

Infinite Sets

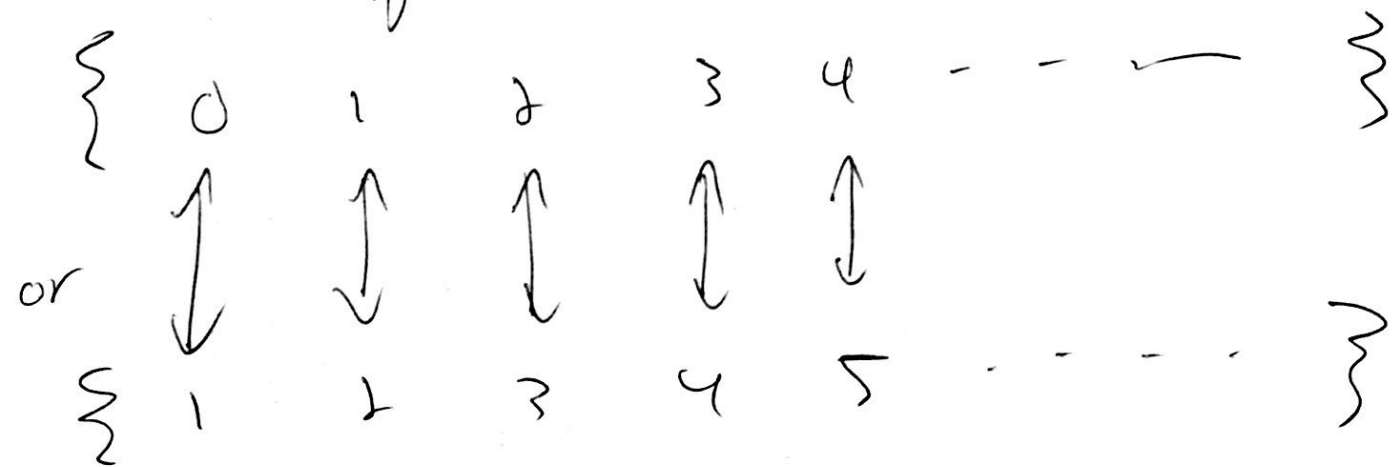
Principle 2 sets have same size if there is a pairing (bijection, or 1-1 onto mapping) between the sets using all of both exactly once.

$$|S| = |T|$$

Principle: A proper subset $S \subset T$ is smaller than T ^{not the whole set}

$$\left. \begin{array}{l} S \subseteq T \\ \text{and} \\ S \neq T \end{array} \right\} \rightarrow |S| < |T|$$

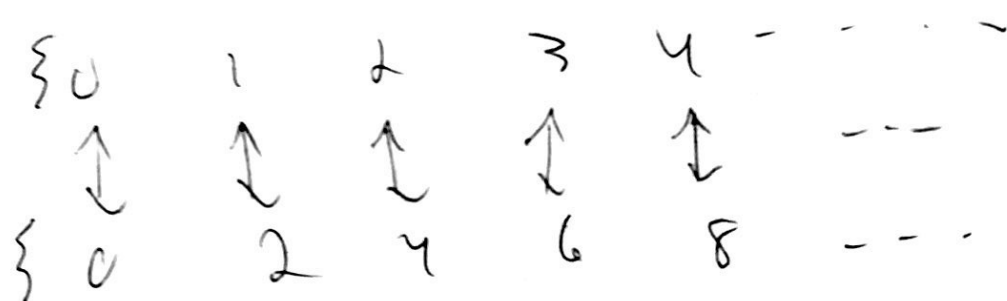
which is bigger?



