Mean-field study of the equation of states of nuclear matter and tidal deformation of neutron star

Bachelor Thesis

Author:
Nguyen Hoang Dang KHOA

Supervisors:
Prof. Dao Tien KHOA
Dr. Ngo Hai TAN

May 9, 2021

Contents

List of Abbreviations

BH   Black hole

EoS   Equation of States

GE   Gravito-electric
GM   Gravito-magnetic
GR   General Relativity
GRB   Gamma-ray burst
GW   Gravitational wave

HF   Hartree-Fock

NM   Nuclear Matter
NN   Nucleon-Nucleon
NS   Neutron Star

TOV   Tolmann-Oppenheimer-Volkoff

List of Tables

List of Figures

Chapter 1

Introduction

Neutron stars (NS) are star-like astronomical objects with mass M on the order of solar mass ($M_\odot$), a radius of $\sim 10 - 12$ km and an average density n several times greater than that of nucleon ($\rho_0 \approx 0.16$ fm^{-3}). They are arguably the densest accessible objects, excluding black holes which we know nothing about inside the event horizon, in the universe [4]. Due to extremely high density, the matter on NS mainly consists of neutrons that are closely packed together with a small percentage of other particles (p, e^-, ...), similar to a atomic nucleus on macroscopic scale. For this reason, they are also the ideal objects for testing physical theories of dense matter and provide connections between different field of physics, i.e. nuclear physics, elementary particle physics and astrophysics [5].

During the NS's formation process, protons (p) and electrons (e^-) combined together to form neutrons, i.e.

$$p + e^- \longrightarrow n + \nu_e \tag{1.1}$$

and the star only holds itself against gravity by its own degeneracy pressure and strong force repulsion, which explains why the matter on NS is neutron-dominant and hence the name "neutron stars". After the NS is formed, energy quickly dissipates through neutrino emission, resulting in a relatively cold NS. In this study, we will only concern with the NS after a considerable time from its formation, when the temperature is considered to be $T = 0$ K.

On a NS, the matter exists as an inhomogenous, low-density *crust* and gradually becomes a more uniform *core* the closer to the NS center as in Figure 1.1. In order to study about the properties of NS matter, the problem have to be approached from the nuclear physics point of view, where we study about *nuclear matter* (NM). For a nuclear system as massive as a NS, we consider one with infinite number of nucleons that are in β-stable state with a small portion of leptons, in which the properties of matter are described using an *equation of states* (EoS), i.e. the relation between different state variables (pressure P, mass-energy density ε, ...) of the system. Ideally, the EoS can be derived from the interactions of quarks under strong force in the framework of quantum chromodynamics. However, due to this having yet to be possible at the moment, the EoS is instead interpreted from a nonrelativistic mean-field study approach with many versions of the realistic density-dependent CDM3Yn interaction models [6,7] using Hartree-Fock (HF) formalism to describe NS matter, which will be implemented further in Chapter 2.

Following the gravitational wave (GW) signals GW170817 [8] and GW190425 [9] from two binary NS mergers observed by LIGO and Virgo laser interferometer in 17th August 2017 and 25th April 2019
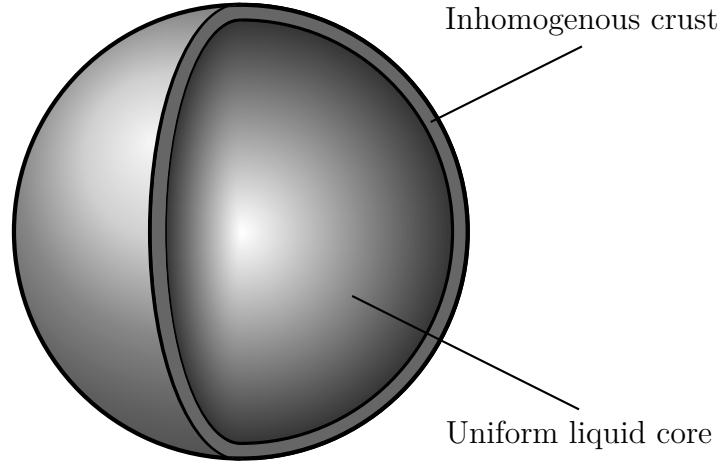
Figure 1.1: Neutron star's structure. The baryon density decreases (from white to dark gray) as we move outward from the NS center.

