University of Science and Technology of Hanoi

Mean-field study of the equation of states of nuclear matter and tidal deformation of neutron star

Bachelor Thesis

Author:
Nguyen Hoang Dang Khoa

Supervisors:
Prof. Dao Tien Khoa
Dr. Ngo Hai Tan

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List of Abbreviations

BH	Black hole
BHF	Brueckner-Hartree-Fock
EoS	Equation of States
GE	Gravito-electric
GM	Gravito-magnetic
GR	General Relativity
GRB	Gamma-ray burst
GW	Gravitational wave
HF	Hartree-Fock
NM	Nuclear Matter
NN	Nucleon-Nucleon
NS	Neutron Star
TOV	Tolmann-Oppenheimer-Volkoff

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Chapter 1

Introduction

Neutron stars (NS) are star-like astronomical objects with mass M on the order of solar mass ($M_\odot$), a radius of $\sim 10-12\ km$ and an average density n several times greater than that of nucleon ($\rho_0 \approx 0.16\ fm^{-3}$). They are arguably the densest accessible objects, excluding black holes which we know nothing about inside the event horizon, in the universe [5]. Due to extremely high density, the matter on NS mainly consists of neutrons that are closely packed together with a small percentage of other particles (p, e^-, ...), similar to a atomic nucleus on macroscopic scale. For this reason, they are also the ideal objects for testing physical theories of dense matter and provide connections between different field of physics, i.e. nuclear physics, elementary particle physics and astrophysics [6].

During the NS's formation process, protons (p) and electrons (e^-) combined together to form neutrons, i.e.

$$p + e^- \longrightarrow n + \nu_e \tag{1.1}$$

and the star only holds itself against gravity by its own degeneracy pressure and strong force repulsion, which explains why the matter on NS is neutron-dominant and hence the name "neutron stars". After the NS is formed, energy quickly dissipates through neutrino emission, resulting in a relatively cold NS. In this study, we will only concern with the NS after a considerable time from its formation, when the temperature is considered to be $T = 0\ K$.

On a NS, the matter exists as an inhomogenous, low-density *crust* and gradually becomes a more uniform *core* the closer to the NS center as in Figure 1.1. In order to study about the properties of NS matter, the problem have to be approached from the nuclear physics point of view, where we study about *nuclear matter* (NM). For a nuclear system as massive as a NS, we consider one with infinite number of nucleons that are in β-stable state with a small portion of leptons, in which the properties of matter are described using an *equation of states* (EoS), i.e. the relation between different state variables (pressure P, mass-energy density ε, ...) of the system. Ideally, the EoS can be derived from the interactions of quarks under strong force in the framework of quantum chromodynamics. However, due to this having yet to be possible at the moment, the EoS of NM is instead interpreted from a nonrelativistic mean-field study approach with several updated versions of the realistic density-dependent CDM3Yn interaction models [7,8] using Hartree-Fock (HF) formalism, which will be implemented further in Chapter 2.

Following the gravitational wave (GW) signals GW170817 [9] and GW190425 [10] from two binary NS mergers observed by LIGO and Virgo laser interferometer in 17$^{\text{th}}$ August 2017 and 25$^{\text{th}}$ April 2019

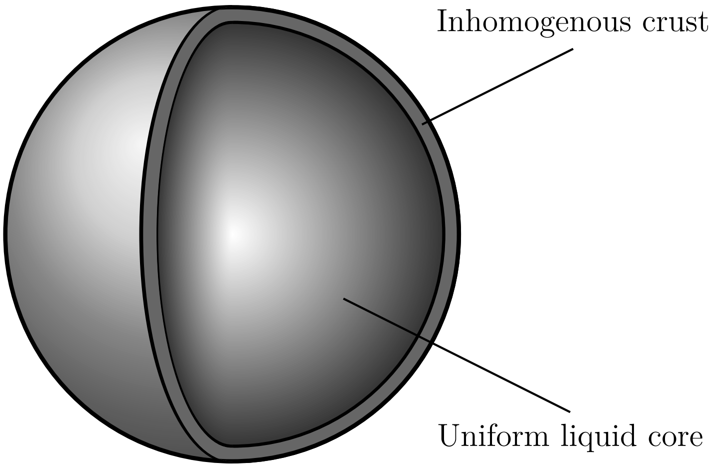

Figure 1.1: Neutron star's overall structure. The baryon density decreases (from white to dark gray) as we move outward from the NS center.

respectively, the tidal deformation of the NS can be further constrained, as well as the mass M and radius R of the NS [11]. The NS merger event is illustrated as in Figure 1.2.

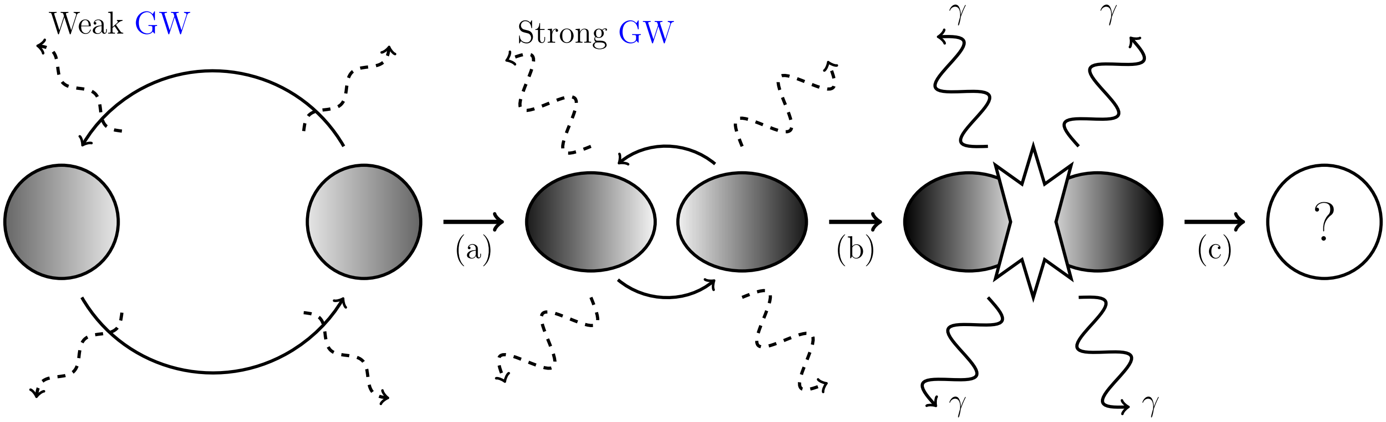

Figure 1.2: Illustration of binary NS merger. (a) The two companion NSs orbit about each others, while gradually losing energy through weak GW and come closer with time. (b) As the two NS get closer, they accelerate and emit stronger GW until (c) colliding, which results in a *kilonova*, characterized by a short *gamma ray burst* (GRB). The product of the merger has yet been decided to be a black hole (BH) or another NS.

