Given an implication $P \to Q$

we define $Q \to P$ to be the converse of $P \to Q$

we define $\neg Q \to \neg P$ to be the contrapositive

The contrapositive is equivalent. So $P \to Q$ iff $\neg Q \to \neg P$

$\neg P \vee Q$ $\qquad$ $\neg\neg Q \vee \neg P$

(The converse is not equivalent)

$P \to Q$ and $Q \to P$ iff $P \Leftrightarrow Q$

Direct Proof (the natural argument form of the deduction method)

Assume the premises and argue to the conclusion.

Show that the product of two even numbers is always even.

$[\text{even}(x) \wedge \text{even}(y) \rightarrow \text{even}(xy)]$

Proof Let x and y be two even numbers. Let j and k be numbers such that $x = 2j$ and $y = 2k$. Then

$xy = (2j)(2k) = 2(2jk)$ which is even since $2jk$ is a number.

<u>Show</u> if n^2 is odd then n is odd.

<u>idea</u> consider the contrapositive

if n is ~~not odd~~ even then n^2 is ~~not odd~~ even

$\Longrightarrow$ a special case of previous slide.

<u>Proof by contradiction</u> to show Q, assume $\neg Q$ and derive

<u>any</u> contradiction

Example. Prove there are no natural numbers p & q

s.t. $\sqrt{2} = p/q$

Suppose not. [Suppose ~~no~~ such p & q exist, for contradiction]

So $2 = p^2/q^2$ and so $2q^2 = p^2$

So p^2 has an even # of 2's in its factorization, and

$2q^2$ " " odd " " " " " " .

So we have an number that is both even and odd. #

Technically proof by contradiction subsumes direct proof and direct proof of the contrapositive.

Goal $P \to Q$

	Assume	Show
Direct proof	P	Q
D.P. of contrapositive	$\neg Q$	$\neg P$
Proof by contradiction	$\neg(P \to Q)$ $P \wedge \neg Q$	any contradiction (includes Q since $\neg Q$ known includes $\neg P$ since P known)

Showing equivalence between P_1 P_2 $\cdots$ P_n ?

Show $P_1 \leftrightarrow P_2$ by showing $P_1 \rightarrow P_2 \land P_2 \rightarrow P_1$.

$P_2 \leftrightarrow P_3$

[then $P_1 \leftrightarrow P_3$ would be implied]

$\vdots$

$P_{n-1} \leftrightarrow P_n$

total $2(n-1)$ $\rightarrow$ proofs

Alternative

Show $P_1 \rightarrow P_2 \land P_2 \rightarrow P_3 \land \cdots \land P_{n-1} \rightarrow P_n \rightarrow P_1$

total n $\rightarrow$ proofs

2 Fallacies involving the converse.

Affirming the conclusion

$(P \to Q) \wedge Q \not\vDash P$

Denying the conclusion

$P \to Q \wedge \neg P \not\vDash \neg Q$

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another fallacy: circular reasoning.
[sneakily assuming the goal]

Prove n is even:

Let k be a number such that $n = 2k$. Therefore
n is even.