§3.1 二维随机变量与分布函数 §3.2 边缘分布及独立性 §3.3 函数的分布

一 选择填空题

<table>
<tr><td>1</td><td>2</td><td>3</td><td colspan="3">4</td><td colspan="2">5</td></tr>
<tr><td>B</td><td>0.5</td><td>5/18</td><td>5/9</td><td>1/4</td><td>5/8</td><td>0.4</td><td>0.1</td></tr>
<tr><td colspan="2">6</td><td>7</td><td colspan="3">8</td><td colspan="2">9</td></tr>
<tr><td>3/4</td><td>7/16</td><td>1</td><td colspan="2">$(\pi-2)/4\pi$</td><td>1/2</td><td colspan="2">$1-11e^{-10}$</td></tr>
</table>

二 计算题

1. $P\{X_3=0\}=2p(1-p)$ $P\{X_3=1\}=1-2p(1-p)=2p^2-2p+1$

$P\{X_1=0,X_3=0\}=P\{X_1=0\}P\{X_2=1\}=p(1-p)$

$p(1-p)=(1-p)2p(1-p)$ $p=0.5$

2.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">V</td><td></td></tr>
<tr><td>1</td><td>2</td><td></td></tr>
<tr><td rowspan="2">U</td><td>1</td><td>0.36</td><td>0</td><td>0.36</td></tr>
<tr><td>2</td><td>0.48</td><td>0.16</td><td>0.64</td></tr>
<tr><td colspan="2"></td><td>0.84</td><td>0.16</td><td>1</td></tr>
</table>

3.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Y</td><td rowspan="2"></td></tr>
<tr><td>3</td><td>4</td><td>5</td></tr>
<tr><td rowspan="3">X</td><td>3</td><td>0</td><td>1/6</td><td>1/12</td><td>1/4</td></tr>
<tr><td>4</td><td>1/6</td><td>1/6</td><td>1/6</td><td>1/2</td></tr>
<tr><td>5</td><td>1/12</td><td>1/6</td><td>0</td><td>1/4</td></tr>
<tr><td colspan="2"></td><td>1/4</td><td>1/2</td><td>1/4</td><td>1</td></tr>
</table>

$X+Y\sim\begin{pmatrix}7 & 8 & 9\\ 1/3 & 1/3 & 1/3\end{pmatrix}$ $\min(X,Y)\sim\begin{pmatrix}3 & 4\\ 1/2 & 1/2\end{pmatrix}$

4. $P\{X\ge 1\}=\int_1^2 x/2\,\mathrm{d}x=\frac{3}{4}$

$P\{\min(X,Y)<1\}=1-P\{\min(X,Y)\ge 1\}=1-(P\{X\ge 1\})^2=7/16$

$0.75=P\{A\cup B\}=1-(P\{X<a\})^2$

$0.5=P\{X<a\}=\int_0^a x/2\,\mathrm{d}x=a^2/4$ $a=\sqrt{2}$

$f_Z(z)=\int_{-\infty}^{+\infty} f(x,z-x)\mathrm{d}x=\frac{1}{4}\int_0^z x(z-x)\mathrm{d}x=z^3/24$

5. $f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,\mathrm{d}y=\begin{cases}\int_0^{2x}\mathrm{d}y=2x\,, & 0\le x\le 1\\ 0, & 其他\end{cases}$

$$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,\mathrm{d}x=\begin{cases}\int_{y/2}^{1}dx=1-\frac{y}{2}, & 0\le y\le 2\\ 0, & 其他\end{cases}$$

$$P\{X\le 0.5\}=\int_0^{0.5}2x\mathrm{d}x=\frac{1}{4}\qquad P\{X\le 0.5,Y\le 0.5\}=\int_0^{0.5}\mathrm{d}y\int_{y/2}^{0.5}\mathrm{d}x=\frac{3}{16}$$

$$P\{Y\le 0.5|X\le 0.5\}=\frac{P\{X\le 0.5,Y\le 0.5\}}{P\{X\le 0.5\}}=\frac{3}{4}$$

