSecret<TMCG_CardSecret>& *ss*, const TMCG_PublicKeyRing& *ring*, bool *TimingAttackProtection* =true) [Method on SchindelhauerTMCG]

This method shuffles a given stack *s* according to the previously created stack secret *ss* (see TMCG_CreateStackSecret). The result of the shuffle is returned in *s2*. The parameter *ss* MUST be a fresh stack secret which has NEVER been involved in a shuffle operation before. The parameters *s* and *ss* MUST be of the same size. The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. The protection against timing attacks is turned on, if *TimingAttackProtection* is set to true.

void TMCG_MixStack (const TMCG_Stack<VTMF_Card>& *s*, TMCG_Stack<VTMF_Card>& *s2*, const TMCG_StackSecret<VTMF_CardSecret>& *ss*, BarnettSmartVTMF_dlog* *vtmf*, bool *TimingAttackProtection* =true) [Method on SchindelhauerTMCG]

This method shuffles a given stack *s* according to the previously created stack secret *ss* (see TMCG_CreateStackSecret). The result of the shuffle is returned in *s2*. The parameter *ss* MUST be a fresh stack secret which has NEVER been involved in a shuffle operation before. The parameters *s* and *ss* MUST be of the same size. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The protection against timing attacks is turned on, if *TimingAttackProtection* is set to `true`.

`void TMCG_ProveStackEquality (const TMCG_Stack<TMCG_Card>&` *s*, `const TMCG_Stack<TMCG_Card>&` *s2*, `const TMCG_StackSecret<TMCG_CardSecret>&` *ss*, `bool` *cyclic*, `const TMCG_PublicKeyRing&` *ring*, `size_t` *index*, `std::istream&` *in*, `std::ostream&` *out*) [Method on `SchindelhauerTMCG`]

This method should be called by the prover after `TMCG_MixStack` to show that he performed the shuffle operation correctly. The parameters *s*, *s2*, and *ss* are the input, the result, and the used stack secret of `TMCG_MixStack`, respectively. Of course, the parameters *s*, *s2*, and *ss* MUST be of the same size. The parameter *cyclic* determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. The parameter *index* is from the interval $[0, k-1]$ and contains the position of the provers public key in the container *ring*. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`void TMCG_ProveStackEquality (const TMCG_Stack<VTMF_Card>&` *s*, `const TMCG_Stack<VTMF_Card>&` *s2*, `const TMCG_StackSecret<VTMF_CardSecret>&` *ss*, `bool` *cyclic*, `BarnettSmartVTMF_dlog*` *vtmf*, `std::istream&` *in*, `std::ostream&` *out*) [Method on `SchindelhauerTMCG`]

This method should be called by the prover after `TMCG_MixStack` to show that he performed the shuffle operation correctly. The parameters *s*, *s2*, and *ss* are the input, the result, and the used stack secret of `TMCG_MixStack`, respectively. Of course, the parameters *s*, *s2*, and *ss* MUST be of the same size. The parameter *cyclic* determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively.

`void TMCG_ProveStackEquality_Groth (const TMCG_Stack<VTMF_Card>&` *s*, `const TMCG_Stack<VTMF_Card>&` *s2*, `const TMCG_StackSecret<VTMF_CardSecret>&` *ss*, `BarnettSmartVTMF_dlog*` *vtmf*, `GrothVSSHE*` *vsshe*, `std::istream&` *in*, `std::ostream&` *out*) [Method on `SchindelhauerTMCG`]

This is a method like above. The only difference is that the more efficient shuffle verification protocol of Groth is used. Thus *vsshe* is a pointer to an initialized instance of `GrothVSSHE`. The rest of the arguments are the same.

void TMCG_ProveStackEquality_Hoogh [Method on SchindelhauerTMCG]
(const TMCG_Stack<VTMF_Card>& *s*, const TMCG_Stack<VTMF_Card>& *s2*, const TMCG_StackSecret<VTMF_CardSecret>& *ss*, BarnettSmartVTMF_dlog* *vtmf*, HooghSchoenmakersSkoricVillegasVRHE* *vrhe*, std::istream& *in*, std::ostream& *out*)

This is a method like above. The only difference is that the more efficient rotation verification protocol [HSSV09] is used. Thus *vrhe* is a pointer to an initialized instance of HooghSchoenmakersSkoricVillegasVRHE. The rest of the arguments are the same.

