

Formalizing computation : Turing Machines

infinite tape starts with the input on it

... | 1 | 1 | 1 | a | a | a | b | b | b | ...



with a finite-state controller that can read/write/move  
the tape head ← the program

# of steps of computation = # of transitions of the controller

— unambiguous

For us : imagine # of assembly language instructions is  
the # of basic steps (a bit ambiguous)

A time bound on a program is a function specifying the number of basic steps allowed for each input size.

$f: \mathbb{N} \rightarrow \mathbb{N}$  where  $f(|w|)$  gives # of steps allowed for input  $w$ .

A time bound is polynomial if  $f(x) = \sum_{i=0}^k a_i x^i$  for some  $a_i \in \mathbb{R}$   
 $k \in \mathbb{N}$

we will consider a program to run in polynomial time if there is a polynomial time bound for the program further within for all inputs. (a worst case time bound)

A language  $L$  is decidable in polynomial time iff there is some program  $P$  that says yes to input  $w$  exactly when  $w \in L$  and  $P$  runs in polynomial time. (always finishes within  $f(|w|)$  for some polynomial bound  $f$ )

The class of all such  $L$  is called  $P$ .

Let " $\varphi; p$ " be a string encoding of a Boolean formula  $\varphi$  and a truth assignment  $p$

The language  $\text{FORMULA\_EVAL} = \{ \varphi; p \mid \varphi \text{ is true under } p \}$

— can be decided in polynomial time. (e.g. use recursion and solve recurrence)

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related question:

FORMULA-SATISFIABILITY =  $\{ \psi \mid \text{There is } p \text{ s.t. } \psi \text{ is true} \}$   
-SAT under  $p$

obvious algorithm:

try every  $p$  to see if  $\psi$  is true

How many truth assignments ~~there~~ are there?

$\sim 2^{|\psi|}$  exponential in  $|\psi|$

FORMULA-SAT  $\in P$  is OPEN.