The Magnetic Furnace: Examining Fully Convective Dynamos and the Influence of Rotation

Kyle Augustson[1], S. Mathis[1], A. S. Brun[1], & J. Toomre[2]

[1]CEA, [2]JILA

CoolStars 19 – Variability of Solar/Stellar Magnetic Activity

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Fully Convective Dynamos in Rapidly Rotating Regimes

- Liquid Cores of Rocky Planets
- Giant Planets with Deep Convection Zones
- Low Mass M Dwarfs
- Red Giant He Burning Cores
- Cores of Main-Sequence Intermediate and High Mass Stars

Questions Potentially Addressed with Global-Scale Simulations

What differences arise between HD and MHD?

Impact for 1-D models

How do a greater luminosity and a changing rotation impact a dynamo?

+ Magnetostrophy and possible Rossby number scaling

+ Sensitivities of core dynamo models to diffusion

e.g. Low-Pm vs. High-Pm

Can superequipartition states be sustained?

What are the limiting behaviors of the system?

How superequipartition can they become?!

Some Simple Considerations for Scaling Laws

- Consider a statistically steady state with the following force balance for a non-rotating system:

$$\rho \mathbf{v} \cdot \nabla \mathbf{v} \approx \frac{1}{4\pi} \boldsymbol{\nabla} \times \mathbf{B} \times \mathbf{B}.$$

- Further, let

$$\ell_v = Pm \ell_B$$

- Then, the equipartition magnetic field should roughly be

$$\frac{4\pi \ell_B}{\ell_v} \rho v^2 \approx B^2 \implies B_{\text{eq}} \approx \left[\frac{4\pi \rho v^2}{Pm} \right]^{1/2}$$

Some Simple Considerations for Scaling Laws

- Extend this statistically-steady force balance to a rotating system:

$$\alpha\rho\mathbf{v}\cdot\nabla\mathbf{v}+2\rho\mathbf{v}\times\hat{\Omega}_0 \approx \frac{1}{4\pi}\nabla\times\mathbf{B}\times\mathbf{B},$$

- Then, the super-equipartition magnetic field may scale as

$$\Rightarrow \frac{\alpha}{\ell}\rho v^2+2\rho v\Omega_0 \approx \frac{B^2}{4\pi\ell},$$

$$\Rightarrow \frac{B^2}{8\pi} \approx \frac{1}{2}\rho v^2\left(\alpha+2\ell\Omega_0/v\right),$$

$$\Rightarrow \frac{ME}{KE} \approx \alpha+Ro^{-1}.$$

Some Simple Considerations for Scaling Laws

- An alternative approach from Davidson 2013: the MAC Balance

$$\left|\int_{R_C} \beta \overline{T'\boldsymbol{u}} \cdot \boldsymbol{g} dV\right| = \int_{R_C} \overline{\boldsymbol{J}^2} \Big/ \rho\sigma dV \sim \frac{\boldsymbol{J}^2}{\rho\sigma} V_C \sim \frac{B^2}{\rho\mu} \frac{\lambda}{\ell_{\min}^2} V_C,$$

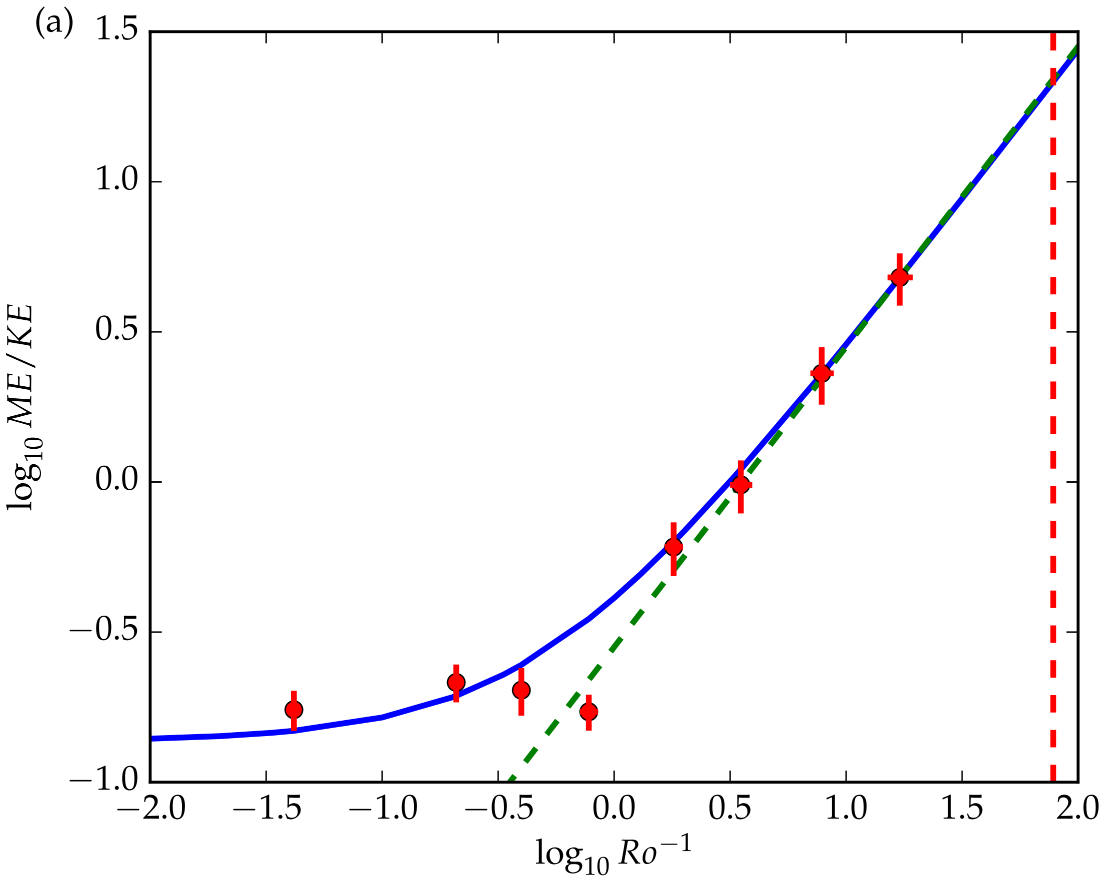
- Or

$$f(\mathrm{Pr}_m, Ro) \frac{1}{V_C} \left|\int_{R_C} \beta \overline{T'\boldsymbol{u}} \cdot \boldsymbol{g} dV\right| \sim \frac{B^2}{\rho\mu} \omega_{\mathrm{small}},$$

