ECE285 Assignment 1: Neural Network in NumPy

Use this notebook to build your neural network by implementing the following functions in the python files under `ece285/algorithms` directory:

1. `linear.py`
2. `relu.py`
3. `softmax.py`
4. `loss_func.py`

You will be testing your 2 layer neural network implementation on a toy dataset.

TO SUBMIT: PDF of this notebook with all the required outputs and answers.

In [ ]:
```python
# Setup
import matplotlib.pyplot as plt
import numpy as np

from ece285.layers.sequential import Sequential
from ece285.layers.linear import Linear
from ece285.layers.relu import ReLU
from ece285.layers.softmax import Softmax
from ece285.layers.loss_func import CrossEntropyLoss
from ece285.utils.optimizer import SGD

%matplotlib inline
plt.rcParams["figure.figsize"] = (10.0, 8.0)  # set default size of plots

# For auto-reloading external modules
# See http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

We will use the class Sequential as implemented in the file assignment2/layers/sequential.py to build a layer by layer model of our neural network. Below we initialize the toy model and the toy random data that you will use to develop your implementation.

In [ ]:
```python
# Create a small net and some toy data to check your implementations.
# Note that we set the random seed for repeatable experiments.

input_size = 4
hidden_size = 10
num_classes = 3  # Output
num_inputs = 10  # N


def init_toy_model():
    np.random.seed(0)
    l1 = Linear(input_size, hidden_size)
    l2 = Linear(hidden_size, num_classes)

    r1 = ReLU()
    softmax = Softmax()
    return Sequential([l1, r1, l2, softmax])


def init_toy_data():
    np.random.seed(0)
    X = 10 * np.random.randn(num_inputs, input_size)
    y = np.random.randint(num_classes, size=num_inputs)
    # y = np.array([0, 1, 2, 2, 1])
    return X, y


net = init_toy_model()
X, y = init_toy_data()
```

Forward Pass: Compute Scores (20%)

Implement the forward functions in Linear, Relu and Softmax layers and get the output by passing our toy data X The output must match the given output scores

In [ ]:
```
scores = net.forward(X)
print("Your scores:")
print(scores)
print()
print("correct scores:")
correct_scores = np.asarray(
    [
        [0.33333514, 0.33333826, 0.33332661],
        [0.3333351, 0.33333828, 0.33332661],
        [0.3333351, 0.33333828, 0.33332662],
        [0.3333351, 0.33333828, 0.33332662],
        [0.33333509, 0.33333829, 0.33332662],
        [0.33333508, 0.33333829, 0.33332662],
        [0.33333511, 0.33333828, 0.33332661],
        [0.33333512, 0.33333827, 0.33332661],
        [0.33333508, 0.33333829, 0.33332662],
        [0.33333511, 0.33333828, 0.33332662],
    ]
)
print(correct_scores)

# The difference should be very small. We get < 1e-7
print("Difference between your scores and correct scores:")
print(np.sum(np.abs(scores - correct_scores)))
```

```
Your scores:
[[0.33333514 0.33333826 0.33332661]
 [0.3333351  0.33333828 0.33332661]
 [0.3333351  0.33333828 0.33332662]
 [0.3333351  0.33333828 0.33332662]
 [0.33333509 0.33333829 0.33332662]
 [0.33333508 0.33333829 0.33332662]
 [0.33333511 0.33333828 0.33332661]
 [0.33333512 0.33333827 0.33332661]
 [0.33333508 0.33333829 0.33332662]
 [0.33333511 0.33333828 0.33332662]]

correct scores:
[[0.33333514 0.33333826 0.33332661]
 [0.3333351  0.33333828 0.33332661]
 [0.3333351  0.33333828 0.33332662]
 [0.3333351  0.33333828 0.33332662]
 [0.33333509 0.33333829 0.33332662]
 [0.33333508 0.33333829 0.33332662]
 [0.33333511 0.33333828 0.33332661]
 [0.33333512 0.33333827 0.33332661]
 [0.33333508 0.33333829 0.33332662]
 [0.33333511 0.33333828 0.33332662]]
Difference between your scores and correct scores:
8.799388540037256e-08
```

Forward Pass: Compute loss given the output scores from the previous step (10%)

Implement the forward function in the loss_func.py file, and output the loss value. The loss value must match the given loss value.

In [ ]:
```
Loss = CrossEntropyLoss()
loss = Loss.forward(scores, y)
correct_loss = 1.098612723362578
print(loss)
# should be very small, we get < 1e-12
print("Difference between your loss and correct loss:")
print(np.sum(np.abs(loss - correct_loss)))
```

