First-Order Predicate Logic (FOL)

Symbols

1. **Constant symbols** — usually $c, d, c_1, c_2, \dots, d_1, d_2 \dots$
2. **Variables** — usually $x, y, z, \dots x_1, x_2, \dots, y_1, y_2 \dots$
3. **Function symbols** — usually $f, g, h, \dots f_1, f_2, \dots, g_1, g_2, \dots$
4. **Predicate symbols** (relation symbols) — usually $P, Q, \dots, P_1, P_2, \dots, Q_1, Q_2, \dots$
5. **Connectives** — $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$
6. **Quantifiers** — $\forall$ and $\exists$
7. **Punctuation** — '(', ')', and ','

Formulae

1. **Atomic formula**
   Expression of the form $P(t_1, \dots, t_n)$, where P is an n-ary predicate symbol and each t_i is a term ($1 \leq i \leq \mathrm{n}$).

   ⚠ For $n = 2$ you may use infix notation $(t_1 \ P \ t_2)$
2. **Negation**
   If α is a formula, then $(\neg\alpha)$ is a formula.
3. **Binary connective**
   If α, β are formulae, and ❆ is a binary connective, then α ❆ β is a formula.
4. **Quantifiers**
   If α is a formula, and x is a variable, then each of $(\forall x \ \alpha)$ and $(\exists x \ \alpha)$ is a formula.
5. **Closure**
   Nothing else is a formula.

Term

1. **Atomic term**
   Any constant symbol or variable symbol.
2. **Function application**
   $t_1, \dots, t_n$ are terms, f is an n-ary function symbol, then $f(t_1, \dots, t_n)$ is a term.

   ⚠ For $n = 2$ you may use infix notation $(t_1 \ f \ t_2)$
3. **Closure**
   Nothing else is a term.

Semantics

A. **Interpretation $\mathcal{I}$**
   - **Domain/Universe**
     Non-empty set $\mathrm{dom}(\mathcal{I})$
   - **Constants**
     For each constant symbol c, a member $c^{\mathcal{I}} \in \mathrm{dom}(\mathcal{I})$
   - **Functions**
     For each function symbol $f^{(i)}$, an i-ary function $f^{\mathcal{I}}$
   - **Relations**
     For each relation symbol $R^{(i)}$, an i-ary relation $R^{\mathcal{I}}$

B. **Environment E**
   Assigns a value to each variable

C. **Satisfaction of Formulas**
   An iterpretation $\mathcal{I}$ and an environment E **satisfy** a formula α, denoted $\mathcal{I} \vDash_E \alpha$, if $\alpha^{(\mathcal{I}, E)} = \mathtt{T}$, or $\mathcal{I} \vDash \alpha$, if $\alpha^{(\mathcal{I}, E)} = \mathtt{T}$ for all environments