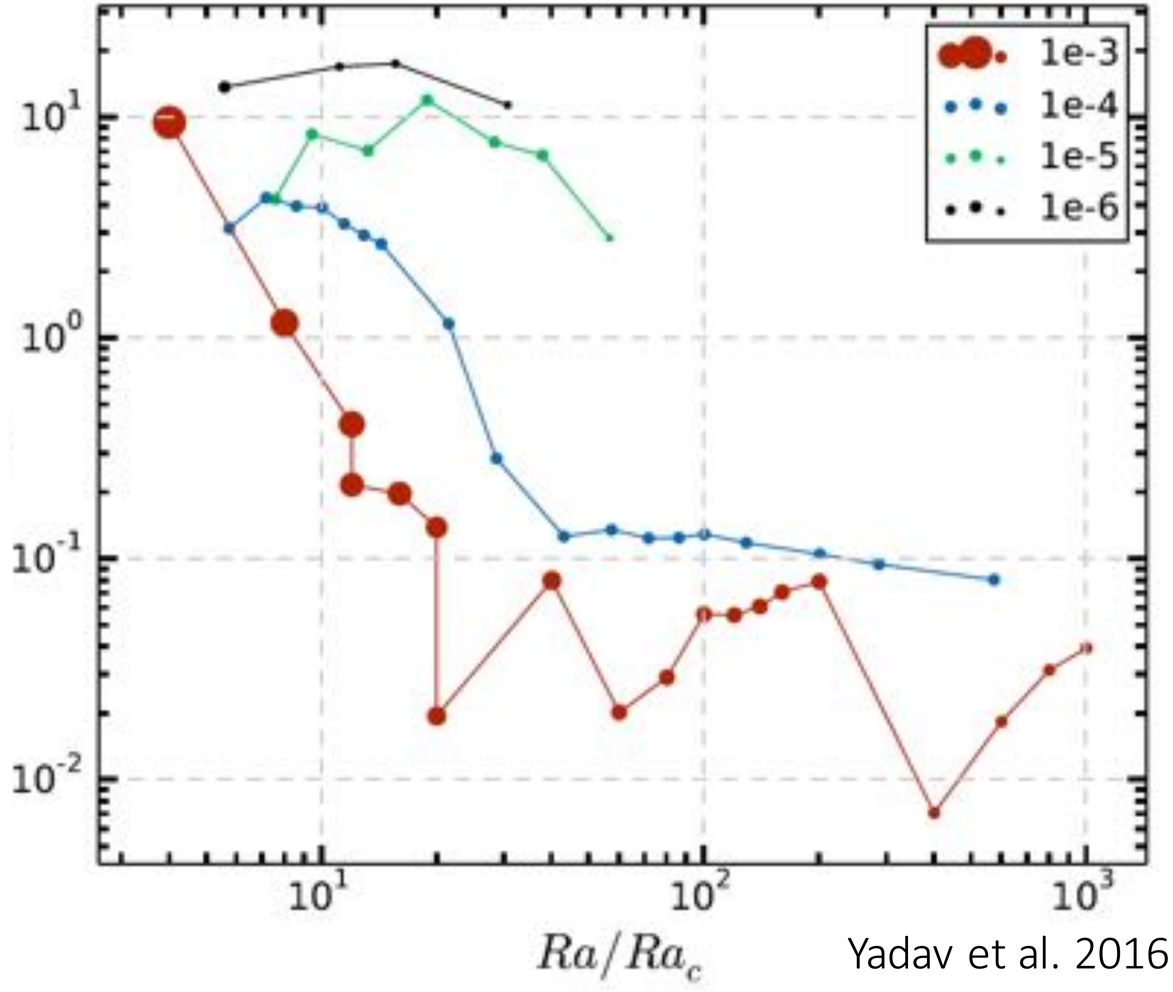
- Predicts that super-equipartition magnetic field may scale as

$$\Rightarrow \frac{ME}{KE} \approx \alpha + Ro^{-1/2}.$$

Some Simple Considerations for Scaling Laws



Augustson et al. 2016



Yadav et al. 2016

The ASH Code

ASH (Anelastic Spherical Harmonic) code

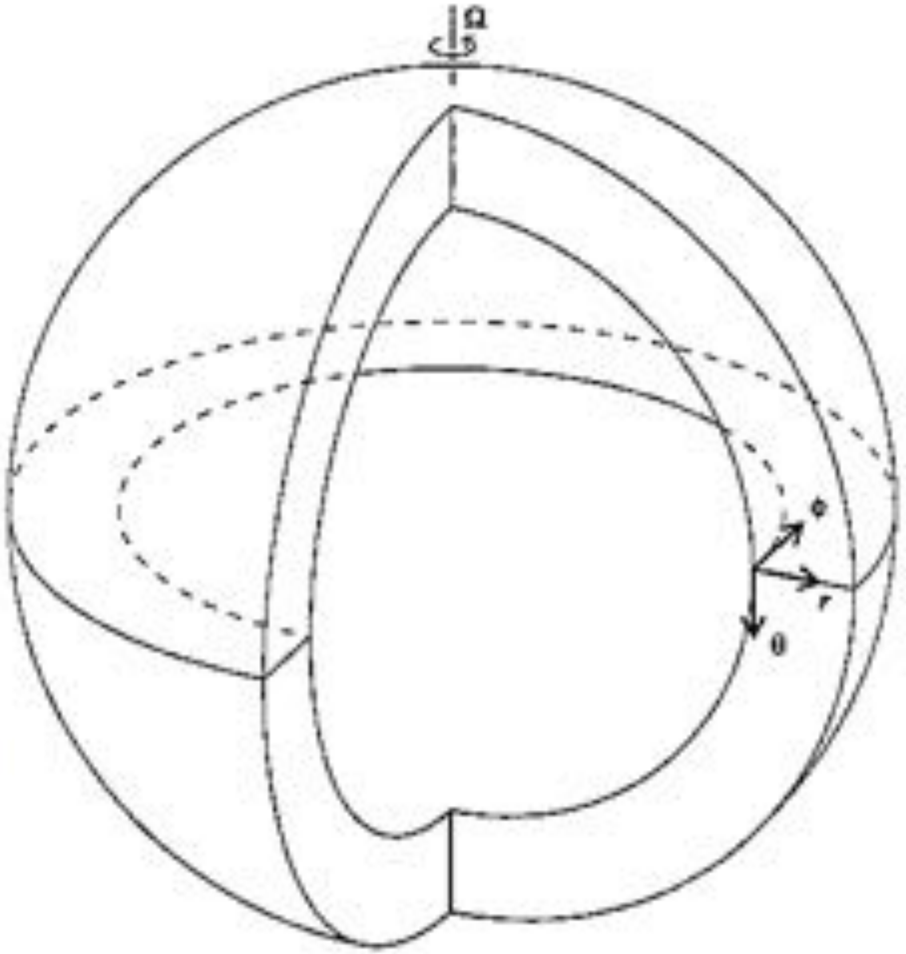
- Parallel pseudospectral code
- Spherical harmonic & Chebyshev or Finite-difference decomposition
- Semi-implicit time-stepping
- Realistic stratification
- Including a stratified stable layer
- Magnetism

