

INDEPENDENT-SET =

IND-SET =  $\{ \langle G, k \rangle \mid \text{There } \exists \text{ a size } k \text{ subset of vertices in } G \text{ with no edges} \}$

— Ex. show IND-SET  $\in$  NP

Let's show that any solution to IND-SET can be applied to CLIQUE problems by converting CLIQUE problems suitably.

Specifically we give a <sup>polynomial-time computable</sup> function  $\tau: \Sigma^* \rightarrow \Sigma^*$  s.t.

$w \in \text{CLIQUE}$  iff  $\tau(w) \in \text{IND-SET}$

" $G, k$ " —  $\tau()$  is a reduction from CLIQUE to IND-SET

so: instead of looking for a clique in  $G, k$ , convert to  $G', k'$  and look for an IND-SET.

$\tau(\langle G, k \rangle) = \tau(\langle V, E, k \rangle) = \langle V, \bar{E}, k \rangle$  achieves above claim

A language  $L_1$  polynomially reduces to a language  $L_2$ ,  
written  $L_1 \leq_p L_2$ , if there is polynomially computable  
 $\tau: \Sigma^* \rightarrow \Sigma^*$  on strings s.t.

$w \in L_1$  iff  $\tau(w) \in L_2$

immed. consequence: if  $L_2 \in P$  then  $L_1 \in P$   
ie.  $L_1$  is no harder than  $L_2$  to solve  
hence  $L_1 \leq_p L_2$

Example: given new language  $L_2$ , if you show a  $\tau$  achieving  
FORMULA-SAT  $\leq_p L_2$  then  $L_2 \in P$  <sup>NOT KNOWN TRUE</sup> ~~OR~~ OR  $P=NP$  is  
resolved.

Harder example Show  $\text{FORMULA\_SAT} \leq_p \text{CLIQUE}$ .

actually show  $3\text{-SAT} \leq_p \text{CLIQUE}$

$$3\text{-SAT} = \{ \text{"}\varphi\text{"} \mid \varphi \text{ is satisfiable and is in 3-CNF} \}$$

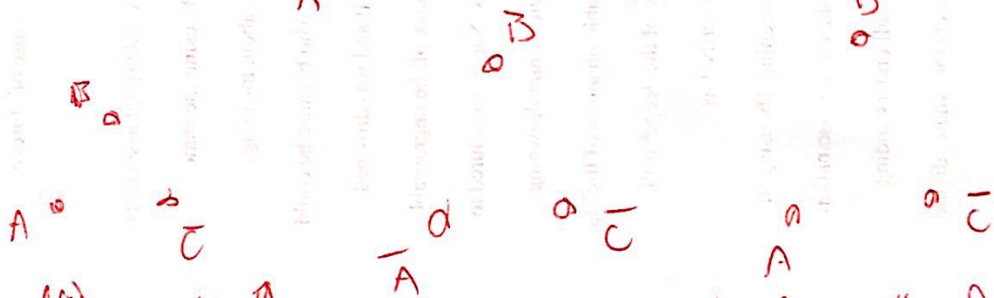
3 A formula is 3-CNF (3-conjunctive normal form) iff

it is a conjunction<sup>(AND)</sup> of "3-clauses" (arbitrarily many)  $(A \vee \bar{B} \vee C)$

A 3-clause is a disjunction (OR) of exactly 3 literals  $\rightarrow$

A literal is a variable or a negated variable.  $A \quad \overline{A}$   
3  $\overline{B}$

e.g.  $(A \vee B \vee \bar{C}) \wedge (\bar{A} \vee B \vee \bar{C}) \wedge (A \vee B \vee \bar{C}) \wedge \dots$



no edge in the  $\Delta$  for each clause, set  $h = \#$  of clauses