

THE PROBLEM

Variational Bayesian inference frequently relies on Monte Carlo (MC) integration when intractable expectations appear in the evidence lower bound (ELBO). Variational inference evaluates these expectations wrt the proposed posterior approximations. In many cases, such as in Variational Auto-Encoders, we maximize the ELBO via gradient methods, however, to do so in conjunction MC integration requires that samples from the posterior approximations are differentiable wrt their respective parameters. Closed-form “reparameterization tricks” unfortunately only exist for a subset of distribution families (e.g. Gaussian).

INVERSE CDF SAMPLING

For continuous univariate distribution, inverse CDF sampling allows us to reparameterize a sample from a Uniform distribution to the target distribution:

$$\epsilon \sim \text{Uniform}(0, 1)$$

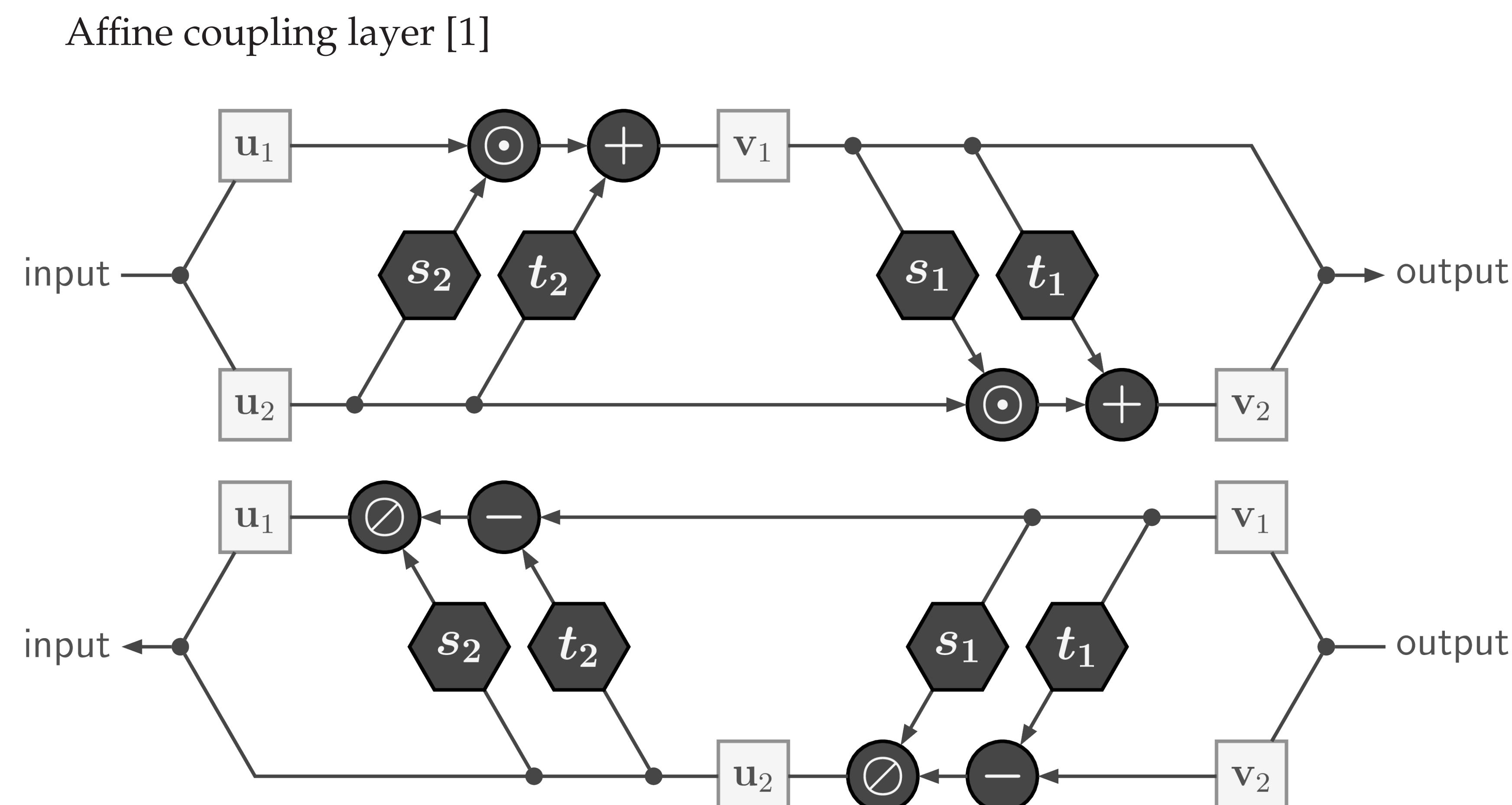
$$z = F^{-1}(\epsilon; \theta)$$

Unfortunately, not all continuous univariate distributions have a closed form quantile function, F^{-1} . In these cases, $\frac{\partial z}{\partial \theta}$ does not exist, prohibiting gradient methods.