Apparently, the EoS of high-density NM plays the most important role in deciding the macroscopic properties of NS. In particular, given the EoS of the crust from the compressible liquid drop model and by using the EoS of the uniform NS core from the result of the HF calculation of cold β-stable NM, the gravitational mass and radius of the star can be decided by the framework of General Relativity (GR) [1, 2], i.e. the Tolman-Oppenheimer-Volkoff (TOV) equation, which will in turn be compared to the observational astrophysical constraints to deduce the most suitable EoS of the constituent NS in this system.

In addition, due to the enormous mass, each NS possesses powerful gravitational field and therefore, they tend to "stretch out" their companion under the tidal effect as in Figure 1.2, while orbiting spirally toward each others and dissipating energy under the form of GW. Particularly, the shape and mass-energy distribution of the NS are tidally deformed from its supposedly spherical shape, resulting in nonzero multipole moments [12–14]. The NS's reaction, i.e. how strongly it deforms when being under a tidal field, is expressed in terms of the *tidal Love numbers* k_l of several orders l, where in this study, we will evaluate the Love number of NS up to the 4^{th} order, i.e. $l = 2, 3, 4$ [15]. Apparently, the tidal Love number depends heavily on the EoS of matter and this dependence will be further emphasized in Chapter 3. For NS, the central density can be up to $6\rho_0$ and possesses a Love number of order ~ 0.1, while our Earth has that of 0.3. In a recent study, the Love number was calculated for spinning black holes, which showed that even with nearly infinite density, they still posess a small Love number of 0.002 [16].

Furthermore, under small perturbation of spacetime, the tidal field can also be separated into two components: the *gravito-electric* (GE) and *gravito-magnetic* (GM) terms [14] that are analogous to that in eletromagnetic field. As a result, the deformation of the NS, i.e. Love numbers, in the perturbed tidal field can also be categorized into the corresponding GE (k_l) and GM (j_l) components [15], whose result will be presented in more details in Chapter 3. To sum up, this study is dedicated to:

- Include the spin polarization effect to the existing CDM3Yn models [1],
- Assess the dependence of NS's gravitational mass and radius to different EoS,
- Investigate the sensitivity of GE and GM tidal deformability and Love numbers to NM properties,
- Compare the NS's above properties to the astrophysical constraints obtained experimentally.

Chapter 2

Hartree-Fock Formalism of Nuclear Matter

2.1 Nucleon-Nucleon Interaction

Due to the lack of a exact theory to describe the nucleon-nucleon (NN) interaction, a model need to be imposed and fit with experimental measurement or theoretical calculation results. Plus, for a system as massive as a NS, deducing the EoS using the *ab initio* method, i.e. solving the Schrödinger equation over all particles, is simply impossible, therefore an *effective interaction* must be used [17]. In this section, we only limit ourselves to two-body interaction, thus, the NN potential can be expressed in the form of

$$v = v(\boldsymbol{r}, \boldsymbol{r'}, \boldsymbol{p}, \boldsymbol{p'}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) \tag{2.1}$$

where the primed and unprimed variables indicate the properties of 2 nucleons respectively, in which $\boldsymbol{r}$ is the particle's position, $\boldsymbol{p}$ is its momentum, $\boldsymbol{\sigma}$ is its intrinsic spin and $\boldsymbol{\tau}$ is its isospin.

The functional form of v in (2.1) cannot freely take any form but is constrained by many invariance requirements [17]

- **Translational invariance:** The NN potential should only depend on the *relative position* of the two particle but not their explicit positions, thus we can reduce (2.1) to

$$v = v(\boldsymbol{r} - \boldsymbol{r'}, \boldsymbol{p}, \boldsymbol{p'}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) = v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{p'}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) \tag{2.2}$$

  with at the last expression, we redefine $\boldsymbol{r}$ as the relative position vector.

- **Galilei invariance:** The potential should also be invariant under transformation between inertial frame of reference, which requires that only the relative momentum $\boldsymbol{p} - \boldsymbol{p'}$ is depended, i.e.

$$v = v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) \tag{2.3}$$

  where here we denote $\boldsymbol{p}$ as the relative momentum.

- **Rotational invariance:** The potential should be constructed such that the total angular momentum is zero.

- **Isospin invariance:** The NN interaction potential needs to be invariant under rotation in isospin space, in other words, it can only depend on the isospin-independent terms and the terms with $\boldsymbol{\tau} \cdot \boldsymbol{\tau'}$. Therefore, we can split the potential into

$$v = v_0(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}) + v_1(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma'})\hat{\boldsymbol{\tau}} \cdot \hat{\boldsymbol{\tau}}' \tag{2.4}$$

- **Parity invariance:** The NN interaction potential is also expected to be invariant under the action of parity operator, i.e. changing the sign of spatial coordinates

$$v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) = v(-\boldsymbol{r}, -\boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) \tag{2.5}$$

- **Time reversal invariance:** Finally, the interaction should stay the same after switching the time arrow direction

$$v(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) = v(\boldsymbol{r}, -\boldsymbol{p}, -\boldsymbol{\sigma}, -\boldsymbol{\sigma'}, \boldsymbol{\tau}, \boldsymbol{\tau'}) \tag{2.6}$$

