Theory of Vibration

Analysis of the dynamic behavior of an offshore wind turbine

October 12, 2023

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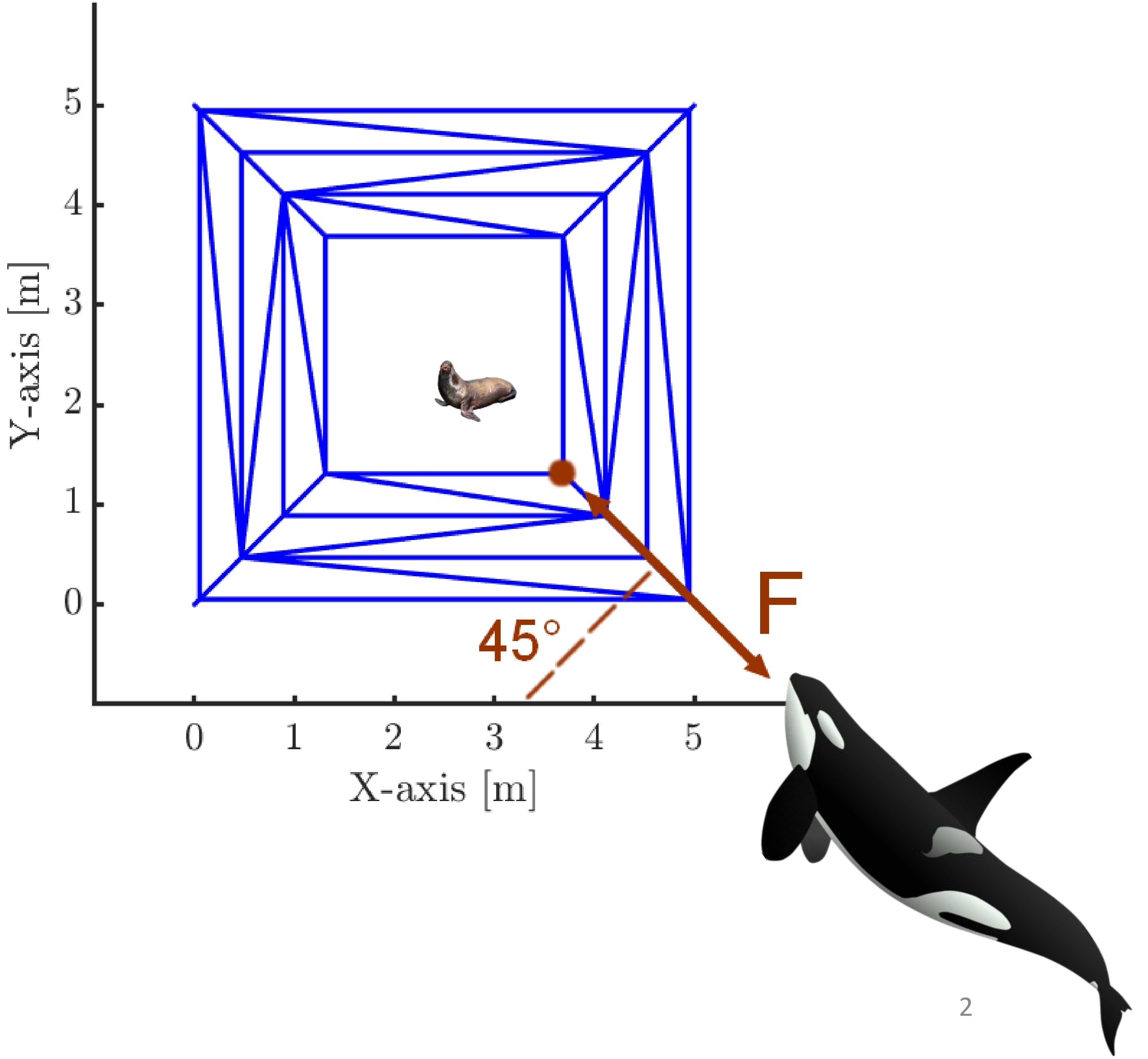
Transient response to a whale attack

