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A Brief Introduction to Quantum Annealing

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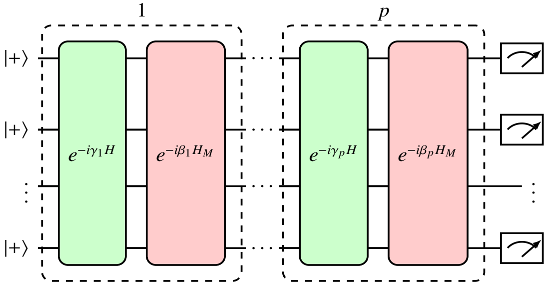
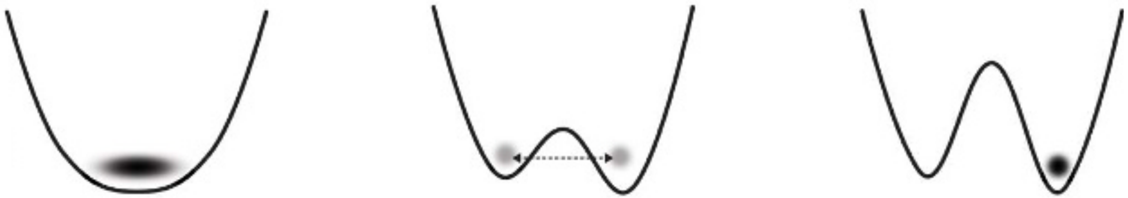
Optimization With Quantum Computers?

Gate Based: Variational Methods

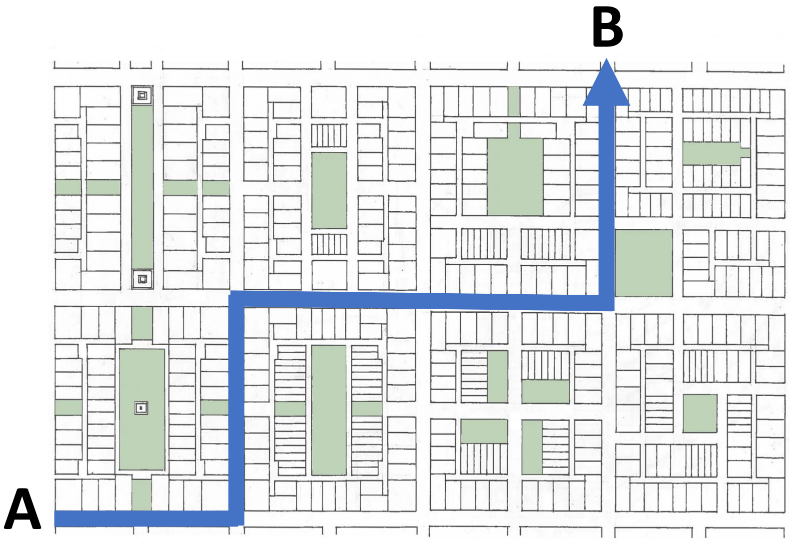


Analog: Quantum Annealing

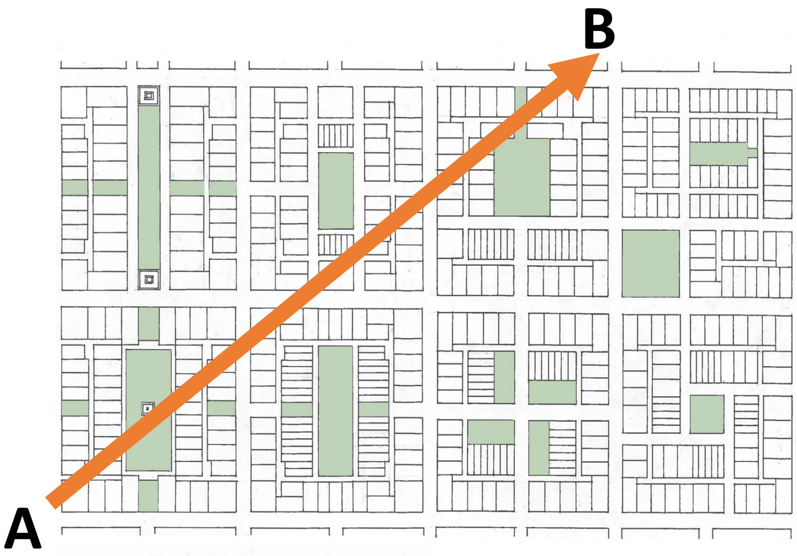


Why Analog?

It's fast!*



Gate Based



Analog

What is Quantum Annealing?

