University of Science and Technology of Hanoi

Mean-field study of the equation of states of nuclear matter and tidal deformation of neutron star

Bachelor Thesis

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May 8, 2021

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List of Abbreviations

BH	Black hole
EoS	Equation of States
GE	Gravito-electric
GM	Gravito-magnetic
GR	General Relativity
GRB	Gamma-ray burst
GW	Gravitational wave
HF	Hartree-Fock
NM	Nuclear Matter
NN	Nucleon-Nucleon
NS	Neutron Star
TOV	Tolmann-Oppenheimer-Volkoff

Chapter 1

Introduction

Neutron stars (NS) are star-like astronomical objects with mass M on the order of solar mass ($M_\odot$), a radius of $\sim 10 - 12\ km$ and an average density n several times greater than that of nucleon ($\rho_0 \approx 0.16\ fm^{-3}$). They are arguably the densest accessible objects, excluding black holes which we know nothing about inside the event horizon, in the universe [1]. Due to extremely high density, the matter on NS mainly consists of neutrons that are closely packed together with a small percentage of other particles (p, e^-, ...), similar to a atomic nucleus on macroscopic scale. For this reason, they are also the ideal objects for testing physical theories of dense matter and provide connections between different field of physics, i.e. nuclear physics, elementary particle physics and astrophysics [2].

During the NS's formation process, protons (p) and electrons (e^-) combined together to form neutrons, i.e.

$$p + e^- \longrightarrow n + \nu_e \tag{1.1}$$

and the star only holds itself against gravity by its own degeneracy pressure and strong force repulsion, which explains why the matter on NS is neutron-dominant and hence the name "neutron stars". After the NS is formed, energy quickly dissipates through neutrino emission, resulting in a relatively cold NS. In this study, we will only concern with the NS after a considerable time from its formation, when the temperature is considered to be $T = 0\ K$.

In order to study about the properties of NS matter, the problem have to be approached from the nuclear physics point of view, where we study about *nuclear matter* (NM). For a nuclear system as massive as a NS, we consider one with infinite number of nucleons that are in β-stable state with a small portion of leptons, in which the properties of matter are described using an *equation of states* (EoS), i.e. the relation between different state variables (pressure P, mass-energy density ε, ...) of the system. Ideally, the EoS can be derived from the interactions of quarks under strong force in the framework of quantum chromodynamics. However, due to this having yet to be possible at the moment, the EoS is instead interpreted from a nonrelativistic mean-field study approach with many versions of the realistic density-dependent CDM3Yn interaction models [3, 4] using Hartree-Fock (HF) formalism, in both *spin saturated* and *spin polarized* case, to describe NS matter. On a NS, the matter exists as an inhomogenous, low-density *crust* and gradually becomes a more uniform *core* the closer to the NS center as in Figure 1.1.

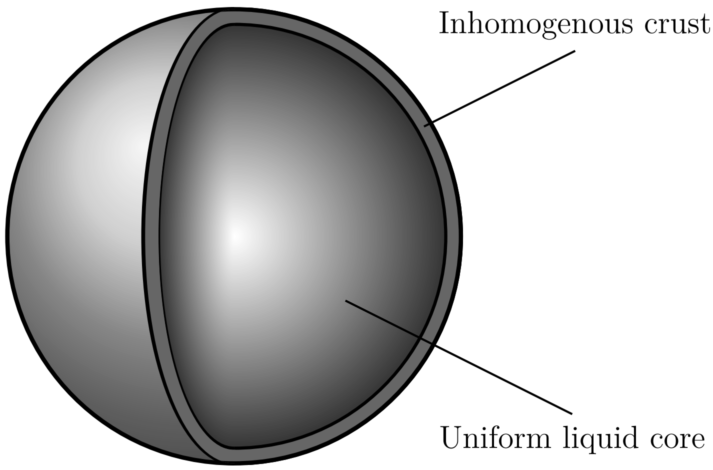

Figure 1.1: Neutron star's structure. The baryon density decreases (from white to dark gray) as we move outward from the NS center.