$$\partial \mathbf{v}/\partial t = -\mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v} - \boldsymbol{\nabla}\varpi + S c_{\mathrm{P}}^{-1}\mathbf{g} - \Lambda\hat{\mathbf{r}} + 2\mathbf{v}\times\boldsymbol{\Omega}_0$$

$$+ (4\pi\overline{\rho})^{-1}(\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B} + \overline{\rho}^{-1}\boldsymbol{\nabla}\cdot\mathcal{D},$$

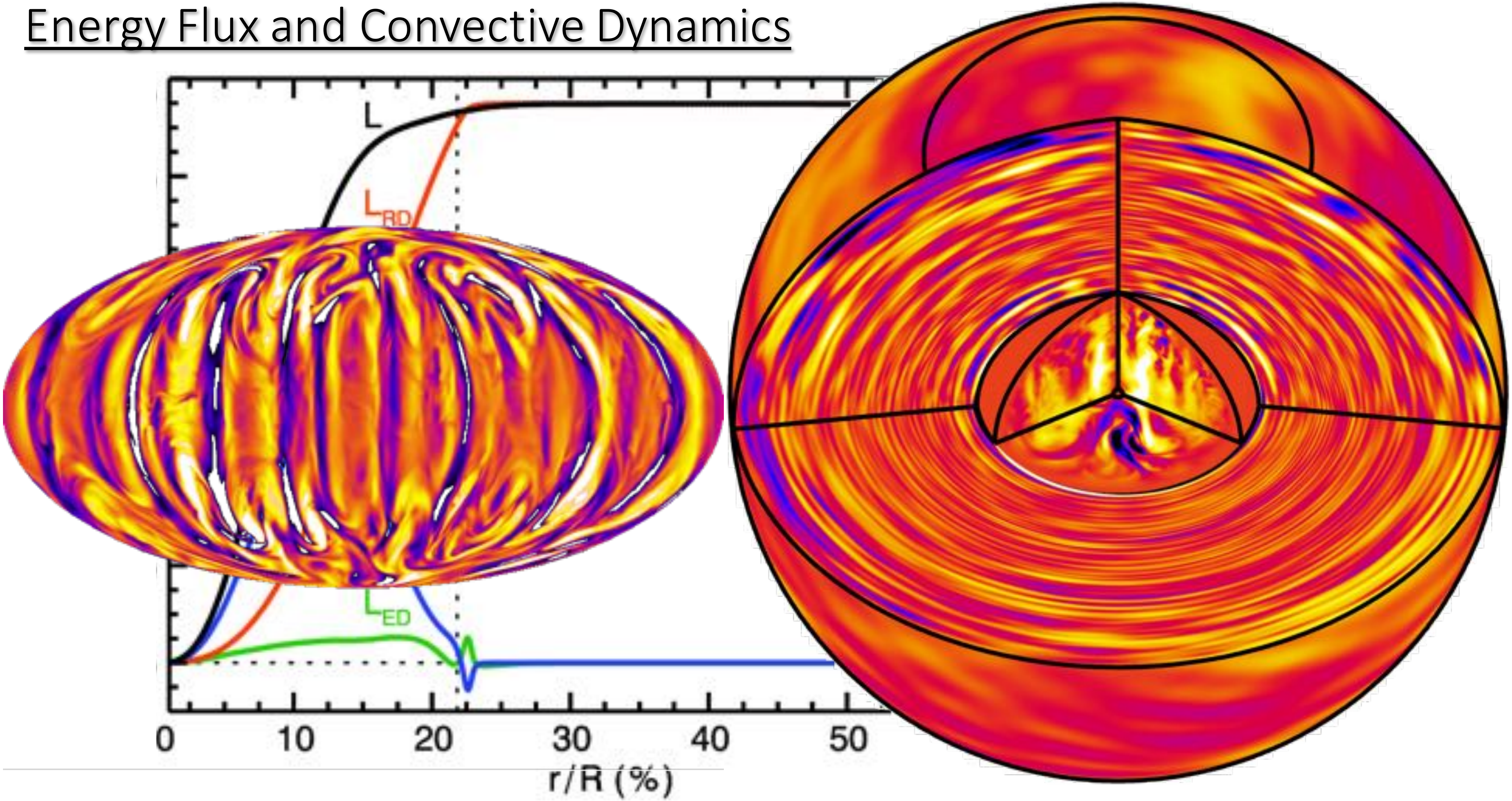
$$\partial \mathbf{B}/\partial t = \boldsymbol{\nabla}\times[\mathbf{v}\times\mathbf{B} - \eta\boldsymbol{\nabla}\times\mathbf{B}],$$

$$\partial S/\partial t = -\mathbf{v}\cdot\boldsymbol{\nabla}(\overline{S}+S) + (\overline{\rho}\overline{T})^{-1}[\nabla\cdot\mathbf{q} + \Phi + \epsilon],$$