```
1.098612723362578
Difference between your loss and correct loss:
0.0
```

Backward Pass (40%)

Implement the rest of the functions in the given files. Specifically, implement the backward function in all the 4 files as mentioned in the files. Note: No backward function in the softmax file, the gradient for softmax is jointly calculated with the cross entropy loss in the loss_func.backward function.

You will use the chain rule to calculate gradient individually for each layer. You can assume that this calculated gradeint then is passed to the next layers in a reversed manner due to the Sequential implementation. So all you need to worry about is implementing the gradient for the current layer and multiply it will the incoming gradient (passed to the backward function as dout) to calculate the total gradient for the parameters of that layer.

We check the values for these gradients by calculating the difference, it is expected to get difference < 1e-8.

In [ ]:
```python
# No need to edit anything in this block ( 20% of the above 40% )
net.backward(Loss.backward())

gradients = []
for module in net._modules:
    # print(module)
    for para, grad in zip(module.parameters, module.grads):
        assert grad is not None, "No Gradient"
        # Print gradients of the linear layer
        # print(grad)
        print(grad.shape)
        gradients.append(grad)

# Check shapes of your gradient. Note that only the linear layer has parameters
# (4, 10) -> Layer 1 W
# (10,)   -> Layer 1 b
# (10, 3) -> Layer 2 W
# (3,)    -> Layer 2 b
```

```
(4, 10)
(10,)
(10, 3)
(3,)
```

In [ ]:
```python
# No need to edit anything in this block ( 20% of the above 40% )
grad_w1 = np.array(
    [
        [
            -6.24320917e-05,
            3.41037180e-06,
            -1.69125969e-05,
            2.41514079e-05,
            3.88697976e-06,
            7.63842314e-05,
            -8.88925758e-05,
            3.34909890e-05,
            -1.42758303e-05,
            -4.74748560e-06,
        ],
        [
            -7.16182867e-05,
            4.63270039e-06,
            -2.20344270e-05,
            -2.72027034e-06,
            6.52903437e-07,
            8.97294847e-05,
            -1.05981609e-04,
            4.15825391e-05,
            -2.12210745e-05,
            3.06061658e-05,
        ],
        [
            -1.69074923e-05,
            -8.83185056e-06,
            3.10730840e-05,
            1.23010428e-05,
            5.25830316e-05,
            -7.82980115e-06,
            3.02117990e-05,
            -3.37645284e-05,
            6.17276346e-05,
            -1.10735656e-05,
```

```
        ],
        [
            -4.35902272e-05,
            3.71512704e-06,
            -1.66837877e-05,
            2.54069557e-06,
            -4.33258099e-06,
            5.72310022e-05,
            -6.94881762e-05,
            2.92408329e-05,
            -1.89369767e-05,
            2.01692516e-05,
        ],
    ]
)
grad_b1 = np.array(
    [
        -2.27150209e-06,
        5.14674340e-07,
        -2.04284403e-06,
        6.08849787e-07,
        -1.92177796e-06,
        3.92085824e-06,
        -5.40772636e-06,
        2.93354593e-06,
        -3.14568138e-06,
        5.27501592e-11,
    ]
)

grad_w2 = np.array(
    [
        [1.28932983e-04, 1.19946731e-04, -2.48879714e-04],
        [1.08784150e-04, 1.55140199e-04, -2.63924349e-04],
        [6.96017544e-05, 1.42748410e-04, -2.12350164e-04],
        [9.92512487e-05, 1.73257611e-04, -2.72508860e-04],
        [2.05484895e-05, 4.96161144e-05, -7.01646039e-05],
        [8.20539510e-05, 9.37063861e-05, -1.75760337e-04],
        [2.45831715e-05, 8.74369112e-05, -1.12020083e-04],
        [1.34073379e-04, 1.86253064e-04, -3.20326443e-04],
        [8.86473128e-05, 2.35554414e-04, -3.24201726e-04],
        [3.57433149e-05, 1.91164061e-04, -2.26907376e-04],
    ]
)

grad_b2 = np.array([-0.1666649, 0.13333828, 0.03332662])

difference = (
    np.sum(np.abs(gradients[0] - grad_w1))
    + np.sum(np.abs(gradients[1] - grad_b1))
    + np.sum(np.abs(gradients[2] - grad_w2))
    + np.sum(np.abs(gradients[3] - grad_b2))
)
print("Difference in Gradient values", difference)
```