bool TMCG_VerifyStackEquality (const [Method on SchindelhauerTMCG]
TMCG_Stack<TMCG_Card>& *s*, const TMCG_Stack<TMCG_Card>& *s2*, bool *cyclic*, const TMCG_PublicKeyRing& *ring*, std::istream& *in*, std::ostream& *out*)

This method should be executed by the verifier to check whether or not a shuffle operation was performed correctly. The parameters *s* and *s2* are the input and the result of TMCG_MixStack, respectively. Of course, the parameters *s* and *s2* should be of the same size. The parameter *cyclic* determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter *ring* is a container with exactly $k$ public keys, where $k$ is the number given to the constructor of this class. The input/output protocol messages from and to the prover are transmitted on the streams *in* and *out*, respectively. This method returns true, if the shuffle operation was successfully verified.

bool TMCG_VerifyStackEquality (const [Method on SchindelhauerTMCG]
TMCG_Stack<VTMF_Card>& *s*, const TMCG_Stack<VTMF_Card>& *s2*, bool *cyclic*, BarnettSmartVTMF_dlog* *vtmf*, std::istream& *in*, std::ostream& *out*)

This method should be executed by the verifier to check whether or not a shuffle operation was performed correctly. The parameters *s* and *s2* are the input and the result of TMCG_MixStack, respectively. Of course, the parameters *s* and *s2* should be of the same size. The parameter *cyclic* determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter *vtmf* is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams *in* and *out*, respectively. This method returns true, if the shuffle operation was successfully verified.

bool TMCG_VerifyStackEquality_Groth [Method on SchindelhauerTMCG]
(const TMCG_Stack<VTMF_Card>& *s*, const TMCG_Stack<VTMF_Card>& *s2*, BarnettSmartVTMF_dlog* *vtmf*, GrothVSSHE* *vsshe*, std::istream& *in*, std::ostream& *out*)

This is a method like above. The only difference is that the more efficient shuffle verification protocol of Groth is used. Thus *vsshe* is a pointer to an initialized instance of GrothVSSHE. The rest of the arguments and the returned values are the same.

bool TMCG_VerifyStackEquality_Hoogh [Method on SchindelhauerTMCG]
(const TMCG_Stack<VTMF_Card>& *s*, const TMCG_Stack<VTMF_Card>& *s2*, BarnettSmartVTMF_dlog* *vtmf*, HooghSchoenmakersSkoricVillegasVRHE* *vrhe*, std::istream& *in*, std::ostream& *out*)

This is a method like above. The only difference is that the more efficient rotation verification protocol [HSSV09] is used. Thus *vrhe* is a pointer to an initialized instance of HooghSchoenmakersSkoricVillegasVRHE. The rest of the arguments and the returned values are the same.

~SchindelhauerTMCG () [Destructor on SchindelhauerTMCG]

This destructor releases all occupied resources.

# 3 Examples

The following examples explain most of the steps that are necessary to create a secure and fair card game with LibTMCG. We consider an application with five permanent players (denoted by $P_0$, $P_1$, $P_2$, $P_3$, and $P_4$) and a regular deck of 52 different cards. For convenience only the more efficient card encoding scheme of Barnett and Smart [BS03] is described. Additionally, we complete our exposition with code fragments which show the usage of the fast shuffle verification protocol due to Groth [Gr05].

Throughout the remaining pages we suppose that all players are pairwise connected by authenticated communication channels. These channels are organized in input resp. output streams, where `input_stream[i]` resp. `output_stream[i]` denote the corresponding `std::istream` resp. `std::ostream` instance for the communication with player $P_i$.[1]

## 3.1 Library Initialization

The very first step that should be performed is the initialization of LibTMCG. That can simply be done by calling the function `init_libTMCG` and evaluating the return code.

```
if (!init_libTMCG())
  std::cerr << "Initialization of LibTMCG failed!" << std::endl;
```

Additionally, in the most cases it is useful to check the installed library version by using the function `version_libTMCG`.

