



UNIVERSITY OF SCIENCE AND TECHNOLOGY OF HANOI

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# Mean-field study of the equation of states of nuclear matter and tidal deformation of neutron star

Bachelor Thesis

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# List of Abbreviations

**EoS** Equation of States

**GE** Gravito-electric

**GM** Gravito-magnetic

**GR** General Relativity

**HF** Hartree-Fock

**NM** Nuclear Matter

**NN** Nucleon-Nucleon

**NS** Neutron Star

**TOV** Tolmann-Oppenheimer-Volkoff

# Chapter 1

## Introduction

Neutron stars (NS) are star-like astronomical objects with mass  $M$  on the order of solar mass ( $M_\odot$ ), a radius of  $\sim 10 - 12 \text{ km}$  and an average density  $n$  several times greater than that of nucleon ( $\rho_0 \approx 0.16 \text{ fm}^{-3}$ ). They are arguably the densest accessible objects, excluding black holes which we know nothing about inside the event horizon, in the universe [1]. Because of its extremely high density, NS can trap even weakly interacting particles like neutrinos, as the neutrino's mean free path  $\lambda_\nu \approx (\sigma_\nu \rho)^{-1} \sim 10 \text{ cm}$ , i.e. its average traveling distance between successive interactions [2], is much lower than the radius  $R$  of the star, where  $\sigma_\nu \approx 10^{-40} \text{ cm}^2$  is the neutrino's cross section [3]. The matter on NS is highly condensed and mainly consists of neutrons that are closely packed together, similar to a atomic nucleus on macroscopic scale. For this reason, they are also the ideal objects for testing physical theories of dense matter and provide connections between different field of physics, i.e. nuclear physics, elementary particle physics and astrophysics [4].

During the NS's formation process, protons ( $p$ ) and electrons ( $e^-$ ) combined together to form neutrons, i.e.

$$p + e^- \longrightarrow n + \nu_e \quad (1.1)$$

and the star only holds itself against gravity by its own degeneracy pressure and strong force repulsion. This process explains why the matter on NS is neutron-dominant. Once  $\lambda_\nu > R$ , the star becomes transparent with neutrinos and energy is further carried away, accelerate the cooling process of the NS. In this study, we will only concern with the NS after a considerable time from its formation, when its temperature is considered to be  $T = 0 \text{ K}$ .

In order to study about the properties of NS matter, we have to approach from the nuclear physics framework, where we study about *nuclear matter* (NM). For a nuclear system as massive as a NS, we consider one with infinite number of nucleons that are in  $\beta$ -stable state with a small portion of leptons, in which the properties of matter are described using an *equation of states* (EoS), i.e. the relation between different state variables (pressure  $P$ , mass-energy density  $\varepsilon$ , ...) of the system. Ideally, the EoS can be derived from the interactions of quarks under strong force in the framework of quantum chromodynamics. However, due to this having yet to be possible at the moment, the EoS is instead interpreted from a nonrelativistic mean-field study approach with many versions of the realistic density-dependent CDM3Yn interactions [5, 6] using Hartree-Fock (HF) formalism, in both spin saturated (CDM3Y3, 4, 6) and polarized (CDM3Y8) case, to describe NS matter.

Following the event of gravitational wave direct observation GW170817 from a binary NS merger by LIGO and Virgo laser interferometer [7], the tidal deformation of a NS can be detected, which leads to a further constraint of mass  $M$  and radius  $R$  of the NS [8]. On the other hand, with using the EoS obtained in the HF calculation, we can get the mass and radius of the star by the framework of General Relativity (GR) [9, 10], which will in turn be compared to the observational astrophysical constraint to deduce the most suitable EoS of the constituent NS in this system.

In a binary system of NS rotating around each others, since each NS possesses powerful gravitational field, they tend to deform their companion under the tidal effect, while orbiting spirally toward each others and dissipating energy under the form of gravitational waves. Particularly, the shape and mass-energy distribution of the NS are tidally deformed, resulting in nonzero multipole moments [11–13]. This deformation is expressed in terms of the *tidal Love numbers*  $k_l$  of several orders  $l$ , where in the following chapters, we will evaluate the Love number of NS up until the 4<sup>th</sup> order, i.e.  $l = 2, 3, 4$  [14]. The tidal Love number depends heavily on the EoS of matter. For NS, the center density can be up to  $6\rho_0$  and possesses a Love number of order  $\sim 0.1$ , while our Earth has that of 0.3. In a recent study, the Love number was calculated for spinning black holes, which showed that even with nearly infinite density, they still possess a small Love number of 0.002 [15].