Following the gravitational wave (GW) signals GW170817 [5] and GW190425 [6] from two binary NS mergers observed by LIGO and Virgo laser interferometer in 17$^{\text{th}}$ August 2017 and 25$^{\text{th}}$ April 2019 respectively, the tidal deformation of the NS can be further constrained, as well as the mass M and radius R of the NS [7]. The NS merger event is illustrated as in Figure 1.2.

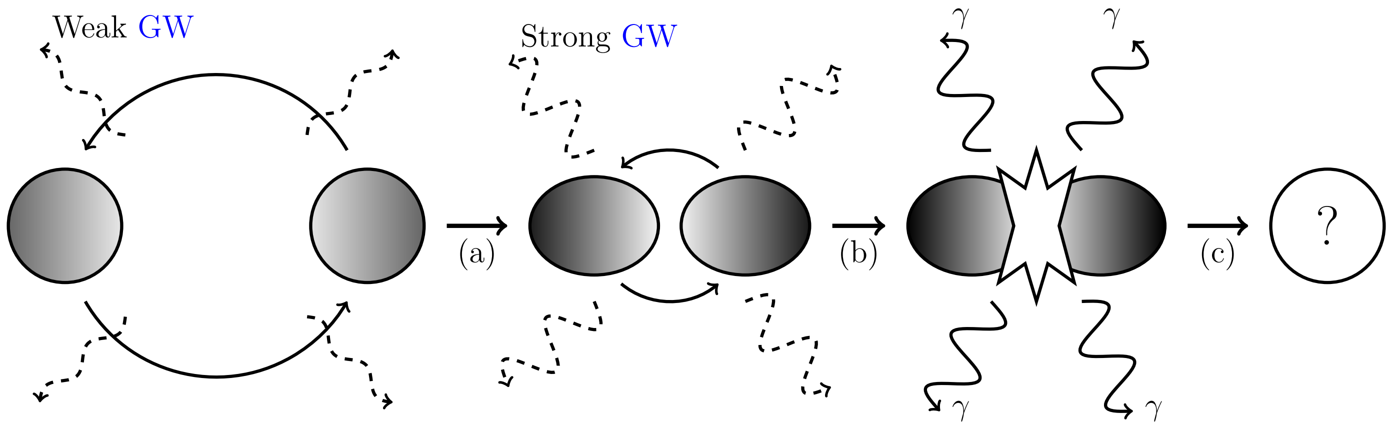

Figure 1.2: Illustration of binary NS merger. (a) The two companion NSs orbit about each others, while gradually losing energy through weak GW and come closer with time. (b) As the two NS get closer, they accelerate and emit stronger GW until (c) colliding, which results in a *kilonova*, characterized by a short *gamma ray burst* (GRB). The product of the merger hasn't been decided to be a black hole (BH) or another NS yet.

Apparently, the EoS of high-density NM plays the most important role in deciding the macroscopic properties of NS. In particular, by using the EoS from the result of the HF calculation of cold β-stable NM, the gravitational mass and radius of the star can be decided by the framework of General Relativity (GR) [8, 9], i.e. the Tolman-Oppenheimer-Volkoff (TOV) equation, which will in turn be compared to the observational astrophysical constraints to deduce the most suitable EoS of the constituent NS in this system.

In addition, due to the enormous mass, each NS possesses powerful gravitational field and therefore, they tend to "stretch out" their companion under the tidal effect as in Figure 1.2, while orbiting spirally toward each others and dissipating energy under the form of GW. Particularly, the shape and mass-energy distribution of the NS are tidally deformed from its supposedly spherical shape, resulting in nonzero multipole moments [10–12]. The NS's reaction against this tidal field, i.e. how strongly it deforms, is expressed in terms of the *tidal Love numbers* k_l of several orders l, where in the following chapters, we will evaluate the Love number of NS up until the 4$^{\text{th}}$ order, i.e. $l = 2, 3, 4$ [13]. The tidal Love number depends heavily on the EoS of matter. For NS, the center density can be up to $6\rho_0$ and possesses a Love number of order ~ 0.1, while our Earth has that of 0.3. In a recent study, the Love number was calculated for spinning black holes, which showed that even with nearly infinite density, they still posess a small Love number of 0.002 [14].