```
Difference in Gradient values 7.70191643436727e-09
```

Train the complete network on the toy data. (30%)

To train the network we will use stochastic gradient descent (SGD), we have implemented the optimizer for you. You do not implement any more functions in the python files. Below we implement the training procedure, you should get yourself familiar with the training process. Specifically looking at which functions to call and when.

Once you have implemented the method and tested various parts in the above blocks, run the code below to train a two-layer network on toy data. You should see your training loss decrease below 0.01.

In [ ]:

```
# Training Procedure
# Initialize the optimizer. DO NOT change any of the hyper-parameters here or above.
# We have implemented the SGD optimizer class for you here, which visits each layer sequentially to
# get the gradients and optimize the respective parameters.
# You should work with the given parameters and only edit your implementation in the .py files

epochs = 1000
```

```python
optim = SGD(net, lr=0.1, weight_decay=0.00001)

epoch_loss = []
for epoch in range(epochs):
    # Get output scores from the network
    output_x = net(X)
    # Calculate the loss for these output scores, given the true labels
    # print(output_x, y)
    loss = Loss.forward(output_x, y)
    # print(loss)
    # Initialize your gradients to None in each epoch
    optim.zero_grad()
    # Make a backward pass to update the internal gradients in the layers
    net.backward(Loss.backward())
    # call the step function in the optimizer to update the values of the params with the gradients
    optim.step()
    # Append the loss at each iteration
    epoch_loss.append(loss)
    print("Epoch {}, loss={:3f}".format(epoch + 1, epoch_loss[-1]))

    if (epoch + 1) % 50 == 0:
        print("Epoch {}, loss={:3f}".format(epoch + 1, epoch_loss[-1]))
```

```
Epoch 1, loss=1.098613
Epoch 2, loss=1.094024
Epoch 3, loss=1.089737
Epoch 4, loss=1.085733
Epoch 5, loss=1.081994
Epoch 6, loss=1.078505
Epoch 7, loss=1.075248
Epoch 8, loss=1.072208
Epoch 9, loss=1.069372
Epoch 10, loss=1.066726
Epoch 11, loss=1.064257
Epoch 12, loss=1.061952
Epoch 13, loss=1.059799
Epoch 14, loss=1.057785
Epoch 15, loss=1.055899
Epoch 16, loss=1.054124
Epoch 17, loss=1.052445
Epoch 18, loss=1.050839
Epoch 19, loss=1.049277
Epoch 20, loss=1.047713
Epoch 21, loss=1.046081
Epoch 22, loss=1.044279
Epoch 23, loss=1.042147
Epoch 24, loss=1.039435
Epoch 25, loss=1.035774
Epoch 26, loss=1.030637
Epoch 27, loss=1.023365
Epoch 28, loss=1.013337
Epoch 29, loss=1.000400
Epoch 30, loss=0.985502
Epoch 31, loss=0.970780
Epoch 32, loss=0.958224
Epoch 33, loss=0.948146
Epoch 34, loss=0.939687
Epoch 35, loss=0.932411
Epoch 36, loss=0.926597
Epoch 37, loss=0.920919
Epoch 38, loss=0.914630
Epoch 39, loss=0.908446
Epoch 40, loss=0.902708
Epoch 41, loss=0.895794
Epoch 42, loss=0.889273
Epoch 43, loss=0.882132
Epoch 44, loss=0.875647
Epoch 45, loss=0.870536
Epoch 46, loss=0.861775
Epoch 47, loss=0.855117
Epoch 48, loss=0.848626
Epoch 49, loss=0.839904
Epoch 50, loss=0.832706
Epoch 50, loss=0.832706
Epoch 51, loss=0.827365
Epoch 52, loss=0.815973
Epoch 53, loss=0.810844
Epoch 54, loss=0.802194
Epoch 55, loss=0.790601
Epoch 56, loss=0.783502
Epoch 57, loss=0.770632
Epoch 58, loss=0.760160
Epoch 59, loss=0.749472
Epoch 60, loss=0.739295
Epoch 61, loss=0.732478
Epoch 62, loss=0.719142
Epoch 63, loss=0.713844
Epoch 64, loss=0.700037
Epoch 65, loss=0.693629
Epoch 66, loss=0.689958
Epoch 67, loss=0.674298
Epoch 68, loss=0.659023
Epoch 69, loss=0.647852
Epoch 70, loss=0.638275
Epoch 71, loss=0.628526
Epoch 72, loss=0.622772
Epoch 73, loss=0.617140
Epoch 74, loss=0.608889
Epoch 75, loss=0.601315
```