Minimize the eigenvalue of a “target” function

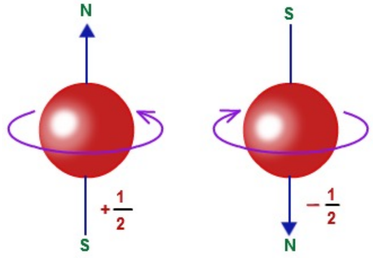
$$H = A(s)H_{\text{initial}} + B(s)H_{\text{target}}$$

Target is often the Ising Model

$$H_{\text{target}} = \underbrace{\sum_i h_i\sigma_i + \sum_{i<j} J_{ij}\sigma_i\sigma_j}_{\text{Classical}} \quad \text{where} \quad \sigma_i \in \{-1, 1\}$$

Why The Ising Model?

Directly realizable by some quantum systems



Quadratic Unconstrained Binary Optimization (QUBO)

QUBO

$$\min: \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} x_i x_j + \sum_{i \in \mathcal{N}} \boldsymbol{c}_i x_i$$

s.t.

$$x_i \in \{0, 1\} \; \forall i \in \mathcal{N}$$

$$\sigma_i = 2x_i - 1$$

Ising Model

$$\min: \sum_{i \in \mathcal{N}} \boldsymbol{c}_i \sigma_i + \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} \sigma_i \sigma_j$$

$$\text{s.t.: } \sigma_i \in \{-1, 1\} \; \forall i \in \mathcal{N}$$

$$x_i = \frac{\sigma_i + 1}{2}$$

What Kind of Problems Can I Encode?

Many Common Problems of Interest!*

Number Partitioning
Graph Partitioning
Cliques
Binary Integer Linear Programming
Exact Cover
Set Packing
Vertex Cover

3SAT
Minimal Maximal Matching
Set Cover
Integer Knapsack
Graph Coloring
Clique Cover
Job Sequencing

Hamiltonian Paths/Cycles
Minimal Spanning Tree
Steiner Trees
Directed Feedback Vertex Set
Feedback Edge Set
Graph Isomorphism

And more…

https://www.frontiersin.org/articles/10.3389/fphy.2014.00005/full

*With polynomial overhead in a lot of cases

How Do I Encode My Problem?

$$\sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma^z|0\rangle = |0\rangle$$

$$\sigma^z|1\rangle = -|1\rangle$$

$$|1\rangle \leftrightarrow -1 \leftrightarrow |\downarrow\rangle$$

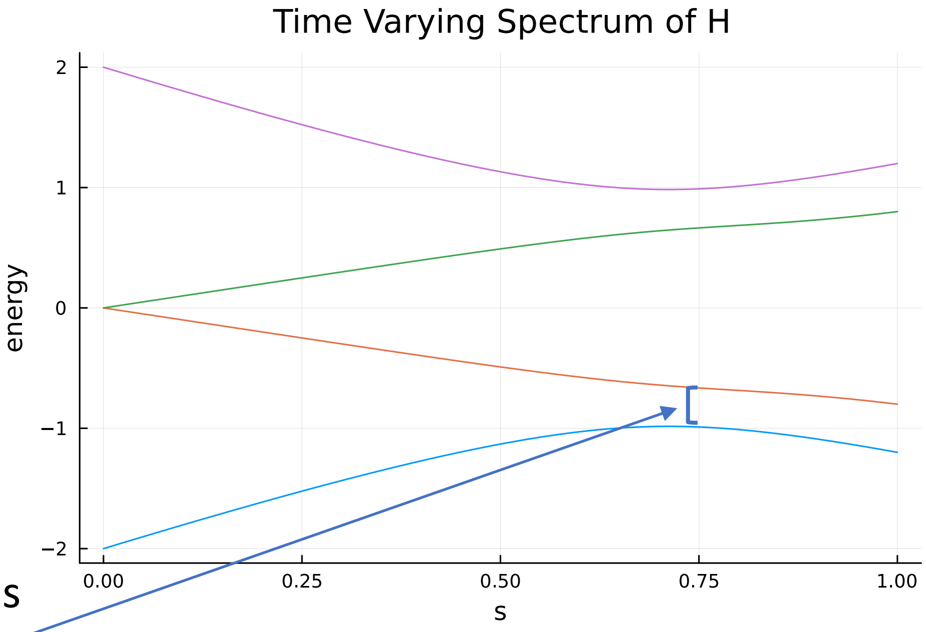
$$|0\rangle \leftrightarrow +1 \leftrightarrow |\uparrow\rangle$$

$$\Rightarrow \quad H_{\mathrm{target}} = \sum_i h_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z$$

How Do I Solve My Problem?

