Coding of numbers as ~~sets~~ sets

$0 = \{\} = \emptyset$

$1 = \{0\} = \{\emptyset\} = \{\{\}\}$ idea: code n as $\{0, 1, 2, \ldots, n-1\}$

$2 = \{0, 1\} = \{\emptyset, \{\emptyset\}\}$

$3 = \{0, 1, 2\} = \ldots$

(can code real numbers using Dedekind cuts of rationals)

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operations on sets

union $\cup$ $\quad X \cup Y$ is the set of all of members of either X or Y

intersection $\cap$

[sets do not allow duplicate members]

$X \cap Y$ . . . . both X and Y

complement X' = all "universe elements" not in X

(Independence of the continuum hypothesis)

Set ~~Iden~~ Identities

$(X \cap Y) \cap Z = X \cap (Y \cap Z)$

$X \cap Y = Y \cap X$

$X \cap Y = (X' \cup Y')'$ when $X \subseteq U$ $Y \subseteq U$ for provided U

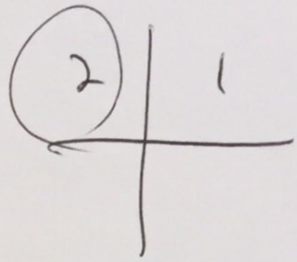
To prove $X = Y$, prove $X \subseteq Y$ and $Y \subseteq X$

$\equiv$

every $z \in Y$ is in X

so, to show $Y \subseteq X$, consider arbitrary $z \in Y$, show $z \in X$

Ordered pairs & Tuples



Given ~~2~~ sets A and B,

cartesian ~~cross~~ product

$A \times B$ is the set of ordered pairs

$$\{(x,y) \mid x \in A, y \in B\}$$

reals $\mathbb{R}^- \times \mathbb{R}^+ = 2^{nd}$ quadrant

integers $\mathbb{Z}$ = set of all integers

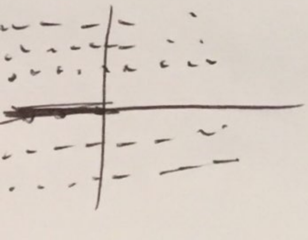
$\mathbb{Z} \setminus \{0\}$ = set of nonzero integers

$\mathbb{Z} - \{0\}$

$$\mathbb{Z} \times (\mathbb{Z} - \{0\}) \approx \mathbb{Q}$$

sort of like

rationals



$\{(-1,-1), (-1,-2), (-1,-3) \cdots$

$\vdots$

$\}$

$A^1 = A$

$A^2 = A \times A$

$A^3 = A \times A \times A$

ordered tuples

$A^n = A \times A \times \cdots \times A$

$\mathbb{P}$ = set of all people

$\cancel{\mathbb{P} \times \mathbb{P}}$ $\mathbb{P} \times \mathbb{P}$ set of pairs of people

$F \subseteq \mathbb{P} \times \mathbb{P}$

F → Facebook Friend relationship

$(x, y) \in \mathbb{F}$

iff

x is friends with y

A set of pairs from a domain set D, i.e. a subset of $D \times D$ is called a binary relation on D.

so F is a binary relation on $\mathbb{R}$

Let S be the smallest set such that

$0 \in S$

for any $n \in S$, $n+1 \in S$.

$\Longrightarrow$ defines S to be $\mathbb{N}$

Whatever we take S to be, clearly we want

$$S = \boxed{\{x \mid x=0 \text{ or there is } n \in S \text{ such that } x = n+1\}}$$

also we want the smallest such S.

0'th approximation to S: $S=\emptyset \Longrightarrow S=\{0\}$

1st " " S: $S=\{0\} \Longrightarrow S=\{0,1\}$

2nd " " S: $S=\{0,1\} \Longrightarrow S=\{0,1,2\}$

$\vdots$

Take S to be $\bigcup_{i\in\mathbb{N}}$ (i^{th} approximation of S)