Under small perturbation, on the GR framework, the tidal field can also be further separated into two components: the *gravito-electric* (GE) and *gravito-magnetic* (GM) [13]. As a result, the deformation of the NS, i.e. Love numbers, in the perturbed tidal field can also be categorized into the corresponding GE and GM components [14], the evaluation and calculation result will be presented in later chapters.

# Chapter 2

## Hartree-Fock Calculation

### 2.1 Nucleon-Nucleon Interaction

Due to the lack of a exact theory to describe the nucleon-nucleon (NN) interaction, a model need to be imposed and fit with experimental measurement or theoretical calculation results. For a system as massive as a NS, deducing the EoS using the *ab initio* method, i.e. solving the Schrödinger equation over all particles, is simply impossible, therefore an *effective interaction* must be used [16]. In this section, we only limit ourselves to two-body interaction, thus, the NN potential can be expressed in the form of

$$v = v(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \quad (2.1)$$

where the primed and unprimed variables indicate the properties of 2 nucleons respectively, in which  $\mathbf{r}$  is the particle's position,  $\mathbf{p}$  is its momentum,  $\boldsymbol{\sigma}$  is its intrinsic spin and  $\boldsymbol{\tau}$  is its isospin.

The functional form of  $v$  in (2.1) cannot freely take any form but is constrained by many invariance requirements [16]

- **Translational invariance:** The NN potential should only depend on the *relative position* of the two particle but not their explicit positions, thus we can reduce (2.1) to

$$v = v(\mathbf{r} - \mathbf{r}', \mathbf{p}, \mathbf{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(\mathbf{r}, \mathbf{p}, \mathbf{p}', \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \quad (2.2)$$

with at the last expression, we redefine  $\mathbf{r}$  as the relative position vector.

- **Galilei invariance:** The potential should also be invariant under transformation between inertial frame of reference, which requires that only the relative momentum  $\mathbf{p} - \mathbf{p}'$  is depended, i.e.

$$v = v(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \quad (2.3)$$

where here we denote  $\mathbf{p}$  as the relative momentum.

- **Rotational invariance:** The potential should be constructed such that the total angular momentum is zero.

- **Isospin invariance:** The isospin  $\tau$  only enters (2.3) through the *isospin operator*  $\hat{\tau}$  and  $\hat{\tau}'$ . The NN interaction potential needs to be invariant under rotation in isospin space, in other words, it can only depend on the isospin-independent terms, the terms with  $\tau \cdot \tau'$  and their higher powers. Coincide the isospin operator matrices with the Pauli matrices (since by definition they differ by only a conversion scaling factor), we have

$$[\hat{\tau}_m, \hat{\tau}_n] = 2i \sum_k \epsilon_{mnk} \hat{\tau}_k \quad \text{and} \quad \{\hat{\tau}_m, \hat{\tau}_n\} = 2\delta_{mn} \quad (2.4)$$

adding the two equations together, we get the term

$$\hat{\tau}_m \hat{\tau}_n = \delta_{mn} + i \sum_k \epsilon_{mnk} \hat{\tau}_k \quad (2.5)$$

Then we obtain the dot product

$$(\hat{\tau} \cdot \hat{\tau}')^2 = \sum_m \delta_{mm} - \sum_{mnkk'} \epsilon_{mnk} \epsilon_{mnk'} \hat{\tau}_k \hat{\tau}'_{k'} = 3 - 2\hat{\tau} \cdot \hat{\tau}' \quad (2.6)$$

Therefore, we can split the potential into

$$v = v_0(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}') + v_1(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}') \hat{\tau} \cdot \hat{\tau}' \quad (2.7)$$

- **Parity invariance:** The NN interaction potential is also expected to be invariant under the action of parity operator, i.e. changing the sign of spatial coordinates

$$v(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(-\mathbf{r}, -\mathbf{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \quad (2.8)$$

