Predicate Logic Models / Interpretations

- we will build models from relational databases.

- A _relational_ _database_ is a domain set D and

a relation over D for each relation symbol in the language.

└ given in answer to "question $\emptyset$"

(each relation symbol has a known arity : # of expected arguments)

a subset of D^n for relation of arity n.
└ set of n-tuples drawn from D,

Relational Database, interpreting $>_2, \geq_2, =_2,$ and prime$_1$

Examples

$$D = \{0, 1, 2\}$$

$>(x,y)$

$$x >_y = \{(1,0), (2,0), (2,1)\}$$

$$x \geq_y = \{(0,0), (1,1), (2,2), (1,0), (2,0), (2,1)\}$$

$$|D^2| = |D \times D| = 9$$

$$|\mathcal{P}(D^2)| = 2^9$$

choices for a binary relation over this D

$$x =_y = \{(0,0), (1,1), (2,2)\quad\}$$

$$\text{prime}(x) = \{(2)\} \stackrel{?}{=} \{2\}$$

Symbols being interpreted by the database

Defining Predicate Logic

Q0) What symbols will we use?

- a set of relation symbols, each with given a<u>rity</u>.
  - $Likes(x,y)$ $Tall(x)$ $>(x,y)$
- a set of constant symbols
  - John Mary 5
- a set of variables
  - $x, y, z \cdots$

- the familiar Boolean operation symbols $\land$ $\lor$ $\lnot$ $\rightarrow$ $\leftrightarrow$

- $\forall$, $\exists$
  - "for all"   "there exists"

Q1] what are the formulas of predicate logic?

A term is either a constant symbol or a variable.

An atomic formula is the application of a relation symbol to the appropriate number of terms given by the arity of the relation symbol R(Mary, John)

$$>(5,0) \qquad >(5,x) \qquad Tall(Mary) \qquad Prime(5)$$
$$5>x$$

A formula is either
- an atomic formula, or
- a Boolean combination of formulas   Prime(x) → Likes(mary,x)
- a quantified formula $\forall x \phi$ or $\exists x \phi$ for any variable x and smaller formula ϕ

$\forall x$ Prime(x) → Likes(Mary,x)

Likes(mary, $\exists x$)