## 3.2 Session Initialization and Key Generation

In the next step we create an instance of the class `SchindelhauerTMCG`. The first parameter determines the number of protocol iterations $t$ which upper-bounds the cheating probability by $2^{-t}$. In our example the used value `64` defines a maximum cheating probability of $5.421010862 \cdot 10^{-20}$ which is reasonable small for our purposes. The second parameter passes the number of players to the instance which is simply `5` in our case. The last argument defines the number of bits that are necessary to encode all card types in a binary representation. The given value `6` allows the encoding of $2^6 = 64$ different card types at maximum. This is enough to form our deck of 52 cards.

```
SchindelhauerTMCG *tmcg = new SchindelhauerTMCG(64, 5, 6);
```

We would like to use the more efficient encoding scheme of Barnett and Smart, thus we create an instance of `BarnettSmartVTMF_dlog`. However, a particular player has to act as a *leader* who performs the generation of the group $G$. In our case $P_0$ will be the session leader. First, he executes the constructor of `BarnettSmartVTMF_dlog`.

```
BarnettSmartVTMF_dlog *vtmf = new BarnettSmartVTMF_dlog();
```

[1] We assume that the players are ordered in a natural way such that we can use the same nomenclature.

Afterwards he checks the generated group $G$ and sends the public parameters to all other players (corresponding stream indices are `1`, `2`, `3`, and `4`, respectively).

```
if (!vtmf->CheckGroup())
  std::cerr << "Group G was not correctly generated!" << std::endl;
for (size_t i = 1; i < 5; i++)
  vtmf->PublishGroup(output_stream[i]);
```

The other players receive the group parameters from $P_0$ and use them to initialize their corresponding instances of `BarnettSmartVTMF_dlog`. It is very important that they also check, whether the group $G$ was correctly generated by the leader.

```
BarnettSmartVTMF_dlog *vtmf =
  new BarnettSmartVTMF_dlog(input_stream[0]);
if (!vtmf->CheckGroup())
  std::cerr << "Group G was not correctly generated!" << std::endl;
```

Afterwards the key generation protocol is carried out. First, every player generates his own VTMF key. The secret key material is stored internally and will never be exposed.

```
vtmf->KeyGenerationProtocol_GenerateKey();
```

Then every player $P_j$ sends the public part of his VTMF key along with a non-interactive zero-knowledge proof of knowledge to each other player. The appended proof shows that he indeed knows the corresponding secret key. However, due to the non-interactive nature of this proof we have to be careful, if same group $G$ is used again.

```
for (size_t i = 0; i < 5; i++)
{
  if (i != j)
    vtmf->KeyGenerationProtocol_PublishKey(output_stream[i]);
}
```

After sending $P_j$ receives the public keys. Simultaneously he checks, whether the keys are correctly generated, and updates the common public key $h$.

```
for (size_t i = 0; i < 5; i++)
{
  if (i != j)
  {
    if (!vtmf->KeyGenerationProtocol_UpdateKey(input_stream[i]))
      std::cerr << "Public key was not correctly generated!" << std::endl;
  }
}
```

Finally, every player must finalize the key generation protocol.

```
vtmf->KeyGenerationProtocol_Finalize();
```

If we want to use the more efficient shuffle verification protocol of Groth, then $P_0$ must also create an instance of `GrothVSSHE`. The first argument determines the maximum stack size of which the correctness of a shuffle will be proven. The other parameters are obtained from the former created VTMF instance `vtmf`. It is important that the key generation protocol has been finalized before the common public key $h$ (i.e. `vtmf->h`) is passed.

```
GrothVSSHE *vsshe = new GrothVSSHE(52, vtmf->p, vtmf->q, vtmf->k,
  vtmf->g, vtmf->h);
```

Again, $P_0$ will send the public parameters of the VSSHE instance to all other players.

```
for (size_t i = 1; i < 5; i++)
  vsshe->PublishGroup(output_stream[i]);
```

The other players receive these parameters from the leader and use them to initialize their corresponding instances of `GrothVSSHE`. Again, it is important to check, whether the parameters were correctly chosen by the leader.

```
GrothVSSHE *vsshe = new GrothVSSHE(52, input_stream[0]);
if (!vsshe->CheckGroup())
  std::cerr << "VSSHE was not correctly generated!" << std::endl;
if (mpz_cmp(vtmf->h, vsshe->com->h))
  std::cerr << "VSSHE: Common public key does not match!" << std::endl;
if (mpz_cmp(vtmf->q, vsshe->com->q))
  std::cerr << "VSSHE: Subgroup order does not match!" << std::endl;
if (mpz_cmp(vtmf->p, vsshe->p) || mpz_cmp(vtmf->q, vsshe->q) ||
  mpz_cmp(vtmf->g, vsshe->g) || mpz_cmp(vtmf->h, vsshe->h))
    std::cerr << "VSSHE: Encryption scheme does not match!" << std::endl;
```