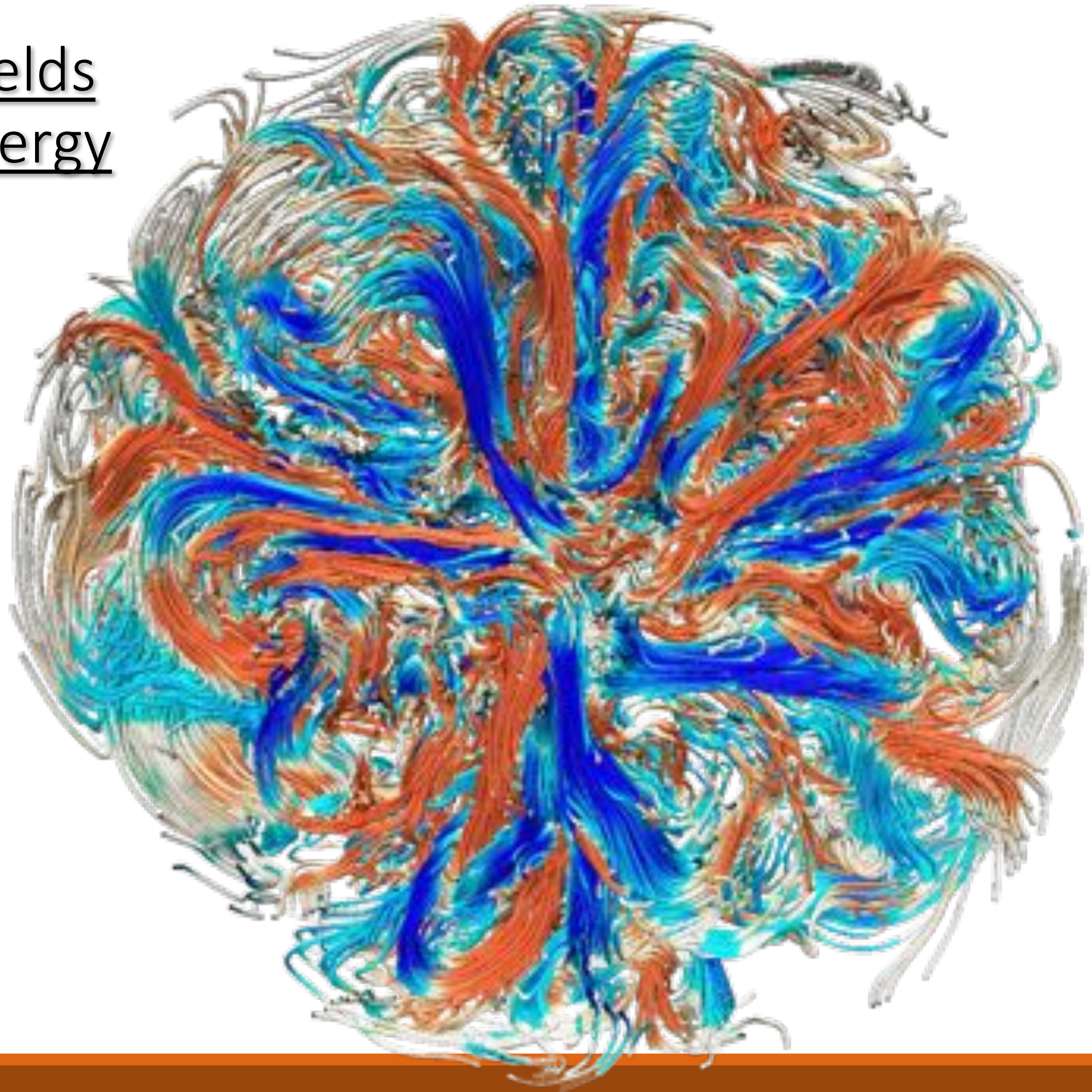


Clune et al. 1999; Miesch et al. 2000; Brun et al. 2004

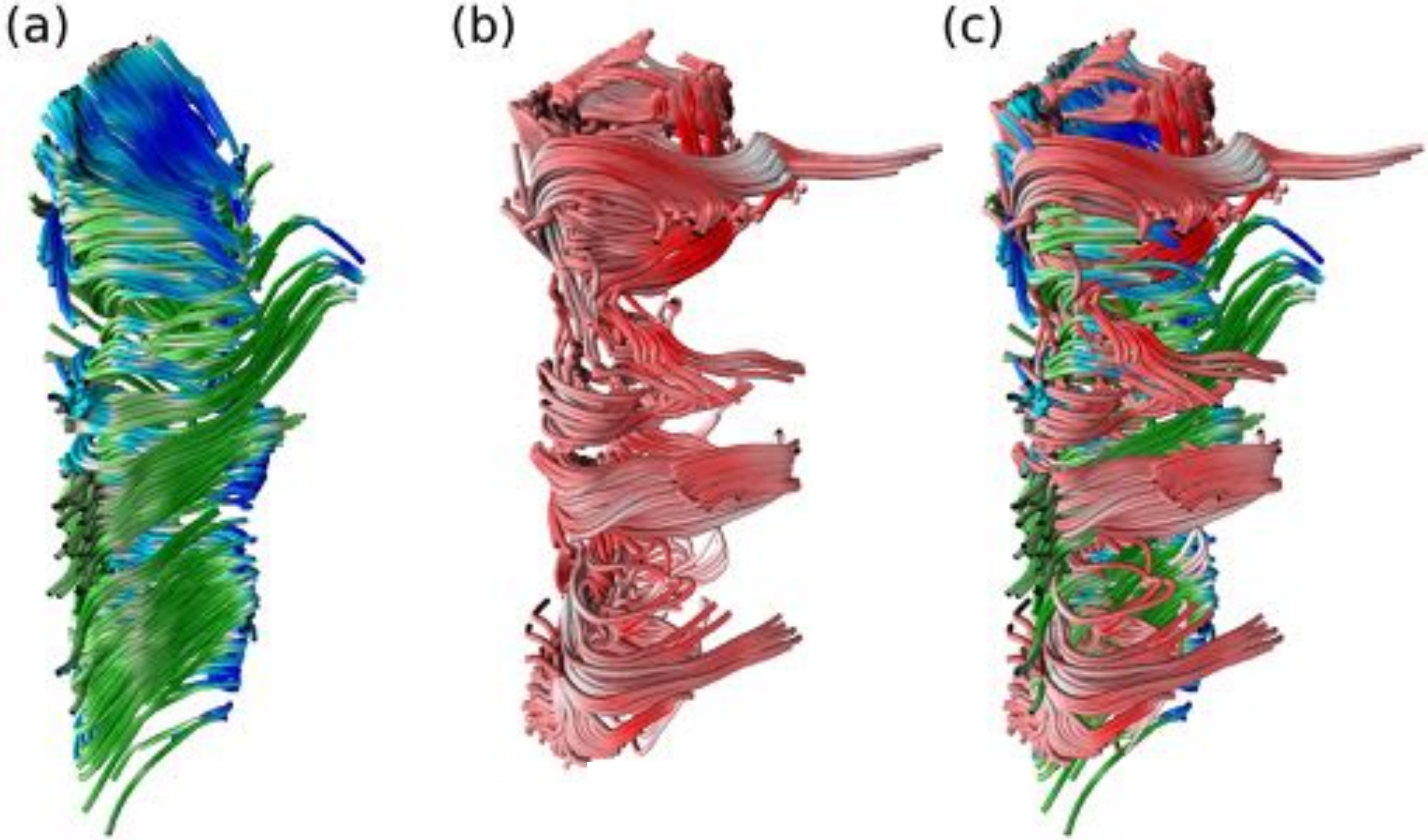
Energy Flux and Convective Dynamics



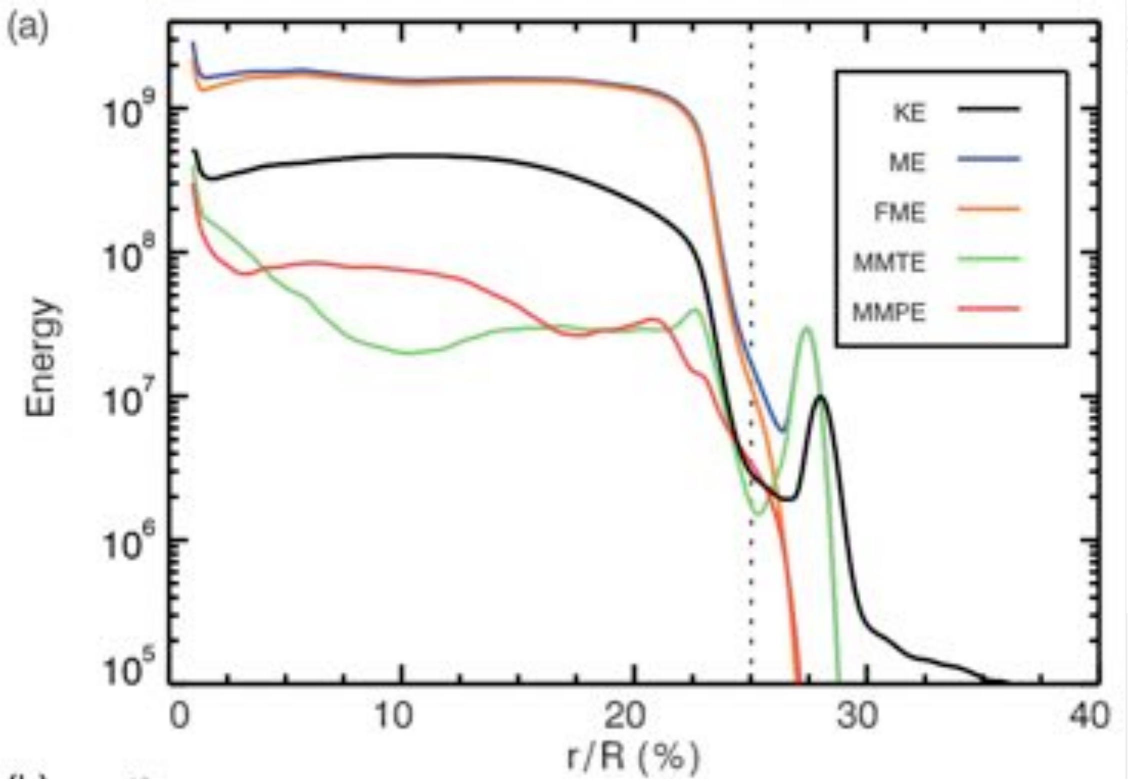
