

Natural Deduction Inference Rules for Propositional Logic

Basic Inference Rules

0 Reflexivity

$$\frac{\alpha}{\alpha}$$

2 \wedge -introduction

$$\frac{\alpha \quad \beta}{\alpha \wedge \beta}$$

4 \rightarrow -introduction

$$\frac{\boxed{\begin{array}{c} \alpha \\ \vdots \\ \beta \end{array}}}{\alpha \rightarrow \beta}$$

5 \vee -introduction

$$\frac{\alpha}{\alpha \vee \beta} \quad \frac{\alpha}{\beta \vee \alpha}$$

7 \neg -introduction

$$\frac{\boxed{\begin{array}{c} \alpha \\ \vdots \\ \perp \end{array}}}{\neg \alpha}$$

1 $\neg\neg$ -elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

3 \wedge -elimination

$$\frac{\alpha \wedge \beta}{\alpha} \quad \frac{\alpha \wedge \beta}{\beta}$$

5 \rightarrow -elimination

$$\frac{\alpha \rightarrow \beta \quad \alpha}{\beta}$$

6 \vee -elimination

$$\frac{\alpha_1 \vee \alpha_2 \quad \boxed{\begin{array}{c} \alpha_1 \\ \vdots \\ \beta \end{array}} \quad \boxed{\begin{array}{c} \alpha_2 \\ \vdots \\ \beta \end{array}}}{\beta}$$

8 \neg -elimination
(\perp -introduction)

$$\frac{\alpha \quad \neg\alpha}{\perp}$$

Derived Inference Rules

A \perp -elimination

$$\frac{\perp}{\alpha}$$

B Modus Tollens (MT)

$$\frac{\alpha \rightarrow \beta \quad \neg\beta}{\neg\alpha}$$

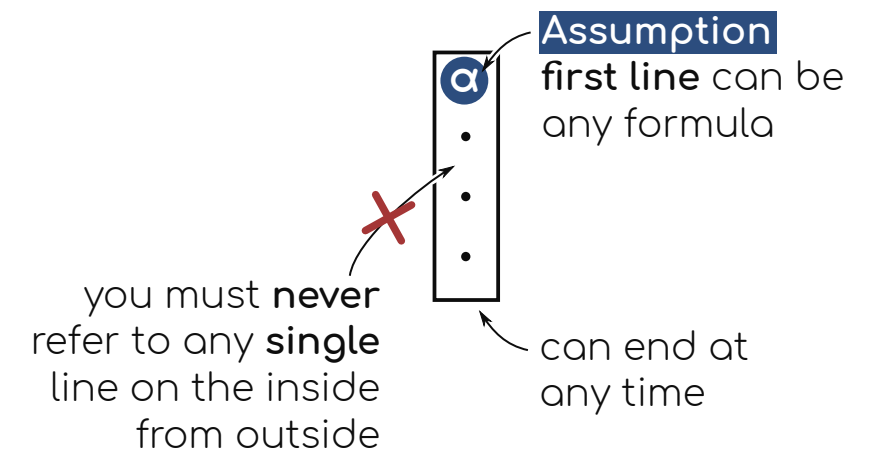
C Law of Excluded Middle (LEM)

$$\frac{\emptyset}{\alpha \vee \neg\alpha}$$

D Double-Negation Introduction

$$\frac{\alpha}{\neg\neg\alpha}$$

Subproofs



! All subproofs must be closed by the end of the proof

Soundness & Completeness

Soundness

"All formulae derived by ND are entailments"

$$\Sigma \vdash_{ND} \varphi \Rightarrow \Sigma \models \varphi$$

$$\Sigma \not\vdash_{ND} \varphi \Leftarrow \Sigma \not\models \varphi$$

Completeness

"All formulae that are entailments can be derived by ND"

$$\Sigma \models \varphi \Rightarrow \Sigma \vdash_{ND} \varphi$$

$$\Sigma \not\models \varphi \Leftarrow \Sigma \not\vdash_{ND} \varphi$$