$\leq$ vs. $<$

A reflexive antisymmetric transitive relation is called a partial ordering. (for ordering)

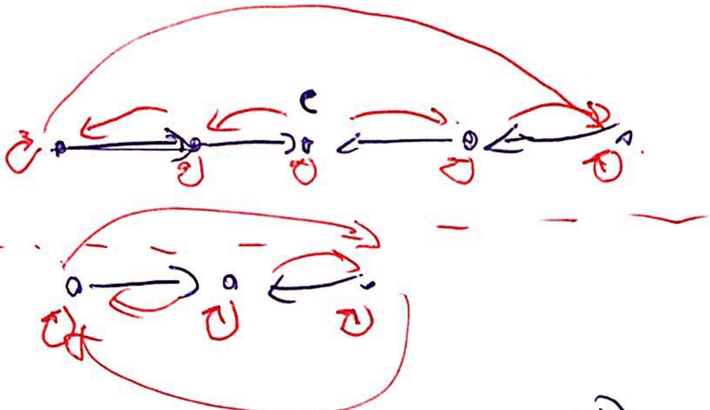
A reflexive symmetric transitive relation (rst) is called an equivalence relation. (for grouping)

read about Hasse diagram for drawing partial orders.

in a partial order:

- a greatest element x is an element s.t. $\forall y \in D\ R(y,x)$
  - if there is one, it is unique due to antisymmetry
- a maximal element x is one such that $\neg \exists y \in D\ R(x,y)$
  - there can be more than one, and always at least 1
    in finite D.
- correspondingly least and minimal.
- the ordering is total if $\forall xy\ R(x,y) \vee R(y,x)$

Equivalence relation pairs represent "same group"



$C(5,2)+5$ arcs

$C(3,2)+3$

the <u>group</u> of an element $x \in D$ under an equivalence relation R is called the <u>equivalence class</u> of x, (under R), written $|x|_R$ $[x]_R$

so, e.g., if $R(a,b)$ then $|a|_R = |b|_R$

Thm Any equivalence relation R describes a partition of the domain D into blocks $\underbrace{\{y \in D \mid R(x,y)\}}_{\text{the group containing } x} = |x|_R$ for each x

A partition P of a set D is a set of subsets of D, $P = \{B_0, B_1, \ldots B_k\}$ where each $B_i \subseteq D$, that union to D, $\bigcup_{i=0..k} B_i = D$, and do not overlap:

$\forall i,j \; B_i \cap B_j = \emptyset$ when $i \neq j$

So the thm says the set of equivalence classes $\{|x|_R \mid x \in D\}$ of an equivalence relation is a partition of D.

Conversely, any partition $P = \{B_0 \ldots B_k\}$ describes an equivalence relation $R = \{(x,y) \mid \exists i \; x \in B_i \text{ and } y \in B_i\}$ exercise: write proof that R is an equiv. relation.

A ~~The~~ closure of a binary relation R under a property or collection of properties is a set containing R with the property (properties) s.t. no subset has the property.

the smallest extension of R to get the property.

- may not exist. e.g. antisymmetric closure of R that is not already antisymmetric.

- may be more than one: ~~there~~ are many total-closures of an empty partial order (just identity loops (x,x))

But unique and defined for reflexivity, symmetry, transitivity, and any combination of these.