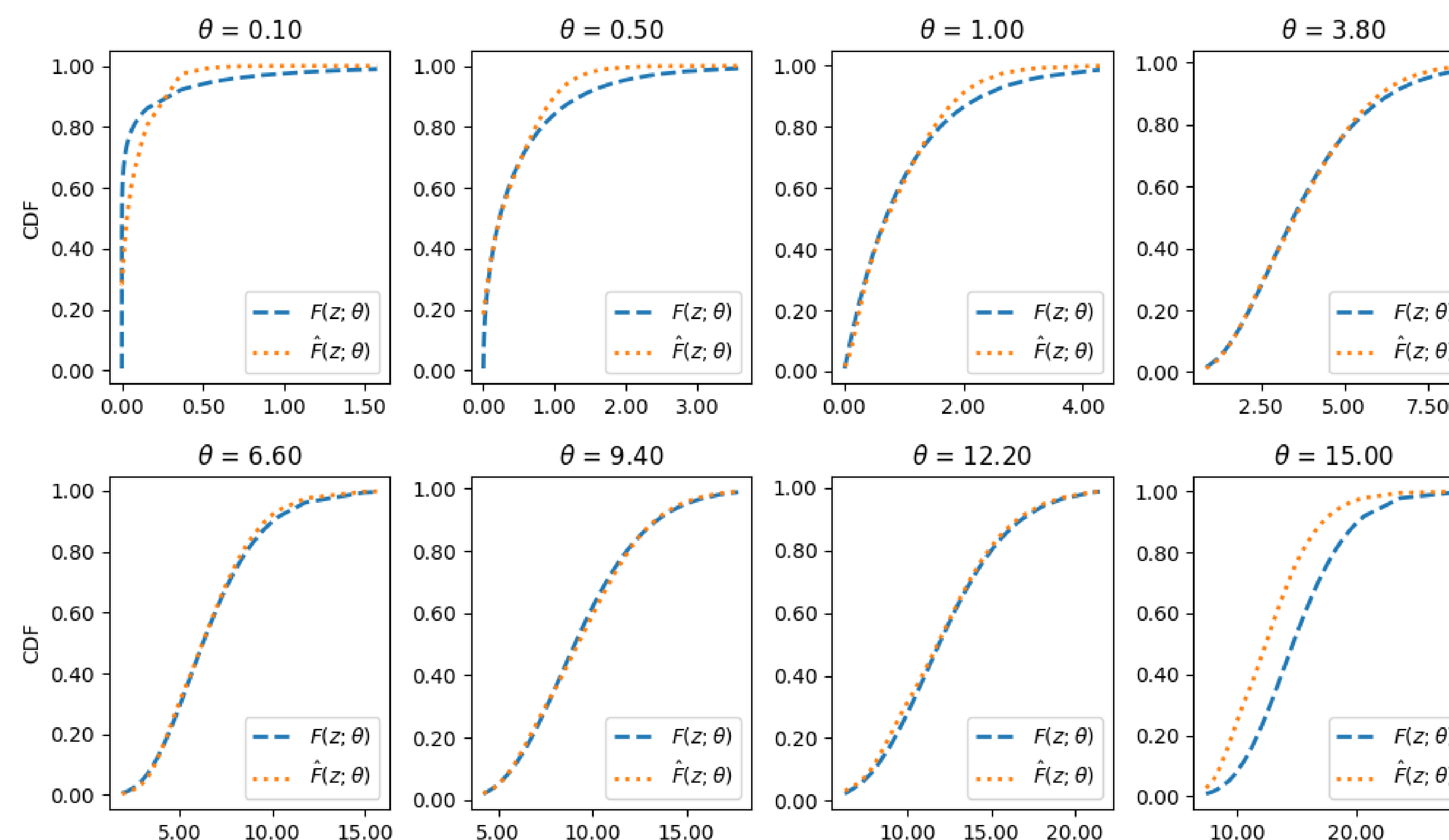
OUR CONTRIBUTIONS

We leverage invertible neural network (INN) blocks [1] to approximate continuous univariate CDF's, allowing for inverse CDF sampling while ensuring the existence of $\frac{\partial z}{\partial \theta}$. Because the INN blocks are analytically invertible, our approximation maintains *monotonicity* and *one-to-oneness* thus ensuring: $z = F^{-1}(F(z; \theta); \theta)$.