Under small pertubation, in the pertubation theory of GR, the tidal field can also be further separated into two components: the *gravito-electric* (GE) and *gravito-magnetic* (GM) [12] that are analogous to the eletromagnetic field. As a result, the deformation of the NS, i.e. Love numbers, in the pertubed tidal field can also be categorized into the corresponding GE (k_l) and GM (j_l) components [13], the evaluation and calculation result will be presented in later chapters.

In summary, this study is dedicated to

- Assess the dependence of NS's gravitational mass and radius to different EoS,
- Investigate the sensitivity of GE and GM tidal deformability and Love numbers to NM properties,
- Compare the NS's above properties to the astrophysical constraints obtained experimentally.

Chapter 2

Hartree-Fock Formalism

2.1 Nucleon-Nucleon Interaction

Due to the lack of a exact theory to describe the nucleon-nucleon (NN) interaction, a model need to be imposed and fit with experimental measurement or theoretical calculation results. For a system as massive as a NS, deducing the EoS using the *ab initio* method, i.e. solving the Schrödinger equation over all particles, is simply impossible, therefore an *effective interaction* must be used [15]. In this section, we only limit ourselves to two-body interaction, thus, the NN potential can be expressed in the form of

$$v = v(\boldsymbol{r}, \boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.1}$$

where the primed and unprimed variables indicate the properties of 2 nucleons respectively, in which $\boldsymbol{r}$ is the particle's position, $\boldsymbol{p}$ is its momentum, $\boldsymbol{\sigma}$ is its intrinsic spin and $\boldsymbol{\tau}$ is its isospin.

The functional form of v in (2.1) cannot freely take any form but is constrained by many invariance requirements [15]

- **Translational invariance:** The NN potential should only depend on the *relative position* of the two particle but not their explicit positions, thus we can reduce (2.1) to

$$v = v(\boldsymbol{r} - \boldsymbol{r}', \boldsymbol{p}, \boldsymbol{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.2}$$

  with at the last expression, we redefine $\boldsymbol{r}$ as the relative position vector.

- **Galilei invariance:** The potential should also be invariant under transformation between inertial frame of reference, which requires that only the relative momentum $\boldsymbol{p} - \boldsymbol{p}'$ is depended, i.e.

$$v = v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.3}$$

  where here we denote $\boldsymbol{p}$ as the relative momentum.

- **Rotational invariance:** The potential should be constructed such that the total angular momentum is zero.

- **Isospin invariance:** The isospin $\boldsymbol{\tau}$ only enters (2.3) through the *isospin operator* $\hat{\boldsymbol{\tau}}$ and $\hat{\boldsymbol{\tau}}'$. The NN interaction potential needs to be invariant under rotation in isospin space, in other words, it can only depend on the isospin-independent terms, the terms with $\boldsymbol{\tau} \cdot \boldsymbol{\tau}'$ and their higher powers. Coincide the isospin operator matrices with the Pauli matrices (since by definition they differ by only a conversion scaling factor), we have

$$[\hat{\tau}_m, \hat{\tau}_n] = 2i \sum_k \epsilon_{mnk} \hat{\tau}_k \quad \text{and} \quad \{\hat{\tau}_m, \hat{\tau}_n\} = 2\delta_{mn} \tag{2.4}$$

adding the two equations together, we get the term

$$\hat{\tau}_m \hat{\tau}_n = \delta_{mn} + i \sum_k \epsilon_{mnk} \hat{\tau}_k \tag{2.5}$$

