§6.1 点估计 §6.2 估计量的评选标准

一 选择填空题

1	2	3	4		5			6	7
D	1 3	0	1234	1	μ^2+1	$1/n$	1	$\bar{X}-1$	$10/11$

二 计算题

1. $\frac{3}{4}\times p+\frac{1}{4}\times\frac{1}{3}=\frac{3}{10}$ $\quad \hat{p}=\frac{13}{45}$

2. $E(\hat{\theta}_1)=E\left(\frac{2X_1+4X_3}{3}\right)=\frac{2}{3}E(X_1)+\frac{4}{3}E(X_2)=\frac{2}{3}\times\frac{\theta}{2}+\frac{4}{3}\times\frac{\theta}{2}=\theta$

$$Y=\max\{X_1,X_2,X_3\} \qquad F_Y(x)=\left(F_X(x)\right)^3 \qquad f_Y(x)=\begin{cases}\frac{3x^2}{\theta^3}, & 0\le x\le\theta\\ 0, & \text{其他}\end{cases}$$

$$E(Y)=\int_0^{\theta}\frac{3x^3}{\theta^3}\mathrm{d}x=\frac{3\theta}{4} \qquad E(\hat{\theta}_2)=E\left(\frac{3}{4}Y\right)=\theta$$

$$D(\hat{\theta}_1)=D\left(\frac{2X_1+4X_3}{3}\right)=\frac{4}{9}D(X_1)+\frac{16}{9}D(X_2)=\frac{4}{9}\times\frac{\theta^2}{12}+\frac{16}{9}\times\frac{\theta^2}{12}=\frac{5\theta^2}{27}$$

$$E(Y^2)=\int_0^{\theta}\frac{3x^4}{\theta^3}\mathrm{d}x=\frac{3\theta^2}{5} \qquad D(Y)=\frac{3\theta^2}{80}$$

$$D(\hat{\theta}_2)=D\left(\frac{3}{4}Y\right)=\frac{9}{16}D(Y)=\frac{\theta^2}{15}<D(\hat{\theta}_1)$$

3. $L(\theta)=2\theta^7(1-\theta)^7$ $\qquad \hat{\theta}=0.5$

$$X\sim\begin{pmatrix}-1 & 0 & 1\\ 0.25 & 0.5 & 0.25\end{pmatrix} \qquad E(X)=0 \qquad D(X)=0.5$$

$$\bar{X}=\frac{X_1+X_2+\cdots+X_{50}}{50}\sim N(0,0.01)$$

$$P\left\{\left|\frac{X_1+X_2+\cdots+X_{50}}{50}\right|\le 0.1\right\}=P\left\{\left|\frac{X_1+X_2+\cdots+X_{50}}{5}\right|\le 1\right\}=0.68$$

4. $L(\theta)=\prod_{i=1}^{n}\frac{1}{\theta}e^{-X_i/\theta}=\theta^{-n}e^{-n\bar{X}/\theta}$

$$\ln L(\theta)=-n\ln\theta-n\bar{X}/\theta$$

$$\frac{\mathrm{d}\ln L(\theta)}{\mathrm{d}\theta}=-\frac{n}{\theta}+\frac{n\bar{X}}{\theta^2}=0 \qquad \hat{\theta}_1=\bar{X}$$

$$F(x)=\begin{cases}1-e^{-x/\theta}, & x\geq 0\\ 0, & x<0\end{cases}$$

$$f_{X_{(1)}}(x)=n\left(1-F(x)\right)^{n-1}f(x)=\frac{n}{\theta}e^{-nx/\theta}=\frac{1}{\theta/n}e^{-\frac{x}{\theta/n}},\qquad (x\geq 0)$$

$$f_{X_{(1)}}(x)=\begin{cases}\dfrac{1}{\theta/n}e^{-\frac{x}{\theta/n}}, & x\geq 0\\ 0, & x<0\end{cases}\qquad \therefore X_{(1)}\sim \mathrm{Exp}(\frac{\theta}{n})$$

$$E(\hat{\theta}_1)=E(\bar{X})=E(X)=\int_0^{+\infty}\frac{x}{\theta}e^{-x/\theta}\mathrm{d}x=\theta$$

$$E(\hat{\theta}_2)=nE(X_{(1)})=n\int_0^{+\infty}\frac{x}{\theta/n}e^{-\frac{x}{\theta/n}}\mathrm{d}x=n\frac{\theta}{n}=\theta$$