- **Time reversal invariance:** Finally, the interaction should stay the same after switching the time arrow direction

$$v(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}, \boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') = v(\mathbf{r}, -\mathbf{p}, -\boldsymbol{\sigma}, -\boldsymbol{\sigma}', \boldsymbol{\tau}, \boldsymbol{\tau}') \quad (2.9)$$

Having the above considerations, developing further the M3Y-Paris interaction [17–19]

$$v = v_{00}(r) + v_{10}(r) \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' + v_{01}(r) \boldsymbol{\tau} \cdot \boldsymbol{\tau}' + v_{11}(r) (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') (\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \quad (2.10)$$

by adding a density-dependent form factor to each term gives the CDM3Yn interaction model

$$\begin{aligned} v(\rho, r) = & F_{00}(\rho) v_{00}(r) + F_{10}(\rho) v_{10}(r) \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' \\ & + F_{01}(\rho) v_{01}(r) \boldsymbol{\tau} \cdot \boldsymbol{\tau}' + F_{11}(\rho) v_{11}(r) (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') (\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \end{aligned} \quad (2.11)$$

where each radial term is the superposition of 3 Yukawa potentials with different parameters

$$v_{\sigma\tau}(r) = \sum_{k=1}^3 Y_{\sigma\tau}(k) \frac{\exp(-\mu_k r)}{\mu_k r} \quad (2.12)$$

and the form factor is adapted the form [9, 10, 20]

$$F_{\sigma\tau}(\rho) = C_{\sigma\tau} [1 + \alpha_{\sigma\tau} \exp(-\beta_{\sigma\tau} \rho) + \gamma_{\sigma\tau} \rho] \quad (2.13)$$

## 2.2 Equation of States of Nuclear Matter

The total HF energy of the system can be expressed as

$$E = \sum_{\sigma\tau} \sum_{\mathbf{k}}^{k_F^{\sigma\tau}} \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} \sum_{\mathbf{k}\sigma\tau} \sum_{\mathbf{k}'\sigma'\tau'} \left[ \langle \mathbf{k}\sigma\tau, \mathbf{k}'\sigma'\tau' | v^D | \mathbf{k}\sigma\tau, \mathbf{k}'\sigma'\tau' \rangle + \langle \mathbf{k}\sigma\tau, \mathbf{k}'\sigma'\tau' | v^{EX} | \mathbf{k}'\sigma\tau, \mathbf{k}\sigma'\tau' \rangle \right] \quad (2.14)$$

where the single-particle wave function is plane wave

$$|\mathbf{k}\sigma\tau\rangle = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}} \chi_\sigma \chi_\tau \quad (2.15)$$

$\Omega$  being the spatial volume of the system,  $k_F^{\sigma\tau} = (6\pi^2 \rho_{\sigma\tau})^{1/3}$  is the Fermi momentum corresponding to spin  $\sigma$  and isospin  $\tau$ ,  $v^{D(EX)}$  is the direct (exchange) part of the interaction determined from the singlet and triplet-even (and odd) of the central NN force. The direct and exchange interaction is then

$$\begin{aligned} v^{D(EX)}(\rho_b, r) = & F_{00}^{D(EX)}(\rho_b) v_{00}^{D(EX)}(r) + F_{10}^{D(EX)}(\rho_b) v_{10}^{D(EX)}(r) \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}' \\ & + F_{01}^{D(EX)}(\rho_b) v_{01}^{D(EX)}(r) \boldsymbol{\tau} \cdot \boldsymbol{\tau}' \\ & + F_{11}^{D(EX)}(\rho_b) v_{11}^{D(EX)}(r) (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') (\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \end{aligned} \quad (2.16)$$

and

$$v_{\sigma\tau}^{D(EX)}(r) = \sum_{k=1}^3 Y_{\sigma\tau}^{D(EX)}(k) \frac{\exp(-\mu_k r)}{\mu_k r} \quad (2.17)$$

with the Yukawa strengths given in Table 2.2 and the density-dependent form factor parameters are in Table 2.1. Note that in (2.16),  $\rho_b$  denotes the *baryon density*, this will be used in order to distinguish with the lepton density in the later section.

Table 2.1: CDM3Yn interaction's parameters [9, 21]. The version CDM3Y3, 4, 6 don't have the 10 and 11 terms due to them being *spin saturated*, i.e. the spin dependent terms vanishes effectively.

