CSC4140 Assignment I

Computer Graphics

February 14, 2022

Learn to use VirtualBox and Mathematic Review

This assignment is 5% of the total mark.

Strict Due Date: 11:59PM, Feb 14^{th}, 2022

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This assignment represents my own work in accordance with University regulations.

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1 Basic vector operations

1.1 Vector definition

The definition of vectors v and w, represented in a form normalized to Cartesian coordinates, are shown in figure 1.

```
normalized v is
0.333333
     0.5
0.666667
normalized w is
   0
0.25
 0.5
```

Figure 1: Definitions of v, w

1.2 Vector add

The result after adding v and w is shown in figure 2

```
normalized v+w is
0.333333
    0.75
 1.16667
```

Figure 2: v+w

1.3 Vector inner product

The result of the inner product of v and w is shown in figure 3

```
normalized v inner product w is
0.458333
```

Figure 3: v dot product w

1.4 Vector cross product

The result of v cross w and w cross v are shown in figure 4.

```
normalized v cross w is
0.0833333
-0.166667
0.0833333
```

(a) v cross product w

```
normalized w cross v is
-0.0833333
  0.166667
-0.0833333
```

(b) w cross product v

Figure 4: Cross products

2 Basic matrix operations

2.1 Matrix definition

The result of the definitions of matrices i and j are shown in figure 5.

```
i is
  1   2   3   4
  5   6   7   8
  9  10  11  12
 13  14  15  16
```

(a) Definition of matrix i

```
j is
  4   3   2   1
  8   7   6   5
 12  11  10   9
 16  15  14  13
```

(b) Definition of matrix j

Figure 5: Definition of matrices i, j

2.2 Matrix add

The result of i+j is shown in figure 6.

```
i+j is
  5   5   5   5
 13  13  13  13
 21  21  21  21
 29  29  29  29
```

Figure 6: i+j

2.3 Matrix multiplication with matrix

The result of i*j is shown in figure 7.

```
i*j is
120 110 100  90
280 254 228 202
440 398 356 314
600 542 484 426
```

Figure 7: i*j

2.4 Matrix multiplication with vector

The result of i*v, where v is the vector defined in 1.1, is shown in figure 8.

```
i*v is
 22
 52
 82
112
```

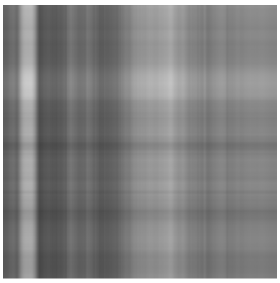
Figure 8: i*v

3 SVD of "Lenna"

3.1 SVD with different number of singular values

The results of "Lenna" converted to greyscale image, preserving 1 singular value, preserving 10 singular values, preserving 50 singular values, are shown in figure 9 (S denotes the number of singular values preserved. By default, larger singular values are kept in priority).

(a) Greyscale image of Lenna



(b) Result when $S = 1$

(c) Result when $S = 10$

(d) Result when $S = 50$

Figure 9: Results under different number of singular values (S)

Note that as the number of singular values preserved increases, the resulting image is having more and more characteristics of the original greyscale image and as $S = 50$, the clarity (the ability to recognize Lenna) of the resulting image is comparable to that of the original image.

This can be explained by the outer products' representation of the original image shown in equation 1, where u_i, v_i represents column vectors of matrices U, V, respectively. σ_i represents the ith singular values and r denotes the rank of matrix A. Each outer product produces an eigenimage (say, a specific geometric feature of the original image) with a corresponding singular value as luminance (the degree in which the feature is presented). Hence, as S increases, the number of eigenimages preserved for A increases, equipping the resulting image with characteristics that are more closed to the original image (which has all singular values preserved).

We also notice that even though there are 512 singular values in total, preserving only 50 singular values gives an image comparable in clarity (of recognizing Lenna) to the original image. The reason is that larger singular values, together with their corresponding outer products, contribute more significantly to the overall image quality (clarity, etc) than smaller singular values. In other words, 50 large singular values contribute mostly to the image clarity, making the remaining smaller 462 singular values redundant if the minor loss of clarity is tolerable.

$$A = U\Sigma V^T = \sum_{i=1}^{r} \sigma_i u_i {v_i}^T \tag{1}$$

4 Basic tranformation

4.1 Rotation with a given point

The results of the point, before and after rotation, are shown in figure 4.1.

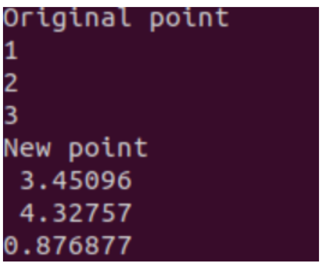


Figure 10: Points before and after the rotation