§2.1 一维随机变量与分布函数 §2.2 一维离散型随机变量

一 选择填空题

1	2	3	4	5		6
D	9/64	0.5	3/4	1	1/4	0.68

二 计算题

1. $P\{X=k\}=C_3^k 0.4^k 0.6^{3-k}$，$k=0,1,2,3$

$P\{X=0\}=0.1$　$P\{X=1\}=0.6$　$P\{X=2\}=0.3$

2. $P\{X=k\}=0.4\times 0.6^{k-1}$,　$k=1,2,\cdots$

$P\{X=1\}=0.4$　$P\{X=2\}=0.3$　$P\{X=3\}=0.2$　$P\{X=4\}=0.1$

3. $P\{X=k\}=C_{k-1}^3 0.4^4 0.6^{k-4}$，$k=4,5,\cdots$

4. $p=0.2\times 0.5=0.1$　$\lambda=np=10$

$P\{X=k\}=\dfrac{10^k e^{-10}}{k!},k=0,1,\cdots$

$P\{X\geq 2\}=1-11e^{-10}$

5. $P\{X_1=k\}=C_{20}^k 0.6^k 0.4^{20-k}$，$k=0,1,\cdots,20$

$P\{X_1\geq 1\}=1-P\{X=0\}=1-0.4^{20}$

$P\{X_1=k_1,X_2=k_2,X_3=k_3\}=\dfrac{20!}{k_1!\,k_2!\,k_3!}0.6^{k_1}0.3^{k_2}0.1^{k_3}$

6. $P\{X=k\}=pq^{k-1}$,　$k=1,2,\cdots$　　$(q=1-p)$

$P\{X\text{为偶数}\}=p(q+q^3+\cdots)=p\dfrac{q}{1-q^2}=\dfrac{1-p}{2-p}$

$(\Rightarrow)$若X服从几何分布，则：

$P\{X>n\}=p(q^n+q^{n+1}+\cdots\cdots)=p\dfrac{q^n}{1-q}=q^n$

$\therefore P\{X>n+m|X>m\}=\dfrac{P\{X>n+m\}}{P\{X>m\}}=\dfrac{q^{n+m}}{q^m}=q^n=P\{X>n\}$

$(\Leftarrow)$若$\forall n,m\in N$，$P\{X>n+m|X>m\}=P\{X>n\}$, 即$P\{X>n+m\}=P\{X>m\}P\{X>n\}$

取$n,m=0$，　$\therefore P\{X>0\}=1$

设$P\{X>1\}=q$,　则$P\{X>n+1\}=P\{X>n\}P\{X>1\}=qP\{X>n\}$

$\therefore P\{X>n\}$是首项为q，公比为q的几何数列，$P\{X>n\}=q^n$

$\therefore P\{X=k\}=P\{X>k-1\}-P\{X>k\}=q^{k-1}-q^k=pq^{k-1}$

§2.3 一维连续型随机变量

一 选择填空题

1	2	3	4	5	6	7		
B	C	A	A	4	0.2	1	0.5	5/8

8		9		10	
175	196.8	0.6826	0.9544	0.37	0.37

二 计算题

1. $1=\int_0^1 ax\mathrm{d}x+\int_1^2(2-x)\mathrm{d}x=(a+1)/2$， $a=1$

$x<0,\ F(x)=0,\qquad 0\le x<1,\ F(x)=x^2/2,$

$1\le x<2,\ F(x)=2x-x^2/2-1,\qquad x\ge 2,\quad F(x)=1$

$P\{0.5<X<1.5\}=F(1.5)-F(0.5)=0.75$

2. $1=\lim\limits_{x\to+\infty}\left(a+be^{-\frac{x^2}{2}}\right)=a,\quad 0=\lim\limits_{x\to 0}\left(a+be^{-\frac{x^2}{2}}\right)=a+b,\quad a=1,b=-1$

$f(x)=\begin{cases}xe^{-\frac{x^2}{2}}, & x\ge 0\\ 0, & x<0\end{cases}\qquad P\{0<X<\sqrt{2}\}=1-e^{-1}\approx 0.63$

