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ECE 302: Probabilistic Methods in Electrical and Computer Engineering
Fall 2019
Instructor: Prof. A. R. Reibman

PURDUE UNIVERSITY

Homework 1

Fall 2019
(Due Monday September 2 at 11:59pm)

Homework is due on **Monday September 2 at 11:59pm** on Gradescope. No late homework will be accepted. Include a brief description of all sources of information you used (including other people), not counting the text, handouts, or material posted on the web page, or state "I did not receive help on this homework". You do not need to reference any material presented in class or on the course web-site, in the textbook, nor Prof. Reibman nor TA Chengzhang Zhong.

Topics: Sample space, events, set theory; reading in Chapters 1 and Section 2.1-2.2.

Statement: I did not give or receive help on this assignment

Exercise 1.
The space S and three of its subsets are given by $S = \{n \in \mathbb{Z} : 0 \le n \le 11\}$, $A = \{1,3,4,5,9\}$, and $B = \{4,7,9,11\}$, and $C = \{1,3,9,11\}$.
Find $A \cap B \cap C$, $A^c \cap B, A - C$, and $(A - B) \cup B$.

$A \cap B \cap C = \{9\}$

$A^c \cap B = \{7, 11\}$

$A - C = \{4, 5\}$

$(A - B) \cup B = \{1,3,4,5,7,9,11\}$

if $N_1 == 2$, N_2 can only be 1 or 2

Exercise 2.
Four marbles, numbered 1,2,3 and 4 are placed in a box. One of the marbles is drawn randomly from the box and its number, N_1 is noted. (So, N_1=1,2,3, or 4.) An integer N_2 is then selected at random from the values $1, \ldots, N_1$. The outcome of this experiment is the ordered pair (N_1, N_2), where N_1 denotes the marble and N_2 is just a number.

(a) Write the sample space of the experiment.

$S = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(4,4)\}$

(b) Write the event "Marble 2 is selected".

{ Marble 2 is selected } $= \{(2,1),(2,2)\}$

(c) Write the event "$N_2 = 3$".

$\{ N_2 = 3 \} = \{(3,3),(4,3)\}$

(d) Write the event "Marble 2 is selected and $N_2 = 3$".

{ Marble 2 is selected and $N_2 = 3$} = $\emptyset$, empty set

Exercise 3.

An integrated circuit (IC) factory has three machines, X, Y, and Z. Test one IC from each machine, and observe if each is *acceptable (a)* or *fails (f)*. Thus, an observation is a sequence of the three test results from each machine. For example, the observation that the circuit from Z fails while the circuits from X and Y pass is aaf.

(a) What is the sample space?

$S = \{$aaa, aaf, afa, aff, faa, faf, ffa, fff$\}$

(b) What are the elements of the sets
$Z_F = \{$circuit from Z fails$\}$ and
$X_A = \{$circuit from X is acceptable$\}$?

$Z_F = \{$fff, aaf, aff, faf$\}$

$X_A = \{$aaa, aaf, aff, afa$\}$

(c) Are Z_F and X_A mutually exclusive? Are Z_F and X_A collectively exhaustive?

Mutually Exclusive? No, aff and aaf are in common

Collectively Exhaustive? No, the ~~[illegible]~~ union of the two subsets is not the sample space.

(d) What are the elements of the sets
$C = \{$more than one circuit is acceptable$\}$ and
$D = \{$at least two circuits fail$\}$?

$C = \{$aaa, aaf, afa, faa$\}$

$D = \{$aff, faf, ffa, fff$\}$

(e) Are C and D mutually exclusive? Are C and D collectively exhaustive?

Mutually Exclusive? yes

Collectively Exhaustive? yes

Exercise 4. (FROM EXAM 1, FALL 2015)
For each of the following relations, determine which is valid for arbitrary events A, B, and C. (Note: to be true "for arbitrary events", it must be true for any such event. Use a Venn diagram if it is helpful.)
(On the exam, this was a True/False question. For this homework, show whether it is true or false.)

(a) $(A \cup B \cup C)^c = A^c \cup B^c \cup C^c$

(b) $(A \cap B) \cup (A \cap B^c) \cup (A^c \cap B) = (A^c \cap B^c)^c$

(c) $(A - B) - C = A - (B - C)$.

(d) $(A \cup B) \cap (A^c \cup B^c) = (A \cap B^c) \cup (A^c \cap B) \cup (A^c \cap B \cap C^c)$

Hint: be careful, item (d) is tricky.

(a) $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$ $\quad \therefore$ False

(b) $(A \cap B) \cup (A \cap B^c) \cup (A^c \cap B) = (A \cup B) = (A^c \cap B^c)^c$

$A \cap \underbrace{(B \cup B^c)}$

$A \cap S$

$= A \cup (A^c \cap B)$

$= (A \cup A^c) \cap (A \cup B)$

$= S \cap (A \cup B)$

$= (A \cup B)$

$\therefore$ True

(c) $(A-B) - C = A - (B - C)$

$(A \cap B^c) - C = A - (B \cap C^c)$

$A \cap (B^c \cap C^c) \neq A \cap (B^c \cup C)$

$\therefore$ False

(d) $(A \cup B) \cap \underbrace{(A^c \cup B^c)}_{X^c} = \underbrace{(A \cap B^c) \cup (A^c \cap B)}_{\text{items only in B or A ie } (A-B)+(B-A)} \cup (A^c \cap B \cap C^c)$

$(A \cup B) - X \neq X \cup A \cup B^c \cup C$

$\therefore$ False