6. $f(x,y)=f_X(x)f_Y(y)=\begin{cases}0.5e^{-0.5y}, & 0\le x\le 1,y>0\\ 0, & 其他\end{cases}$

$$P\{Y\le X^2\}=\int_0^1\mathrm{d}x\int_0^{x^2}0.5e^{-0.5y}\mathrm{d}y=\int_0^1\left(1-e^{-\frac{x^2}{2}}\right)\mathrm{d}x$$

$$=1-\sqrt{2\pi}\int_0^1\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\mathrm{d}x=1-0.3413\sqrt{2\pi}$$

7. $P\{X+Y=k\}=\sum_{i=0}^{k}P\{X=i\}P\{Y=k-i\}=\sum_{i=0}^{k}C_{n_1}^{i}p^i(1-p)^{n_1-i}\,C_{n_2}^{k-i}p^{k-i}(1-p)^{n_2-k+i}$

$$=\sum_{i=0}^{k}C_{n_1}^{i}\,C_{n_2}^{k-i}p^k(1-p)^{n_1+n_2-k}=C_{n_1+n_2}^{k}p^k(1-p)^{n_1+n_2-k}$$

$$P\{X+Y=k\}=\sum_{i=0}^{k}P\{X=i\}P\{Y=k-i\}=\sum_{i=0}^{k}\frac{{\lambda_1}^i}{i!}e^{-\lambda_1}\frac{{\lambda_2}^{k-i}}{(k-i)!}e^{-\lambda_2}$$

$$=\frac{e^{-(\lambda_1+\lambda_2)}}{k!}\sum_{i=0}^{k}\frac{k!}{i!\,(k-i)!}{\lambda_1}^i{\lambda_2}^{k-i}=\frac{(\lambda_1+\lambda_2)^k}{k!}e^{-(\lambda_1+\lambda_2)}$$

$$f_{X+Y}(t)=\int_{-\infty}^{+\infty}\varphi(x)\varphi(t-x)\mathrm{d}x=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\exp\left\{-\frac{x^2}{2}-\frac{(t-x)^2}{2}\right\}\mathrm{d}x$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\exp\left\{-\left(x-\frac{t}{2}\right)^2-\frac{t^2}{4}\right\}\mathrm{d}x=\frac{1}{2\pi}\exp\left(-\frac{t^2}{4}\right)\int_{-\infty}^{+\infty}\exp\left\{-\left(x-\frac{t}{2}\right)^2\right\}\mathrm{d}x$$

$$=\frac{1}{2\pi}\exp\left(-\frac{t^2}{4}\right)\int_{-\infty}^{+\infty}e^{-y^2}\mathrm{d}y=\frac{1}{\sqrt{2\pi}\sqrt{2}}\exp\left(-\frac{t^2}{2\left(\sqrt{2}\right)^2}\right)$$

8. $f_{X+Y}(t)=\int_{-\infty}^{+\infty}f(x,t-x)\,\mathrm{d}x=\lambda^2\int_0^t e^t\mathrm{d}x=\begin{cases}\lambda^2te^{-\lambda t}, & t>0\\ 0, & t\le 0\end{cases}$

$E(X_1+X_2)=2E(X_1)=2/\lambda$

$$F_2(t)=1-\left(1-F_X(t)\right)^2=\begin{cases}1-e^{-2\lambda t}, & t>0\\ 0, & t\le 0\end{cases}\qquad f_2(t)=\begin{cases}2\lambda e^{-2\lambda t}, & t>0\\ 0, & t\le 0\end{cases}$$