So for infinite sets we keep the pairing principle and abandon the proper subset principle:

$$|\mathbb{N}| = |\mathbb{N} - \{0\}|$$

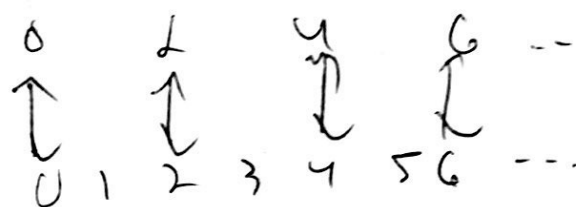
Which is bigger? \mathbb{N} vs. evens?
 $\{0, 2, 4, \dots\}$



Same size.

we say $|S| \leq |T|$ if there is a pairing between S and a subset of T .

$$|\{0, 2, 4, 6, \dots\}| \leq |\mathbb{N}|$$

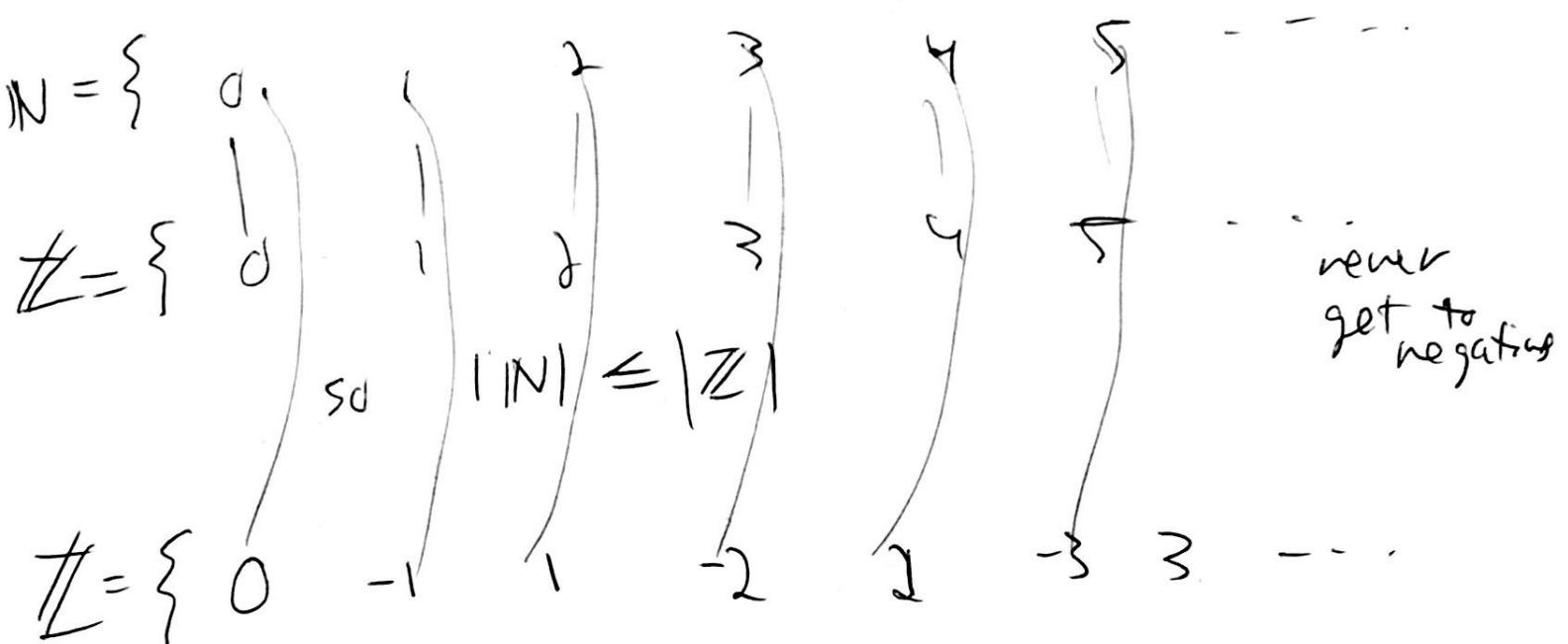


Then if $|S| \leq |T|$ and $|T| \leq |S|$

Then $|S| = |T|$. (Schröder-Bernstein Theorem)

(if S can be paired w/ a subset of T and
 T " " " " " " " " S then
 S and T can be fully paired off)

Show $|N| = |Z|$



so $|N| = |Z|$