Example $T(n) = 3T(n-1) - 2T(n-2)$
$T(0) = 1 \quad T(1) = 2$

Step 1 Compute the characteristic equation by subbing $T(n) = r^n$
will have same degree as $T(n)$, here 2.

ex. $r^2 - 3r + 2 = 0$

Step 2 Identify $r_1 \dots r_n$ the distinct roots of the char. eqn.
(if possible) ex $r_1 = 1 \quad r_2 = 2$

Step 3 write ~~general~~ form of the family of all solutions to the recursive equation (i.e. w/o base cases incorporated)

$$T(n) = \sum_{i=1}^{n} a_i r_i^n$$

ex. $a_1 1^n + a_2 2^n = T(n)$

Step 4 Choose $a_1 \dots a_n$ to match the n base cases

ex: see prev. slide

Step 5 write closed form combining steps 3 & 4.

Ex. with repeated roots → $\boxed{\begin{array}{l} T(n) = -2T(n-1) - T(n-2) \\ T(0)=2,\ T(1)=4 \end{array}}$

Step 1 Find char. eqn.

$$r^n = -2r^{n-1} - r^{n-2}$$

$$\boxed{r^2 + 2r + 1 = 0}$$

Step 2 Find roots $r_1 \cdots r_k$ for $k \leq n$

and multiplicities $m_1 \cdots m_k$

for the roots

Note $m_1 + m_2 + \cdots + m_k = n$

$$(r+1)^2 = 0$$

$$r_1 = -1$$

$$m_1 = 2$$

Step 3 write the form for the family of solutions to the recursive eqn.

$$\boxed{T(n) = \sum_{i=1}^{k} p_i(n) r_i^n}$$

where $p_i(n)$ has degree $m_i - 1$

i.e. $p_i(n) = \sum_{j=0}^{m_i - 1} c_{ji} n^j$

★ still n constants c_{ji}

example

$$\boxed{T(n) = (c_0 n + c_1)(-1)^n}$$

steps 4 & 5 unchanged.

Example: $T(0) = (c_0 \cdot 0 + c_1)(-1)^0 = c_1 = 2$

$T(1) = (c_0 \cdot 1 + c_1)(-1)^1 = -(c_0 + c_1)$

$= -(c_0 + 2) = 4$

$c_0 = -6$

so: $\boxed{T(n) = (2 - 6n)(-1)^n}$

Solving Non-homogeneous recurrences:

when $F(n) = s^n p(n)$ is the non-homogeneous part.

Observation: The difference between any 2 solutions to the recursive eqn solves the homogeneous part!

Corollary: If we have one solution $P(n)$ to the recursive equation, then the family of all solutions is $T(n) = T_H(n) + P(n)$ where $T_H(n)$ is the family of solutions to the homogeneous part.

a particular solution

$$\underbrace{\boxed{T(n) = 2T(n-1)}}_{\text{Homogeneous Part } T_H} + \underbrace{n^2}_{F(n)}$$

$T(1) = 1$

$F(n) = 1^n \cdot n^2 = n^2$

so $s = 1$

$p(n) = n^2$ degree 2

Try: $T(n) = 2T(n-1) + 1$

so $F(n) = 1^n \cdot 1$ $\quad s=1$ $\quad p(n) = 1$ degree 0

Theorem: if $F(n) = s^n p(n)$ and s has multiplicity m as a root of the characteristic equation for T_H, then there exists a particular solution of the form

$m = 0$ if s not a root

$$P(n) = n^m p'(n) s^n$$

where $p'(n)$ has same degree as p.

Find constants in $p'(n)$ by plugging $P(n)$ into original recursive eqn. [Not using base cases].