## 3.3 Operations on Cards

Now we are ready to perform several operations on cards. We start with some basic stuff which might be of interest in particular situations. However, it is often more convenient to work directly with stacks, as explained later.

### 3.3.1 Creating an Open Card

The creation of an open card is very simple. The following code creates a card of type 7.

```
VTMF_Card c;
tmcg->TMCG_CreateOpenCard(c, vtmf, 7);
```

### 3.3.2 Masking and Re-masking of a Card

Now the previously created card `c` will be masked to hide its type. Then `cc` is sent to $P_1$.

```
VTMF_Card cc;
VTMF_CardSecret cs;
tmcg->TMCG_CreateCardSecret(cs, vtmf);
tmcg->TMCG_MaskCard(c, cc, cs, vtmf);
out_stream[1] << cc << std::endl;
```

$P_1$ receives the card `cc`, re-masks them, and sends the result `ccc` back to the player $P_0$. Further he proves that the masking operation was performed correctly.

```
VTMF_Card cc, ccc;
VTMF_CardSecret ccs;
in_stream[0] >> cc;
if (!in_stream[0].good())
  std::cerr << "Read or parse error!" << std::endl;
tmcg->TMCG_CreateCardSecret(ccs, vtmf);
tmcg->TMCG_MaskCard(cc, ccc, ccs, vtmf);
out_stream[0] << ccc << std::endl;
tmcg->TMCG_ProveMaskCard(cc, ccc, ccs, vtmf, in_stream[0], out_stream[0]);
```

$P_0$ receives the card, verifies the proof, and sends the card to all other players.

```
VTMF_Card ccc;
in_stream[1] >> ccc;
if (!tmcg->TMCG_VerifyMaskCard(cc, ccc, vtmf, in_stream[1], out_stream[1]))
  std::cerr << "Verification failed!" << std::endl;
for (size_t i = 1; i < 5; i++)
  out_stream[i] << ccc << std::endl;
```

Finally, all other players receive and store the masked card `ccc`.

### 3.3.3 Opening a Masked Card

Suppose that $P_1$ would like to know the type of the masked card `ccc`. Of course, $P_0$ could simply reveal it, but that isn't verifiable. Anyway, if all players cooperate, then $P_1$ can compute the type in a verifiable way. First, every player (except $P_1$) will execute the following code.

```
tmcg->TMCG_ProveCardSecret(ccc, vtmf, in_stream[1], out_stream[1]);
```

On the other hand, $P_1$ will execute the following commands exactly in the given order. Finally, he obtain the card type in the variable `type`.

```
tmcg->TMCG_SelfCardSecret(ccc, vtmf);
for (size_t i = 0; i < 5; i++)
{
  if (i == 1)
    continue;
  if (!tmcg->TMCG_VerifyCardSecret(ccc, vtmf, in_stream[i], out_stream[i]))
    std::cerr << "Verification failed!" << std::endl;
}
type = tmcg->TMCG_TypeOfCard(ccc, vtmf);
```

## 3.4 Operations on Stacks

There exist a lot of basic operations on stacks, e.g. pushing a card to a stack or importing a stack. These functions are to simple for explaining them here, but they are used implicitly. However, a short description can be found in the API part of the manual (see `TMCG_Stack` and `TMCG_OpenStack`).

### 3.4.1 Creating the Deck

A quite common operation is the creation of a card deck. The deck will initially be represented by an open stack (see `TMCG_OpenStack`) called `deck`. Every player creates his own deck which consists of 52 different open cards in our example.

```
TMCG_OpenStack<VTMF_Card> deck;
for (size_t type = 0; type < 52; type++)
{
  VTMF_Card c;
  tmcg->TMCG_CreateOpenCard(c, vtmf, type);
  deck.push(type, c);
}
```

Note that this card deck must be consistent for all players, that means, the order of the open cards must be exactly the same for all players.

