Name: Andrew St. Pierre PU ID: 00279-01932

ECE 302: Probabilistic Methods in Electrical and Computer Engineering
Fall 2019
Instructor: Prof. A. R. Reibman

PURDUE
UNIVERSITY

Homework 2

Fall 2019
(Due Wednesday September 11 at 11:59pm)

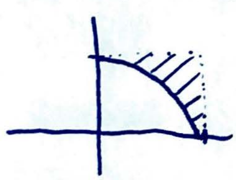
Homework is due on **Wednesday September 11 at 11:59pm** on Gradescope. No late homework will be accepted. Include a brief description of all sources of information you used (including other people), not counting the text, handouts, or material posted on the web page, or state "I did not receive help on this homework". You do not need to reference any material presented in class or on the course web-site, in the textbook, nor Prof. Reibman nor TA Chengzhang Zhong.

Statement: I did not receive help on this homework.

Topics: Probability mappings; Conditional probability (Chapter 2.4); Independent events; sequential experiments (Chapters 2.5, 2.6.1, 2.6.5).

Exercise 1. SIMILAR TO BUT NOT TEXTBOOK PROBLEM 2.38
Two numbers (x, y) are selected at random from the interval $[0, 1]$.

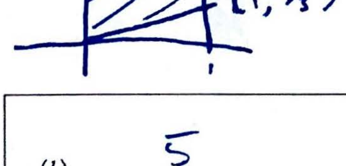
(a) Find the probability that the pair of numbers are outside the unit circle.



$$\frac{1-\frac{\pi}{4}}{1} = 1-\frac{\pi}{4} \approx 0.21$$

(*a*) $1-\frac{\pi}{4} \approx 0.21$

(b) Find the probability that $3y > x$.



$$1-\left(\frac{1}{2}\right)(1)\left(\frac{1}{3}\right) = 1-\frac{1}{6} = \frac{5}{6}$$

(*b*) $\frac{5}{6}$

Exercise 2.
Mobile phones perform *handoffs* as they move from one cell to another. During a call, a phone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it last more than three minutes, or short S. The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
S	0.4	–	0.1

↑
0.1

(a) What is the probability $P(H_0)$ that a phone makes no handoffs?

$$P(H_0) = \frac{0.1 + 0.4}{1}$$

(a) 0.5

(b) What is the probability a call is short?

$$P(S) = 0.4 + 0.1 + 0.1$$

(b) 0.6

(c) What is the probability a call is long or there are at least two handoffs?

$$P(L \cup H_2) = P(L) + P(H_2) - P(L \cap H_2)$$

(c) 0.5

Exercise 3.
A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up one unit with probability p or it moves down one unit with probability $1-p$. The changes on different days are assumed to be independent.

(a) What is the probability that after two days the stock will be at its original price?

(b) What is the probability that after three days the stock's price will have increased by one unit?

(c) Given that after three days the stock's price has increased by one unit, what is the probability that it went up on the first day?

R_i = price rise on day "i"

F_i = price fall on day "i"

(a) $$P((R_1 \cap F_2) \cup (F_1 \cap R_2)) = P(R_1)P(F_2) + P(F_1)P(R_2)$$
$$= p(1-p) + (1-p)p = 2p(1-p)$$

(b) $$P((R_1 \cap R_2 \cap F_3) \cup (R_1 \cap F_2 \cap R_3) \cup (F_1 \cap R_2 \cap R_3))$$
$$= P(R_1)P(R_2)P(F_3) + P(R_1)P(F_2)P(R_3) + P(F_1)P(R_2)P(R_3)$$
$$= (p)(p)(1-p) + (p)(1-p)(p) + (1-p)(p)(p) = 3p^2(1-p)$$

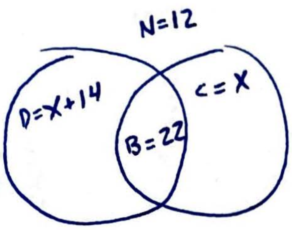
(c) $$P(R_1 \mid (R_1 \cap R_2 \cap F_3) \cup (R_1 \cap F_2 \cap R_3) \cup (F_1 \cap R_2 \cap R_3)) = \frac{2}{3}$$