Magnetic Fields and Their Energy

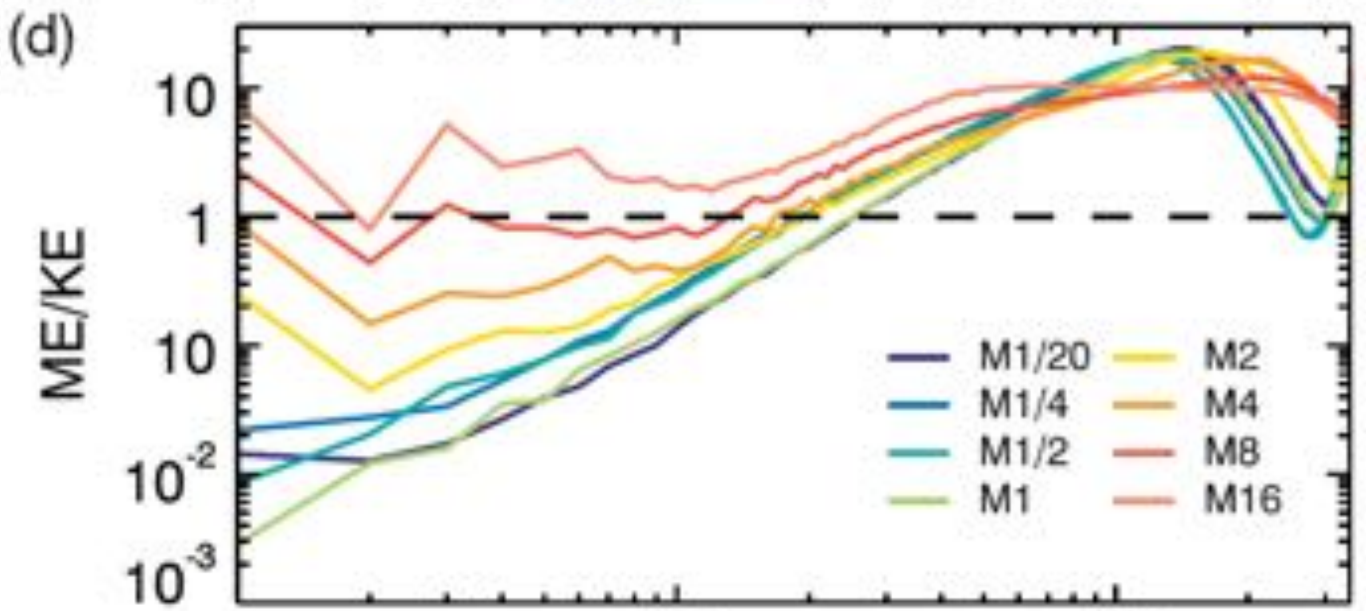


Magnetic Fields and Their Energy



Superequipartition Across Resolved Scales

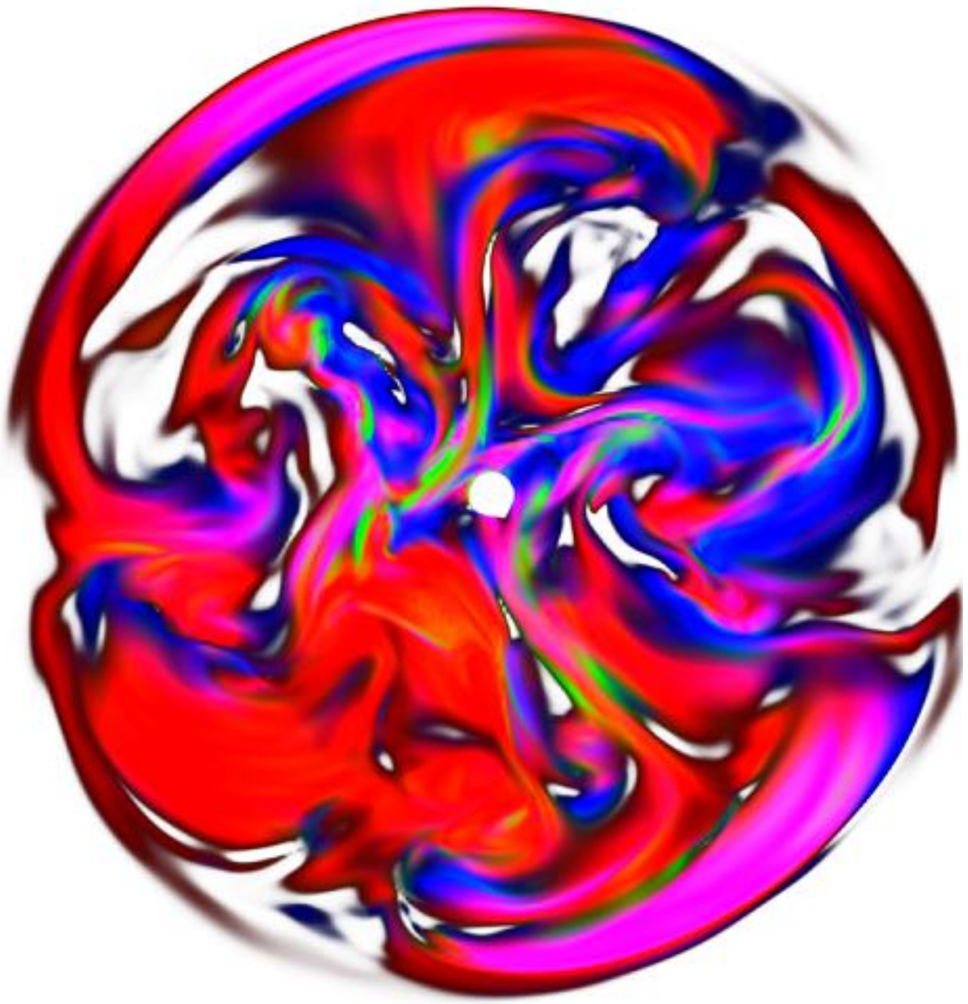