Having the above considerations, developing further the M3Y-Paris interaction, which was used by the HF study of NM [1, 2, 18, 19] and the folding model study of NN scattering [20, 21],

$$v = v_{00}(r) + v_{10}(r)\boldsymbol{\sigma} \cdot \boldsymbol{\sigma'} + v_{01}(r)\boldsymbol{\tau} \cdot \boldsymbol{\tau'} + v_{11}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma'})(\boldsymbol{\tau} \cdot \boldsymbol{\tau'}) \tag{2.7}$$

by adding a density-dependent form factor to each term gives the CDM3Yn interaction model

$$\begin{aligned} v(\rho, r) = {} & F_{00}(\rho)v_{00}(r) + F_{10}(\rho)v_{10}(r)\boldsymbol{\sigma} \cdot \boldsymbol{\sigma'} \\ & + F_{01}(\rho)v_{01}(r)\boldsymbol{\tau} \cdot \boldsymbol{\tau'} + F_{11}(\rho)v_{11}(r)(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma'})(\boldsymbol{\tau} \cdot \boldsymbol{\tau'}) \end{aligned} \tag{2.8}$$

where each radial term is the superposition of 3 Yukawa potentials

$$v_{\sigma\tau}(r) = \sum_{k=1}^{3} Y_{\sigma\tau}(k)\frac{\exp(-\mu_k r)}{\mu_k r} \tag{2.9}$$

and the form factor $F_{\sigma\tau}(\rho)$ shared the functional form [1, 2, 20, 22]

$$F_{\sigma\tau}(\rho) = C_{\sigma\tau}[1 + \alpha_{\sigma\tau}\exp(-\beta_{\sigma\tau}\rho) + \gamma_{\sigma\tau}\rho] \tag{2.10}$$

with parameters given in Table 2.1. The parameters of F_{00} were adjusted to give the corresponding incompressibility K of symmetric NM at saturation density ρ_0 and the binding energy $E_0 \approx 15.8\ MeV$, while the 10 term is modified from [22] to reproduce $E_{sym}(\rho_0) \approx 30\ MeV$, $L \approx 50\ MeV$ and to be in agreement with the ab-initio results [23, 24] at higher density [1]. On the other hand, the spin-dependent terms, 10 and 11, are hereby included in the 5 models by fine tuning the parameters to yield the same result as the Brueckner-Hartree-Fock (BHF) study of spin polarized NM [4] as in Figure 2.1.

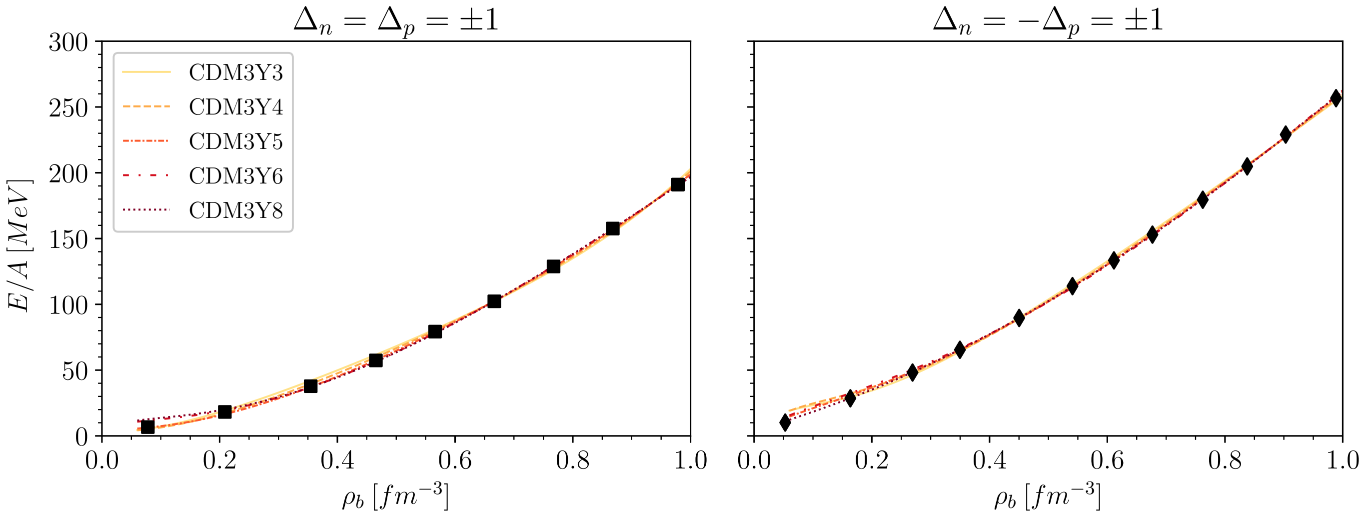

Figure 2.1: Energy per baryon E/A of symmetric NM by the 5 CDM3Yn models compared to BHF result [4]. The diamond and square represent the BHF result for $\Delta_n = -\Delta_p = \pm 1$ and $\Delta_n = \Delta_p = \pm 1$ respectively with Δ_τ being the baryon spin polarity.

Table 2.1: CDM3Yn interaction's parameters [1].

Interaction	$\sigma\tau$	$C_{\sigma\tau}$	$\alpha_{\sigma\tau}$	$\beta_{\sigma\tau}$ (fm^3)	$\gamma_{\sigma\tau}$ (fm^3)	K (MeV)
CDM3Y3	00	0.2985	3.4528	2.6388	-1.5	217
	01	0.2343	5.3336	6.4738	4.3172	
	10	0.3890	3.5635	-2.6717	20.3624	
	11	0.8802	4.0433	12.3262	0.3662	
CDM3Y4	00	0.3052	3.2998	2.3180	-2.0	228
	01	0.2129	6.3581	7.0584	5.6091	
	10	0.2593	6.0016	-2.3377	18.8725	
	11	0.8329	3.5941	9.2012	0.2690	
CDM3Y5	00	0.2728	3.7367	1.8294	-3.0	241
	01	0.2204	6.6146	7.9910	6.0040	
	10	0.4106	5.6265	-1.6698	-1.9866	
	11	0.6815	2.5833	5.1700	0.2578	
CDM3Y6	00	0.2658	3.8033	1.4099	-4.0	252
	01	0.2313	6.6865	8.6775	6.0182	
	10	0.5186	9.9402	1.6698	2.9799	
	11	0.6058	3.1947	4.4512	0.0822	
CDM3Y8	00	0.2658	3.8033	1.4099	-4.3	257
	01	0.2643	6.3836	9.8950	5.4249	
	10	0.5997	9.1900	0.7514	-4.7181	
	11	0.3786	3.9435	2.7012	0.3512	