Slowly change from H_{initial} to H_{target}

Adiabatic Theorem: If you start in a minimal eigenvector of your initial Hamiltonian and change your system slowly enough, you will finish in the minimal eigenvector of your target Hamiltonian!



Time depends on this gap

What Should I Start With?

Start with easy initial problem

$$H_{\text{initial}} = -\sum_i \sigma_i^x$$

Known minimal eigenvector

$$|\psi_0\rangle = \bigotimes_i |+\rangle_i \quad \text{where} \quad |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

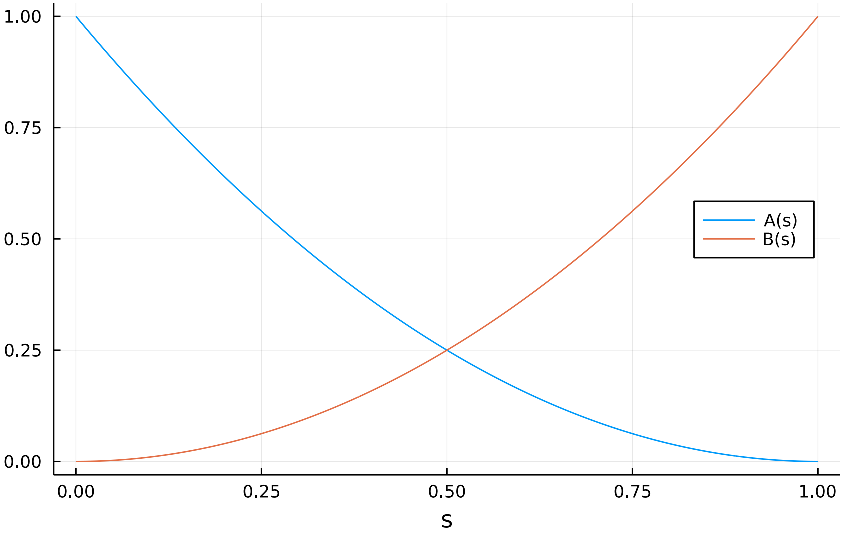
How Should I Interpolate?

Subject to annealing schedules $A(s)$ and $B(s)$

Required that $|A(0)| \gg |B(0)|$ and $|A(1)| \ll |B(1)|$

$$s = \frac{t}{t_{\text{total}}}$$

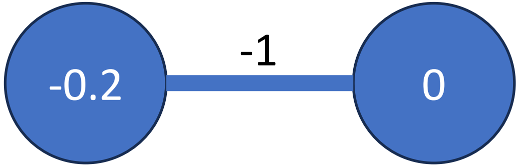
$$t \in [0, t_{\text{total}}]$$



Bringing It All Together

$$H(s) = -A(s)\sum_i \sigma_i^x + B(s)\left(\sum_i h_i \sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z \sigma_j^z\right)$$