- Attack of killer whales on the structure
- **Synchronous excitation** in the form of a sine wave $F = A\sin(\omega t)$
  - Parameters of the sine can be computed with the mass and velocity of the whale tail, and the impact properties:
  - **m = 1 [t] ; v = 25 [km/h] ; f = 1 [Hz] ; t_{impact} = 0.05 [s] ; 85 [%] of the momentum is conserved**



Modal superposition: concept

- The **exact** dynamic response to a loading $\mathbf{p}(t)$ can be computed using **modal expansion**
- In that case, eigenmodes $\mathbf{x}_r$ are used as a basis to represent the response of the structure $\mathbf{q}(t)$

Total number of modes

$$\mathbf{q}(t) = \sum_{r=1}^{n} \eta_r(t)\mathbf{x}_r$$

Eigenmodes of the structure

Modal superposition: concept

- $\eta_r(t)$ is such that:

$$\ddot{\eta}_r + 2\varepsilon_r\omega_r\dot{\eta}_r + \omega_r^2\eta_r = \Phi_r(t)$$

with $\Phi_r(t) = \dfrac{\mathbf{x}_r^T\mathbf{p}(t)}{\mu_r}$ and $\mu_r = \mathbf{x}_r^T\mathbf{M}\mathbf{x}_r$

- A general solution of this second order differential equation is:

$$\eta_r(t) = e^{-\varepsilon_r\omega_r t}\left(A\cos\left(\omega_r^d t\right) + B\sin\left(\omega_r^d t\right)\right) + \boxed{\int_0^t \Phi_r(\tau)h(t-\tau)d\tau}$$

A and B can be determined from the initial conditions **(jacket initially at rest)**

$A = 0$

$B = 0$

PAR CDT

$\eta(0) = 0$ & $\dfrac{d\eta}{dt} = 0$

$\hookrightarrow B \sin\left(\omega_r^d \, 0{,}05\right) = -\int_0^{0{,}05} \phi_1(t)\, h(t-\tau)\, d\tau$

Modal superposition: concept

- Impulse response function in the damped case:

$$h(t) = \frac{1}{\omega_r^d} e^{-\varepsilon_r \omega_r t} \sin\left(\omega_r^d t\right) \qquad t > 0$$

- ω_r^d is the *r*-th damped frequency: $\omega_r^d = \omega_r \sqrt{1-\varepsilon^2}$
- ω_r [rad/s] is the *r*-th eigenfrequency (computed in part 1)
- ε_r [-] is the *r*-th damping ratio

Mode displacement method

Computation time too high if all the modes are included in the sum

Assumption: The response lies on a **subspace of the total basis** so that only **some modes** participate to the response

→ k modes are retained where k < n

$$\mathbf{q}(t) = \sum_{r=1}^{k} \eta_r(t)\mathbf{x}_r$$

Mode acceleration method

- Only **inertia and viscous forces** are expanded in the truncated modal basis

$$\mathbf{K}\mathbf{q} = \mathbf{p} - \mathbf{M}\ddot{\mathbf{q}} - \mathbf{C}\dot{\mathbf{q}}$$

- The final solution obtained in this case is:

$$\mathbf{q}(t) = \underbrace{\sum_{r=1}^{k} \eta_r(t)\mathbf{x}_r}_{\text{Mode displacement method}} + \left(\underbrace{\mathbf{K}^{-1}\mathbf{p}(t)}_{\text{Static response of the complete structure}} - \underbrace{\sum_{r=1}^{k} \frac{\Phi_r(t)}{\omega_r^2}\mathbf{x}_r}_{\text{Static response for modes 1 to } k \text{, already included in the mode displacement method}} \right)$$

Mode displacement method

Static response of the complete structure

Static response for modes 1 to k, **already included** in the mode displacement method

Part 2A: Transient response with mode superposition methods

- Compute the **damping matrix** using the **proportional damping** assumption.
- Compute an approximate solution using the **mode displacement method**.
- Compute an approximate solution using the **mode acceleration method**.
- **Compare** the results and discuss the **convergence** in terms of the number of modes included in the superposition.

Recommended deadline: November 16, 2023

Good work!