2.2 Equation of States of Nuclear Matter

In HF formalism, the total HF energy of the system can be expressed as

$$E_{HF} = \sum_{\sigma\tau} \sum_{\boldsymbol{k}}^{k_F^{\sigma\tau}} \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} \sum_{\boldsymbol{k}\sigma\tau} \sum_{\boldsymbol{k'}\sigma'\tau'} \Big[\Big\langle \boldsymbol{k}\sigma\tau, \boldsymbol{k'}\sigma'\tau' \Big| v^D \Big| \boldsymbol{k}\sigma\tau, \boldsymbol{k'}\sigma'\tau' \Big\rangle + \Big\langle \boldsymbol{k}\sigma\tau, \boldsymbol{k'}\sigma'\tau' \Big| v^{EX} \Big| \boldsymbol{k'}\sigma\tau, \boldsymbol{k}\sigma'\tau' \Big\rangle \Big] \tag{2.11}$$

where the single-particle wave function is plane wave

$$|\boldsymbol{k}\sigma\tau\rangle = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\sqrt{\Omega}} \chi_\sigma \chi_\tau \tag{2.12}$$

Ω being the spatial volume of the system, $k_F^{\sigma\tau} = (6\pi^2 \rho_{\sigma\tau})^{1/3}$ is the Fermi momentum corresponding to spin σ and isospin τ, $v^{D(EX)}$ is the direct (exchange) part of the interaction determined from the singlet and triplet-even (odd) of the central NN force. Adopting the same functional form of (2.8), the direct and exchange interaction is written as

$$v^{D(EX)}(\rho_b, r) = F_{00}(\rho_b) v_{00}^{D(EX)}(r) + F_{10}(\rho_b) v_{10}^{D(EX)}(r) \boldsymbol{\sigma}\cdot\boldsymbol{\sigma'} + F_{01}(\rho_b) v_{01}^{D(EX)}(r) \boldsymbol{\tau}\cdot\boldsymbol{\tau'} + F_{11}(\rho_b) v_{11}^{D(EX)}(r) (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma'})(\boldsymbol{\tau}\cdot\boldsymbol{\tau'}) \tag{2.13}$$

and

$$v_{\sigma\tau}^{D(EX)}(r) = \sum_{k=1}^{3} Y_{\sigma\tau}^{D(EX)}(k) \frac{\exp(-\mu_k r)}{\mu_k r} \tag{2.14}$$

with the Yukawa strengths given in Table 2.2 and the density-dependent form factor parameters are in Table 2.1. Note that in (2.13), ρ_b denotes the *baryon density*, this will be used in order to distinguish with the lepton density in the later section.

Table 2.2: Yukawa strengths of the M3Y-Paris interaction [2, 3].

k	μ_k (fm^{-1})	Y_{00}^D (MeV)	Y_{10}^D (MeV)	Y_{01}^D (MeV)	Y_{11}^D (MeV)	Y_{00}^{EX} (MeV)	Y_{10}^{EX} (MeV)	Y_{01}^{EX} (MeV)	Y_{11}^{EX} (MeV)
1	4.0	11061.625	938.875	313.625	−969.125	−1524.25	−3492.75	−4118.0	−2210.0
2	2.5	−2537.5	−36.0	223.5	450.0	−518.75	795.25	1054.75	568.75
3	0.7072	0.0	0.0	0.0	3.4877	−7.8474	2.6157	2.6157	−0.8719

Multiply (2.11) with Ω^{-1}, the energy density of the NM is separated into the kinetic term ε_{kin} and the potential terms $\varepsilon_{\sigma\tau}$, i.e.

$$\varepsilon_{HF} = \frac{E_{HF}}{\Omega} = \varepsilon_{kin} + F_{00}(\rho_b)\varepsilon_{00} + F_{01}(\rho_b)\varepsilon_{01} + F_{10}(\rho_b)\varepsilon_{10} + F_{11}(\rho_b)\varepsilon_{11} \tag{2.15}$$

The final expressions of each terms of the energy density are

$$\varepsilon_{kin} = \frac{3}{10}\sum_{\sigma\tau}\frac{\hbar^2(k_F^{\sigma\tau})^2}{m_\tau}\rho_{\sigma\tau} \tag{2.16}$$

$$\varepsilon_{00} = \frac{1}{2}\left[\rho_b^2 J_{00}^D + \int A_{00}^2 v_{00}^{EX}(r)\, d^3r\right] \tag{2.17}$$

$$\varepsilon_{10} = \frac{1}{2}\left[\rho_b^2 J_{10}^D\left(\Delta_n\frac{1+\delta}{2} + \Delta_p\frac{1-\delta}{2}\right)^2 + \int A_{10}^2 v_{10}^{EX}(r)\, d^3r\right] \tag{2.18}$$

$$\varepsilon_{01} = \frac{1}{2}\left[\rho_b^2 J_{01}^D\delta^2 + \int A_{01}^2 v_{01}^{EX}(r)\, d^3r\right] \tag{2.19}$$

$$\varepsilon_{11} = \frac{1}{2}\left[\rho_b^2 J_{11}^D\left(\Delta_n\frac{1+\delta}{2} - \Delta_p\frac{1-\delta}{2}\right)^2 + \int A_{11}^2 v_{11}^{EX}(r)\, d^3r\right] \tag{2.20}$$

where $\Delta_\tau = (\rho_{\uparrow\tau} - \rho_{\downarrow\tau})/\rho_\tau$ is the polarity of nucleon, $\delta = (\rho_n - \rho_p)/\rho_b$ is the asymmetry of NM, $J_{\sigma\tau}^D = \int v_{\sigma\tau}^D(r)\, d^3r$ is the volume integral of the direct interaction and

$$\begin{aligned}
A_{00} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}r) + \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}r) + \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}r) + \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}r)\\
A_{10} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}r) - \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}r) + \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}r) - \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}r)\\
A_{01} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}r) + \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}r) - \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}r) - \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}r)\\
A_{11} &= \rho_{\uparrow n}\hat{j}_1(k_F^{\uparrow n}r) - \rho_{\downarrow n}\hat{j}_1(k_F^{\downarrow n}r) - \rho_{\uparrow p}\hat{j}_1(k_F^{\uparrow p}r) + \rho_{\downarrow p}\hat{j}_1(k_F^{\downarrow p}r)
\end{aligned} \tag{2.21}$$

with $\hat{j}_1(x) = 3j_1(x)/x$ and $j_1(x)$ being the 1st order spherical Bessel function.

