Equality and Peano Axioms in Predicate Logic

Equality Axioms

EQ1 $\forall x\ (x = x)$

"Equality introduction"

EQ2 $\forall x\, \forall y \left(x = y \rightarrow \left(\alpha[x/z] \rightarrow \alpha[y/z]\right)\right)$

"Equality elimination"

EQsymm $\left(\forall x\, \left(\forall y\, \left((x = y) \rightarrow (y = x)\right)\right)\right)$

"Equality is symmetric"

EQtrans(k) $$\frac{(t_1 = t_2) \quad (t_2 = t_3) \quad \cdots \quad (t_k = t_{k+1})}{(t_1 = t_{k+1})}$$

"Transitivity of equality/equality chains"

EQsubs(r) $$\frac{(t_1 = t_2)}{(r[t_1/z] = r[t_2/z])}$$

"Equality substitution"

Peano Axioms

NUMBER LINE

P1 $\left(\forall x\, \left(\neg(s(x) = 0)\right)\right)$

"Zero is not a successor"

P2 $\left(\forall x\, \left(\forall y\, \left((s(x) = s(y)) \rightarrow (x = y)\right)\right)\right)$

"Nothing has two predecessors"

ADDITION

P3 $\left(\forall x\, \left((x + 0) = x\right)\right)$

"Zero is neutral w.r.t. addition"

P4 $\left(\forall x\, \left(\forall y\, \left((x + s(y)) = s((x + y))\right)\right)\right)$

"Adding a successor yields the successor of adding the number"

MULTIPLICATION

P5 $\left(\forall x\, \left((x \times 0) = 0\right)\right)$

"Multiplying by zero yields zero"

P6 $\left(\forall x\, \left(\forall y\, \left((x \times s(y)) = ((x \times y) + x)\right)\right)\right)$

"Multiplication by a successor"

P7 $\left(\varphi[0/v] \rightarrow \left(\left(\forall v\, (\varphi \rightarrow \varphi[s(v)/v])\right) \rightarrow (\forall v\ \varphi)\right)\right)$

"Induction"

https://github.com/astoeckel/cs245-f18/