respectively, the tidal deformation of the NS can be further constrained, as well as the mass M and radius R of the NS [10]. The NS merger event is illustrated as in Figure 1.2.
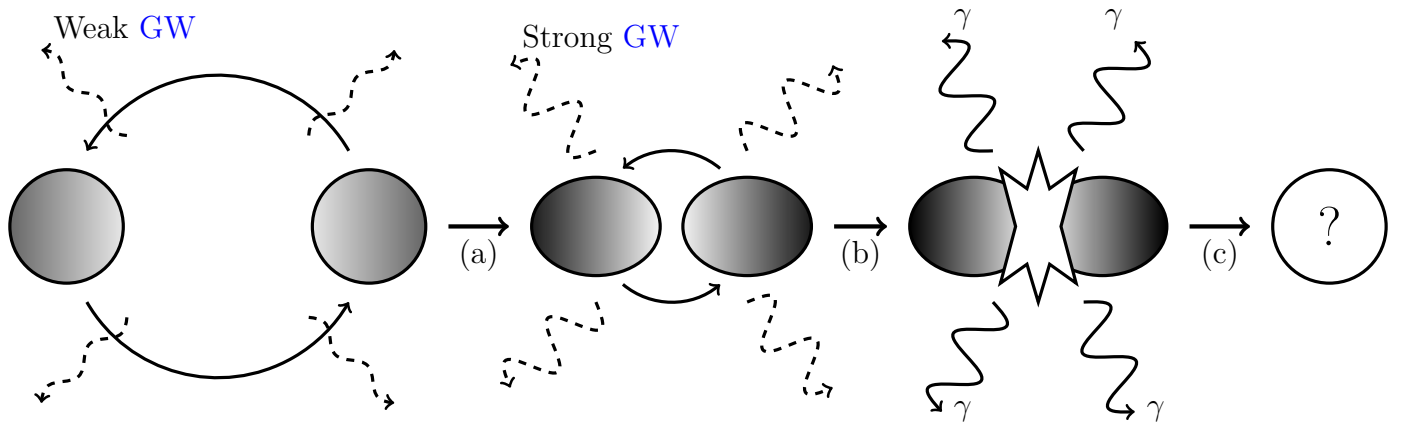
Figure 1.2: Illustration of binary NS merger. (a) The two companion NSs orbit about each others, while gradually losing energy through weak GW and come closer with time. (b) As the two NS get closer, they accelerate and emit stronger GW until (c) colliding, which results in a *kilonova*, characterized by a short *gamma ray burst* (GRB). The product of the merger hasn't been decided to be a black hole (BH) or another NS yet.

Apparently, the EoS of high-density NM plays the most important role in deciding the macroscopic properties of NS. In particular, given the EoS of the crust from the compressible liquid drop model and

by using the EoS of the uniform NS core from the result of the HF calculation of cold β-stable NM, the gravitational mass and radius of the star can be decided by the framework of General Relativity (GR) [1, 11], i.e. the Tolman-Oppenheimer-Volkoff (TOV) equation, which will in turn be compared to the observational astrophysical constraints to deduce the most suitable EoS of the constituent NS in this system.