2.3 β-Stable Nuclear Matter

After the HF calculation, we were able to obtain a numerical HF energy density $\varepsilon(\rho_n, \rho_p, \Delta_n, \Delta_p)$. However, it is in fact impossible for a NS to exist while consisting of purely nucleon. In order to compensate for this issue, leptons (e^- and μ^-) have to be introduced to the matter constituents and the $npe\mu$ matter has to satisfy the *β-stable* condition, i.e.

- Charge balance

$$\rho_p = \rho_e + \rho_\mu \tag{2.22}$$

- Chemical potential balance

$$\mu_n - \mu_p = \mu_e = \mu_\mu \tag{2.23}$$

  where $\mu_i = \frac{\partial\varepsilon}{\partial\rho_i}$ $(i = n, p, e, \mu)$ is the chemical potential of the i particle.

The total energy density of the $npe\mu$ matter is thus

$$\varepsilon = \varepsilon_{HF} + \rho_n m_n c^2 + \rho_p m_p c^2 + \varepsilon_e + \varepsilon_\mu \tag{2.24}$$

which leads to the nucleon chemical potential of the form

$$\mu_\tau(\rho_n, \rho_p, \Delta_n, \Delta_p) = \frac{\partial \varepsilon}{\partial \rho_\tau} = \frac{\partial \varepsilon_{HF}}{\partial \rho_\tau} + m_\tau c^2 \tag{2.25}$$

Let $\hat{\mu} = \mu_n - \mu_p$ be the leptons' chemical potential, (2.22) is equivalent to[1]

$$3\pi^2(\hbar c)^3\rho_p - \hat{\mu}^3 - \left[\hat{\mu}^2 - (m_\mu c^2)^2\right]^{3/2}\theta(\hat{\mu} - m_\mu c^2) = 0 \tag{2.26}$$

from which the proton fraction $x_p = \rho_p/\rho_b$ can be obtained as shown in Figure 4.1. Furthermore, under strong magnetic field like that of a magnetar, we can approximate $\Delta_n \approx -\Delta_p \approx \Delta$ and reduce the EoS to depend on just the baryon polarity Δ alone, and the more baryon polarized, the stronger the magnetic field of the NS.

For a fixed value of Δ, we are able to obtain a density function of the form $\rho_n(\rho_b, \Delta)$ and $\rho_p(\rho_b, \Delta)$, which in turn gives the lepton chemical potential $\hat{\mu}(\rho_b, \Delta) = \hat{\mu}(\rho_n, \rho_p)$ On the other hand, the leptons' densities are then

$$\rho_e(\rho_b, \Delta) = \frac{\hat{\mu}^3(\rho_b, \Delta)}{3\pi^2(\hbar c)^3} \quad \text{and} \quad \rho_\mu(\rho_b, \Delta) = \frac{\left[\hat{\mu}^2(\rho_b, \Delta) - (m_\mu c^2)^2\right]^{3/2}}{3\pi^2(\hbar c)^3}\theta(\hat{\mu}(\rho_b, \Delta) - m_\mu c^2) \tag{2.27}$$

Consider the e^- and μ^- to be systems of relativistic Fermi gas, then their respective energy densities and pressure contributions are ($l = e, \mu$)

$$\varepsilon_l(\rho_b, \Delta) = \frac{2}{(2\pi)^3}\int_0^{[3\pi^2\rho_l(\rho_b,\Delta)]^{1/3}} \sqrt{\hbar^2c^2k^2 + m_l^2c^4}\, d^3\mathbf{k} \tag{2.28}$$

and

$$P_l(\rho_b, \Delta) = \frac{1}{3}\frac{2}{(2\pi)^3}\int_0^{[3\pi^2\rho_l(\rho_b,\Delta)]^{1/3}} \frac{\hbar^2c^2k^2}{\sqrt{\hbar^2c^2k^2 + m_l^2c^4}}\, d^3\mathbf{k} \tag{2.29}$$

Plus, from the HF formalism with NM, the baryon pressure is given by

$$P_b = \rho_b^2\frac{\partial(\varepsilon_{HF}/\rho_b)}{\partial \rho_b} \tag{2.30}$$

Finally, we obtain the total energy density dependence on baryon density as

$$\varepsilon(\rho_b, \Delta) = \varepsilon_{HF}(\rho_b, \Delta) + \rho_n(\rho_b, \Delta)m_nc^2 + \rho_p(\rho_b, \Delta)m_pc^2 + \varepsilon_e(\rho_b, \Delta) + \varepsilon_\mu(\rho_b, \Delta) \tag{2.31}$$

and the total pressure of NS matter

$$P(\rho_b, \Delta) = P_b(\rho_b, \Delta) + P_e(\rho_b, \Delta) + P_\mu(\rho_b, \Delta) \tag{2.32}$$

and this completes the EoS of cold β-stable NS matter (Figure 4.2 and 4.3).

[1] $\theta(x)$ is the Heaviside function, i.e. it returns 1 for $x \geq 0$ and 0 otherwise.