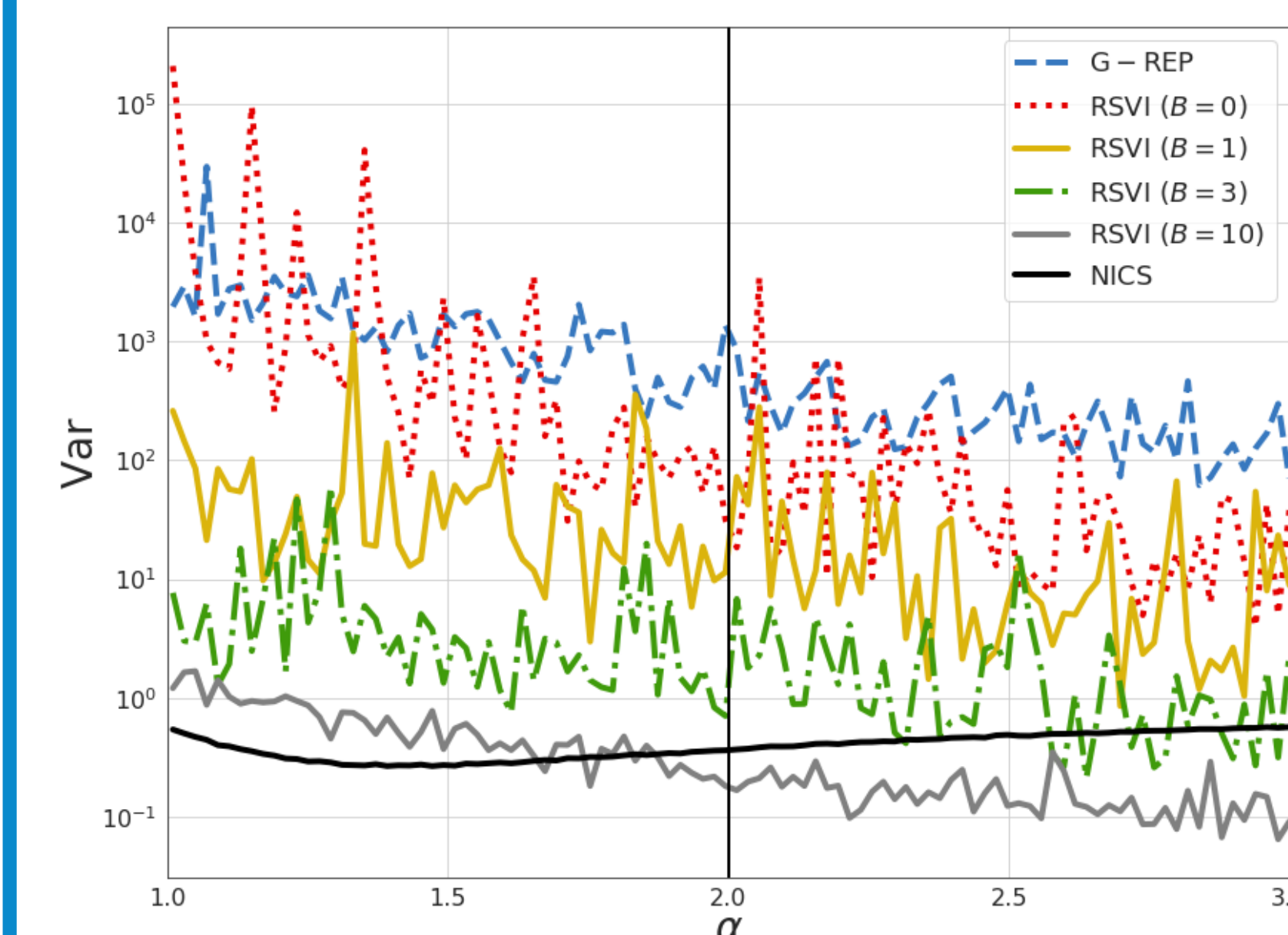
INVERTIBLE NEURAL NETWORK



CDF APPROXIMATION FOR GAMMA DISTRIBUTION



GRADIENT REPARAMETERIZATION



RESULTS

	Perplexity Scores	
	Softmax	NGS
NVLDA	1099.96	1103.77
ProDLDA	1161.69	1099.94

REFERENCES & CODE

[1] Laurent Dinh, Jascha Sohl-Dickstein, and Samy Bengio. Density estimation using real NVP. *CoRR*, abs/1605.08803, 2016.

Source code available at:
github.com/astirn/neural-inverse-cdf-sampling