$E(\mathrm{Min}(X_1,X_2))=1/2\lambda$

$$F_3(t)=\left(F_X(t)\right)^2=\begin{cases}\left(1-e^{-\lambda t}\right)^2, & t>0\\ 0, & t\le 0\end{cases}\qquad f_3(t)=\begin{cases}2\lambda\left(e^{-\lambda t}-e^{-2\lambda t}\right), & t>0\\ 0, & t\le 0\end{cases}$$

$$E(\mathrm{Max}(X_1,X_2))=2\int_0^{+\infty}\lambda t e^{-\lambda t}\mathrm{d}x-\int_0^{+\infty}2t\lambda e^{-2\lambda t}\mathrm{d}x=2/\lambda-1/2\lambda=3/2\lambda$$

9. $f_1(t)=\begin{cases}t, & 0\le t<1\\ 2-t, & 1\le t<2\\ 0, & \text{其他}\end{cases}\qquad E(X_1+X_2)=2E(X_1)=1$

$f_2(t)=\begin{cases}2t, & 0\le t\le 1\\ 0, & \text{其他}\end{cases}\qquad E(\mathrm{Max}(X_1,X_2))=2/3$

$f_3(t)=\begin{cases}2-2t, & 0\le t\le 1\\ 0, & \text{其他}\end{cases}\qquad E(\mathrm{Min}(X_1,X_2))=1/3$

$f_4(t)=\begin{cases}2-2t, & 0\le t\le 1\\ 0, & \text{其他}\end{cases}\qquad E(|X_1-X_2|)=1/3$

§3.5 多维随机变量函数的数字特征及性质 §3.6 协方差，相关系数和矩

一 选择填空题

<table>
<tr><td>1</td><td colspan="3">2</td><td>3</td><td colspan="2">4</td></tr>
<tr><td>D</td><td colspan="3">A</td><td>1234</td><td>$F^2(x)$</td><td>$2F(x)-F^2(x)$</td></tr>
<tr><td>5</td><td colspan="3">6</td><td>7</td><td colspan="2">8</td></tr>
<tr><td>17</td><td>2</td><td>2</td><td>1/2</td><td>1</td><td colspan="2">−1</td></tr>
</table>

二 计算题

1. $P\{X=1\}=P(A)$ $P\{X=0\}=P(\bar{A})$ $P\{Y=1\}=P(B)$ $P\{Y=0\}=P(\bar{B})$

$E(X)=P\{X=1\}=P(A)$ $E(Y)=P\{Y=1\}=P(B)$ $E(XY)=P\{X=1,Y=1\}=P(AB)$

$E(XY)=E(X)E(Y)\Leftrightarrow P(AB)=P(A)P(B)$

2.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Y</td><td rowspan="2"></td></tr>
<tr><td>**3**</td><td>**4**</td><td>**5**</td></tr>
<tr><td rowspan="3">X</td><td>**1**</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.6</td></tr>
<tr><td>**2**</td><td>0</td><td>0.1</td><td>0.2</td><td>0.3</td></tr>
<tr><td>**3**</td><td>0</td><td>0</td><td>0.1</td><td>0.1</td></tr>
<tr><td colspan="2"></td><td>0.1</td><td>0.3</td><td>0.6</td><td>1</td></tr>
</table>

$E(X)=1.5 \quad E(Y)=4.5 \quad E(XY)=6.9$ ， $\mathrm{Cov}(X,Y)=0.15$

3.

		Y			
		0	**1**	**2**	
X	**0**	0	0.2	0.1	0.3
	1	0.2	0.4	0	0.6
	2	0.1	0	0	0.1
		0.3	0.6	0.1	1

		Y			
		0	**1**	**2**	
X	**0**	1/25	4/25	4/25	9/25
	1	4/25	8/25	0	12/25
	2	4/25	0	0	4/25
		9/25	12/25	4/25	1

$E(X)=E(Y)=0.8,\quad E(X^2)=E(Y^2)=1$

$D(X)=D(Y)=0.36$ ， $E(XY)=0.4$ ， $\rho_{XY}=-2/3$

$$P\{X=1|X+Y=2\}=\frac{P\{X=1,Y=1\}}{P\{X+Y=2\}}=\frac{1}{2}$$

4. $$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)\,\mathrm{d}y=\begin{cases}\int_0^x 12y^2\mathrm{d}y=4x^3, & 0\le x\le 1\\ 0, & 其他\end{cases}$$

