Sample Final Exam

ECE 369
Name: Solution

Read all of the following information before starting the exam:

1. **NOTE:** Unanswered questions are worth 25% credit, rounded down. Writing any answer loses this "free" credit, which must be earned back by the quality of the answer. If you wish a question to be treated as unanswered, but you have written on it, clearly write "DO NOT GRADE" in the answer area. In a multi-part question, unanswered *parts* are worth 25%. This is an option only for parts that are numbered or lettered on the exam: you may not create your own "parts" for this purpose.
2. Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
3. No calculators, or materials other than pen/pencil and blank paper are allowed except those we distribute during the exam. This is a closed book closed notes exam.
4. Please keep your written answers brief; be clear and to the point. Points may be deducted for rambling and for incorrect or irrelevant statements.
5. There are 6 problems. Each of the problems is approximately equal in value, except where indicated. Multi-part problems divide the problem score approximately equally among the parts.
6. Good luck!

1.

Argue that in any group, no element can have two distinct inverse elements. Be sure you point out where you use each property of a group in this argument.

Let x_1 and x_2 be two distinct inverses of an element, say x in a group, G. We show that they must be the same.
Let i be the identity element of the group, G.

$x_1 \oplus x = i = x \oplus x_1$(inverse definition)
$x_2 \oplus x = i = x \oplus x_2$(inverse definition)

$x_1 = i \oplus x_1$ (identity)
$= (x_2 \oplus x) \oplus x_1$ (identity)
$= x_2 \oplus (x \oplus x_1)$ (associativity)
$= x_2 \oplus i$ (identity)
$= x_2$

2.

a. What homogeneous recurrence has the following characteristic equation?

$$(x-2)(x^2+2x+1) = x^3 - 3x - 2 = 0$$

T(n) = 3T(n-2) + 2T(n-3)

b. Suppose a recurrence has the homogeneous part given in part a above and non-homogeneous part $f(n) = 3$. Find a particular solution for the recurrence.
The non-homogeneous recurrence is

T(n) = 3T(n-2) + 2T(n-3) + 3

Since the non-homogeneous part is of the form

$$f(n) = b_0 s^n = 3.(1)^n$$

and s $(= 1)$ is not a root (2 or -1), the particular solution guess is c_o.
Substituting guess in the non-homogenous recurrance, we have

$$c_0 = 3c_0 + 2c_0 + 3$$

, which gives $c_0 = -3/4$.
The particular solution is T(n)=$-3/4$.

c. Write a parameterized closed form for the family of solutions to the recurrence described in part b.

$$\alpha_1 2^n + \alpha_2(-1)^n + \alpha_3 n(-1)^n - 3/4$$

3.

a. Give the reflexive transitive closure of the relation $R = \{(1,0),(2,2),(0,2)\}$ over the domain $D = \{0,1,2,3\}$.

Reflexive transitive closure = $\{(0,0)(1,1)(3,3)(1,2),(1,0),(2,2),(0,2)\}$

b. For which of the following properties is the closure of a relation under the property always well defined?

- Transitivity - yes
- Irreflexivity - no (if a relation has pairs such as (a,a), irreflexivity closure is not well-defined since such pairs cannot be removed by closure operation.)
- Symmetry - yes
- Anti-symmetry - no (if a relation already has (a,b) and (b,a) for two distinct elements a and b of the domain, anti-symmetry closure is not well-defined since those pairs cannot be removed by closure operation.)
- Reflexivity - yes

c. What properties are required for a relation to be an *equivalence relation*?

Reflexivity, Symmetry, Transitivity

4.

a. Define the symbol $\models$ in expression $\Sigma \models \phi$.

Every model of Σ is a model of ϕ.

b. Describe a method for determining if $\Sigma \models \phi$ for Boolean logic. Comment precisely on the runtime cost of the method.

In Boolean logic, models of Σ correspond to different truth (0/1) values of the variables. Therefore, construct a truth table with a line for each possible assignment of truth values to the variables, in other words, a line for each model in Σ. For every model (truth assignment from the truth table) that makes Σ true, check if it makes ϕ true as well. If no model exists that make Σ true but ϕ false, then output Σ entails ϕ. Otherwise, output Σ does not entail ϕ.

Since we have to check all the models that make Σ true, the runtime in the worst case be 2^n, where n is the number of variables in Σ.

5.

Precisely define different languages, one with each of the following properties. Assume that a deterministic automaton must have a transition specified for every state and character, but a non-deterministic automaton may omit transitions that would go to a dead state (and omit that dead state).

a. The language is undecidable:

HALTING-PROBLEM = {(P,I) | P encodes a C program and I encodes the input to the C program and P halts on input I}

b. The language is decidable but cannot be defined by any regular expression:

$L_{not-regular} = \{a^n b^n | n \in \mathcal{N}\}$ = the set of strings with equal numbers of a's and b's

c. The language can be recognized by a one state finite automaton:

Since a single-state finite automaton (FSA) must accept all strings or reject all strings according to whether that state is a "final" state.

$L_{all} = \{\Sigma^*\}$ = the set of all strings over Σ

$L_{nothing} = \{\}$ = the empty language

(Note: Either of the two languages above will fetch full points for this question.)

d. The language can be recognized by a one state nondeterministic finite automaton, but not by a one state finite automaton:

$L_{empty-string} = \{\lambda\}$ = the language with only one string, the empty string.

(Note: $L_{empty-string}$ is a different language from $L_{nothing}$. The former accepts one string, which is empty string but the latter accepts nothing.).

6.

a. Write a nondeterministic automaton in normal form for the regular language

$$(abb \vee bab \vee ab)^*$$

.

M=(S,I,F,R)
where
S={s0,s1,s2,s3}, I={a,b}, F={s0}
R={(s0,a,s1),(s0,b,s2),(s1,b,s0),(s1,b,s3),(s2,a,s3),(s3,b,s0)}

Now, convert it into "normal form", which requires no IN arcs for the initial state, no OUT arcs for the final state, and only one final state.

M-normal=(S,I,F,R)
where
S={si, s0,s1,s2,s3, sf}, I={a,b}, F={sf}
R={(si,λ,s0), (s0,λ,sf) (s0,a,s1),(s0,b,s2),(s1,b,s0),(s1,b,s3),(s2,a,s3),(s3,b,s0)}

(continued)

Problem 5 (continued)

b. Given the following nondeterministic automaton, eliminate state 2 using the process for converting automata into regular expressions. Draw the resulting generalized automaton. You do not need to eliminate any other states or complete the conversion.
M=(S,I,F,f_S), where S=s0,s1,s2,f, I=0,1, F=f, and
f_S(s0,0)=s1
f_S(s0,1)=s2
f_S(s1,1)=s1
f_S(s1,0)=s2
f_S(s2,0)=s1
f_S(s2,1)=s2
f_S(s2,λ)=f
λ is the empty string.

Homework 8 problem E2

c. Given the same automaton provided in part b (not your answer), convert the automaton into an equivalent deterministic automaton using the process used in class to prove that every nondeterministic automaton has a deterministic equivalent.

Homework 8 problem E3