```
Epoch 76, loss=0.590965
Epoch 77, loss=0.588483
Epoch 78, loss=0.580111
Epoch 79, loss=0.580237
Epoch 80, loss=0.579590
Epoch 81, loss=0.575265
Epoch 82, loss=0.583167
Epoch 83, loss=0.569299
Epoch 84, loss=0.567051
Epoch 85, loss=0.558750
Epoch 86, loss=0.570455
Epoch 87, loss=0.550303
Epoch 88, loss=0.556647
Epoch 89, loss=0.528476
Epoch 90, loss=0.523165
Epoch 91, loss=0.503276
Epoch 92, loss=0.508981
Epoch 93, loss=0.493973
Epoch 94, loss=0.512211
Epoch 95, loss=0.477031
Epoch 96, loss=0.483331
Epoch 97, loss=0.456475
Epoch 98, loss=0.472357
Epoch 99, loss=0.443569
Epoch 100, loss=0.454687
Epoch 100, loss=0.454687
Epoch 101, loss=0.429413
Epoch 102, loss=0.441666
Epoch 103, loss=0.413838
Epoch 104, loss=0.443494
Epoch 105, loss=0.407146
Epoch 106, loss=0.444991
Epoch 107, loss=0.403453
Epoch 108, loss=0.436572
Epoch 109, loss=0.397764
Epoch 110, loss=0.439797
Epoch 111, loss=0.422846
Epoch 112, loss=0.526893
Epoch 113, loss=0.553876
Epoch 114, loss=0.728167
Epoch 115, loss=0.582263
Epoch 116, loss=0.547702
Epoch 117, loss=0.416240
Epoch 118, loss=0.453693
Epoch 119, loss=0.381732
Epoch 120, loss=0.362521
Epoch 121, loss=0.324897
Epoch 122, loss=0.338047
Epoch 123, loss=0.342623
Epoch 124, loss=0.326819
Epoch 125, loss=0.392589
Epoch 126, loss=0.292861
Epoch 127, loss=0.314854
Epoch 128, loss=0.235608
Epoch 129, loss=0.227234
Epoch 130, loss=0.207276
Epoch 131, loss=0.220121
Epoch 132, loss=0.208397
Epoch 133, loss=0.249017
Epoch 134, loss=0.226502
Epoch 135, loss=0.284894
Epoch 136, loss=0.233826
Epoch 137, loss=0.259128
Epoch 138, loss=0.195131
Epoch 139, loss=0.177242
Epoch 140, loss=0.156995
Epoch 141, loss=0.147287
Epoch 142, loss=0.142642
Epoch 143, loss=0.139515
Epoch 144, loss=0.136126
Epoch 145, loss=0.135978
Epoch 146, loss=0.130053
Epoch 147, loss=0.127937
Epoch 148, loss=0.122477
Epoch 149, loss=0.120641
Epoch 150, loss=0.118350
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Epoch 150, loss=0.118350
Epoch 151, loss=0.117382
Epoch 152, loss=0.112601
Epoch 153, loss=0.111264
Epoch 154, loss=0.106784
Epoch 155, loss=0.105601
Epoch 156, loss=0.105261
Epoch 157, loss=0.103690
Epoch 158, loss=0.099740
Epoch 159, loss=0.097608
Epoch 160, loss=0.094972
Epoch 161, loss=0.094597
Epoch 162, loss=0.093439
Epoch 163, loss=0.092069
Epoch 164, loss=0.090610
Epoch 165, loss=0.087682
Epoch 166, loss=0.085767
Epoch 167, loss=0.084664
Epoch 168, loss=0.083181
Epoch 169, loss=0.082531
Epoch 170, loss=0.081955
Epoch 171, loss=0.079550
Epoch 172, loss=0.078298
Epoch 173, loss=0.077149
Epoch 174, loss=0.075943
Epoch 175, loss=0.074927
Epoch 176, loss=0.074027
Epoch 177, loss=0.074400
Epoch 178, loss=0.072842
Epoch 179, loss=0.071362
Epoch 180, loss=0.070140
Epoch 181, loss=0.069109
Epoch 182, loss=0.068189
Epoch 183, loss=0.067438
Epoch 184, loss=0.066542
Epoch 185, loss=0.065878
Epoch 186, loss=0.065619
Epoch 187, loss=0.065236
Epoch 188, loss=0.064023
Epoch 189, loss=0.063155
Epoch 190, loss=0.062142
Epoch 191, loss=0.061411
Epoch 192, loss=0.060913
Epoch 193, loss=0.060049
Epoch 194, loss=0.059368
Epoch 195, loss=0.059265
Epoch 196, loss=0.058653
Epoch 197, loss=0.057884
Epoch 198, loss=0.057724
Epoch 199, loss=0.056640
Epoch 200, loss=0.055911
Epoch 200, loss=0.055911
Epoch 201, loss=0.055581
Epoch 202, loss=0.054974
Epoch 203, loss=0.054336
Epoch 204, loss=0.053731
Epoch 205, loss=0.053237
Epoch 206, loss=0.052698
Epoch 207, loss=0.052610
Epoch 208, loss=0.052149
Epoch 209, loss=0.051605
Epoch 210, loss=0.051018
Epoch 211, loss=0.050427
Epoch 212, loss=0.050176
Epoch 213, loss=0.050089
Epoch 214, loss=0.049610
Epoch 215, loss=0.048936
Epoch 216, loss=0.048407
Epoch 217, loss=0.048042
Epoch 218, loss=0.047693
Epoch 219, loss=0.047232
Epoch 220, loss=0.047006
Epoch 221, loss=0.046590
Epoch 222, loss=0.046342
Epoch 223, loss=0.045836
Epoch 224, loss=0.045452
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Epoch 225, loss=0.045112
Epoch 226, loss=0.044795
Epoch 227, loss=0.044464
Epoch 228, loss=0.044038
Epoch 229, loss=0.043724
Epoch 230, loss=0.043653
Epoch 231, loss=0.043454
Epoch 232, loss=0.042933
Epoch 233, loss=0.042565
Epoch 234, loss=0.042280
Epoch 235, loss=0.041901
Epoch 236, loss=0.041709
Epoch 237, loss=0.041528
Epoch 238, loss=0.041169
Epoch 239, loss=0.040963
Epoch 240, loss=0.040683
Epoch 241, loss=0.040507
Epoch 242, loss=0.040178
Epoch 243, loss=0.039961
Epoch 244, loss=0.039609
Epoch 245, loss=0.039391
Epoch 246, loss=0.039159
Epoch 247, loss=0.038825
Epoch 248, loss=0.038546
Epoch 249, loss=0.038335
Epoch 250, loss=0.038039
Epoch 250, loss=0.038039
Epoch 251, loss=0.037768
Epoch 252, loss=0.037602
Epoch 253, loss=0.037523
Epoch 254, loss=0.037224
Epoch 255, loss=0.036954
Epoch 256, loss=0.036836
Epoch 257, loss=0.036784
Epoch 258, loss=0.036457
Epoch 259, loss=0.036186
Epoch 260, loss=0.035966
Epoch 261, loss=0.035764
Epoch 262, loss=0.035549
Epoch 263, loss=0.035444
Epoch 264, loss=0.035394
Epoch 265, loss=0.035143
Epoch 266, loss=0.034874
Epoch 267, loss=0.034649
Epoch 268, loss=0.034474
Epoch 269, loss=0.034245
Epoch 270, loss=0.034032
Epoch 271, loss=0.033865
Epoch 272, loss=0.033680
Epoch 273, loss=0.033470
Epoch 274, loss=0.033298
Epoch 275, loss=0.033177
Epoch 276, loss=0.032968
Epoch 277, loss=0.032779
Epoch 278, loss=0.032638
Epoch 279, loss=0.032442
Epoch 280, loss=0.032259
Epoch 281, loss=0.032117
Epoch 282, loss=0.032015
Epoch 283, loss=0.032165
Epoch 284, loss=0.031888
Epoch 285, loss=0.031721
Epoch 286, loss=0.031566
Epoch 287, loss=0.031528
Epoch 288, loss=0.031292
Epoch 289, loss=0.031094
Epoch 290, loss=0.030956
Epoch 291, loss=0.030888
Epoch 292, loss=0.030697
Epoch 293, loss=0.030521
Epoch 294, loss=0.030383
Epoch 295, loss=0.030241
Epoch 296, loss=0.030111
Epoch 297, loss=0.029963
Epoch 298, loss=0.029830
Epoch 299, loss=0.029668
```

