Sample Final Exam

ECE 369
Name: ______________________________

Read all of the following information before starting the exam:

1. **NOTE:** Unanswered questions are worth 25% credit, rounded down. Writing any answer loses this “free” credit, which must be earned back by the quality of the answer. If you wish a question to be treated as unanswered, but you have written on it, clearly write “DO NOT GRADE” in the answer area. In a multi-part question, unanswered *parts* are worth 25%. This is an option only for parts that are numbered or lettered on the exam: you may not create your own “parts” for this purpose.
2. Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
3. No calculators, or materials other than pen/pencil and blank paper are allowed except those we distribute during the exam. This is a closed book closed notes exam.
4. Please keep your written answers brief; be clear and to the point. Points may be deducted for rambling and for incorrect or irrelevant statements.
5. There are 5 problems. Each of the problems is approximately equal in value, except where indicated. Multi-part problems divide the problem score approximately equally among the parts.
6. Good luck!

1.

Argue that in any group, no element can have two distinct inverse elements. Be sure you point out where you use each property of a group in this argument.

2.

a. What homogeneous recurrence has the following characteristic equation?

b. Suppose a recurrence has the homogeneous part given in part a above and non-homogeneous part $f(n) = 3$. Find a particular solution for the recurrence.

c. Write a parameterized closed form for the family of solutions to the recurrence described in part b.

3.

a. Give the reflexive transitive closure of the relation $R = \{(1,0), (2,2), (0,2)\}$ over the domain $D = \{0,1,2,3\}$.

b. For which of the following properties is the closure of a relation under the property always well defined?

- Transitivity
- Irreflexivity
- Symmetry
- Anti-symmetry
- Reflexivity

c. What properties are required for a relation to be an *equivalence relation*?

4.

a. Define the symbol $\models$ in expression $\Sigma \models \phi$.

b. Describe a method for determining if $\Sigma \models \phi$ for Boolean logic. Comment precisely on the runtime cost of the method.

5.

Precisely define different languages, one with each of the following properties. Assume that a deterministic automaton must have a transition specified for every state and character, but a non-deterministic automaton may omit transitions that would go to a dead state (and omit that dead state).

a. The language is undecidable:

b. The language is decidable but cannot be defined by any regular expression:

c. The language can be recognized by a one state finite automaton:

d. The language can be recognized by a one state nondeterministic finite automaton, but not by a one state finite automaton:

6.

a. Write a nondeterministic automaton in normal form for the regular language

$$(abb \wedge bab \wedge ab)^*$$

.

(continued)

Problem 5 (continued)

b. Given the following nondeterministic automaton, eliminate state 2 using the process for converting automata into regular expressions. Draw the resulting generalized automaton. You do not need to eliminate any other states or complete the conversion.
M=(S,I,F,f_S), where S=s0,s1,s2,f, I=0,1, F=f, and
f_S(s0,0)=s1
f_S(s0,1)=s2
f_S(s1,1)=s1
f_S(s1,0)=s2
f_S(s2,0)=s1
f_S(s2,1)=s2
f_S(s2,λ)=f
λ is the empty string.

c. Given the same automaton provided in part b (not your answer), convert the automaton into an equivalent deterministic automaton using the process used in class to prove that every nondeterministic automaton has a deterministic equivalent.