In addition, due to the enormous mass, each NS possesses powerful gravitational field and therefore, they tend to "stretch out" their companion under the tidal effect as in Figure 1.2, while orbiting spirally toward each others and dissipating energy under the form of GW. Particularly, the shape and mass-energy distribution of the NS are tidally deformed from its supposedly spherical shape, resulting in nonzero multipole moments [12–14]. The NS's reaction, i.e. how strongly it deforms when being under a tidal field, is expressed in terms of the *tidal Love numbers* k_l of several orders l, where in this study, we will evaluate the Love number of NS up until the 4th order, i.e. $l = 2, 3, 4$ [15]. The tidal Love number depends heavily on the EoS of matter. For NS, the central density can be up to $6\rho_0$ and possesses a Love number of order ~ 0.1, while our Earth has that of 0.3. In a recent study, the Love number was calculated for spinning black holes, which showed that even with nearly infinite density, they still posess a small Love number of 0.002 [16].

Furthermore, under small pertubation of spacetime, the tidal field can also be further separated into two components: the *gravito-electric* (GE) and *gravito-magnetic* (GM) terms [14] that are analogous to that in eletromagnetic field. As a result, the deformation of the NS, i.e. Love numbers, in the pertubed tidal field can also be categorized into the corresponding GE (k_l) and GM (j_l) components [15], the evaluation and calculation result will be presented in Chapter 3. In summary, this study is dedicated to:

- Assess the dependence of NS's gravitational mass and radius to different EoS,

- Investigate the sensitivity of GE and GM tidal deformability and Love numbers to NM properties,

- Compare the NS's above properties to the astrophysical constraints obtained experimentally.

Chapter 2

Hartree-Fock Formalism of Nuclear Matter

2.1   Nucleon-Nucleon Interaction

Due to the lack of a exact theory to describe the nucleon-nucleon (NN) interaction, a model need to be imposed and fit with experimental measurement or theoretical calculation results. Plus, for a system as massive as a NS, deducing the EoS using the *ab initio* method, i.e. solving the Schrödinger equation over all particles, is simply impossible, therefore an *effective interaction* must be used [17]. In this section, we only limit ourselves to two-body interaction, thus, the NN potential can be expressed in the form of

$$v = v(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.1}$$

where the primed and unprimed variables indicate the properties of 2 nucleons respectively, in which $\boldsymbol{r}$ is the particle's position, $\boldsymbol{p}$ is its momentum, $\boldsymbol{\sigma}$ is its intrinsic spin and $\boldsymbol{\tau}$ is its isospin.

The functional form of v in (2.1) cannot freely take any form but is constrained by many invariance requirements [17]

- **Translational invariance:** The NN potential should only depend on the *relative position* of the two particle but not their explicit positions, thus we can reduce (2.1) to

$$v = v(\boldsymbol{r} - \boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.2}$$

  with at the last expression, we redefine $\boldsymbol{r}$ as the relative position vector.

- **Galilei invariance:** The potential should also be invariant under transformation between inertial frame of reference, which requires that only the relative momentum $\boldsymbol{p} - \boldsymbol{p}'$ is depended, i.e.

$$v = v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.3}$$

  where here we denote $\boldsymbol{p}$ as the relative momentum.

- **Rotational invariance:** The potential should be constructed such that the total angular momentum is zero.

- **Isospin invariance:** The NN interaction potential needs to be invariant under rotation in isospin space, in other words, it can only depend on the isospin-independent terms and the terms with $\boldsymbol{\tau} \cdot \boldsymbol{\tau}'$. Therefore, we can split the potential into

$$v = v_0(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}') + v_1(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}')\hat{\boldsymbol{\tau}} \cdot \hat{\boldsymbol{\tau}}' \tag{2.4}$$

- **Parity invariance:** The NN interaction potential is also expected to be invariant under the action of parity operator, i.e. changing the sign of spatial coordinates

$$v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(-\boldsymbol{r}, -\boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.5}$$

- **Time reversal invariance:** Finally, the interaction should stay the same after switching the time arrow direction

$$v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(\boldsymbol{r}, -\boldsymbol{p}, -\boldsymbol{\sigma}, -\boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.6}$$