```
Epoch 300, loss=0.029528
Epoch 300, loss=0.029528
Epoch 301, loss=0.029408
Epoch 302, loss=0.029255
Epoch 303, loss=0.029114
Epoch 304, loss=0.029005
Epoch 305, loss=0.028858
Epoch 306, loss=0.028718
Epoch 307, loss=0.028612
Epoch 308, loss=0.028480
Epoch 309, loss=0.028342
Epoch 310, loss=0.028228
Epoch 311, loss=0.028112
Epoch 312, loss=0.028042
Epoch 313, loss=0.028009
Epoch 314, loss=0.027869
Epoch 315, loss=0.027722
Epoch 316, loss=0.027666
Epoch 317, loss=0.027744
Epoch 318, loss=0.027542
Epoch 319, loss=0.027385
Epoch 320, loss=0.027244
Epoch 321, loss=0.027115
Epoch 322, loss=0.027015
Epoch 323, loss=0.026908
Epoch 324, loss=0.026801
Epoch 325, loss=0.026707
Epoch 326, loss=0.026660
Epoch 327, loss=0.026527
Epoch 328, loss=0.026404
Epoch 329, loss=0.026311
Epoch 330, loss=0.026192
Epoch 331, loss=0.026078
Epoch 332, loss=0.025984
Epoch 333, loss=0.025883
Epoch 334, loss=0.025771
Epoch 335, loss=0.025673
Epoch 336, loss=0.025583
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Epoch 945, loss=0.008047
Epoch 946, loss=0.008039
Epoch 947, loss=0.008030
Epoch 948, loss=0.008021
Epoch 949, loss=0.008012
Epoch 950, loss=0.008003
Epoch 950, loss=0.008003
Epoch 951, loss=0.007994
Epoch 952, loss=0.007985
Epoch 953, loss=0.007977
Epoch 954, loss=0.007968
Epoch 955, loss=0.007959
Epoch 956, loss=0.007951
Epoch 957, loss=0.007943
Epoch 958, loss=0.007935
Epoch 959, loss=0.007927
Epoch 960, loss=0.007918
Epoch 961, loss=0.007909
Epoch 962, loss=0.007901
Epoch 963, loss=0.007892
Epoch 964, loss=0.007884
Epoch 965, loss=0.007875
Epoch 966, loss=0.007867
Epoch 967, loss=0.007858
Epoch 968, loss=0.007850
Epoch 969, loss=0.007844
```