(a): $2p(1-p)$

(b): $3p^2(1-p)$

(c): $\frac{2}{3}$

Exercise 4. (FROM EXAM 1, FALL 2016)
Among the Purdue students taking ECE 302 this semester, some like dogs, some like cats, some like both, and some like neither. Let D be the set of Purdue ECE 302 students who like dogs, and C be the set who like cats.
A study shows that 22% like both cats and dogs, and 12% like neither. The probability a student likes dogs exceeds the probability a student likes cats by 0.14. What is the probability a randomly chosen student likes cats?



$N + D + C = 100$

$(22 + x) + (22 + x + 14) = 88$

$\therefore\ 2x = 30$

$x = 15$

$\therefore$ 37 % like cats

Answer : 37 % like cats

Exercise 5.

Suppose among all six-letter English words, a word is picked at random (i.e., each six-letter word has the same probability of being picked). Which event is more probable: That the selected word has an "n" as its fifth letter, or that the selected word ends in "ing"?

Answer :	The selected word is more probable to have an "n" as its fifth letter. ex. "begins"

Exercise 6. FROM EXAM 1, FALL 2015

You have 4 otherwise identical cans of soda (also known as pop or cola), except you know that 1 was shaken up about 10 minutes ago, while the other 3 have been stable for hours. (You have lost track of which can is which.) The probability of the shaken can splattering when opened is 4/5, and the probability of a stable can splattering when opened is 1/3.

(a) If you choose one can at random and open it, what is the probability of it splattering?

(b) If you open a can and it splatters, what is the probability that it was the shaken can?

(You may leave your answers in fractional form.)

(a) $\left(\frac{1}{4}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{3}\right) = \frac{9}{20} = 0.45$

(b) $$\frac{P(\text{splat} \cap \text{shook})}{P(\text{splat})} = \frac{\left(\frac{1}{4}\right)\left(\frac{4}{5}\right)}{\left(9/20\right)} = 0.44$$

(a): 0.45

(b): 0.44

Exercise 7. (From Exam 1 Spring 2016; Note: this counts as 2 problems)
Suppose 3 boxes contain Red, Green, and Blue marbles, denoted R, G, B, respectively.
Box 1 has 3 Red, 4 Green, and 3 Blue.
Box 2 has 8 Red, 1 Green, and 1 Blue.
Box 3 has 0 Red, 4 Green, and 1 Blue.

Suppose a box is chosen at random, and then a marble is selected from the box.

(a) If box 1 is selected, what is the probability a Green marble is drawn?

(b) What is the probability a Blue marble is drawn from Box 3?

(c) What is the probability a Red marble is drawn?

(d) Suppose a Red marble is drawn. What is the probability it came from Box 2?

Box_1

R: 3
G: 4
B: 3 T=10

Box_2

R: 8
G: 1
B: 1 T=10

Box_3

R: 0
G: 4
B: 1 T=5

(a) $P(G \mid \text{Box}_1) = 3/10$

(b) $P(\text{Box}_3 \cap B) = 1/30$

(c) $P((B_1 \cap R) \cup (\text{Box}_2 \cap R)) = \frac{3}{30} + \frac{8}{30} = \frac{11}{30}$

(d) $P(B_2 \mid R) = \frac{\frac{8}{30}}{\frac{11}{30}} = \frac{8}{11}$

(a): $3/10$

(b): $1/30$

(c): $\frac{11}{30}$

(d): $\frac{8}{11}$

Exercise 8.
An experiment consists of picking one of two urns at random and then selecting a ball from the urn and noting its color (black or white). Let A be the event "urn 1 is selected" and B the event "a black ball is observed." Under what conditions are A and B independent?
(Hint: recall that independence is a mathematical definition, so determine what mathematical conditions are necessary, and how that translates into the world.)

A = "urn 1 is observed"

B = "a black ball is observed"

A and B are independent iff $P(A \mid B) = P(A)$

and

$P(B \mid A) = P(B)$