Interaction	$\sigma\tau$	$C_{\sigma\tau}$	$\alpha_{\sigma\tau}$	$\beta_{\sigma\tau}$ ( $fm^3$ )	$\gamma_{\sigma\tau}$ ( $fm^3$ )
CDM3Y3	00	0.2985	3.4528	2.6388	-1.5
	01	0.1574	9.7016	16.2704	11.9946
CDM3Y4	00	0.3052	3.2998	2.3180	-2.0
	01	0.1318	11.7739	16.0279	15.1987
CDM3Y6	00	0.2658	3.8033	1.4099	-4.0
	01	0.1824	8.8819	16.4346	10.8703
CDM3Y8	00	0.2658	3.8033	1.4099	-4.3
	01	0.2463	6.3836	10.2566	6.3549
	10	0.2161	3.7510	-3.3396	9.9329
	11	0.7572	1.9967	33.2012	0.2989



Table 2.2: Yukawa strengths of the M3Y-Paris interaction [9, 22].

$k$	$\mu_k$ ( $fm^{-1}$ )	$Y_{00}^D$ ( $MeV$ )	$Y_{10}^D$ ( $MeV$ )	$Y_{01}^D$ ( $MeV$ )	$Y_{11}^D$ ( $MeV$ )	$Y_{00}^{EX}$ ( $MeV$ )	$Y_{10}^{EX}$ ( $MeV$ )	$Y_{01}^{EX}$ ( $MeV$ )	$Y_{11}^{EX}$ ( $MeV$ )
1	4.0	11061.625	938.875	313.625	-969.125	-1524.25	-3492.75	-4118.0	-2210.0
2	2.5	-2537.5	-36.0	223.5	450.0	-518.75	795.25	1054.75	568.75
3	0.7072	0.0	0.0	0.0	3.4877	-7.8474	2.6157	2.6157	-0.8719

Multiply (2.14) with  $\Omega^{-1}$ , the energy density of the NM is separated into the kinetic term  $\varepsilon_{kin}$  and the potential terms  $\varepsilon_{\sigma\tau}$ , i.e.

$$\varepsilon = \frac{E}{\Omega} = \varepsilon_{kin} + F_{00}(\rho_b)\varepsilon_{00} + F_{01}(\rho_b)\varepsilon_{01} + F_{10}(\rho_b)\varepsilon_{10} + F_{11}(\rho_b)\varepsilon_{11} \quad (2.18)$$

The final expressions of each terms of the energy density are

$$\varepsilon_{kin} = \frac{3}{10} \sum_{\sigma\tau} \frac{\hbar^2 (k_F^{\sigma\tau})^2}{m_\tau} \rho_{\sigma\tau} \quad (2.19)$$

$$\varepsilon_{00} = \frac{1}{2} \left[ \rho_b^2 J_{00}^D + \int A_{00}^2 v_{00}^{EX}(r) d^3r \right] \quad (2.20)$$

$$\varepsilon_{10} = \frac{1}{2} \left[ \rho_b^2 J_{10}^D \left( \Delta_n \frac{1+\delta}{2} + \Delta_p \frac{1-\delta}{2} \right)^2 + \int A_{10}^2 v_{10}^{EX}(r) d^3r \right] \quad (2.21)$$

$$\varepsilon_{01} = \frac{1}{2} \left[ \rho_b^2 J_{01}^D \delta^2 + \int A_{01}^2 v_{01}^{EX}(r) d^3r \right] \quad (2.22)$$

$$\varepsilon_{11} = \frac{1}{2} \left[ \rho_b^2 J_{11}^D \left( \Delta_n \frac{1+\delta}{2} - \Delta_p \frac{1-\delta}{2} \right)^2 + \int A_{11}^2 v_{11}^{EX}(r) d^3r \right] \quad (2.23)$$

where  $\Delta_\tau = (\rho_{\uparrow\tau} - \rho_{\downarrow\tau})/\rho_\tau$  is the polarity of nucleon,  $\delta = (\rho_n - \rho_p)/\rho_b$  is the asymmetry of NM,  $J_{\sigma\tau}^D = \int v_{\sigma\tau}(r) d^3r$  is the volume integral of the direct interaction and