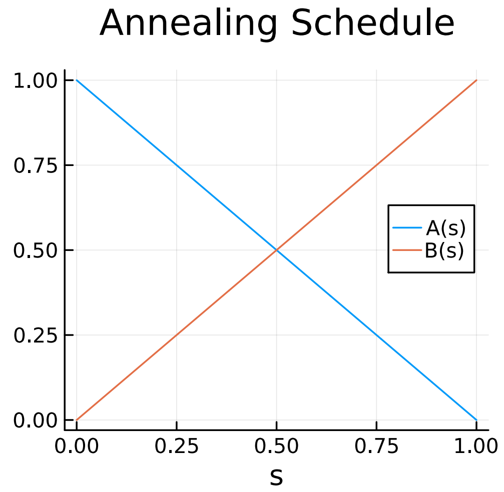
Example

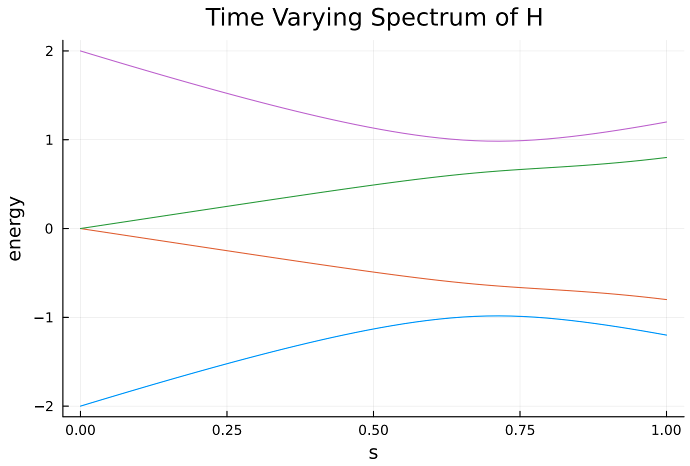


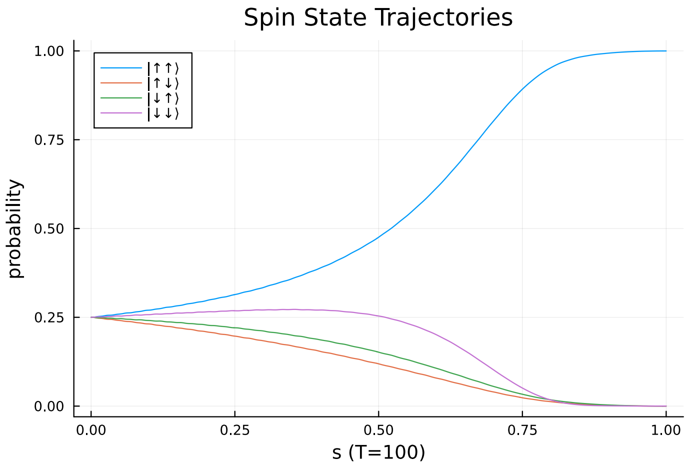
$$\min : -0.2\sigma_1 - \sigma_1\sigma_2$$
$$\text{s.t. } \sigma_1, \sigma_2 \in \{-1, 1\}$$

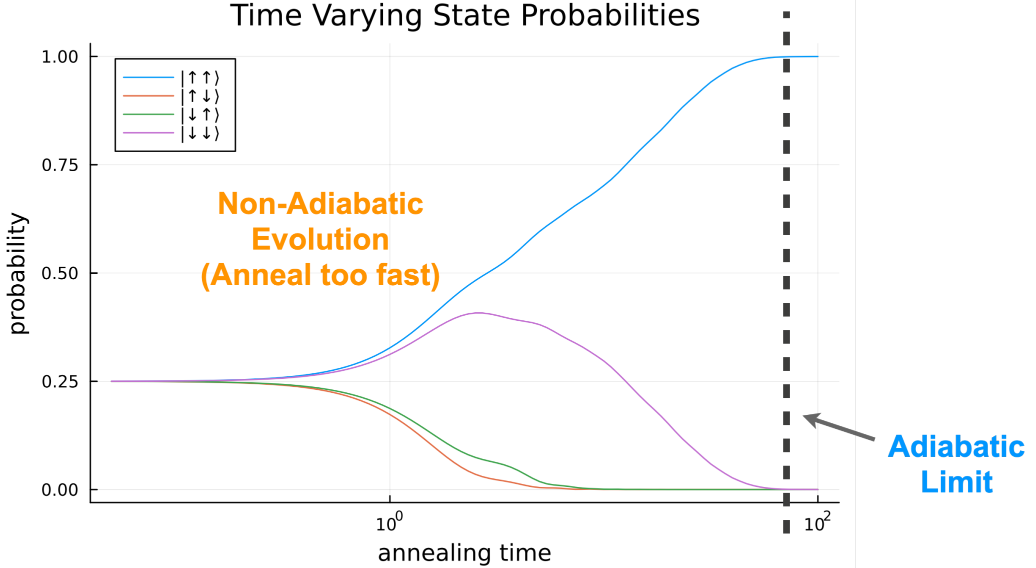
$$H(s) = (1-s)\left(-\sigma_1^x - \sigma_2^x\right) + s\left(-0.2\sigma_1^z - \sigma_1^z\sigma_2^z\right)$$

label	σ_1	σ_2	$f(\sigma)$
$\vert\uparrow\uparrow\rangle$	1	1	-1.2
$\vert\uparrow\downarrow\rangle$	-1	1	1.2
$\vert\downarrow\uparrow\rangle$	1	-1	0.8
$\vert\downarrow\downarrow\rangle$	-1	-1	-0.8



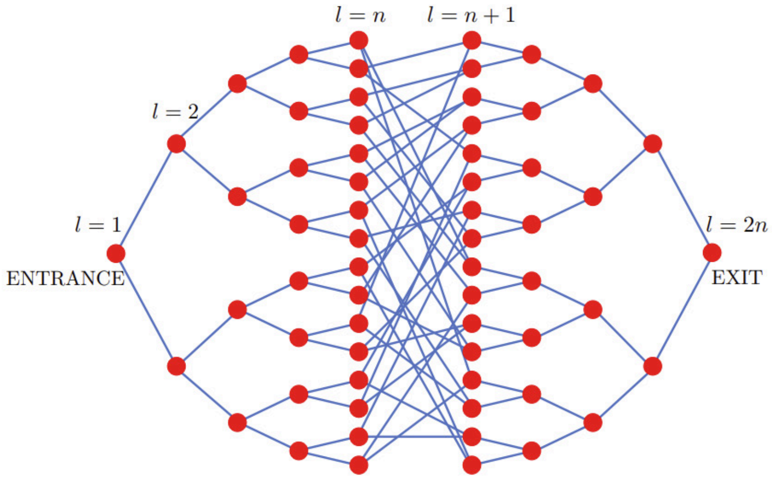

Time Varying Spectrum of H
energy
s


Spin State Trajectories
|↑↑⟩
|↑↓⟩
|↓↑⟩
|↓↓⟩
probability
s (T=100)