Chapter 3

Neutron Star Properties

3.1 Tolman-Oppenheimer-Volkoff Equation

Suppose the NS to be static and spherically symmetric, the metric elements are then [25]

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = e^{2\nu(r)}c^2\mathrm{d}t^2 - e^{2\lambda(r)}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 \tag{3.1}$$

Consider the NS matter to be perfect fluid, we have the energy-momentum tensor as

$$T^{\mu\nu} = -Pg^{\mu\nu} + (P+\varepsilon)u^\mu u^\nu \tag{3.2}$$

where $u^\mu = dx^\mu/d\tau$ is the local fluid 4-velocity. Solving the Einstein's field equation [25]

$$G^{\mu\nu} = -\frac{8\pi G}{c^4}T^{\mu\nu} \tag{3.3}$$

gives the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\varepsilon(P)m}{c^2r^2}\left(1+\frac{P}{\varepsilon(P)}\right)\left(1+\frac{4\pi Pr^3}{mc^2}\right)\left(1-\frac{2Gm}{c^2r}\right)^{-1} \tag{3.4}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = \frac{4\pi r^2\varepsilon(P)}{c^2} \tag{3.5}$$

where $\varepsilon(P)$ is the EoS obtained from the CDM3Yn interaction calculated previously. Additional boundary conditions are

$$P(0) = P_c; \qquad P(R) = 0; \qquad m(0) = 0; \qquad m(R) = M$$

and by varying the center pressure P_c, a relation of the gravitational mass M and radius R of the NS can be obtained.

3.2 Gravito-electric and Gravito-magnetic Tidal Deformation

In close orbit with another compact companion in a binary system, the NS is tidally deformed by strong gravitational interaction. Analogous to the classical theory of electromagnetism, the tidal field experienced by it can be decomposed into 2 types: the *gravito-electric* and *gravito-magnetic* components with respective *relativistic tidal moment* [14]

$$\mathcal{E}_L = \partial_{L-1} E_{a_l} \qquad \text{and} \qquad \mathcal{M}_L = c^2 \partial_{L-1} B_{a_l} \tag{3.6}$$

where E_{a_l} and B_{a_l} are the a_l component of the externally generated local GE and GM field, L represents the multi-index $(a_1, a_2, \ldots, a_l)$ and l being the order of the moment. As a result, the deformation of NS is parameterized by the GE and GM *tidal deformabilities* λ_l and σ_l, i.e. in leading order [14]

$$\mathcal{Q}_L = \lambda_l \mathcal{E}_L, \tag{3.7}$$
$$\mathcal{S}_L = \sigma_l \mathcal{M}_L \tag{3.8}$$

with $\mathcal{Q}_L$ being the induced mass multipole moment, i.e. the deviation of the mass distribution from spherically symmetry at order l, while $\mathcal{S}_L$ is the current multipole moment in adiabatic approximation [14, 15]. From the deformabilities, the dimensionless GE and GM *tidal Love numbers* are defined as [15]

$$k_l = \frac{1}{2}(2l-1)!!\frac{G\lambda_l}{R^{2l+1}} \quad \text{and} \quad j_l = 4(2l-1)!!\frac{G\sigma_l}{R^{2l+1}} \tag{3.9}$$

These parameters are related to the GE and GM *tidal deformability parameters* as

$$\Lambda_l = \frac{2}{(2l-1)!!} k_l \left(\frac{c^2 R}{GM}\right)^{2l+1} \tag{3.10}$$
$$\Sigma_l = \frac{1}{4(2l-1)!!} j_l \left(\frac{c^2 R}{GM}\right)^{2l+1} \tag{3.11}$$

which can be potentially extracted from the signal of GW. In order to properly calculate these parameters, let $H_l(r)$ and $\tilde{H}_l(r)$ be small perturbations of the static metric. These functions have to satisfy [15]

$$\begin{aligned} H_l''(r) + H_l'(r)\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1} &\left\{\frac{2}{r} - \frac{2Gm(r)}{c^2 r^2} - \frac{4\pi G}{c^4} r[\varepsilon(r) - P(r)]\right\} \\ + H_l(r)\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1} &\left\{\frac{4\pi G}{c^4}\left[5\varepsilon(r) + 9P(r) + c^2 \frac{\mathrm{d}\varepsilon}{\mathrm{d}P}[\varepsilon(r) + P(r)]\right]\right. \\ &\left. - \frac{l(l+1)}{r^2} - 4\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}\left[\frac{Gm(r)}{c^2 r^2} + \frac{4\pi G}{c^4} r P(r)\right]^2\right\} = 0 \end{aligned} \tag{3.12}$$

for GE perturbations and

$$\begin{aligned} \tilde{H}_l''(r) - \tilde{H}_l'(r)\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1} &\frac{4\pi G}{c^4} r\left[P(r) + \varepsilon(r)\right] \\ - \tilde{H}_l(r)\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1} &\left\{\frac{l(l+1)}{r^2} - \frac{4Gm(r)}{c^2 r^3} + \theta \frac{8\pi G}{c^4}\left[P(r) + \varepsilon(r)\right]\right\} = 0 \end{aligned} \tag{3.13}$$

for GM perturbations; the value of $\theta = 1$ is for static fluid while irrotational fluid adopts the value $\theta = -1$. These two equations are integrated along with the TOV equation (3.4). In addition, we have the compactness parameters $C = GM/(Rc^2)$ and define

$$y_l = \frac{RH_l'(R)}{H_l(R)} \quad \text{and} \quad \tilde{y}_l = \frac{R\tilde{H}_l'(R)}{\tilde{H}_l(R)}. \tag{3.14}$$

The explicit expressions of the first few orders of the GE and GM Love numbers are