3. $P\{X<1500\}=\int_{1000}^{1500}1000x^{-2}\mathrm{d}x=\dfrac{1}{3}$

$P\{Y=k\}=C_5^k(1/3)^k(2/3)^{5-k}$， $k=0,1,2,3,4,5$

$P\{Y\ge 2\}=1-(2/3)^5-5/3\times(2/3)^4=131/243$

4. $P\{X=k\}=(0.2t)^k e^{-0.2t}/k!$， $k=0,1,\cdots$

$P\{X\ge 1\}=1-P\{X=0\}=1-e^{-0.2\times 6}=1-e^{-1.2}$

$t<0,\ F(t)=0,\qquad t\ge 0,\ F(t)=1-P\{T\ge t\}=1-e^{-0.2t}$

$f(t)=\begin{cases}0.2e^{-0.2t}, & t\ge 0\\ 0, & t<0\end{cases}$

§2.4 一维随机变量函数的分布

1. $x>0,\quad F(x)=P\{X\le x\}=1-e^{-x}$

$P\{X\le a+1|X>a\}=\dfrac{P\{a<X\le a+1\}}{P\{X>a\}}=\dfrac{F(a+1)-F(a)}{1-F(a)}=1-e^{-1}$

$y\le 0,\quad F_Y(y)=0,\qquad y\ge 1,\quad F_Y(y)=1,$

$0<y<1,\quad F_Y(y)=P\{Y\le y\}=P\{1-e^{-X}\le y\}=P\{X\le -\ln(1-y)\}=y$

$f_Y(y)=\begin{cases}1, & 0<y<1\\ 0, & \text{其他}\end{cases}$

$P\{e^{-X}\le 0.8\}=P\{1-e^{-X}\ge 0.2\}=0.8$

2. $y<a,\ F(y)=0,\qquad y>b,\ F(y)=1,$

$a<y<b,\quad F(y)=P\{Y_1\le y\}=P\{a+(b-a)X\le y\}=P\{X\le\dfrac{y-a}{b-a}\}=\dfrac{y-a}{b-a}$

$$f(y)=\begin{cases}\dfrac{1}{b-a}, a<y<b\\ 0, \quad\quad 其他\end{cases}$$

$y\le 0$，$F(y)=0$， $y>0$，$F(y)=P\{Y_2\le y\}=P\{-\ln X\le y\}=P\{X\ge e^{-y}\}=1-e^{-y}$

$$f(y)=\begin{cases}e^{-y}, & y>0\\ 0, & y\le 0\end{cases}$$

3. $F_Y(x)=P\{Y\le x\}=P\{aX+b\le x\}=\Phi\left(\frac{x-b}{a}\right)$

$$f_Y(x)=\varphi\left(\frac{x-b}{a}\right)\frac{1}{a}=\frac{1}{\sqrt{2\pi}a}\exp\left\{-\frac{(x-b)^2}{2a^2}\right\}$$

$F_Y(x)=P\{Y\le x\}=P\{X^2\le x\}$

$x\le 0$，$F_Y(x)=0$

$x>0, F_Y(x)=P\{-\sqrt{x}\le X\le\sqrt{x}\}=\Phi(\sqrt{x})-\Phi(-\sqrt{x})=2\Phi(\sqrt{x})-1$

$$f_Y(x)=2\varphi(\sqrt{x})\frac{1}{2\sqrt{x}}=\begin{cases}\dfrac{1}{\sqrt{2\pi x}}e^{-\frac{x}{2}}, & x>0\\ 0, & x\le 0\end{cases}$$

$F_Y(x)=P\{Y\le x\}=P\{e^X\le x\}$ $x\le 0$，$F_Y(x)=0$

$x>0, F_Y(x)=P\{X\le\ln x\}=\Phi(\ln x)$

$$f_Y(x)=\varphi(\ln x)\frac{1}{x}=\begin{cases}\dfrac{1}{x\sqrt{2\pi}}\exp\left\{-\dfrac{(\ln x)^2}{2}\right\}, & x>0\\ 0, & x\le 0\end{cases}$$

§2.5 一维随机变量的数字特征

一 选择填空题

1	2	3	4	5	6		7
B	B	B	A	C	0.5	5	1
8	9	10	11	12	13		14
1 / 2	√(2 / π)	0.6826	$1/e$	8 / 9	2 / 3		A

二 计算题

1. $P\{X=1\}=20/36$ $P\{X=2\}=15/36$ $P\{X=3\}=1/36$ $E(X)=53/36$

$P\{Y=0\}=1/8$ $P\{Y=1\}=3/8$ $P\{Y=2\}=3/8$ $P\{Y=3\}=1/8$ $E(Y)=1.5$

$P\{X_i=k\}=1/6$，$k=1,2,3,4,5,6$，$i=1,2,3$ $E(X_i)=7/2$ $D(X_i)=35/12$

$E(Z)=21/2$ $D(Z)=35/4$

2. $p=2/3\times 1/2=1/3$

$P\{X=0\}=1/3$ $P\{X=10\}=1/6$ $P\{X=20\}=1/6$ $P\{X=30\}=1/3$ $E(X)=15$

3. $P\{甲\ 2:0\ 胜\}=0.6\times 0.5=0.3$ $P\{甲\ 2:1\ 胜\}=0.6\times 0.5\times 0.6+0.4\times 0.5\times 0.6=0.3$