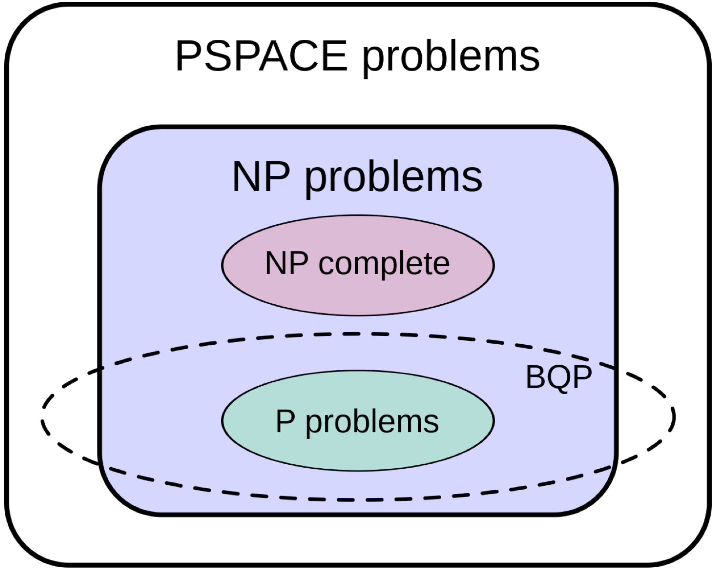

Time Varying State Probabilities
|↑↑⟩
|↑↓⟩
|↓↑⟩
|↓↓⟩
probability
annealing time
Non-Adiabatic Evolution (Anneal too fast)
Adiabatic Limit

Can I solve NP Hard Problems?