Having the above considerations, developing further the M3Y-Paris interaction [18–20]

$$v = v_{00}(r) + v_{10}(r)\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + v_{01}(r)\boldsymbol{\tau} \cdot \boldsymbol{\tau}' + v_{11}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}')(\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \tag{2.7}$$

by adding a density-dependent form factor to each term gives the CDM3Yn interaction model

$$\begin{aligned} v(\rho, r) = F_{00}(\rho)v_{00}(r) &+ F_{10}(\rho)v_{10}(r)\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' \\ &+ F_{01}(\rho)v_{01}(r)\boldsymbol{\tau} \cdot \boldsymbol{\tau}' + F_{11}(\rho)v_{11}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}')(\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \end{aligned} \tag{2.8}$$

where each radial term is the superposition of 3 Yukawa potentials with different parameters

$$v_{\sigma\tau}(r) = \sum_{k=1}^{3} Y_{\sigma\tau}(k)\frac{\exp(-\mu_k r)}{\mu_k r} \tag{2.9}$$

and the form factor is adapted the form [1, 11, 21]

$$F_{\sigma\tau}(\rho) = C_{\sigma\tau}[1 + \alpha_{\sigma\tau} \exp(-\beta_{\sigma\tau}\rho) + \gamma_{\sigma\tau}\rho] \tag{2.10}$$

2.2 Equation of States of Nuclear Matter

In HF formalism, the total HF energy of the system can be expressed as

$$\begin{aligned} E = \sum_{\sigma\tau} \sum_{k}^{k_F^{\sigma\tau}} \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} \sum_{k\sigma\tau} \sum_{k'\sigma'\tau'} \Big[&\langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau' | v^D | \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau' \rangle \\ &+ \langle \boldsymbol{k}\sigma\tau, \boldsymbol{k}'\sigma'\tau' | v^{EX} | \boldsymbol{k}'\sigma\tau, \boldsymbol{k}\sigma'\tau' \rangle \Big] \end{aligned} \tag{2.11}$$

where the single-particle wave function is plane wave

$$|\boldsymbol{k}\sigma\tau\rangle = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\sqrt{\Omega}}\chi_\sigma\chi_\tau \tag{2.12}$$

Ω being the spatial volume of the system, $k_F^{\sigma\tau} = (6\pi^2\rho_{\sigma\tau})^{1/3}$ is the Fermi momentum corresponding to spin σ and isospin τ, $v^{D(EX)}$ is the direct (exchange) part of the interaction determined from the singlet and triplet-even (and odd) of the central NN force. Adopting the same functional form of (2.8), the direct and exchange interaction is written as

$$
\begin{aligned}
v^{D(EX)}(\rho_b, r) = {} & F_{00}^{D(EX)}(\rho_b)v_{00}^{D(EX)}(r) + F_{10}^{D(EX)}(\rho_b)v_{10}^{D(EX)}(r)\boldsymbol{\sigma}\cdot\boldsymbol{\sigma'} \\
& + F_{01}^{D(EX)}(\rho_b)v_{01}^{D(EX)}(r)\boldsymbol{\tau}\cdot\boldsymbol{\tau'} \\
& + F_{11}^{D(EX)}(\rho_b)v_{11}^{D(EX)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma'})(\boldsymbol{\tau}\cdot\boldsymbol{\tau'})
\end{aligned}
\tag{2.13}
$$

and

$$
v_{\sigma\tau}^{D(EX)}(r) = \sum_{k=1}^{3} Y_{\sigma\tau}^{D(EX)}(k)\frac{\exp(-\mu_k r)}{\mu_k r}
\tag{2.14}
$$

with the Yukawa strengths given in Table 2.2 and the density-dependent form factor parameters are in Table 2.1. Note that in (2.13), ρ_b denotes the *baryon density*, this will be used in order to distinguish with the lepton density in the later section.

Table 2.1: CDM3Yn interaction's parameters [1, 2]. The version CDM3Y3, 4, 6 don't have the 10 and 11 terms due to them being *spin saturated*, i.e. the spin dependent terms vanishes effectively.

