Natural Deduction Inference Rules for Predicate Logic

Basic Inference Rules

Derived Inference Rules

Subproofs



3 A-introduction

$$\mathbf{4}$$
 $\mathbf{\Lambda}$ -elimination

B Law of Excluded Middle (LEM)

→-introduction





$$\frac{(\alpha \to \beta) \quad \alpha}{\beta}$$

v-introduction

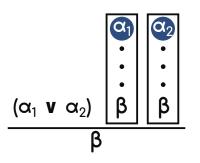
¬-introduction



Proof by Contradiction (PBC)



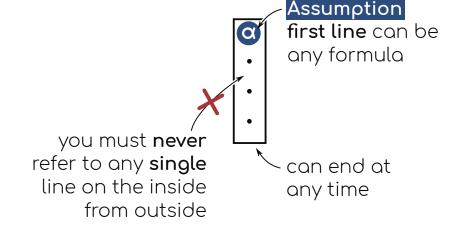


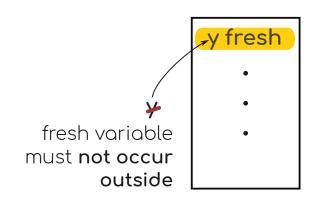


¬-elimination

(⊥-introduction)

Depending on the assignment you may not always be allowed to use all derived rules!





All subproofs must be closed by the end of the proof

Soundness & Completeness

Soundness

"All formulae derived by ND are entailments"

$$\Sigma \vdash_{\text{ND}} \phi \quad \Rightarrow \quad \Sigma \vDash \phi$$

$$\Sigma \not\models \phi \Rightarrow \Sigma \not\vdash_{ND} \phi$$

Basic Inference Rules for Predicate Logic

Variable



y fresh

 $\mathbf{A} \times \mathbf{a}$



∀-elimination

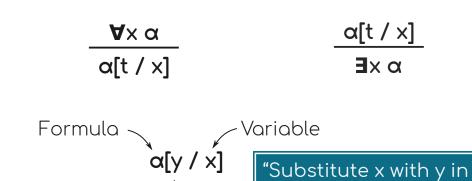


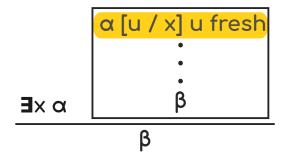
formula α"

3-introduction



3-elimination





Completeness

"All formulae that are entailments can be derived by ND"

$$\Sigma \models \phi \Rightarrow \Sigma \vdash_{ND} \phi$$

$$\Sigma \not\vdash_{\mathsf{ND}} \phi \quad \Rightarrow \quad \Sigma \not\models \phi$$