Not generally in polynomial time



https://arxiv.org/abs/1202.6257



https://en.wikipedia.org/wiki/BQP

Hardware Vendors

PASQAL

200 Qubits

|QuEra⟩

256 Qubits

D:Wave

5500 Qubits

Hardware Considerations

Qubit Count

Qubit Type

Connectivity

Native Hamiltonian

Energy Scale

Programable Annealing Schedules

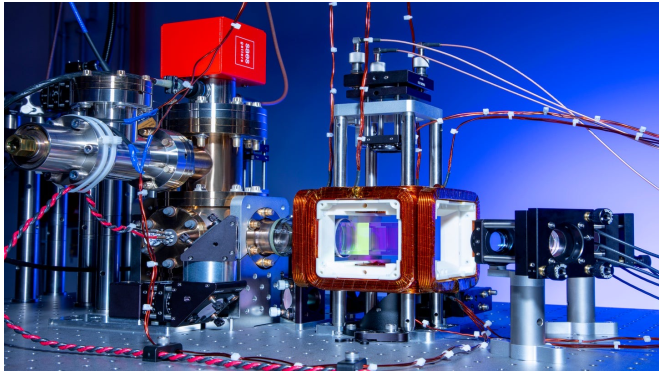
Noise Sources

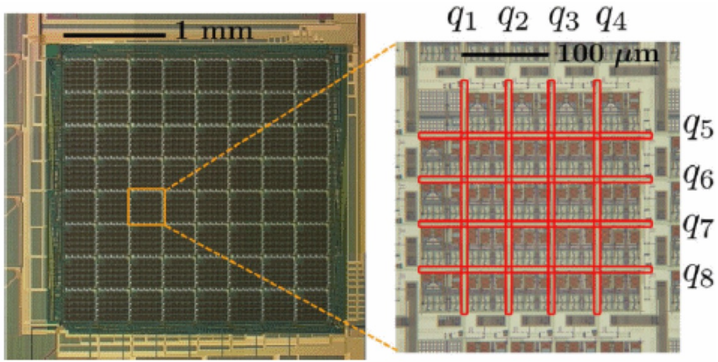
Coherence Times

Alternative Modes of Operation

Shot Rate

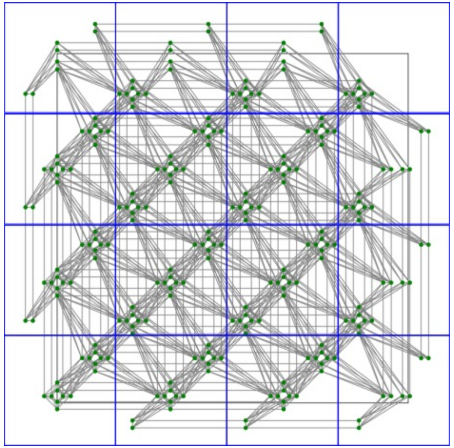
The list goes on…



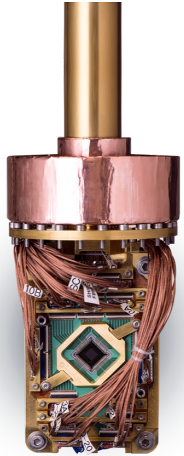


Superconducting Architecture

- Approach used by D-Wave
- Higher energy scale = Faster
- Low coherence times
- Highly connected
- Higher qubit counts
- More programable couplers
- Less programable annealing schedules



https://docs.dwavesys.com/docs/latest/c_gs_4.html#zephyr-graph



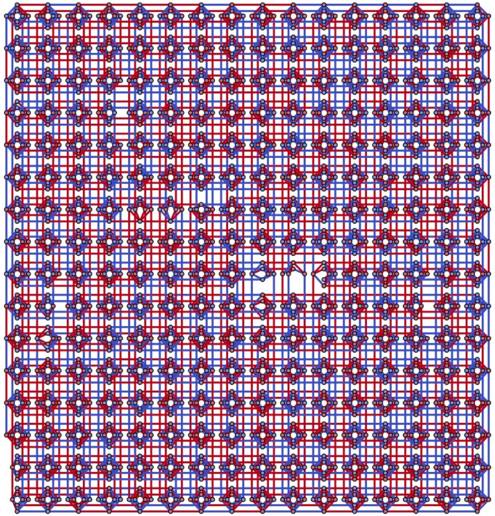
Optimization Achievements of Superconducting Annealers

Minimizing Spin Glasses

5,627 Qubits
40,279 Couplers (quadratic terms)
P16 Sparsity Pattern

$$\min : \sum_{i \in \mathcal{N}} \boldsymbol{c}_i \sigma_i + \sum_{i,j \in \mathcal{E}} \boldsymbol{c}_{ij} \sigma_i \sigma_j$$