$$\begin{aligned} A_{00} &= \rho_{\uparrow n} \hat{j}_1(k_F^{\uparrow n}) + \rho_{\downarrow n} \hat{j}_1(k_F^{\downarrow n}) + \rho_{\uparrow p} \hat{j}_1(k_F^{\uparrow p}) + \rho_{\downarrow p} \hat{j}_1(k_F^{\downarrow p}) \\ A_{10} &= \rho_{\uparrow n} \hat{j}_1(k_F^{\uparrow n}) - \rho_{\downarrow n} \hat{j}_1(k_F^{\downarrow n}) + \rho_{\uparrow p} \hat{j}_1(k_F^{\uparrow p}) - \rho_{\downarrow p} \hat{j}_1(k_F^{\downarrow p}) \\ A_{01} &= \rho_{\uparrow n} \hat{j}_1(k_F^{\uparrow n}) + \rho_{\downarrow n} \hat{j}_1(k_F^{\downarrow n}) - \rho_{\uparrow p} \hat{j}_1(k_F^{\uparrow p}) - \rho_{\downarrow p} \hat{j}_1(k_F^{\downarrow p}) \\ A_{11} &= \rho_{\uparrow n} \hat{j}_1(k_F^{\uparrow n}) - \rho_{\downarrow n} \hat{j}_1(k_F^{\downarrow n}) - \rho_{\uparrow p} \hat{j}_1(k_F^{\uparrow p}) + \rho_{\downarrow p} \hat{j}_1(k_F^{\downarrow p}) \end{aligned} \quad (2.24)$$

A more detailed calculation can be found in the Appendix.

## 2.3 $\beta$ -Stable Nuclear Matter

After the HF calculation, we were able to obtain a numerical HF energy density  $\varepsilon(\rho_n, \rho_p, \Delta_n, \Delta_p)$ . However, it is in fact impossible for a NS to exist while consisting of purely nucleon. In order for the

[NS](#) to exist, leptons ( $e^-$  and  $\mu^-$ ) have to be introduced to the matter constituents and the  $npe\mu$  matter has to be under the condition of  $\beta$ -stable, i.e.

- Charge balance

$$\rho_p = \rho_e + \rho_\mu \quad (2.25)$$

- Chemical potential balance

$$\mu_n - \mu_p = \mu_e = \mu_\mu \quad (2.26)$$

where  $\mu_i$  ( $i = n, p, e, \mu$ ) is the chemical potential of the  $i$  particle.

The total energy density of the  $npe\mu$  matter is thus

$$\varepsilon = \varepsilon_{HF} + \rho_n m_n c^2 + \rho_p m_p c^2 + \varepsilon_e + \varepsilon_\mu \quad (2.27)$$

which leads to the nucleon chemical potential of the form

$$\mu_\tau(\rho_n, \rho_p, \Delta_n, \Delta_p) = \frac{\partial \varepsilon}{\partial \rho_\tau} = \frac{\partial \varepsilon_{HF}}{\partial \rho_\tau} + m_\tau c^2 \quad (2.28)$$

Let  $\hat{\mu} = \mu_n - \mu_p$  be the leptons' chemical potential, (2.25) is equivalent to (the detailed derivation can be found in the Appendix)<sup>1</sup>

$$3\pi^2(\hbar c)^3 \rho_p - \hat{\mu}^3 - [\hat{\mu}^2 - (m_\mu c^2)^2]^{3/2} \theta(\hat{\mu} - m_\mu c^2) = 0 \quad (2.29)$$

In the CDM3Y3, 4, 6 interactions, the [NM](#) is spin saturated, therefore a  $\Delta_n = \Delta_p = 0$  and there are no 10 and 11 terms, while for the interaction CDM3Y8, under strong magnetic field like that of a magnetar, we can approximate  $\Delta_n \approx -\Delta_p \approx \Delta$  and reduce the [EoS](#) to depend on just the baryon polarity  $\Delta$  alone, and the more baryon polarized, the stronger the magnetic field of the [NS](#).