Then we obtain the dot product

$$(\hat{\boldsymbol{\tau}} \cdot \hat{\boldsymbol{\tau}}')^2 = \sum_m \delta_{mm} - \sum_{mnkk'} \epsilon_{mnk} \epsilon_{mnk'} \hat{\tau}_k \hat{\tau}'_{k'} = 3 - 2\hat{\boldsymbol{\tau}} \cdot \hat{\boldsymbol{\tau}}' \tag{2.6}$$

Therefore, we can split the potential into

$$v = v_0(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}') + v_1(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}') \hat{\boldsymbol{\tau}} \cdot \hat{\boldsymbol{\tau}}' \tag{2.7}$$

- **Parity invariance:** The NN interaction potential is also expected to be invariant under the action of parity operator, i.e. changing the sign of spatial coordinates

$$v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(-\boldsymbol{r}, -\boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.8}$$

- **Time reversal invariance:** Finally, the interaction should stay the same after switching the time arrow direction

$$v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(\boldsymbol{r}, -\boldsymbol{p}, -\boldsymbol{\sigma}, -\boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \tag{2.9}$$

Having the above considerations, developing further the M3Y-Paris interaction [16–18]

$$v = v_{00}(r) + v_{10}(r)\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + v_{01}(r)\boldsymbol{\tau} \cdot \boldsymbol{\tau}' + v_{11}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}')(\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \tag{2.10}$$

by adding a density-dependent form factor to each term gives the CDM3Yn interaction model

$$\begin{aligned} v(\rho, r) = {} & F_{00}(\rho) v_{00}(r) + F_{10}(\rho) v_{10}(r) \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' \\ & + F_{01}(\rho) v_{01}(r) \boldsymbol{\tau} \cdot \boldsymbol{\tau}' + F_{11}(\rho) v_{11}(r) (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}')(\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \end{aligned} \tag{2.11}$$

where each radial term is the superposition of 3 Yukawa potentials with different parameters

$$v_{\sigma\tau}(r) = \sum_{k=1}^{3} Y_{\sigma\tau}(k) \frac{\exp(-\mu_k r)}{\mu_k r} \tag{2.12}$$

and the form factor is adapted the form [8, 9, 19]

$$F_{\sigma\tau}(\rho) = C_{\sigma\tau}[1 + \alpha_{\sigma\tau} \exp(-\beta_{\sigma\tau}\rho) + \gamma_{\sigma\tau}\rho] \tag{2.13}$$

2.2 Equation of States of Nuclear Matter

The total HF energy of the system can be expressed as

$$E = \sum_{\sigma\tau} \sum_{\boldsymbol{k}}^{k_F^{\sigma\tau}} \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} \sum_{\boldsymbol{k}\sigma\tau} \sum_{\boldsymbol{k'}\sigma'\tau'} \Big[\langle \boldsymbol{k}\sigma\tau, \boldsymbol{k'}\sigma'\tau' | v^D | \boldsymbol{k}\sigma\tau, \boldsymbol{k'}\sigma'\tau' \rangle \\ + \langle \boldsymbol{k}\sigma\tau, \boldsymbol{k'}\sigma'\tau' | v^{EX} | \boldsymbol{k'}\sigma\tau, \boldsymbol{k}\sigma'\tau' \rangle \Big] \tag{2.14}$$

where the single-particle wave function is plane wave

$$|\boldsymbol{k}\sigma\tau\rangle = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\sqrt{\Omega}} \chi_\sigma \chi_\tau \tag{2.15}$$

Ω being the spatial volume of the system, $k_F^{\sigma\tau} = (6\pi^2 \rho_{\sigma\tau})^{1/3}$ is the Fermi momentum corresponding to spin σ and isospin τ, $v^{D(EX)}$ is the direct (exchange) part of the interaction determined from the singlet and triplet-even (and odd) of the central NN force. The direct and exchange interaction is then