```
Epoch 970, loss=0.007836
Epoch 971, loss=0.007828
Epoch 972, loss=0.007820
Epoch 973, loss=0.007811
Epoch 974, loss=0.007802
Epoch 975, loss=0.007794
Epoch 976, loss=0.007786
Epoch 977, loss=0.007777
Epoch 978, loss=0.007769
Epoch 979, loss=0.007761
Epoch 980, loss=0.007752
Epoch 981, loss=0.007744
Epoch 982, loss=0.007736
Epoch 983, loss=0.007728
Epoch 984, loss=0.007719
Epoch 985, loss=0.007711
Epoch 986, loss=0.007703
Epoch 987, loss=0.007695
Epoch 988, loss=0.007687
Epoch 989, loss=0.007679
Epoch 990, loss=0.007671
Epoch 991, loss=0.007662
Epoch 992, loss=0.007655
Epoch 993, loss=0.007646
Epoch 994, loss=0.007638
Epoch 995, loss=0.007630
Epoch 996, loss=0.007622
Epoch 997, loss=0.007614
Epoch 998, loss=0.007606
Epoch 999, loss=0.007601
Epoch 1000, loss=0.007593
Epoch 1000, loss=0.007593
```

In [ ]:

```
# Test your predictions. The predictions must match the labels
print(net.predict(X))
print(y)
```

```
[2 1 0 1 2 0 0 2 0 0]
[2 1 0 1 2 0 0 2 0 0]

[2 1 0 1 2 0 0 2 0 0]
```

In [ ]:

