Predicate Logic Proof

First, we still have all the rules from Boolean logic.

e.g.

$$\frac{\begin{array}{c}\varphi \\ \varphi \to \psi\end{array}}{\psi}\ (mp)$$

$$\frac{\begin{array}{c}\mathrm{Rich}(\mathrm{Pat}) \\ \mathrm{Rich}(\mathrm{Pat}) \to \neg \mathrm{Homeless}(\mathrm{Pat})\end{array}}{\neg \mathrm{Homeless}(\mathrm{Pat})}$$

$$\frac{\begin{array}{c}\mathrm{Man}(\mathrm{Pat}) \\ \forall x\,(\mathrm{Man}(x) \to \mathrm{mortal}(x))\end{array}}{\mathrm{mortal}(\mathrm{Pat})}\ ??$$

$$\frac{\begin{array}{c}\forall x\ \mathrm{Rich}(x) \\ (\forall x\ \mathrm{Rich}(x)) \to \mathrm{Happy}(\mathrm{Pat})\end{array}}{\mathrm{Happy}(\mathrm{Pat})}$$

	∃ existential	∀ universal	alt notation
instantiation (remove a quantifier)	ei Skolemization $\dfrac{\exists x\, \varphi[x]}{??\ \varphi[c]}$ for ~~new~~ constant c – not in previous lines or premises	$\boxed{\text{ui}}$ $\dfrac{\forall x\, \varphi[x]}{\varphi[c]}$ c is a constant or a variable not quantified in φ.	$\dfrac{\forall x\, \varphi}{[c/x]\varphi}$
generalization (add a quantifier)	$\boxed{\text{eg}}$ $\dfrac{\varphi[c]}{\exists x\, \varphi[x]}$ c does not occur in φ	ug $\dfrac{\varphi[c]}{\forall x\, \varphi[x]}$ where c is a constant we have made NO assumptions about. – c does not occur in any premise $\boxed{\text{– c does not occur in any ei line}}$	

$\varphi[x]$ just means any formula φ possibly containing the variable x.

$\varphi[c]$ means $\varphi[x]$ with all free x's replaced by c.

$[c/x]$ means φ with all free x's replaced by c.

$$\forall n \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$ ---- AHA!

$$\|$$

$$\begin{array}{l} 1+2+\cdots+n \\ \underline{n+\cdots+2+1} \\ (n+1)+\cdots+(n+1) \quad n \text{ times} \\ = n(n+1) \end{array}$$

This works because it begins

"Consider <u>arbitrary</u> $n \in \mathbb{N}$"

and then shows $\varphi(n)$

to conclude $\forall n \in \mathbb{N} \; \varphi(n)$

<u>universal generalization</u>

Premizes: Every human is mortal — 1. $\forall x\ human(x) \rightarrow mortal(x)$

There exists a human — 2. $\exists y\ human(y)$

Prove: There exists a mortal — $\exists z\ mortal(z)$

3. $human(c)$ 2, ei
4. $human(c) \rightarrow mortal(c)$ 1, ui
5. $mortal(c)$ 3, 4, mp
6. $\exists z\ mortal(z)$ 5, eg.