$0.3+0.3=0.6$

$P\{X=2\}=0.3+0.2=0.5 \quad P\{X=3\}=1-0.5=0.5$

$E(X)=2.5 \qquad E(X^2)=13/2 \qquad D(X)=0.25$

4. $0.8=P\{X\le a\}=\int_{-1}^{0}-0.5\mathrm{d}x+\int_{0}^{a}0.25\mathrm{d}x=0.5+0.25a, \quad a=1.2$

$E(|X|)=\int_{-1}^{0}-0.5x\mathrm{d}x+\int_{0}^{2}0.25x\mathrm{d}x=\frac{3}{4} \qquad E(X^2)=\int_{-1}^{0}0.5x^2\mathrm{d}x+\int_{0}^{2}0.25x^2\mathrm{d}x=\frac{5}{6}$

$D(|X|)=E(X^2)-(E(|X|))^2=13/48$

5. $1=\int_{0}^{1}ax^2(1-x)\mathrm{d}x=\frac{a}{12} \quad a=12 \qquad \mu_1=\int_{0}^{1}12x^3(1-x)\mathrm{d}x=\frac{3}{5}$

$f'(x)=12x(2-3x)=0 \quad \mu_2=2/3$

$0.5=\int_{0}^{t}12x^2(1-x)\mathrm{d}x=4t^3-3t^4$

μ_3是$g(t)=6t^4-8t^3+1$的零点，$g'(t)=24t^2(t-1)<0$，且$g(\mu_1)g(\mu_2)<0$

$\therefore \mu_2<\mu_3<\mu_1$

6. $\pi(\theta)=1$, $\qquad f(x|\theta)=C_n^x\theta^x(1-\theta)^{n-x}$

$$\pi(\theta|x)=\frac{3\theta(1-\theta)^2}{\int_0^1 3\theta(1-\theta)^2d\theta}=12\theta(1-\theta)^2$$

$$E(\theta|x)=\int_{0}^{1}12\theta^2(1-\theta)^2d\theta=\frac{2}{5}$$

7. $x<c$, $P\{X\le x|X>c\}=0$

$x\ge c, \qquad P\{X\le x|X>c\}=\frac{P\{x<X\le x\}}{P\{X>c\}}=\frac{F(x)-F(c)}{1-F(c)}$

$x<c, \qquad \frac{\mathrm{d}}{\mathrm{d}x}P\{X\le x|X>c\}=0$

$x\ge c, \qquad \frac{\mathrm{d}}{\mathrm{d}x}P\{X\le x|X>c\}=\frac{\mathrm{d}}{\mathrm{d}x}\frac{F(x)-F(c)}{1-F(c)}=\frac{f(x)}{1-F(c)}$

$$f(x|X>c)=\begin{cases}\dfrac{f(x)}{1-F(c)}, & x\ge c\\ 0, & x<c\end{cases}$$

$$\int_{c}^{+\infty}f(x|X>c)\mathrm{d}x=\int_{c}^{+\infty}\frac{f(x)}{1-F(c)}\mathrm{d}x=\frac{1-F(c)}{1-F(c)}=1$$

$$f(x)=\begin{cases}\dfrac{1}{\mu}e^{-\frac{x}{\mu}}, & x\ge 0\\ 0, & x<0\end{cases} \qquad F(x)=\begin{cases}1-e^{-\frac{x}{\mu}}, & x\ge 0\\ 0, & x<0\end{cases} \qquad f(x|X>c)=\begin{cases}\dfrac{1}{\mu}e^{-\frac{x-c}{\mu}}, & x\ge c\\ 0, & x<c\end{cases}$$

$$E(X|X>c)=\int_c^{+\infty} xf(x|X>c)\mathrm{d}x=\int_c^{+\infty}\frac{x}{\mu}e^{-\frac{x-c}{\mu}}\mathrm{d}x=\int_c^{+\infty}\frac{x-c}{\mu}e^{-\frac{x-c}{\mu}}\mathrm{d}x+\int_c^{+\infty}\frac{c}{\mu}e^{-\frac{x-c}{\mu}}\mathrm{d}x$$

$$=\mu\int_0^{+\infty} te^{-t}\mathrm{d}t+c\int_0^{+\infty} e^{-t}\mathrm{d}t=\mu+c$$