Interaction	$\sigma\tau$	$C_{\sigma\tau}$	$\alpha_{\sigma\tau}$	$\beta_{\sigma\tau}$ (fm^3)	$\gamma_{\sigma\tau}$ (fm^3)
CDM3Y3	00	0.2985	3.4528	2.6388	-1.5
	01	0.1574	9.7016	16.2704	11.9946
CDM3Y4	00	0.3052	3.2998	2.3180	-2.0
	01	0.1318	11.7739	16.0279	15.1987
CDM3Y6	00	0.2658	3.8033	1.4099	-4.0
	01	0.1824	8.8819	16.4346	10.8703
CDM3Y8	00	0.2658	3.8033	1.4099	-4.3
	01	0.2463	6.3836	10.2566	6.3549
	10	0.2161	3.7510	-3.3396	9.9329
	11	0.7572	1.9967	33.2012	0.2989

Table 2.2: Yukawa strengths of the M3Y-Paris interaction [1,3].

k	μ_k (fm^{-1})	Y_{00}^D (MeV)	Y_{10}^D (MeV)	Y_{01}^D (MeV)	Y_{11}^D (MeV)	Y_{00}^{EX} (MeV)	Y_{10}^{EX} (MeV)	Y_{01}^{EX} (MeV)	Y_{11}^{EX} (MeV)
1	4.0	11061.625	938.875	313.625	-969.125	-1524.25	-3492.75	-4118.0	-2210.0
2	2.5	-2537.5	-36.0	223.5	450.0	-518.75	795.25	1054.75	568.75
3	0.7072	0.0	0.0	0.0	3.4877	-7.8474	2.6157	2.6157	-0.8719

Multiply (2.11) with Ω^{-1}, the energy density of the NM is separated into the kinetic term ε_{kin} and the potential terms $\varepsilon_{\sigma\tau}$, i.e.

$$\varepsilon = \frac{E}{\Omega} = \varepsilon_{kin} + F_{00}(\rho_b)\varepsilon_{00} + F_{01}(\rho_b)\varepsilon_{01} + F_{10}(\rho_b)\varepsilon_{10} + F_{11}(\rho_b)\varepsilon_{11} \tag{2.15}$$

The final expressions of each terms of the energy density are

$$\varepsilon_{kin} = \frac{3}{10} \sum_{\sigma\tau} \frac{\hbar^2 (k_F^{\sigma\tau})^2}{m_\tau} \rho_{\sigma\tau} \tag{2.16}$$

$$\varepsilon_{00} = \frac{1}{2}\left[\rho_b^2 J_{00}^D + \int A_{00}^2 v_{00}^{EX}(r)\, d^3r \right] \tag{2.17}$$

$$\varepsilon_{10} = \frac{1}{2}\left[\rho_b^2 J_{10}^D \left(\Delta_n \frac{1+\delta}{2} + \Delta_p \frac{1-\delta}{2} \right)^2 + \int A_{10}^2 v_{10}^{EX}(r)\, d^3r \right] \tag{2.18}$$

$$\varepsilon_{01} = \frac{1}{2}\left[\rho_b^2 J_{01}^D \delta^2 + \int A_{01}^2 v_{01}^{EX}(r)\, d^3r \right] \tag{2.19}$$

$$\varepsilon_{11} = \frac{1}{2}\left[\rho_b^2 J_{11}^D \left(\Delta_n \frac{1+\delta}{2} - \Delta_p \frac{1-\delta}{2} \right)^2 + \int A_{11}^2 v_{11}^{EX}(r)\, d^3r \right] \tag{2.20}$$

where $\Delta_\tau = (\rho_{\uparrow\tau} - \rho_{\downarrow\tau})/\rho_\tau$ is the polarity of nucleon, $\delta = (\rho_n - \rho_p)/\rho_b$ is the asymmetry of NM, $J_{\sigma\tau}^D = \int v_{\sigma\tau}(r)\, d^3r$ is the volume integral of the direct interaction and

$$\begin{aligned}
A_{00} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) + \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) + \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) + \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}) \\
A_{10} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) - \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) + \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) - \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}) \\
A_{01} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) + \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) - \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) - \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}) \\
A_{11} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) - \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) - \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) + \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p})
\end{aligned} \tag{2.21}$$

with $\hat{j}_1(x) = 3j_1(x)/x$ and $j_1(x)$ being the 1st order spherical Bessel function.

