Thm There is no enumeration of $\mathbb{R}$.

Pf Consider an enumeration of numbers in $[0,1)$

position

$$\begin{array}{ll}
0: & 0.\,d_{00}\ d_{01}\ d_{02}\ d_{03}\ \cdots \\
1: & 0.\,d_{10}\ d_{11}\ d_{12}\ d_{13}\ \cdots \\
2: & 0.\,d_{20}\ d_{21}\ d_{22}\ d_{23}\ \cdots \\
3: & 0.\,d_{30}\ d_{31}\ d_{32}\ d_{33}\ \cdots \\
 & \vdots
\end{array}$$

choose digit c_i to be different from d_{ii} and not 0 or 9

Then $0.\,c_0\ c_1\ c_2\ c_3 \cdots$ cannot match any position i since $c_i \neq d_{ii}$

Q. Can same technique construct $0.c_0 c_1 c_2 \cdots$ missing from ~~any~~ a given enumeration of the rationals in $[0,1)$?

A. yes, so that $0.c_0 c_1 c_2 \cdots$ is not rational. !

Thm $|\mathcal{P}(S)| > |S|$ for any set S.

imagine $S = \{s_0, s_1, s_2, \cdots\}$

oops, assumed countability of S here.

S	$\mathcal{P}(S)$: s_0?	s_1?	s_2?	
s_0	b_{00}	b_{01}	b_{02}	$\cdots$
s_1	b_{10}	b_{11}	b_{12}	$\cdots$
s_2	b_{20}	b_{21}	b_{22}	$\cdots$
s_3	$\vdots$			
s_4				
$\vdots$				

bits

flip bits to set a subset not listed.

Instead

$x \in S \mapsto f(x) \in \mathcal{P}$

Given pairing $x \mapsto f(x)$

Define

$\Delta = \{$

See next page

<u>Instead</u> describe pairing between S and $P(S)$

with a function $f : S \to P(S)$ so that each $x \in S$ is paired with $f(x) \in P(S)$.

Then construct missing set

$$\Delta = \{x \in S \mid x \notin f(x)\}$$

easy to show by contradiction there is no $x \in S$ s.t. $f(x) = \Delta$

If there was then either

$$x \in f(x) = \Delta \quad \text{and so } x \notin f(x)$$

or

$$x \notin f(x) = \Delta \quad \text{and so } x \in f(x) \;\#$$

so Δ not ~~enumerated~~ included in range of f, so f does not pair S & $P(S)$

Addition principle. when making ONE of two possible choices, an n-way and m-way choice, there are $m+n$ possible outcomes.

Multiplication principle when making both - - -

- - - — — —

- - - $m \cdot n$ possible outcomes.

How many passwords are 6–8 characters containing at least one digit OR ~~and~~ at least one upper case letter?

$\boxed{\text{\# of 6-char options}}$ + # 7 char options + #8 char options

(no overlap)

then ~~\#~~ multiplying by 6 overcounts because the sets overlap.

_ _ _ _ _ _

upper case or digit

anything

$\cancel{36 \cdot 62^5}$

idea: turn union into intersection using complement (de Morgan's law).

Count all 6 char pwds using no digit or uppercase

U = universe = 6-char pwds, $|U| = 62^6$

bad ones use lowercase only $= 26^6$

good ones $\Rightarrow |U| - 26^6 = 62^6 - 26^6$