Induction Fallacies

Show $\forall n \geq 0 \quad n > 5$

Consider an arbitrary k and assume $k > 5$,
it is easy to see $k+1$ is also greater than 5.

$P(n) \equiv n > 5$

$\boxed{(\forall k \; P(k) \rightarrow P(k+1))}$

inductive case works.

but don't conclude $\forall n \geq 0 \; P(n)$ i.e. $\forall n \geq 0 \; n > 5$.

Because we skipped the base case $P(0)$ i.e. $0 > 5$
which can't be shown.

Induction Fallacies

$\forall_{n\ge1} P(n)$ $\quad$ $P(n) \equiv$ any group of n 369 students go by the same first name.

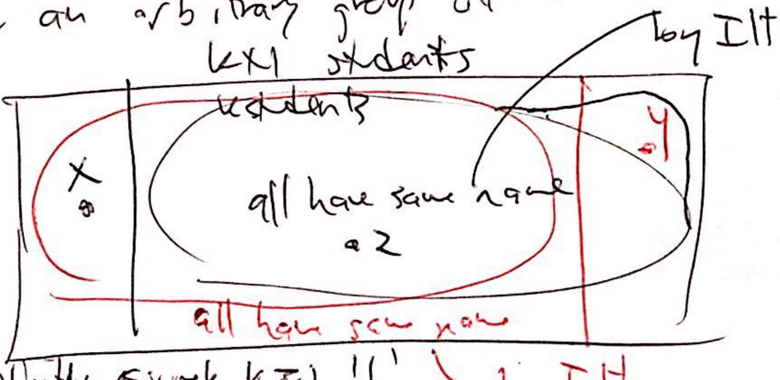
Base case $P(1)$ any group of 1 student all go by the same name

✓

Consider arbitrary $k^{\ge1}$ and suppose for induction $P(k)$, i.e. any group of k 369 students use the same name.

Show $P(k+1)$, i.e. any group of k+1 students does also.

Consider an arbitrary group of k+1 students. Pick student x in group. All others have same name (IH)



Pick y from others, all but y have same name (IH)

Pick z not x or y, then (x and z) and (y and z) share names. so $P(k+1)$

☆ we implicitly assume $k\ge2$!!

2nd principle of M.I.

Show every number can be written as a product of 1 or more primes. (1 if it is prime)

Base case 2 is a product of primes ✓

$P(k) = k$ is a prod. of primes

Ind case Assume k is a prod of primes

Show k+1 is a prod of primes

case 1 k+1 is prime ✓

case 2 $k+1 = a \cdot b$ for $a < k+1$, $b < k+1$

– would be done if we knew a and b were prod of primes

How to get?

idea replace $P(k) \equiv k$ is a prod. of primes

with $Q(k) \equiv$ every number ~~from 2 to k~~ n s.t. $2 \le n < k$ is a product of primes.

$$Q(2) \wedge \forall k \left[2 \le k \wedge \underline{Q(k)} \to \underline{Q(k+1)} \right] \to \forall n\, Q(n) \quad \text{1st principle}$$

$\Downarrow$ easy

$\downarrow$ nothing

$$\forall k \ge 2 \; \underbrace{\boxed{\forall m\; 2 \le m < k \to P(m)}}_{\text{IH}} \to P(k) \quad \to \quad \forall n\, P(n) \quad \text{2nd principle}$$

Proves 2nd principle (strong ind) from 1st principle of M.I.

Again, using 2nd principle. Show every number ≥ 2 can be written as the product of primes.

Consider arb. $k \geq 2$

Suppose for induction that $\boxed{\forall m \; 2 \leq m < k \rightarrow m \text{ can be written as a prod. of primes.}}$

IH.

Show k can be written so.

<u>case 1</u> k is prime. ✓ (includes our "base case")

<u>case 2</u> k is composite. So $k = ab$ for $2 \leq a < k$ and $2 \leq b < k$

Then by IH, a and b are products of primes.

Take all these primes and the multiply to $ab = k$.

□.