Machine Learning Focus on Regression and Planting Other Seeds

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January 19, 2023

Working Example: Predicting Election Results

Goal: Use predictors to forecast Incumbent Vote Share

Potential predictors:

- GDP Growth
- Incumbent Approval
- Stock Market Trends
- Unemployment
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Conjecture: Model relationship in prior elections to predict future election

Model Variables

Input variables $\rightsquigarrow$ Approval, GDP Growth, Unemployment, etc.

- *predictors, independent variables, features, attributes, covariates*
- X; X_1 (Approval), X_2 (GDP Growth), X_3 (Unemployment), etc.

Output variable $\rightsquigarrow$ Incumbent Vote Share

- *response, dependent variable, outcome, target, label*
- Y
  - Quantitative (e.g. 15, 3.14, -82000) $\rightsquigarrow$ Regression Model
  - Categorical (e.g. Republican/Democrat, 0/1, High/Medium/Low) $\rightsquigarrow$ Classification Model

Supervised Learning

We assume some relationship between Y and $X = (X_1, X_2, ..., X_p)$, such that:

$$Y = f(X) + \epsilon$$

- f is some fixed but unknown function of $X_1, X_2, ..., X_p$
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Ultimate Goal: Build an $\hat{f}$ that is as close as possible to f

Why Estimate f?

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We'll focus mostly on prediction in this class.

Test Your Knowledge

Is the scenario a classification or regression problem? Are we most interested in inference or prediction? What are n and p?

We are considering launching a new product and wish to know whether it will be a success or a failure. We collect data on 20 similar products that were previously launched. For each product we have recorded whether it was a success or failure, price charged for the product, marketing budget, competition price, and ten other variables.

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Classification, prediction, $n = 20$, $p = 13$.

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Regression, inference, $n = 500$, $p = 3$.

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Year	**Incumbent Vote Share**	Incumbent Net Approval	GDP Growth
2020	51.1	1.5	3.2
2018	49.1	-2.5	2.3
2016	56.8	22	3.0
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5. Compare predicted response value ($\hat{Y}$) with true response value (Y) for observations in test / validation data to evaluate performance.

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The first thing we need to do is develop a toolkit of methods...

and linear regression will be where we start

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Answer: $f(X)$ is the true function that maps X onto Y. $\hat{f}(X)$ is the estimated / prediction function trained on a sample of data, mapping observed X onto observed Y.

Linear Regression

What is Linear Regression?

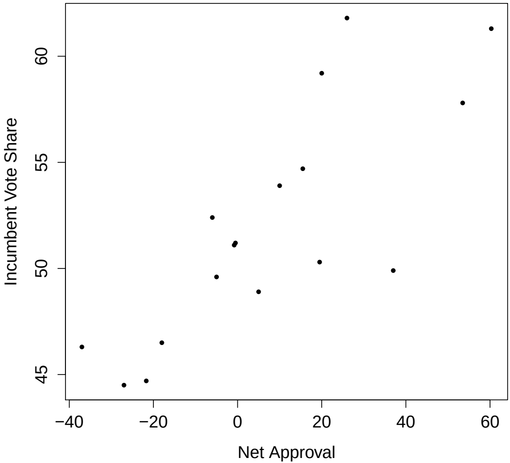
Linear regression is a simple approach for supervised learning.

- Around since 1800s.
- Still a widely used tool for predicting quantitative response.
- Building block for more sophisticated methods.
- We need to understand it before moving on!

Simple linear regression:

- Assumes a linear relationship between quantitative response Y and a **single** variable X.
- Also called bivariate regression.

Bivariate Regression: Geometric Perspective



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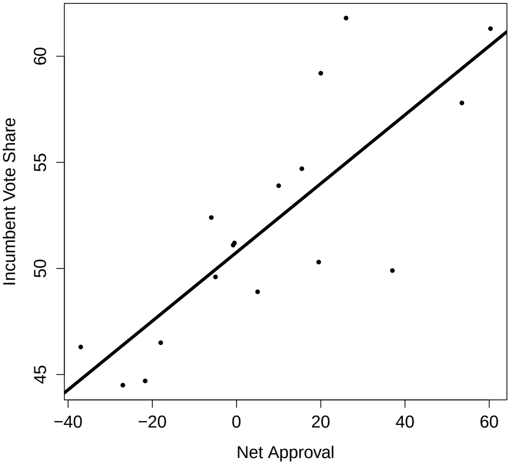

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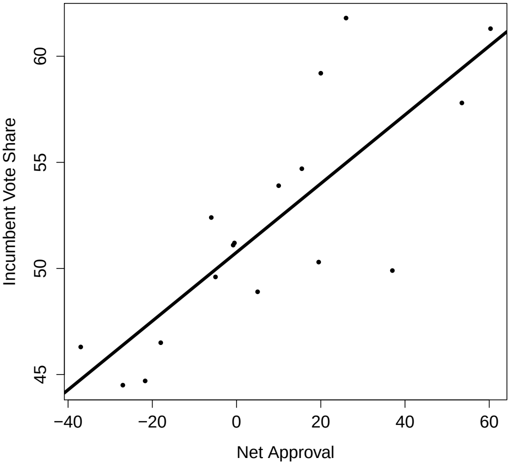
$$y = mx + b$$



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Instead, our training data will be used to estimate β_0 and β_1. By estimating β_0 and β_1, we can then make predictions for $Vote_i$.

That is, we will use our training data to produce estimates, $\hat{\beta}_0$ and $\hat{\beta}_0$, which we'll use to create a prediction function:

$$\widehat{Vote}_i = \hat{\beta}_0 + \hat{\beta}_1 Approval_i$$

Another Preview of Things to Come...

Uncertainty

In this class (and in machine learning applications more broadly), we will often want to not only produce estimates/predictions, but also quantify the uncertainty surrounding those estimates/predictions.

Simple (Univariate) Example of Uncertainty

Gallup Poll ($N \approx 1000$) on Approval of Congress (Nov/Dec, 2022):

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Problem: This calculation is based upon a small sample of the U.S. voting population, so how confident can we be that it reflects the mean approval in the overall population?

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$$[19\%, 25\%]$$

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Where does that uncertainty quantification come from?

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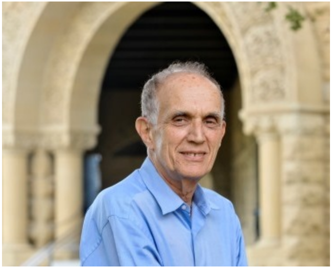
2. **Bootstrap**

Uncertainty and the Bootstrap



Münchhausen O. Herrfurth pinx

Welcome to the Bootstrap World

Real World

Unknown probability distribution

Observed random sample

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$$\downarrow$$

$$\hat{\theta} = s(\mathrm{Z})$$

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- Thus, resampling from Z is the best guide to what can be expected from resampling from P
- Just like we drew Z from P, let's draw a resample Z^* from Z
- If n is sufficiently large, the observed sample Z should be a good approximation of P (i.e. treat Z as $\hat{P}$)

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4. To compute 95% CI, use 2.5/97.5 percentiles of $\{s(Z_1^*), ..., s(Z_B^*)\}$ as the lower/upper bounds (bootstrap percentile CI)

1. Section tomorrow
2. Readings for next Tuesday are listed on syllabus and posted in Files on bCourses site
3. Install/Update `R` and `RStudio` on your computer by next Thursday