$$v^{D(EX)}(\rho_b, r) = F_{00}^{D(EX)}(\rho_b) v_{00}^{D(EX)}(r) + F_{10}^{D(EX)}(\rho_b) v_{10}^{D(EX)}(r) \boldsymbol{\sigma}\cdot\boldsymbol{\sigma'} \\ + F_{01}^{D(EX)}(\rho_b) v_{01}^{D(EX)}(r) \boldsymbol{\tau}\cdot\boldsymbol{\tau'} \\ + F_{11}^{D(EX)}(\rho_b) v_{11}^{D(EX)}(r) (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma'})(\boldsymbol{\tau}\cdot\boldsymbol{\tau'}) \tag{2.16}$$

and

$$v_{\sigma\tau}^{D(EX)}(r) = \sum_{k=1}^{3} Y_{\sigma\tau}^{D(EX)}(k) \frac{\exp(-\mu_k r)}{\mu_k r} \tag{2.17}$$

with the Yukawa strengths given in Table 2.2 and the density-dependent form factor parameters are in Table 2.1. Note that in (2.16), ρ_b denotes the *baryon density*, this will be used in order to distinguish with the lepton density in the later section.

Table 2.1: CDM3Yn interaction's parameters [8, 20]. The version CDM3Y3, 4, 6 don't have the 10 and 11 terms due to them being *spin saturated*, i.e. the spin dependent terms vanishes effectively.

Interaction	$\sigma\tau$	$C_{\sigma\tau}$	$\alpha_{\sigma\tau}$	$\beta_{\sigma\tau}$ (fm^3)	$\gamma_{\sigma\tau}$ (fm^3)
CDM3Y3	00	0.2985	3.4528	2.6388	-1.5
	01	0.1574	9.7016	16.2704	11.9946
CDM3Y4	00	0.3052	3.2998	2.3180	-2.0
	01	0.1318	11.7739	16.0279	15.1987
CDM3Y6	00	0.2658	3.8033	1.4099	-4.0
	01	0.1824	8.8819	16.4346	10.8703
CDM3Y8	00	0.2658	3.8033	1.4099	-4.3
	01	0.2463	6.3836	10.2566	6.3549
	10	0.2161	3.7510	-3.3396	9.9329
	11	0.7572	1.9967	33.2012	0.2989

Table 2.2: Yukawa strengths of the M3Y-Paris interaction [8,21].

k	μ_k (fm^{-1})	Y_{00}^{D} (MeV)	Y_{10}^{D} (MeV)	Y_{01}^{D} (MeV)	Y_{11}^{D} (MeV)	Y_{00}^{EX} (MeV)	Y_{10}^{EX} (MeV)	Y_{01}^{EX} (MeV)	Y_{11}^{EX} (MeV)
1	4.0	11061.625	938.875	313.625	−969.125	−1524.25	−3492.75	−4118.0	−2210.0
2	2.5	−2537.5	−36.0	223.5	450.0	−518.75	795.25	1054.75	568.75
3	0.7072	0.0	0.0	0.0	3.4877	−7.8474	2.6157	2.6157	−0.8719

Multiply (2.14) with Ω^{-1}, the energy density of the NM is separated into the kinetic term ε_{kin} and the potential terms $\varepsilon_{\sigma\tau}$, i.e.

$$\varepsilon = \frac{E}{\Omega} = \varepsilon_{kin} + F_{00}(\rho_b)\varepsilon_{00} + F_{01}(\rho_b)\varepsilon_{01} + F_{10}(\rho_b)\varepsilon_{10} + F_{11}(\rho_b)\varepsilon_{11} \tag{2.18}$$

The final expressions of each terms of the energy density are

$$\varepsilon_{kin} = \frac{3}{10}\sum_{\sigma\tau}\frac{\hbar^2(k_F^{\sigma\tau})^2}{m_\tau}\rho_{\sigma\tau} \tag{2.19}$$

$$\varepsilon_{00} = \frac{1}{2}\left[\rho_b^2 J_{00}^D + \int A_{00}^2 v_{00}^{EX}(r)\, d^3r\right] \tag{2.20}$$

$$\varepsilon_{10} = \frac{1}{2}\left[\rho_b^2 J_{10}^D \left(\Delta_n\frac{1+\delta}{2} + \Delta_p\frac{1-\delta}{2}\right)^2 + \int A_{10}^2 v_{10}^{EX}(r)\, d^3r\right] \tag{2.21}$$