```
# You should be able to achieve a training loss of less than 0.02 (10%)
print("Final training loss", epoch_loss[-1])
```

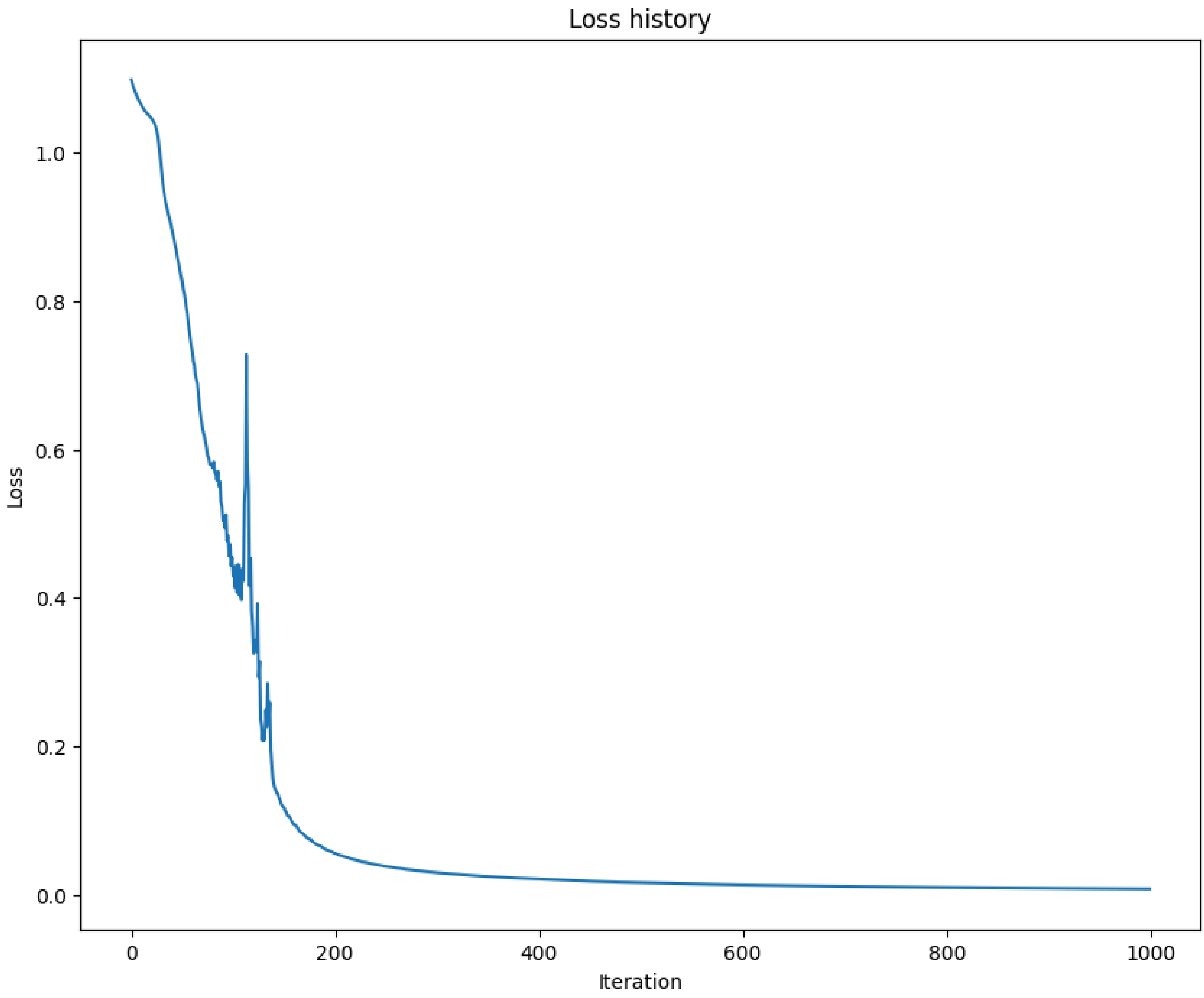
```
Final training loss 0.007593419801731252
```

In [ ]:

```
# Plot the training loss curve. The loss in the curve should be decreasing (20%)
plt.plot(epoch_loss)
plt.title("Loss history")
plt.xlabel("Iteration")
plt.ylabel("Loss")
```

Out[ ]:

```
Text(0, 0.5, 'Loss')
```


Loss history
Loss
Iteration
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000