$$\begin{aligned}
k_2 = {} & \frac{8}{5}C^5(1-2C)^2\left[2(y_2-1)C - y_2 + 2\right]\Big\{2C\Big[4(y_2+1)C^4 + 2(3y_2-2)C^3 \\
& -2(11y_2-13)C^2 + 3(5y_2-8)C - 3(y_2-2)\Big] \\
& +3(1-2C)^2\left[2(y_2-1)C - y_2 + 2\right]\log(1-2C)\Big\}^{-1}, \qquad (3.15) \\
k_3 = {} & \frac{8}{7}C^7(1-2C)^2\Big[2(y_3-1)C^2 - 3(y_3-2)C + y_3 - 3\Big] \\
& \times\Big\{2C\Big[4(y_3+1)C^5 + 2(9y_3-2)C^4 - 20(7y_3-9)C^3 + 5(37y_3-72)C^2 - 45(2y_3-5)C \\
& +15(y_3-3)\Big] + 15(1-2C)^2\Big[2(y_3-1)C^2 - 3(y_3-2)C + y_3 - 3\Big]\log(1-2C)\Big\}^{-1}, \qquad (3.16) \\
k_4 = {} & \frac{32}{147}C^9(1-2C)^2\Big[12(y_4-1)C^3 - 34(y_4-2)C^2 + 28(y_4-3)C - 7(y_4-4)\Big] \\
& \times\Big\{2C\Big[8(y_4+1)C^6 + 4(17y_4-2)C^5 - 12(83y_4-107)C^4 + 40(55y_4-116)C^3 \\
& -10(191y_4-536)C^2 + 105(7y_4-24)C - 105(y_4-4)\Big] + 15(1-2C)^2\Big[12(y_4-1)C^3 \\
& -34(y_4-2)C^2 + 28(y_4-3)C - 7(y_4-4)\Big]\log(1-2C)\Big\}^{-1}, \qquad (3.17) \\
j_2 = {} & \frac{24}{5}C^5\left[2(\tilde{y}_2-2)C - \tilde{y}_2 + 3\right]\Big\{2C\Big[2(\tilde{y}_2+1)C^3 + 2\tilde{y}_2C^2 + 3(\tilde{y}_2-1)C - 3(\tilde{y}_2-3)\Big] \\
& + 3\left[2(\tilde{y}_2-2)C - \tilde{y}_2 + 3\right]\log(1-2C)\Big\}^{-1}, \qquad (3.18) \\
j_3 = {} & \frac{64}{21}C^7\Big[8(\tilde{y}_3-2)C^2 - 10(\tilde{y}_3-3)C + 3(\tilde{y}_3-4)\Big] \\
& \times\Big\{2C\Big[4(\tilde{y}_3+1)C^4 + 10\tilde{y}_3C^3 + 30(\tilde{y}_3-1)C^2 - 15(7\tilde{y}_3-18)C + 45(\tilde{y}_3-4)\Big] \\
& +15\Big[8(\tilde{y}_3-2)C^2 - 10(\tilde{y}_3-3)C + 3(\tilde{y}_3-4)\Big]\log(1-2C)\Big\}^{-1}, \qquad (3.19) \\
j_4 = {} & \frac{80}{147}C^9\Big[40(\tilde{y}_4-2)C^3 - 90(\tilde{y}_4-3)C^2 + 63(\tilde{y}_4-4)C - 14(\tilde{y}_4-5)\Big] \\
& \times\Big\{2C\Big[4(\tilde{y}_4+1)C^5 + 18\tilde{y}_4C^4 + 90(\tilde{y}_4-1)C^3 - 5(137\tilde{y}_4-334)C^2 \\
& +105(7\tilde{y}_4-26)C - 210(\tilde{y}_4-5)\Big] + 15\Big[40(\tilde{y}_4-2)C^3 - 90(\tilde{y}_4-3)C^2 \\
& +63(\tilde{y}_4-4)C - 14(\tilde{y}_4-5)\Big]\log(1-2C)\Big\}^{-1}
\end{aligned}$$

Chapter 4

Results and Discussions

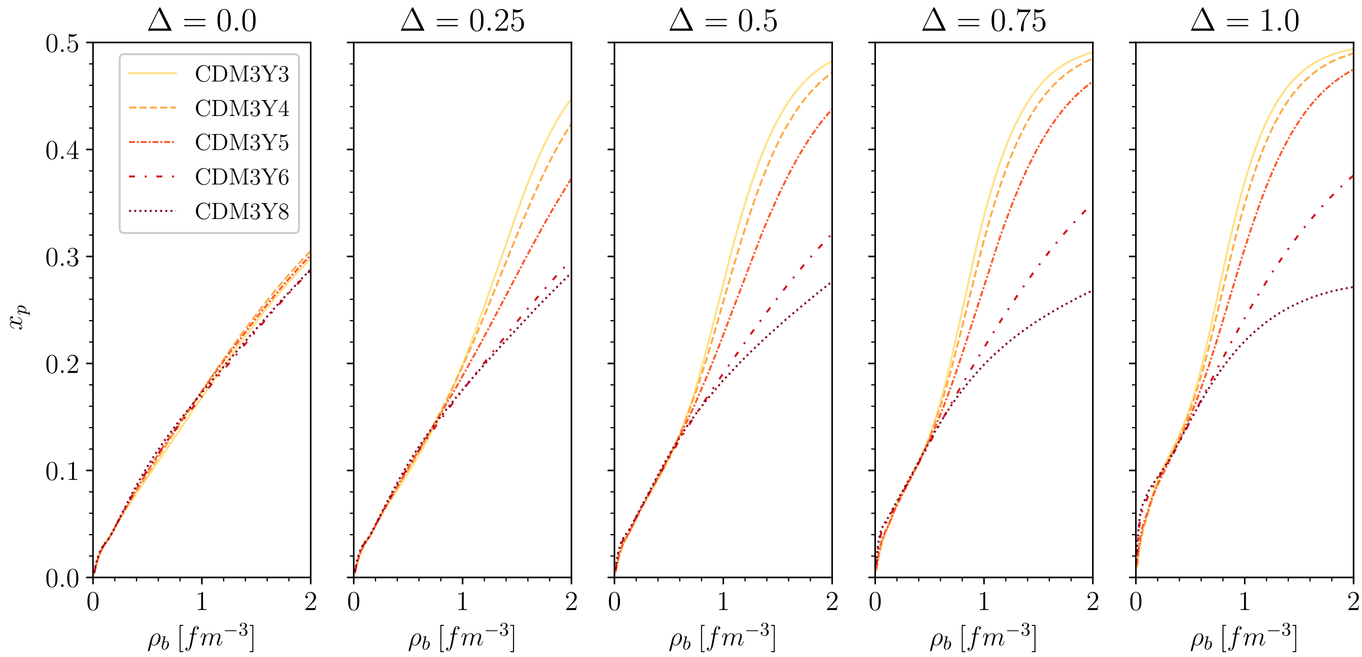

Figure 4.1: Proton fraction x_p of β-stable NM at different baryon density and spin polarity for CDM3Yn interactions.