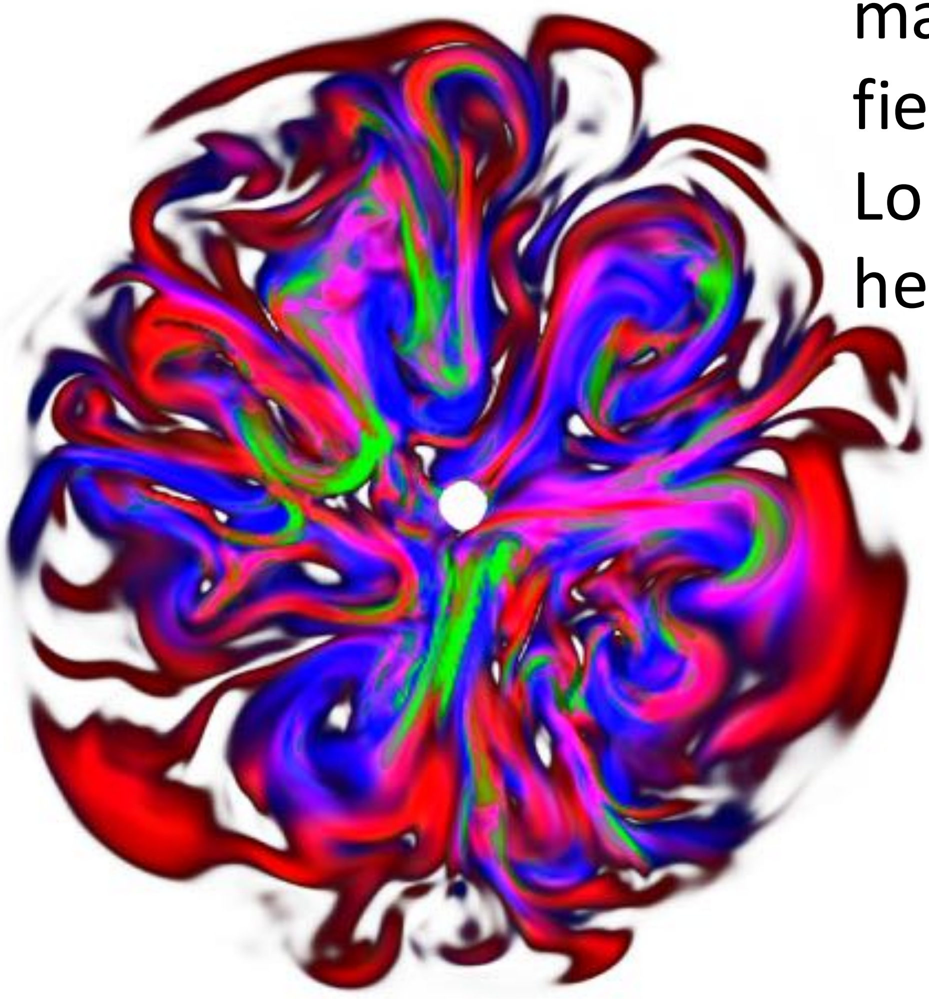
Augustson et al. 2016

How can such states exist?

In these simulations, displacement of magnetic and velocity fields minimizes Lorentz forces on heat-carrying flows.