2.3   β-Stable Nuclear Matter

After the HF calculation, we were able to obtain a numerical HF energy density $\varepsilon(\rho_n, \rho_p, \Delta_n, \Delta_p)$. However, it is in fact impossible for a NS to exist while consisting of purely nucleon. In order for the NS to exist, leptons (e^- and μ^-) have to be introduced to the matter constituents and the $npe\mu$ matter has to be under the condition of β-stable, i.e.

- Charge balance

$$\rho_p = \rho_e + \rho_\mu \tag{2.22}$$

- Chemical potential balance

$$\mu_n - \mu_p = \mu_e = \mu_\mu \tag{2.23}$$

where μ_i $(i = n, p, e, \mu)$ is the chemical potential of the i particle.

The total energy density of the *npeμ* matter is thus

$$\varepsilon = \varepsilon_{HF} + \rho_n m_n c^2 + \rho_p m_p c^2 + \varepsilon_e + \varepsilon_\mu \tag{2.24}$$

which leads to the nucleon chemical potential of the form

$$\mu_\tau(\rho_n, \rho_p, \Delta_n, \Delta_p) = \frac{\partial \varepsilon}{\partial \rho_\tau} = \frac{\partial \varepsilon_{HF}}{\partial \rho_\tau} + m_\tau c^2 \tag{2.25}$$

Let $\hat{\mu} = \mu_n - \mu_p$ be the leptons' chemical potential, (2.22) is equivalent to[1]

$$3\pi^2(\hbar c)^3 \rho_p - \hat{\mu}^3 - \left[\hat{\mu}^2 - (m_\mu c^2)^2\right]^{3/2} \theta(\hat{\mu} - m_\mu c^2) = 0 \tag{2.26}$$

In the CDM3Y3, 4, 6 interactions, the NM is spin saturated, therefore a $\Delta_n = \Delta_p = 0$ and there are no 10 and 11 terms, while for the interaction CDM3Y8, under strong magnetic field like that of a magnetar, we can approximate $\Delta_n \approx -\Delta_p \approx \Delta$ and reduce the EoS to depend on just the baryon polarity Δ alone, and the more baryon polarized, the stronger the magnetic field of the NS.

For a fixed value of Δ, we are able to obtain a density function of the form $\rho_n(\rho_b, \Delta)$ and $\rho_p(\rho_b, \Delta)$, which in turn gives the lepton chemical potential $\hat{\mu}(\rho_b, \Delta) = \hat{\mu}(\rho_n, \rho_p)$ On the other hand, the leptons' densities are then

$$\rho_e(\rho_b, \Delta) = \frac{\hat{\mu}^3(\rho_b, \Delta)}{3\pi^2(\hbar c)^3} \quad \text{and} \quad \rho_\mu(\rho_b, \Delta) = \frac{\left[\hat{\mu}^2(\rho_b, \Delta) - (m_\mu c^2)^2\right]^{3/2}}{3\pi^2(\hbar c)^3} \theta(\hat{\mu}(\rho_b, \Delta) - m_\mu c^2) \tag{2.27}$$

