Exam 1

Mean 51

Std Dev 16

Permutations and Combinations

0 1 2 3 4

How many 3 digit #s can I make? (Permutations)
sequences

P(5,3) – without replacing used digits. (without replacement)

→ 5 choices * 4 choices * 3 choices = 60

How many 3 digit sets can I make? (size 3 sets of digits)
(combinations)

C(5,3) – without replacement, again

$$\frac{5 \cdot 4 \cdot 3 \text{ sequences}}{3 \cdot 2 \cdot 1 \text{ times each set is counted as a sequence}}$$

$\binom{5}{3}$ "5 choose 3"

$P(n,k)$ = # ways to select a sequence of k items from a domain of n items.

$$= \underbrace{n\cdot(n-1)\cdot(n-2)\cdots(n-(k-1))}_{k \text{ factors}} = \boxed{\frac{n!}{(n-k)!}}$$

$n! = 1\cdot 2\cdot 3\cdot 4 \cdots (n-1)\cdot n$

$0! = 1$

$C(n,k)$ = # ways to select a set of k from domain of size n

$$= \frac{P(n,k)}{P(k,k)} = \frac{n!/(n-k)!}{k!/(k-k)!} = \boxed{\frac{n!}{(n-k)!\,k!}}$$

So, e.g. $\frac{30!}{(30-5)!\,5!} = \frac{30!}{(30-25)!\,25!}$

$C(30,5) = C(30,25)$

Binomial Theorem

what does $(x+y)^n$ expand to?

— all terms have the form $x^k y^{n-k}$ for k between 0 and n

$\overbrace{(x+y)\cdots(x+y)}^{n \text{ times}}$

choose x 3 times to get x^3y^{n-3} ⇒ how many ways?

→ there are $C(n,3)$ ways to choose the factors that contribute x

so $C(n,3)x^3y^{n-3}$ is one of the terms in the expansion

$$\boxed{(x+y)^n = \sum_{k=0}^{n} C(n,k)x^k y^{n-k}}$$

$(\underline{x}+y)(x+\underline{y})(x+\underline{y})$

$x^3+3x^2y+3xy^2+y^3$

# of ways to choose x once and y twice:

$(\underline{x}+y)(x+\underline{y})(x+\underline{y})$

$(x+\underline{y})(\underline{x}+y)(x+\underline{y})$

$(x+\underline{y})(x+\underline{y})(\underline{x}+y)$

$C(3,1)$ choices for x $=$ $C(3,2)$ choices for y

example lemma derived from the Binomial Theorem by setting $x=1$ $y=-1$

$$0 = (1+-1)^n = \sum_{k=0}^{n} C(n,k) \cancel{1^k} (-1)^{n-k}$$

$$0 = (-1)^n C(n,0) + (-1)^{n-1} C(n,1) + (-1)^{n-2} C(n,2) + \cdots + C(n,n)$$

$$\boxed{0 = C(n,0) - C(n,1) + C(n,2) - C(n,3) \cdots \pm C(n,n)}$$

Aside

$$C(n,0) + C(n,1) + C(n,2) + \cdots + C(n,n) = ?\ \underline{\underline{2^n}}$$