Sample Second Midterm Exam

ECE 369
Coverage may vary from this semester **Name:** ______________________________

Read all of the following information before starting the exam:

1. **NOTE:** Unanswered questions are worth 25% credit, rounded down. Writing any answer loses this "free" credit, which must be earned back by the quality of the answer. If you wish a question to be treated as unanswered, but you have written on it, clearly write "DO NOT GRADE" in the answer area. In a multi-part question, unanswered *parts* are worth 25%. This is an option only for parts that are numbered or lettered on the exam: you may not create your own "parts" for this purpose.
2. Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
3. No calculators, or materials other than pen/pencil and blank paper are allowed except those we distribute during the exam. This is a closed book closed notes exam.
4. Please keep your written answers brief; be clear and to the point. Points will be deducted for rambling and for incorrect or irrelevant statements. Where algorithms are requested, you may be penalized for inefficient algorithms, and an exponential algorithm may be considered entirely incorrect on the basis of inefficiency alone.
5. Each of the problems is approximately equal in value, except where indicated. Multi-part problems divide the problem score approximately equally among the parts.
6. Good luck!

1. Consider a class of 14 students. Suppose 8 of these are juniors, 5 are seniors and 1 is a sophomore. Suppose I plan to draw a committee of four students. For each question below, explain how you get your result clearly.

a. How many different committees can be drawn from this class?

The number of committees is equal to C(14,4).

b. How many different class makeups are possible for the committee (e.g. all seniors, 2 juniors and 2 seniors, etc.)?

This can be divided into 2 cases.

case 1: When the sophomore is not included.

| _ _ _ _

From 5 positions, we need to choose 1 position for placing the bar.
This gives C(5,1) = 5 ways.

case 2: When the sophomore is included

x | _ _ _

This gives C(4,1) = 4 ways

Totally 9 different class makeups.

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c. How many committees are there where the seniors outnumber the juniors?

If the seniors are to outnumber juniors, there can only be 1 or 0 juniors.
Case 1: 1 junior. The possible committees are: 3 seniors, 1 junior or 2 seniors, 1 junior, 1 sophomore.
This gives C(5,3)*C(8,1) + C(5,2)*C(8,1)
Case 2: 0 juniors. The possible committees are 4 seniors or 3 seniors, 1 sophomore.
This gives C(5,4)+C(5,3)

So adding up the 2 cases we get a total of 175 committees where seniors outnumber juniors.

d. How many committees have exactly one senior?

Exactly one senior means 1 senior and 3 juniors or 1 senior, 2 juniors and 1 sophomore.
This can be done in C(5,1)*C(8,3) + C(5,1)*C(8,2) ways.

e. How many committees have exactly one ~~freshman~~ or exactly one senior?
sophomore

1)
Number of committees with exactly one sophomore or exactly one senior = Number of committees with exactly one sophomore + Number of committees with exactly one senior - Number of committees with exactly one senior and exactly one sophomore. ---- Equation 1 (from inclusion exclusion).

Number of committees with exactly one sophomore = C(13,3).
Number of committees with exactly one senior and one sophomore = C(8,2)*C(5,1).
Apply these two equations to equation 1 along with the result from part d.

2)

The original question's answer is the same as the answer for question 1d.

2. Consider our class of 14 students. Suppose 8 of these are juniors, 5 are seniors and 1 is a sophomore. Suppose I plan to assign four different tasks, one to each of four different students. For each question below, explain how you get your result clearly.

a. How many different ways are there for me to assign the tasks to the class?

P(14,4) ways.

b. How many different class patterns can be assigned (e.g. sophomore to task 1, senior to task 2, and juniors to tasks 3 and 4)?

Without sophomore: choose a 4-permutation with replacement from {Junior, Senior}, 16 ways

With sophomore: place sophomore (4 ways) then choose a 3-permutation with replacement, 4 x 8 = 32 ways.

Total = 48 class patterns.

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c. After I assign the tasks, will there be some class (sophomore, junior, or senior) that is responsible for more than one task? How do you know?

There are more tasks than classes. Therefore there will be some class that is responsible for more than one task. This is by the pigeon hole principle.

d. How many different ways are there for me to assign the tasks that have seniors performing tasks 1 and 4?

This can be done in P(5,2)*P(12,2) ways.

3. State and derive the binomial theorem.

This has been covered in class.

4. Write and solve a recurrence for the Fibonacci numbers $0, 1, 1, 2, 3, 5, 8, \ldots$

This has been covered in class.

5.

Under what conditions do we say that two infinite sets have the same cardinality?

2 infinite sets have the same cardinality if we can find a one-to-one and onto function from one set to the other.

For each of the following pairs of infinite sets, state whether they have the same cardinality or not and prove your answer is correct.

a. The even natural numbers and the natural numbers.

Same cardinality. To prove this, we have to exhibit that there exists a bijection from one to the other.

0 <-> 0
1 <-> 2
2 <-> 4
3 <-> 6 ...

b. The integers and the natural numbers.

Same cardinality.
0 <-> 0
1 <-> 1
2 <-> -1
3 <-> 2
4 <-> -2 ...

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c. The positive rationals and the natural numbers.

Same cardinality.
Positive rationals can be written as a countable union of countable sets.
Since countable union of countable sets is also countable. Positive rationals have the same size as natural numbers.