Consider the e^- and μ^- to be systems of relativistic Fermi gas, then their respective energy densities and pressure contributions are ($l = e, \mu$)

$$\varepsilon_l(\rho_b, \Delta) = \frac{2}{(2\pi)^3} \int_0^{[3\pi^2 \rho_l(\rho_b, \Delta)]^{1/3}} \sqrt{\hbar^2 c^2 k^2 + m_l^2 c^4}\, d^3\mathbf{k} \tag{2.28}$$

and

$$P_l(\rho_b, \Delta) = \frac{1}{3} \frac{2}{(2\pi)^3} \int_0^{[3\pi^2 \rho_l(\rho_b, \Delta)]^{1/3}} \frac{\hbar^2 c^2 k^2}{\sqrt{\hbar^2 c^2 k^2 + m_l^2 c^4}}\, d^3\mathbf{k} \tag{2.29}$$

Plus, from the HF formalism with NM, the baryon pressure is given by

$$P_b = \rho_b^2 \frac{\partial(\varepsilon_{HF}/\rho_b)}{\partial \rho_b} \tag{2.30}$$

Finally, we obtain the total energy density dependence on baryon density as

$$\varepsilon(\rho_b, \Delta) = \varepsilon_{HF}(\rho_b, \Delta) + \rho_n(\rho_b, \Delta)m_n c^2 + \rho_p(\rho_b, \Delta)m_p c^2 + \varepsilon_e(\rho_b, \Delta) + \varepsilon_\mu(\rho_b, \Delta) \tag{2.31}$$

and the total pressure of NS matter

$$P(\rho_b, \Delta) = P_b(\rho_b, \Delta) + P_e(\rho_b, \Delta) + P_\mu(\rho_b, \Delta) \tag{2.32}$$

and this is the final result of the EoS of cold β-stable NS matter.

---

[1] $\theta(x)$ is the Heaviside function, i.e. it returns 1 for $x \geq 0$ and 0 otherwise.

Chapter 3

Neutron Star Properties

3.1 Tolman-Oppenheimer-Volkoff Equation

In the framework of GR, assume the NS to be in a static, isotropic region of spacetime, the metric elements are then

$$c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\nu(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{3.1}$$

Inside the NS, we have [22] the energy-momentum tensor as

$$T^{\mu\nu} = -Pg^{\mu\nu} + (P + \varepsilon)u^\mu u^\nu \tag{3.2}$$

where $u^\mu = dx^\mu/d\tau$ is the local fluid 4-velocity. Solving the Einstein's field equation [22]

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu} \tag{3.3}$$

gives the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP}{dr} = -\frac{G\varepsilon(P)m}{c^2 r^2} \left(1 + \frac{P}{\varepsilon(P)}\right) \left(1 + \frac{4\pi P r^3}{mc^2}\right) \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} \tag{3.4}$$

$$\frac{dm}{dr} = \frac{4\pi r^2 \varepsilon(P)}{c^2} \tag{3.5}$$

where $\varepsilon(P)$ can be obtained from the CDM3Yn interaction calculated previously. Additional boundary conditions are

$$P(0) = P_c; \qquad P(R) = 0; \qquad m(0) = 0; \qquad m(R) = M$$

and by varying the center pressure P_c, a relation of the mass M and radius R of the NS can be obtained.

3.2 Gravito-electric and Gravito-magnetic Tidal Deformation

Pertubing

Chapter 4

Discussions

Chapter 5

Conclusions

Bibliography

[1] Ngo Hai Tan, Dao T Khoa, and Doan Thi Loan. Spin-polarized β-stable neutron star matter: The nuclear symmetry energy and gw170817 constraint. *Physical Review C*, 102(4):045809, 2020.

[2] HOANG Sy Thân. *UFR SCIENTIFIQUE D'ORSAY INSTITUT DE PHYSIQUE NUCLEAIRE D'ORSAY*. PhD thesis, Citeseer, 2010.

[3] N Anantaraman, H Toki, and GF Bertsch. An effective interaction for inelastic scattering derived from the paris potential. *Nuclear Physics A*, 398(2):269–278, 1983.

[4] Gordon Baym and Christopher Pethick. Neutron stars. *Annual Review of Nuclear Science*, 25(1):27–77, 1975.