$$\text{s.t.: } \sigma_i \in \{-1, 1\} \; \forall i \in \mathcal{N}$$

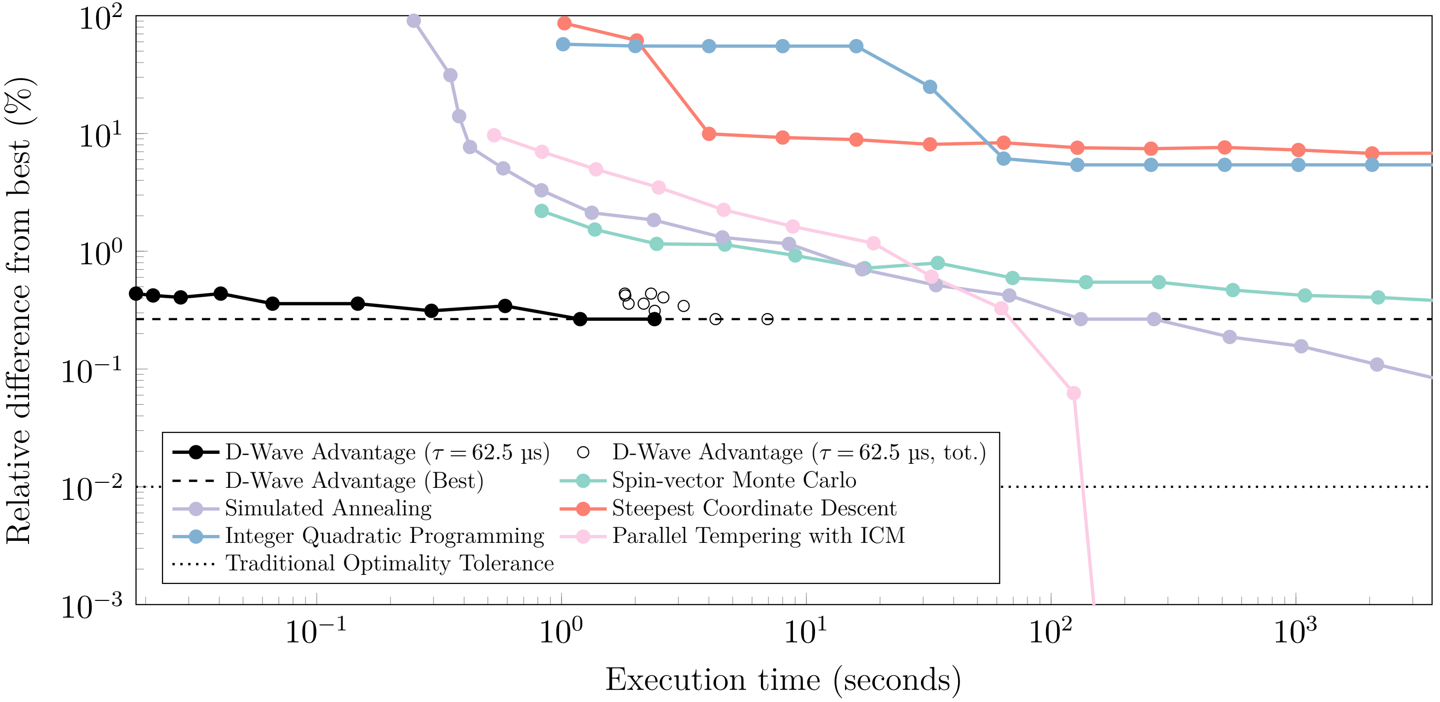


$$P(c_i = 0) = 0.15,\; P(c_i = -1) = 0.85,\; P(c_i = 1) = 0,\; \forall i \in \mathcal{N}$$

$$P(c_{ij} = 0) = 0.35,\; P(c_{ij} = -1) = 0.10,\; P(c_{ij} = 1) = 0.55,\; \forall (i,j) \in \mathcal{P}_{16}$$

https://arxiv.org/pdf/2210.04291

Optimization Achievements of Superconducting Annealers



https://arxiv.org/abs/2210.04291

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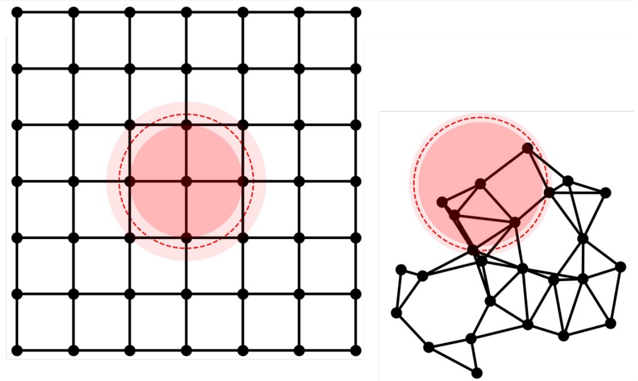
Neutral Atom Architecture

- Approach used by QuEra and Pasqal
- Lower Energy Scale = Slower
- High coherence times
- Spatially Connected
- Lower Qubit Counts
- Less programable couplers
- More programable annealing schedules

PASQAL |QuEra⟩



https://queracomputing.github.io/Bloqade.jl/stable/tutorials/1.blockade/main/

Optimization Achievements of Neutral Atom Annealers

MIS on Unit Disk Graphs

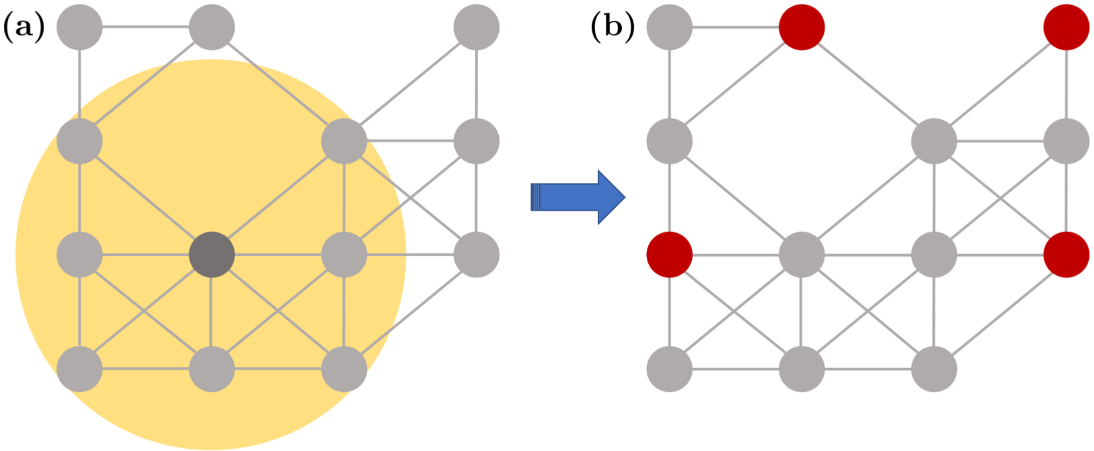
289 Qubits, 894 Couplers, 80% filled King's Graph