### 3.4.2 Shuffling the Deck

Each player must perform a shuffle of the deck, because only such a procedure guarantees that no coalition has influence on the outcome. Thus we build a shuffle chain such that every player shuffles the deck. Consider the following code fragment for the player $P_j$.

The regular stack `s` is initialized with open cards from `deck`. Then each player shuffles the stack (see `TMCG_MixStack`) and proves the correctness of this operation (see `TMCG_ProveStackEquality`). Consequently, every player should verify these proofs (see `TMCG_VerifyStackEquality`). Finally, the stack `s` contains the shuffled result.

```
TMCG_Stack<VTMF_Card> s;
s.push(deck);

for (size_t i = 0; i < 5; i++)
{
  TMCG_Stack<VTMF_Card> s2;
  if (i == j)
  {
    TMCG_StackSecret<VTMF_CardSecret> ss;
    tmcg->TMCG_CreateStackSecret(ss, false, s.size(), vtmf);
    tmcg->TMCG_MixStack(s, s2, ss, vtmf);
    for (size_t i2 = 0; i2 < 5; i2++)
    {
      if (i2 == j)
        continue;
      out_stream[i2] << s2 << std::endl;
      tmcg->TMCG_ProveStackEquality(s, s2, ss, false, vtmf,
        in_stream[i2], out_stream[i2]);
    }
  }
  else
  {
    in_stream[i] >> s2;
    if (!in_stream[i].good())
      std::cerr << "Read or parse error!" << std::endl;
    if (!tmcg->TMCG_VerifyStackEquality(s, s2, false, vtmf,
      in_stream[i], out_stream[i]))
        std::cerr << "Verification failed!" << std::endl;
  }
  s = s2;
}
```

If you want to use the more efficient shuffle verification protocol of Groth, then you must simply replace `TMCG_ProveStackEquality` and `TMCG_VerifyStackEquality` by `TMCG_ProveStackEquality_Groth` and `TMCG_VerifyStackEquality_Groth`, respectively.

### 3.4.3 Drawing a Card from the Deck

Now every player has the same shuffled deck `s` and nobody knows in which order the 52 cards are stacked. Therefore you can simply use any drawing strategy to obtain a players hand. For example, look at the following code that draws two cards from `s` for each player.

```
TMCG_Stack<VTMF_Card> hand[5];
for (size_t i = 0; i < 5; i++)
{
  VTMF_Card c1, c2;
  s.pop(c1), s.pop(c2);
  hand[i].push(c1), hand[i].push(c2);
}
```

Further, probably you want disclose the card types to the corresponding player. Consider the code fragment for the player $P_j$: Every player receives the necessary information from the other players and computes the card types of his hand `hand[j]`. Finally, these types are stored together with the masked cards in the open stack `private_hand`. The example can be modified in a straightforward way to publicly disclose a card from a players hand or from the remaining stack `s`.

```
TMCG_OpenStack<VTMF_Card> private_hand;
for (size_t i = 0; i < 5; i++)
{
  if (i == j)
  {
    for (size_t k = 0; k < hand[j].size(); k++)
    {
      tmcg->TMCG_SelfCardSecret(hand[j][k], vtmf);
      for (size_t i2 = 0; i2 < 5; i2++)
      {
        if (i2 == j)
          continue;
        if (!tmcg->TMCG_VerifyCardSecret(hand[j][k], vtmf,
          in_stream[i2], out_stream[i2]))
            std::cerr << "Verification failed!" << std::endl;
      }
      private_hand.push(tmcg->TMCG_TypeOfCard(hand[j][k], vtmf),
        hand[j][k]);
    }
  }
  else
  {
    for (size_t k = 0; k < hand[i].size(); k++)
    {
      tmcg->TMCG_ProveCardSecret(hand[i][k], vtmf,
        in_stream[i], out_stream[i]);
    }
  }
}
```

## 3.5 Quit a Session

The last step should release all occupied resources.

```
delete vsshe, delete vtmf, delete tmcg;
```

# Appendix A Licenses

## A.1 GNU General Public License

Version 2, June 1991



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# Appendix B General and API Index

## General Index

# API Index

## F



## G



## H



## I



## K



## M



## P



## R



## S



## T

## V