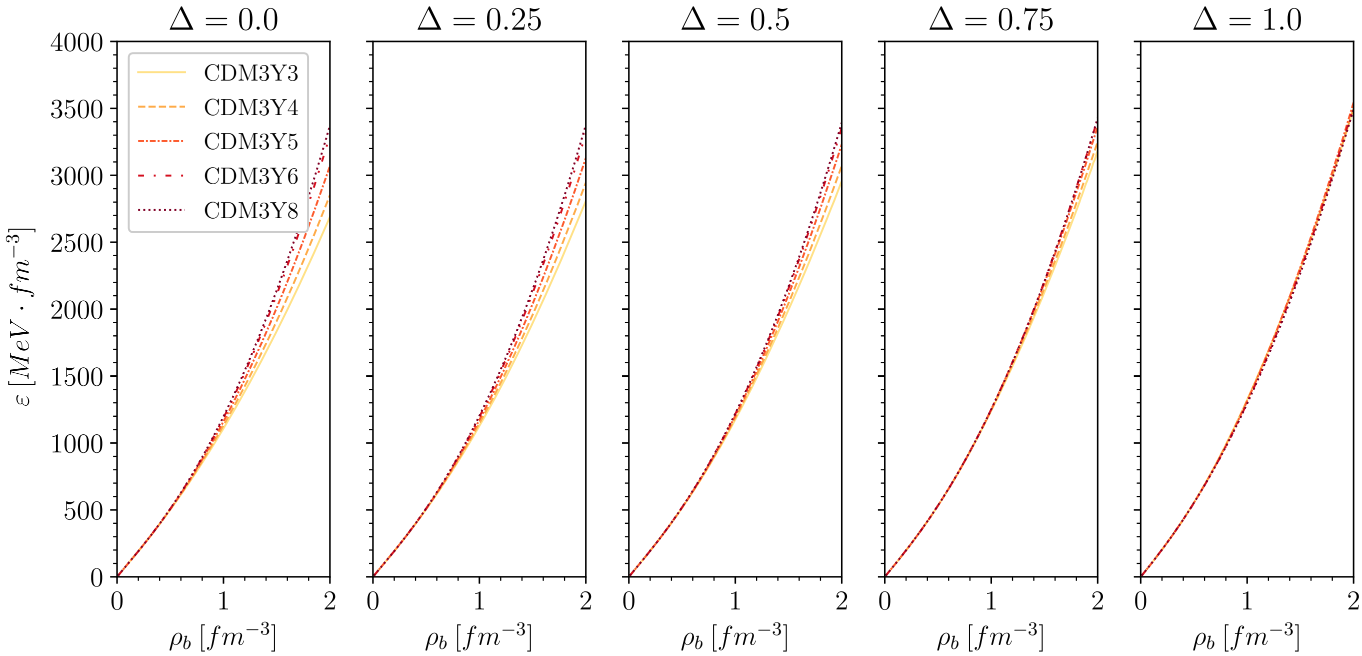

Figure 4.2: Total mass-energy density of β-stable NM at varying spin polarity with different interaction models.

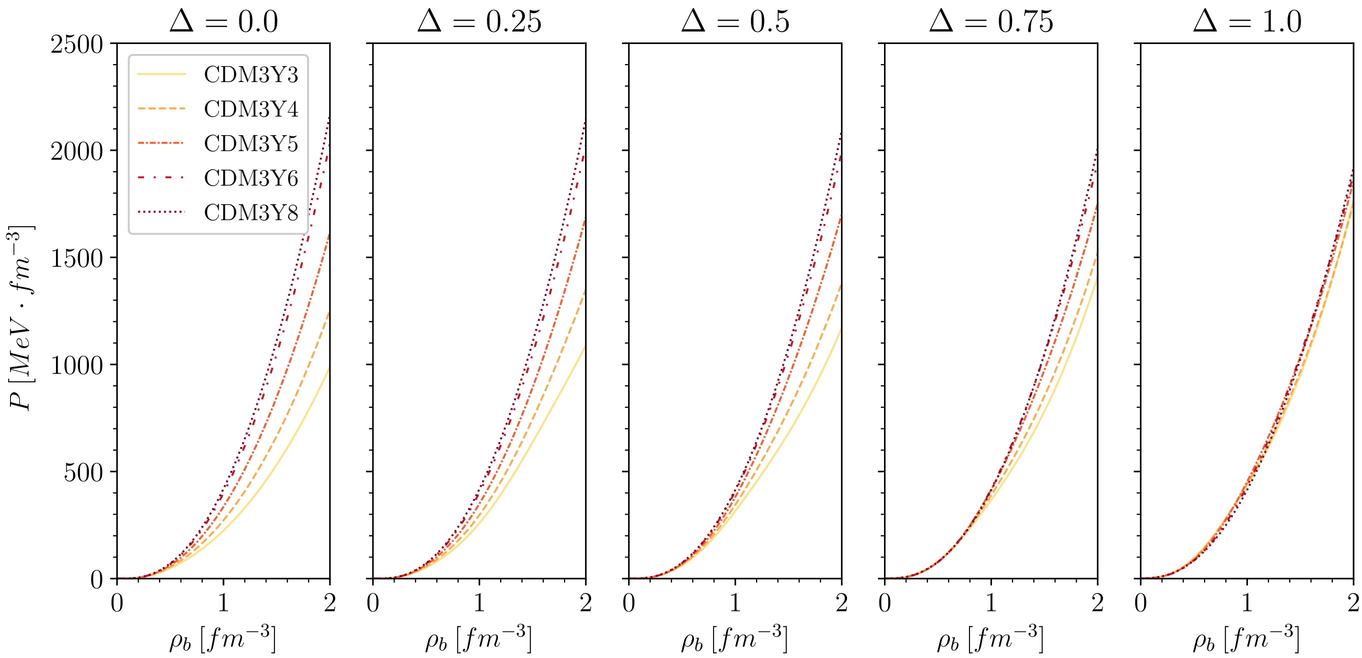

Figure 4.3: Total pressure of β-stable NM at several values of Δ with different CDM3Yn models.

Chapter 5

Conclusions

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