Example:

Given 2 space D and R

$|D| = m \qquad |R| = n$

How many functions are there from D to R?

$D = \{1, 2, 3\}$

$R = \{4, 5\}$

$\rightarrow 2 \times 2 \times 2 \Rightarrow 2^3$

n^m

multiplication principle

R to D n^m

Function $D \mapsto R$

$1 \rightarrow 4$

$2 \rightarrow 4$

$3 \rightarrow 5$

$D = \mathbb{R}$

$R = \mathbb{R}$

$y = x^2$

Example

Given D, R $|D| = m$ $|R| = n$

How many <u>one-to-one</u> functions are from D to R?

$m = n$

[If $m > n$ then it is impossible to have a one-to-one function

if $m \leq n$

$$n \cdot (n-1) \cdot \cdots \cdot (n-m) = \frac{n!}{n!}$$

Pidgeon hole principle

If you have ~~at least~~ more than k items being placed into k bins, then at least 1 bin ~~has~~ contains more than 1 item.

- 2 students in this room are born in the same month.

- Same number of ~~heads~~ hair on heads in Indiana

Generalized Pigeonhole Principle.

If you have n items being placed in k bins,

then at least one bin has

ceiling → $\lceil \frac{n}{k} \rceil$ items.

(rounded up)

50 students

12 months

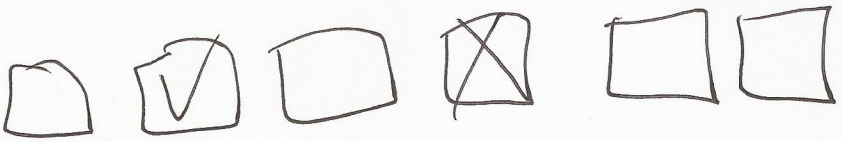
at least $\lceil \frac{50}{12} \rceil = 5$ students are born in the same month

45 beans in 30 bags

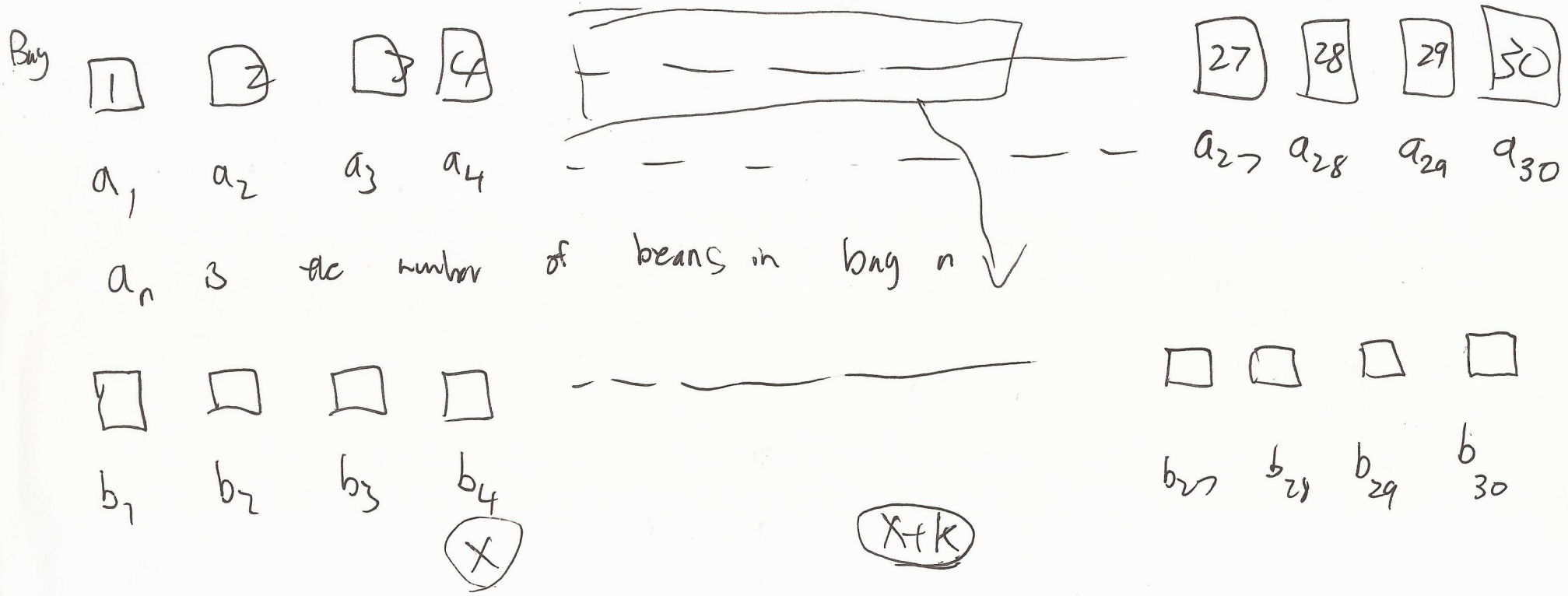
each bag has at least 1 bean.

the bags are lined up in a line

You must only take consecutive bags of beans.



Show that for $0 < k < 15$, you can find a consecutive number of bags that contains exactly k beans.



a_n is the number of beans in bag n

b_n is the number of beans in bags $a_1 a_2 \sim a_n$

a_1+k a_2+k a_3+k a_4+k $\sim\sim$ $a_{29}+k$ $a_{30}+k$

$$a_n+k = b_m$$

we take bags $(m-n+1)$ to bag n

By ~~the~~ pid

we have 60 "slots" of bags between b_{1-n} and $b_{1-n}{}^{tk}$

we only have $45tk = 45t14 = 59$ at most

60 bags, 59 values (at most)

By the pidgeon hole principle, at least 2 bags have same value

You have some socks in your drawer

10 blue socks

8 green socks

1 red socks

5 yellow socks.

What is the minimum number of ~~How many~~ socks do you need to pull out from the drawer to guarantee a pair of matching colors?