[5] James M Lattimer and Maddappa Prakash. The physics of neutron stars. *science*, 304(5670):536–542, 2004.

[6] Dao T Khoa, W Von Oertzen, HG Bohlen, G Bartnitzky, H Clement, Y Sugiyama, B Gebauer, AN Ostrowski, Th Wilpert, M Wilpert, et al. Equation of state for cold nuclear matter from refractive o 16+ o 16 elastic scattering. *Physical review letters*, 74(1):34, 1995.

[7] Dao T Khoa and Do Cong Cuong. Mean-field description of the nucleus-nucleus optical potential. *arXiv preprint arXiv:0708.3711*, 2007.

[8] Benjamin P Abbott, Rich Abbott, TD Abbott, Fausto Acernese, Kendall Ackley, Carl Adams, Thomas Adams, Paolo Addesso, RX Adhikari, VB Adya, et al. Gw170817: observation of gravitational waves from a binary neutron star inspiral. *Physical Review Letters*, 119(16):161101, 2017.

[9] BP Abbott, R Abbott, TD Abbott, S Abraham, F Acernese, K Ackley, C Adams, RX Adhikari, VB Adya, C Affeldt, et al. Gw190425: observation of a compact binary coalescence with total mass $\sim 3.4 m_{\odot}$. *The Astrophysical Journal Letters*, 892(1):L3, 2020.

[10] Benjamin P Abbott, Richard Abbott, TD Abbott, F Acernese, K Ackley, C Adams, T Adams, P Addesso, Rana X Adhikari, Vaishali B Adya, et al. Gw170817: Measurements of neutron star radii and equation of state. *Physical review letters*, 121(16):161101, 2018.

[11] Ngo Hai Tan, Dao T Khoa, and Doan Thi Loan. Equation of state of asymmetric nuclear matter and the tidal deformability of neutron star. *arXiv preprint arXiv:2104.09121*, 2021.

[12] Tanja Hinderer. Tidal love numbers of neutron stars. *The Astrophysical Journal*, 677(2):1216, 2008.

[13] Tanja Hinderer, Benjamin D Lackey, Ryan N Lang, and Jocelyn S Read. Tidal deformability of neutron stars with realistic equations of state and their gravitational wave signatures in binary inspiral. *Physical Review D*, 81(12):123016, 2010.

[14] Thibault Damour and Alessandro Nagar. Relativistic tidal properties of neutron stars. *Physical Review D*, 80(8):084035, 2009.

[15] Loïc Perot and Nicolas Chamel. Role of dense matter in tidal deformations of inspiralling neutron stars and in gravitational waveforms with unified equations of state. *Physical Review C*, 103(2):025801, 2021.

[16] Alexandre Le Tiec and Marc Casals. Spinning black holes fall in love. *Physical Review Letters*, 126(13):131102, 2021.

[17] Walter Greiner and Joachim A Maruhn. *Nuclear models.* Springer, 1996.

[18] H Nakada and M Sato. A method of implementing hartree–fock calculations with zero-and finite-range interactions. *Nuclear Physics A*, 699(3-4):511–537, 2002.

[19] H Nakada. Hartree-fock approach to nuclear matter and finite nuclei with m3y-type nucleon-nucleon interactions. *Physical Review C*, 68(1):014316, 2003.

[20] H Nakada. Erratum: Mean-field approach to nuclear structure with semi-realistic nucleon-nucleon interactions [phys. rev. c 78, 054301 (2008)]. *Physical Review C*, 82(2):029902, 2010.

[21] Dao T Khoa, GR Satchler, and W Von Oertzen. Nuclear incompressibility and density dependent nn interactions in the folding model for nucleus-nucleus potentials. *Physical Review C*, 56(2):954, 1997.

[22] Norman K Glendenning. *Compact stars: Nuclear physics, particle physics and general relativity.* Springer Science & Business Media, 2012.