$$\max \quad \sum_i x_i$$

$$\text{s.t.} \quad x_i + x_j \leq 1 \quad \forall (i,j) \in \mathcal{E}$$

$$x_i \in \{0,1\} \quad i = 1, \ldots, N$$

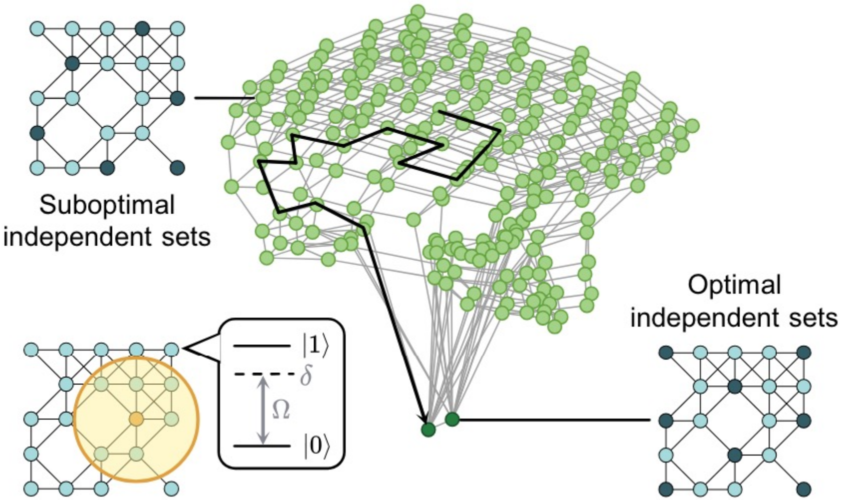
$$H_{\text{MIS}} = -\sum_i \sigma_i^z + P \sum_{(i,j)\in\mathcal{E}} \sigma_i^z \sigma_j^z$$

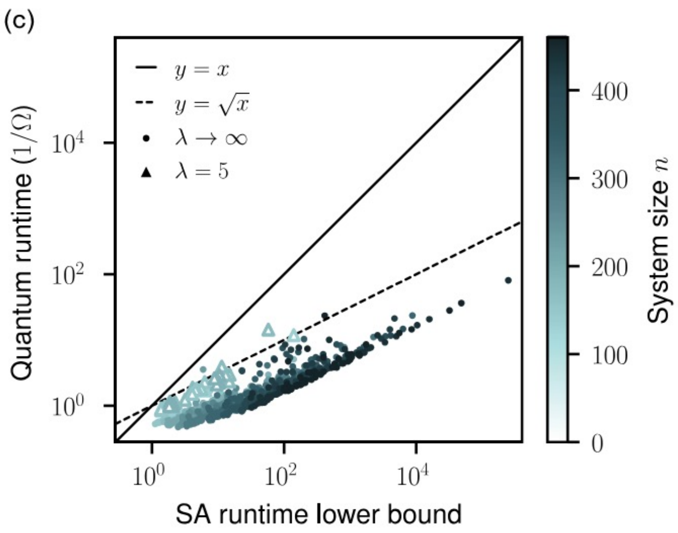


https://arxiv.org/pdf/2307.09442

Optimization Achievements of Neutral Atom Annealers

*Only compares against Markov Chain based methods





https://arxiv.org/abs/2306.13123
https://arxiv.org/pdf/2307.09442

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Why Didn't My Favorite Problem Perform Well?

Several Possible Reasons

- Logical Chains
- Biased Hardware
- Penalty Terms
- Too Much Structure
- Too Few Qubits

⋮

You need to walk before you can run, but that leaves a lot of research opportunity

So What Should I Use?

No clear answer!

Classical optimizers are really good

Hybrid algorithms?

Superconducting annealers are probably more mature quantum optimizers for now

Further Reading

Comprehensive Review Article of Quantum Annealing

- https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.90.015002

D-Wave Quantum Supremacy Demonstration (Quantum Dynamics)

- https://arxiv.org/abs/2403.00910

Neutral Atom MIS Scaling Studies

- https://www.science.org/doi/full/10.1126/science.abo6587
- https://arxiv.org/abs/2307.09442

Speedup over single threaded classical computing on a toy problem using D-Wave

- https://arxiv.org/abs/2210.04291

Comparison of Analog and Gate Based Hardware for simulating Ising Model dynamics

- https://arxiv.org/abs/2402.17667