Equality and Peano Axioms in Predicate Logic

Equality Axioms

EQ1 $\forall x \ (x = x)$

"Equality introduction"

EQ2 $\forall x \, \forall y \left(x = y \to \left(\alpha[x/z] \to \alpha[y/z] \right) \right)$

"Equality elimination"

EQsymm $\Big(\forall x \left(\forall y \left((x = y) \to (y = x) \right) \right) \Big)$

"Equality is symmetric"

EQtrans(k) $\dfrac{(t_1 = t_2) \quad (t_2 = t_3) \quad \cdots \quad (t_k = t_{k+1})}{(t_1 = t_{k+1})}$

"Transitivity of equality/equality chains"

EQsubs(r) $\dfrac{(t_1 = t_2)}{(r[t_1/z] = r[t_2/z])}$

"Equality substitution"

Peano Axioms

NUMBER LINE

P1 $\Big(\forall x \left(\neg (s(x) = 0) \right) \Big)$

"Zero is not a successor"

P2 $\Big(\forall x \left(\forall y \left((s(x) = s(y)) \to (x = y) \right) \right) \Big)$

"Nothing has two predecessors"

ADDITION

P3 $\Big(\forall x \left((x + 0) = x \right) \Big)$

"Zero is neutral w.r.t. addition"

P4 $\Big(\forall x \left(\forall y \left((x + s(y)) = s((x + y)) \right) \right) \Big)$

"Adding a successor yields the successor of adding the number"

MULTIPLICATION

P5 $\Big(\forall x \left((x \times 0) = 0 \right) \Big)$

"Multiplying by zero yields zero"

P6 $\Big(\forall x \left(\forall y \left((x \times s(y)) = ((x \times y) + x) \right) \right) \Big)$

"Multiplication by a successor"

P7 $\Big(\varphi[0/v] \to \left(\left(\forall v \left(\varphi \to \varphi[s(v)/v] \right) \right) \to (\forall v \ \varphi) \right) \Big)$

"Induction"