5. $E(X)=\int_0^{\theta}\frac{2x^2}{\theta^2}\mathrm{d}x=\frac{2\theta}{3}$ $\qquad E(X^2)=\int_0^{\theta}\frac{2x^3}{\theta^2}\mathrm{d}x=\frac{\theta^2}{2}$ $\qquad \hat{\theta}=\frac{3\bar{X}}{2}$

$$E\left(\frac{c}{n}\sum_{i=1}^{n}{X_i}^2\right)=\frac{c}{n}\sum_{i=1}^{n}E({X_i}^2)=\frac{c\theta^2}{2}=\theta^2\qquad c=2$$

$T=\max\{X_1,X_2,X_3\}$ $\qquad F_T(x)=\left(F_X(x)\right)^3$ $\qquad f_Y(x)=\begin{cases}\dfrac{6x^5}{\theta^6}, & 0\leq x\leq\theta\\ 0, & \text{其他}\end{cases}$

$$E\left(\frac{7}{6}T\right)=\frac{7}{6}E(T)=\int_0^{\theta}\frac{6x^6}{\theta^6}\mathrm{d}x=\frac{7}{6}\times\frac{6\theta}{7}=\theta$$

§6.3 区间估计 §6.4 单正态总体均值与方差的区间估计

§7.1 参数假设检验问题概述 §7.2 单正态总体的参数检验

一 选择填空题

1	2		3	4	5	6
(8.2,10.8)	9	(8.608,9.392	A	C	D	A

二 计算题

1. $X_1\sim b(10000,0.2)$ $\quad E(X_1)=n\theta=2000,$ $\quad D(X_1)=n\theta(1-\theta)=1600$

$X_1\sim N(2000,40^2)$ $\qquad P\{X_1\geq 2075\}=P\left\{\frac{X_1-2000}{40}\geq 1.875\right\}=0.031$

$H_0:\theta=0.2\qquad H_1:\theta>0.2$

$P\{X_1\geq 2075|H_0\}=0.031<0.05$

或$(X_1-n\theta_0)/\sqrt{n\theta_0(1-\theta_0)}=(2075-2000)/40=1.875>1.645$

比例显著高于20%

2. $P\{X>65\}=P\left\{\frac{X-50}{10}>1.5\right\}=0.067$ $P\{Y>65\}=P\left\{\frac{Y-47}{10}>1.8\right\}=0.036$

$p=0.5(0.0664+0.0359)=5.1\%$

$H_0:\mu=48$ $H_1:\mu<48$

$t=\sqrt{n}(\bar{x}-\mu_0)/s=-2.25<-1.6896$ A显著偏低（单侧检验）

$H_0:\mu=48$ $H_1:\mu>48$

$t=\sqrt{n}(\bar{x}-\mu_0)/s=2>1.6896$ B显著偏高（单侧检验）

3. $P(115-1.96<\mu<115+1.96)=0.95$

$H_0:\mu=118$ $H_1:\mu<118$ $\sqrt{n}(\bar{x}-\mu_0)/\sigma=-3<-1.65$，有显著差距（单侧检验）

4. $P(18-0.784<\mu<18+0.784)=0.95$

$H_0:\mu=17.6$ $H_1:\mu>17.6$ $\sqrt{n}(\bar{x}-\mu_0)/\sigma=1<1.65$，无显著变化（单侧检验）

5. $P(4.709-0.13<\mu<4.709+0.13)=0.95$

$H_0:\mu=4.585$ $H_1:\mu>4.585$ $\sqrt{n}(\bar{x}-\mu_0)/s=2>1.7531$ 显著提高（单侧检验）

6. $P(69-3.92<\mu<69+3.92)=0.95$

$H_0:\mu=74$ $H_1:\mu\neq 74$ $\left|\sqrt{n}(\bar{x}-\mu_0)/s\right|=2.5>1.96$ 有显著差异（双侧检验）