$$\varepsilon_{01} = \frac{1}{2}\left[\rho_b^2 J_{01}^D \delta^2 + \int A_{01}^2 v_{01}^{EX}(r)\, d^3r\right] \tag{2.22}$$

$$\varepsilon_{11} = \frac{1}{2}\left[\rho_b^2 J_{11}^D \left(\Delta_n\frac{1+\delta}{2} - \Delta_p\frac{1-\delta}{2}\right)^2 + \int A_{11}^2 v_{11}^{EX}(r)\, d^3r\right] \tag{2.23}$$

where $\Delta_\tau = (\rho_{\uparrow\tau} - \rho_{\downarrow\tau})/\rho_\tau$ is the polarity of nucleon, $\delta = (\rho_n - \rho_p)/\rho_b$ is the asymmetry of NM, $J_{\sigma\tau}^D = \int v_{\sigma\tau}(r)\, d^3r$ is the volume integral of the direct interaction and

$$\begin{aligned}
A_{00} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) + \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) + \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) + \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}) \\
A_{10} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) - \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) + \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) - \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}) \\
A_{01} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) + \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) - \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) - \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}) \\
A_{11} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}) - \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}) - \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}) + \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p})
\end{aligned} \tag{2.24}$$

with $\hat{j}_1(x) = 3j_1(x)/x$ and $j_1(x)$ being the 1$^{\text{st}}$ order spherical Bessel function.

2.3 β-Stable Nuclear Matter

After the HF calculation, we were able to obtain a numerical HF energy density $\varepsilon(\rho_n, \rho_p, \Delta_n, \Delta_p)$. However, it is in fact impossible for a NS to exist while consisting of purely nucleon. In order for the

NS to exist, leptons (e^- and μ^-) have to be introduced to the matter constituents and the $npe\mu$ matter has to be under the condition of *β-stable*, i.e.

- Charge balance
$$\rho_p = \rho_e + \rho_\mu \tag{2.25}$$
- Chemical potential balance
$$\mu_n - \mu_p = \mu_e = \mu_\mu \tag{2.26}$$
where μ_i $(i = n, p, e, \mu)$ is the chemical potential of the i particle.

The total energy density of the $npe\mu$ matter is thus

$$\varepsilon = \varepsilon_{HF} + \rho_n m_n c^2 + \rho_p m_p c^2 + \varepsilon_e + \varepsilon_\mu \tag{2.27}$$

which leads to the nucleon chemical potential of the form

$$\mu_\tau(\rho_n, \rho_p, \Delta_n, \Delta_p) = \frac{\partial \varepsilon}{\partial \rho_\tau} = \frac{\partial \varepsilon_{HF}}{\partial \rho_\tau} + m_\tau c^2 \tag{2.28}$$

Let $\hat{\mu} = \mu_n - \mu_p$ be the leptons' chemical potential, (2.25) is equivalent to[1]

$$3\pi^2(\hbar c)^3 \rho_p - \hat{\mu}^3 - \left[\hat{\mu}^2 - (m_\mu c^2)^2\right]^{3/2} \theta(\hat{\mu} - m_\mu c^2) = 0 \tag{2.29}$$

In the CDM3Y3, 4, 6 interactions, the NM is spin saturated, therefore a $\Delta_n = \Delta_p = 0$ and there are no 10 and 11 terms, while for the interaction CDM3Y8, under strong magnetic field like that of a magnetar, we can approximate $\Delta_n \approx -\Delta_p \approx \Delta$ and reduce the EoS to depend on just the baryon polarity Δ alone, and the more baryon polarized, the stronger the magnetic field of the NS.