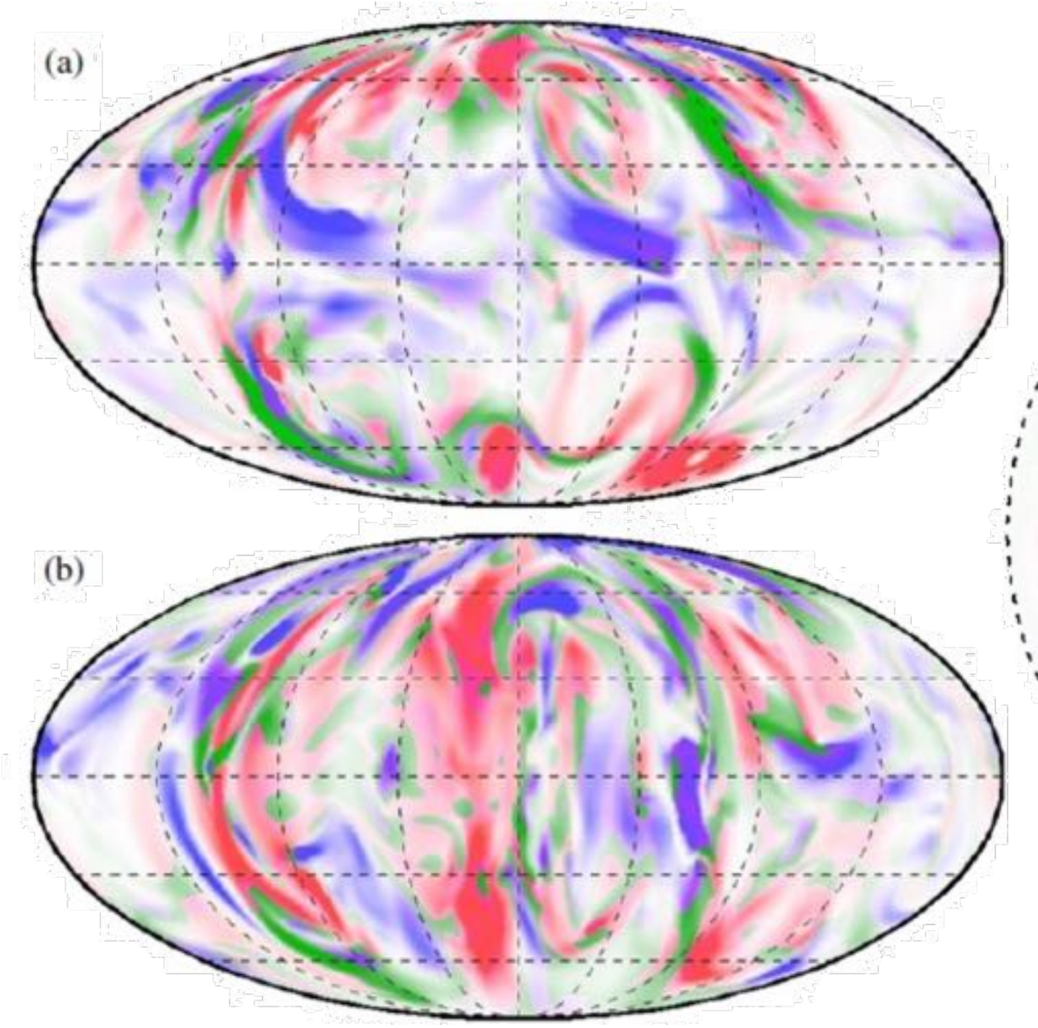


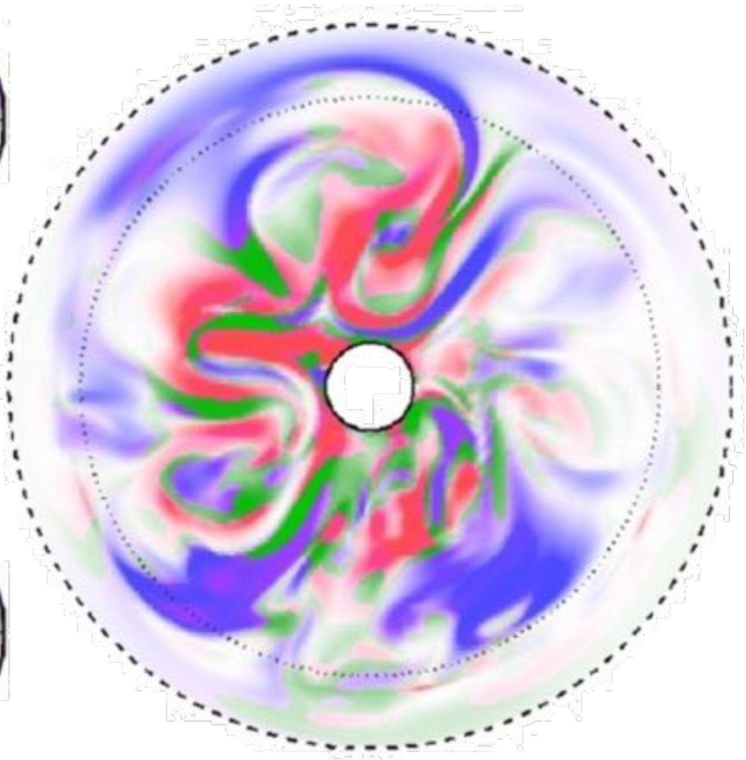
Strong-Field Initial Condition



Weak-Field Initial Condition

How can such states exist?



