Defining Boolean Logic

Step 2 What ~~"worlds"~~ interpretations or "models" are the formulas talking about?

(semantics)

for Boolean logic: a world is an assignment of true/false to each variable in V

$\equiv$

$\boxed{\text{a mapping from } V \text{ to } \{\text{True}, \text{False}\}}$ $\equiv$ a model

i.e. a function f on domain V so that for any variable $v \in V$, $f(v)$ is true or false

example if $V = \{A, B, C\}$ example model A gets False
B gets True
C gets True

There are 2^3 models of this V

Step 3 What formulas are true in what models?

examples

A is true in models that assign true to A

likewise for any $v \in V$

$A \wedge B$ is true in a model iff that model ~~models that~~ assigns both A & B to true.

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

For arbitrary Boolean formulas φ_1 and φ_2 ~~and~~

$\varphi_1 \wedge \varphi_2$ is true in a model iff φ_1 is true in that model and φ_2 is true in that model.

φ_1	φ_2	$\varphi_1 \wedge \varphi_2$
0	0	0
		0
		0
1	1	1

A formula φ is true in a model M iff

— $\varphi \in V$ and V is assigned true by M, or

— $\varphi \equiv \varphi_1 \wedge \varphi_2$ and φ_1 is true in M <u>and</u> φ_2 is true in M

φ_1	φ_2	$\varphi_1 \wedge \varphi_2$

— $\varphi \equiv \varphi_1 \vee \varphi_2$ and φ_1 is true in M <u>OR</u> φ_2 is true in M

φ_1	φ_2	$\varphi_1 \vee \varphi_2$

— <u>$\varphi \equiv \varphi_1 \rightarrow \varphi_2$</u> and φ is true in this truth table, given truth in M of <u>φ_1</u> and <u>φ_2</u>:

φ_1	φ_2	$\varphi_1 \rightarrow \varphi_2$
0	0	1
0	1	1
1	0	0
1	1	1

$$\underbrace{(\qquad\qquad)}_{\substack{\varphi_1 \\ \text{true}}} \rightarrow \underbrace{(\qquad\qquad)}_{\substack{\varphi_2 \\ \text{false}}} \implies \text{so false}$$

— $\varphi \equiv \varphi_1 \leftrightarrow \varphi_2$

φ_1	φ_2	$\varphi_1 \leftrightarrow \varphi_2$
0	0	1
0	1	0
1	0	0
1	1	1

Is $(A \vee B) \rightarrow (A \wedge C)$ true in

$$\boxed{\begin{array}{l} A \mapsto \text{True} \\ B \mapsto \text{False} \\ C \mapsto \text{True} \end{array}} \; M$$

$\varphi_1 \rightarrow \varphi_2$

$1 \qquad 1$

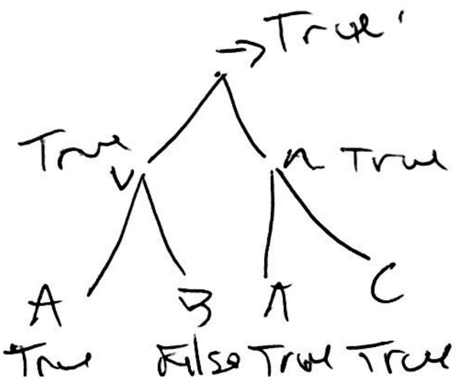
Is $A \vee B$ true in M? Yes

$\varphi_1 \; \varphi_2$ look up $1 \; 0$ in $\vee$ truth table $\Rightarrow 1$

" $A \wedge C$ " " M? Yes

$1 \quad 1$ $\wedge$ truth table

Look up $1,1$ in $\rightarrow$ truth table to get TRUE



Yes