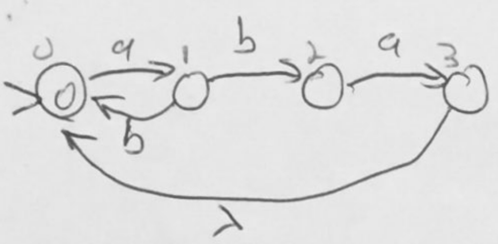
How to convert NFA N into FSA M s.t. $L(N) = L(M)$



$\underline{a}\,\underline{b}\,\underline{a}\,\underline{b}\,\underline{a}\,\underline{b}\,\underline{a}$ ends 0 or 1 or 3

δ^b

$\{0\} \xrightarrow{a} \{1\} \xrightarrow{b} \{0 \text{ or } 2\} \xrightarrow{a} \{0 \text{ or } 1 \text{ or } 3\}$

$\{0 \text{ or } 1 \text{ or } 3\} \xrightarrow{b} \{0 \text{ or } 2\}$

e.g.

$$N = (S_N, s_{0,N}, \Sigma, R, F_N)$$

A deterministic process simulating the NFA over a state set that is the power set (i.e. sets of states)

notation

write $E(p)$ for state p to mean $\{q \mid (p, \lambda) \vdash_N^* (q, \lambda)\}$

so $E(3) = \{3, 0\}$ $E(2) = \{2\}$ $E(1) = \{1\}$ $E(0) = \{0\}$

The desired machine M is $(S_M, s_{0,M}, \Sigma, f, F_M)$

$S_M = \mathcal{P}(S_N)$ "one state in M is a set of states from N"

$s_{0,M} = E(s_{0,N})$ anywhere you could get in N from $s_{0,N}$ reading λ

$f(p,a)$ for $p \in S_M$ i.e. a set of NFA states and $a \in \Sigma$

$$= E\left(\bigcup_{n \in p} \{m \mid R(n,a,m)\}\right)$$

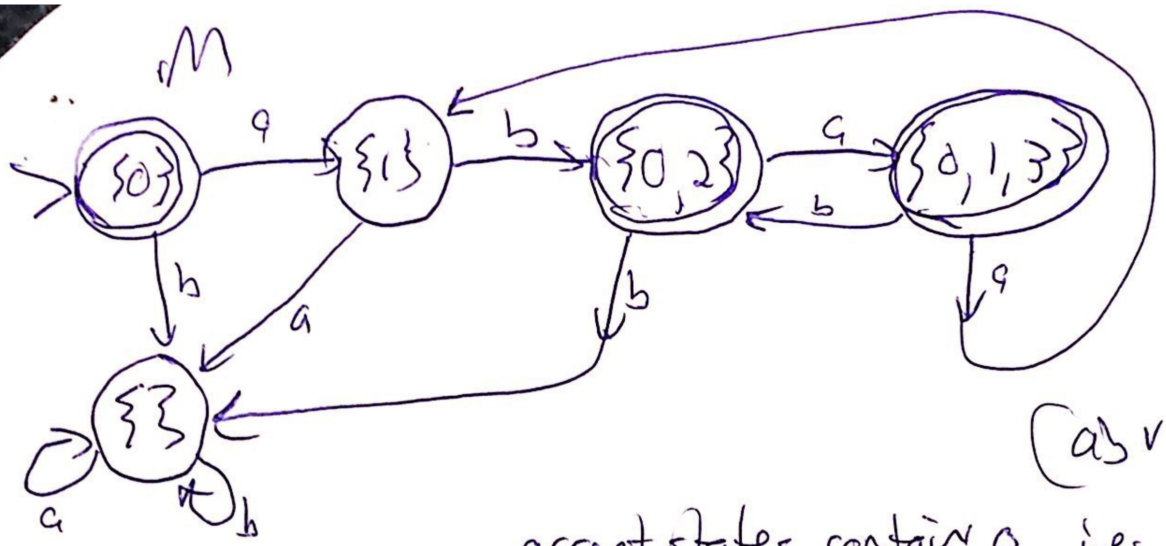
follow all λ transitions

NFA state n

there is NFA transition from n to m

$F_M = \{p \in S_M \mid p \cap F_N \neq \emptyset\}$ an FSA state p accepts iff it overlaps the set of accepting NFA states i.e. it contains an accepting NFA state.

$\boxed{\begin{array}{l}\underline{\text{Claim}}: L(N) = L(M) \\ \text{Pf by induction on length of strings.}\end{array}}$



$(ab \vee aba)^*$

accept states contain 0 i.e. overlap $F_N = \{0\}$

Always close under λ

in general M could have $2^{|S_N|}$ states, exponential blowup.