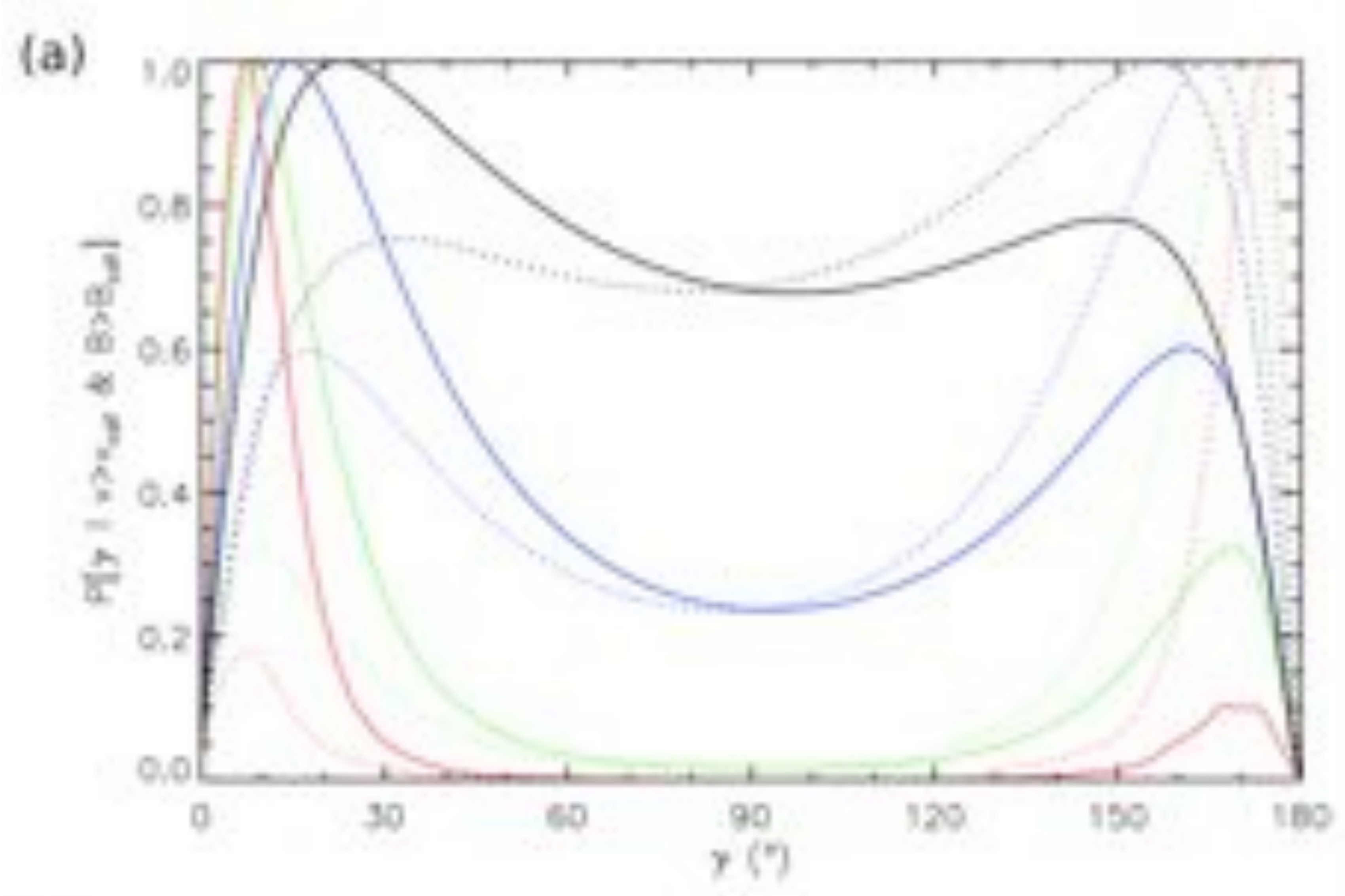


The displaced fields have weak generation, while generation occurs largely in the overlap regions, namely at the edges of the magnetic structures.

So how might the magnetic structures propagate?

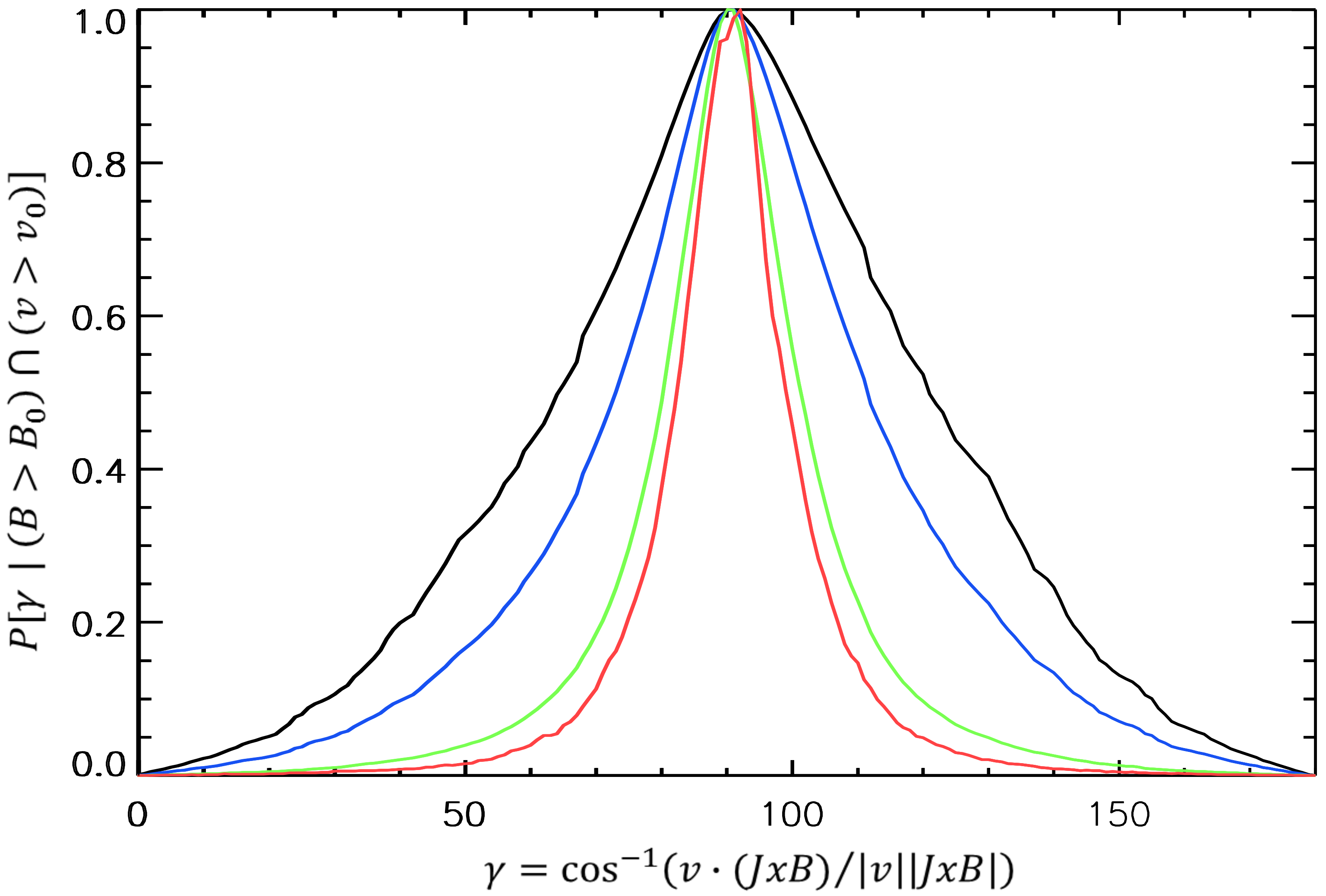
Featherstone et al. 2009

How can such states exist?



Regions of increasing velocity and magnetic field magnitudes are increasing aligned, regulating field generation

How can such states exist?



The velocity field adjusts to minimize the work done by Lorentz forces.

Implications for Rapidly Rotating Objects

Superequipartition convective dynamos are likely above a threshold Rossby number

Such dynamos avoid magnetic quenching through non-local interactions

Minimizing the Lorentz force

Optimizing the induction

Simple Scaling relationships may provide guidance for 1D models regarding MLT and Dynamo behaviors

See the posters of Laura Curie (#182) and

Felix Sainsbury-Martinez (#56)