For a fixed value of Δ, we are able to obtain a density function of the form $\rho_n(\rho_b, \Delta)$ and $\rho_p(\rho_b, \Delta)$, which in turn gives the lepton chemical potential $\hat{\mu}(\rho_b, \Delta) = \hat{\mu}(\rho_n, \rho_p)$ On the other hand, the leptons' densities are then

$$\rho_e(\rho_b, \Delta) = \frac{\hat{\mu}^3(\rho_b, \Delta)}{3\pi^2(\hbar c)^3} \quad \text{and} \quad \rho_\mu(\rho_b, \Delta) = \frac{\left[\hat{\mu}^2(\rho_b, \Delta) - (m_\mu c^2)^2\right]^{3/2}}{3\pi^2(\hbar c)^3} \theta(\hat{\mu}(\rho_b, \Delta) - m_\mu c^2) \tag{2.30}$$

Consider the e^- and μ^- to be systems of relativistic Fermi gas, then their respective energy densities and pressure contributions are $(l = e, \mu)$

$$\varepsilon_l(\rho_b, \Delta) = \frac{2}{(2\pi)^3} \int_0^{[3\pi^2 \rho_l(\rho_b, \Delta)]^{1/3}} \sqrt{\hbar^2 c^2 k^2 + m_l^2 c^4}\, d^3\mathbf{k} \tag{2.31}$$

and

$$P_l(\rho_b, \Delta) = \frac{1}{3} \frac{2}{(2\pi)^3} \int_0^{[3\pi^2 \rho_l(\rho_b, \Delta)]^{1/3}} \frac{\hbar^2 c^2 k^2}{\sqrt{\hbar^2 c^2 k^2 + m_l^2 c^4}}\, d^3\mathbf{k} \tag{2.32}$$

[1] $\theta(x)$ is the Heaviside function, i.e. it returns 1 for $x \geq 0$ and 0 otherwise.

Plus, from the HF formalism with NM, the baryon pressure is given by

$$P_b = \rho_b^2 \frac{\partial(\varepsilon_{HF}/\rho_b)}{\partial \rho_b} \tag{2.33}$$

Finally, we obtain the total energy density dependence on baryon density as

$$\varepsilon(\rho_b, \Delta) = \varepsilon_{HF}(\rho_b, \Delta) + \rho_n(\rho_b, \Delta) m_n c^2 + \rho_p(\rho_b, \Delta) m_p c^2 + \varepsilon_e(\rho_b, \Delta) + \varepsilon_\mu(\rho_b, \Delta) \tag{2.34}$$

and the total pressure of NS matter

$$P(\rho_b, \Delta) = P_b(\rho_b, \Delta) + P_e(\rho_b, \Delta) + P_\mu(\rho_b, \Delta) \tag{2.35}$$

and this is the final result of the EoS of cold β-stable NS matter.

Chapter 3

Neutron Star Properties

3.1 Tolman-Oppenheimer-Volkoff Equation

In the framework of GR, assume the NS to be in a static, isotropic region of spacetime, the metric elements are then

$$c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\nu(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{3.1}$$

Inside the NS, we have [22] the energy-momentum tensor as

$$T^{\mu\nu} = -Pg^{\mu\nu} + (P + \varepsilon) u^\mu u^\nu \tag{3.2}$$

where $u^\mu = dx^\mu / d\tau$ is the local fluid 4-velocity. Solving the Einstein's field equation [22]

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu} \tag{3.3}$$

gives the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{dP}{dr} = -\frac{G\varepsilon(P)m}{c^2 r^2} \left(1 + \frac{P}{\varepsilon(P)}\right) \left(1 + \frac{4\pi P r^3}{mc^2}\right) \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} \tag{3.4}$$

$$\frac{dm}{dr} = \frac{4\pi r^2 \varepsilon(P)}{c^2} \tag{3.5}$$

where $\varepsilon(P)$ can be obtained from the CDM3Yn interaction calculated previously. Additional boundary conditions are

$$P(0) = P_c; \qquad P(R) = 0; \qquad m(0) = 0; \qquad m(R) = M$$

and by varying the center pressure P_c, a relation of the mass M and radius R of the NS can be obtained.

3.2 Gravito-electric and Gravito-magnetic Tidal Deformation

Pertubing

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