For a fixed value of  $\Delta$ , we are able to obtain a density function of the form  $\rho_n(\rho_b, \Delta)$  and  $\rho_p(\rho_b, \Delta)$ , which in turn gives the lepton chemical potential  $\hat{\mu}(\rho_b, \Delta) = \hat{\mu}(\rho_n, \rho_p)$ . On the other hand, the leptons' densities are then

$$\rho_e(\rho_b, \Delta) = \frac{\hat{\mu}^3(\rho_b, \Delta)}{3\pi^2(\hbar c)^3} \quad \text{and} \quad \rho_\mu(\rho_b, \Delta) = \frac{[\hat{\mu}^2(\rho_b, \Delta) - (m_\mu c^2)^2]^{3/2}}{3\pi^2(\hbar c)^3} \theta(\hat{\mu}(\rho_b, \Delta) - m_\mu c^2) \quad (2.30)$$

Consider the  $e^-$  and  $\mu^-$  to be systems of relativistic Fermi gas, then their respective energy densities and pressure contributions are ( $l = e, \mu$ )

$$\varepsilon_l(\rho_b, \Delta) = \frac{2}{(2\pi)^3} \int_0^{[3\pi^2 \rho_l(\rho_b, \Delta)]^{1/3}} \sqrt{\hbar^2 c^2 k^2 + m_l^2 c^4} d^3 \mathbf{k} \quad (2.31)$$

and

$$P_l(\rho_b, \Delta) = \frac{1}{3} \frac{2}{(2\pi)^3} \int_0^{[3\pi^2 \rho_l(\rho_b, \Delta)]^{1/3}} \frac{\hbar^2 c^2 k^2}{\sqrt{\hbar^2 c^2 k^2 + m_l^2 c^4}} d^3 \mathbf{k} \quad (2.32)$$

<sup>1</sup> $\theta(x)$  is the Heaviside function, i.e. it returns 1 for  $x \geq 0$  and 0 otherwise.

Plus, from the HF formalism with NM, the baryon pressure is given by

$$P_b = \rho_b^2 \frac{\partial(\varepsilon_{HF}/\rho_b)}{\partial \rho_b} \quad (2.33)$$

Finally, we obtain the total energy density dependence on baryon density as

$$\varepsilon(\rho_b, \Delta) = \varepsilon_{HF}(\rho_b, \Delta) + \rho_n(\rho_b, \Delta)m_n c^2 + \rho_p(\rho_b, \Delta)m_p c^2 + \varepsilon_e(\rho_b, \Delta) + \varepsilon_\mu(\rho_b, \Delta) \quad (2.34)$$

and the total pressure of NS matter

$$P(\rho_b, \Delta) = P_b(\rho_b, \Delta) + P_e(\rho_b, \Delta) + P_\mu(\rho_b, \Delta) \quad (2.35)$$

and this is the final result of the EoS of cold  $\beta$ -stable NS matter.

# Chapter 3

## Neutron Star Properties

### 3.1 Tolman-Oppenheimer-Volkoff Equation

In the framework of [GR](#), assume the [NS](#) to be in a static, isotropic region of spacetime, the metric elements are then

$$c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\nu(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \quad (3.1)$$

Inside the [NS](#), we have [\[23\]](#) the energy-momentum tensor as

$$T^{\mu\nu} = -Pg^{\mu\nu} + (P + \varepsilon)u^\mu u^\nu \quad (3.2)$$

where  $u^\mu = dx^\mu/d\tau$  is the local fluid 4-velocity. Solving the Einstein's field equation [\[23\]](#)

$$G^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu} \quad (3.3)$$

gives the Tolman-Oppenheimer-Volkoff ([TOV](#)) equation

$$\frac{dP}{dr} = -\frac{G\varepsilon(P)m}{c^2 r^2} \left(1 + \frac{P}{\varepsilon(P)}\right) \left(1 + \frac{4\pi P r^3}{m c^2}\right) \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} \quad (3.4)$$

$$\frac{dm}{dr} = \frac{4\pi r^2 \varepsilon(P)}{c^2} \quad (3.5)$$

where  $\varepsilon(P)$  can be obtained from the CDM3Yn interaction calculated previously. Additional boundary conditions are

$$P(0) = P_c; \quad P(R) = 0; \quad m(0) = 0; \quad m(R) = M$$

and by varying the center pressure  $P_c$ , a relation of the mass  $M$  and radius  $R$  of the [NS](#) can be obtained.

### 3.2 Gravito-electric and Gravito-magnetic Tidal Deformation

Pertubing

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