$$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)\,dx=\begin{cases}\int_y^1 12y^2dx=12y^2(1-y), & 0\le y\le 1\\ 0, & 其他\end{cases}$$

$$E(X)=\int_0^1 4x^4\mathrm{d}x=\frac{4}{5} \qquad E(Y)=\int_0^1 12y^3(1-y)\mathrm{d}y=\frac{3}{5}$$

$$E(XY)=\int_0^1\mathrm{d}x\int_0^x 12xy^3\mathrm{d}y=\frac{1}{2} \qquad \mathrm{Cov}(X,Y)=E(XY)-E(X)E(Y)=\frac{1}{50}$$

5.（未删） $$F_X(x)=\begin{cases}0, & x<1\\ (x-1)/2, & 1\le x\le 3\\ 1, & x>3\end{cases} \qquad f_X(x)=\begin{cases}1/2, & 1\le x\le 3\\ 0, & 其他\end{cases}$$

$$P\{XY\le 3\}=\int_1^3\mathrm{d}x\int_1^{3/x}\frac{1}{4}\mathrm{d}y=\frac{3\ln 3-2}{4}$$

$$F_{Z_1}(z)=\left(F_X(z)\right)^2=\left(\frac{z-1}{2}\right)^2, \qquad f_{Z_1}(z)=\begin{cases}\frac{z-1}{2}, & 1\le z\le 3\\ 0, & 其他\end{cases}$$

$$E(Z_1)=\int_1^3\frac{z(z-1)}{2}\mathrm{d}z=\frac{7}{3}$$

$z<0,\quad F_{Z_2}(z)=0,\quad z>2,\quad F_{Z_2}(z)=1,$

$0\le z\le 2,\qquad F_{Z_2}(z)=P\{Z_2\le z\}=P\{|X-Y|\le z\}=1-\frac{(2-z)^2}{4}$

$$f_{Z_2}(z)=\begin{cases}1-\dfrac{z}{2}, & 0\le z\le 2\\ 0, & \text{其他}\end{cases}\qquad E(Z_2)=\int_0^2 z(1-\frac{z}{2})\mathrm{d}z=\frac{2}{3}$$

6. $P\{X+Y\le 2\}=\iint_{x+y<2} f(x,y)\mathrm{d}x\mathrm{d}y=\dfrac{3}{8}$

$$F_Z(x)=f_X(x)\ f_Y(x)=\begin{cases}0, & x<1\\ x(x-1), & 1\le x\le 1.5\\ x/2\,, & 1.5<x\le 2\\ 1, & x>2\end{cases}\qquad f_Z(x)=\begin{cases}2x-1 & 1\le x\le 1.5\\ 1/2 & 1.5<x\le 2\\ 0 & \text{其他}\end{cases}$$

$p(a)=P\{\min(X,Y)\le a\le \max(X,Y)\}=1-P\{\min(X,Y)>a\}-P\{\max(X,Y)<a\}$

$=1-P\{X>a\}P\{Y>a\}-P\{X<a\}P\{Y<a\}$

$a<0\ or\ a\ge 2$，$p(a)=0$

$0\le a\le 1$，$p(a)=a/2$

$1<a\le 1.5$，$p(a)\ =1-(2-a)(1.5-a)\ -a(a-1)\ =-2\left(a-\frac{9}{8}\right)^2+\frac{17}{32}$

$1.5<a<2$，$p(a)=1-a/2$

7. $P(A_1)=0.25,\quad P(A_2)=0.5,\quad P(A_3)=0.25$

$P(B|A_1)=1,\quad P(B|A_3)=0\qquad P(B|A_2)=P\{\sqrt{X^2+Y^2}<1\}=\pi/4$

$P(B)=\dfrac{\pi+2}{8}$