Lab 6: Dark Matter!

1 Introduction

The *rotation curve* of a galaxy is a measurement of how fast the galaxy rotates as a function of distance from the center. Galaxies don't rotate as solid bodies (like records on a turntable or merry-go-rounds); rather, they rotate *differentially*, with the inner parts moving faster than the outer parts. In this section, you will use Newton's laws to investigate this differential rotation and learn how we can use the rotation curve to unveil rather a strange resident of Galaxies...

2 Differential Rotation in the Solar System

In space, almost everything is orbiting something: the moon orbits Earth, Earth orbits the Sun, the Sun orbits the center of the Milky Way galaxy, etc. All of this motion is due (we suspect) to the force of gravity:

$$F = \frac{GM_1 M_2}{r^2} \tag{1}$$

Masses M_1 and M_2 actually both orbit their common *barycenter*, or center of mass. When more complicated mass distributions are involved (e.g., many orbiting bodies, as in the Solar System or the Milky Way), each mass is subject to the gravitational force of all the mass within its own orbit: for example, the gravitational force on Earth is, properly:

$$F = \frac{GM_{\mathrm{Earth}} \times (M_\odot + M_{\mathrm{Mercury}} + M_{\mathrm{Venus}})}{r^2} \tag{2}$$

where r is the distance between Earth and the center of mass of all four objects. However, the Sun is so much more massive than anything else in the Solar System that we can ignore the contribution of the other planets and write:

$$F = \frac{GM_{\mathrm{Earth}} \times M_\odot}{r^2} \tag{3}$$

where r is the distance between the Earth and the Sun.

In our Solar System, the planets move on very nearly circular orbits. The force on an object on a circular orbit is described by the equation:

$$F = \frac{Mv^2}{r} \tag{4}$$

where M is the orbiting body's mass, v is its velocity, and r is the radius of its orbit. (This is the formula for *centripetal force*.) If we are right, and the force of gravity is really what is making the planets orbit the Sun, then a little bit of algebra will tell us how the velocity of a planet is related to its distance from the Sun.

1. The force of gravity on an orbiting body must balance its centripetal force. Consider a planet (M_{planet}) orbiting the Sun. Set the above two equations (3) and (4) equal to each other and solve for v in terms of $M_{\odot}$, G, and r.

2. Now we have a theoretical prediction for the orbits of the planets. Using information from the tables below, calculate the orbital velocity of each planet (as predicted by the equation you just derived) and record it in your notebook, making two columns: one for the planet's name, and one for their respective calculated orbital velocity. Make sure units check out!

3. Using these values, plot the theoretical rotation curve (in km/s vs. AU) for our Solar System.

4. Using the measured values in Table 1, plot the actual rotation curve (in km/s vs. AU). (Don't forget to label the axes and indicate which plotted curve is which!). Do you think gravity is responsible for the orbits of the planets? Why or why not?

Table 1. Solar System Data

Planet	Semimajor axis (AU)	Orbital Eccentricity	Sidereal Orbital Period (yr)	Average orbital velocity (km/s)	Your calculated velocity (km/s)
Mercury	0.3871	0.2056	0.2408	47.9150	
Venus	0.7233	0.0068	0.6152	35.0435	
Earth	1.0000	0.0167	1.0000	29.8061	
Mars	1.5237	0.0934	1.8809	24.1456	
Jupiter	5.2028	0.0483	11.8622	13.0730	
Saturn	9.5388	0.0560	29.4577	9.65161	
Uranus	19.1914	0.0461	84.0139	6.80864	
Neptune	30.0611	0.0097	164.793	5.43715	

Table 2. Astronomical Constants

constant	value
$M_{\odot}$	1.989×10^{33} g
G	4.46×10^{-21} cm^2 s^{-2} AU g^{-1}
	4.31×10^{-6} km^2 s^{-2} kpc M_{Sun}^{-1}

3 Differential Rotation in a Galaxy

In the last part, we used our knowledge of the mass of the Sun to predict the orbital velocity of the planets. For galaxies, we have to work backwards – we don't know the mass of a galaxy, but we can use the orbits of the stars around the center to deduce what the galaxy's mass must be. Figure 1 shows a measured rotation curve for the galaxy NGC 2742. The positive values of radial velocity are for the stars moving away from us, while the negative velocities are for stars moving towards us. Negative radius values are just used to indicate an opposite side of the galaxy from the positive radius values.

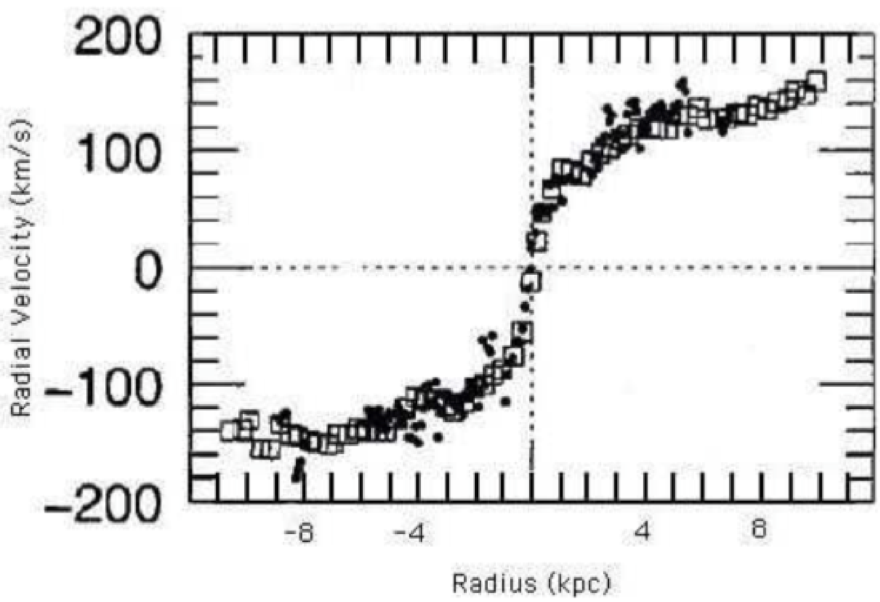

Figure 1: The rotation curve of NGC 2742.

1. Draw a table in your notebook with 5 columns: *Radius (kpc)*, *Rotational Velocity (km/s)*, *Gravitational Mass* ($M_\odot$), *Luminosity* ($L_\odot$), and *Luminous Mass* ($M_\odot$). Select 7 evenly spaced radii from Figure 1, either all positive or all negative, and record them in your table.
2. Use Figure 1 to measure the radial velocity at each of your radii. Record them in your table.
3. Use the equation you derived earlier to solve for enclosed mass M as a function of velocity v, orbital radius r, and the constant G.
4. Solve for the amount of galactic mass within each of the radii you chose. Record these (very big) values in the table, under "gravitational mass" (since this is mass predicted by gravity). Pay attention to units!

Now you have weighed the galaxy and can see that gravity predicts a mass equal to billions and billions of stars. We can compare this to the amount of light from the galaxy, since the light comes from the stars and gas it contains. Figure 2 shows the amount of galactic light that comes from within different radii.

The comparison we're going to make is easier for some galaxies than others. It is harder to figure out how much light is coming from a galaxy the more "edge-on" it is, because if we are looking at the side of the galaxy, a lot of the light is blocked by dust. In contrast, when we look at a galaxy "face-on," we see almost all of its light, but it is harder to accurately determine the galaxy's rotational velocity (because most of the rotational velocity is perpendicular to our line of sight).

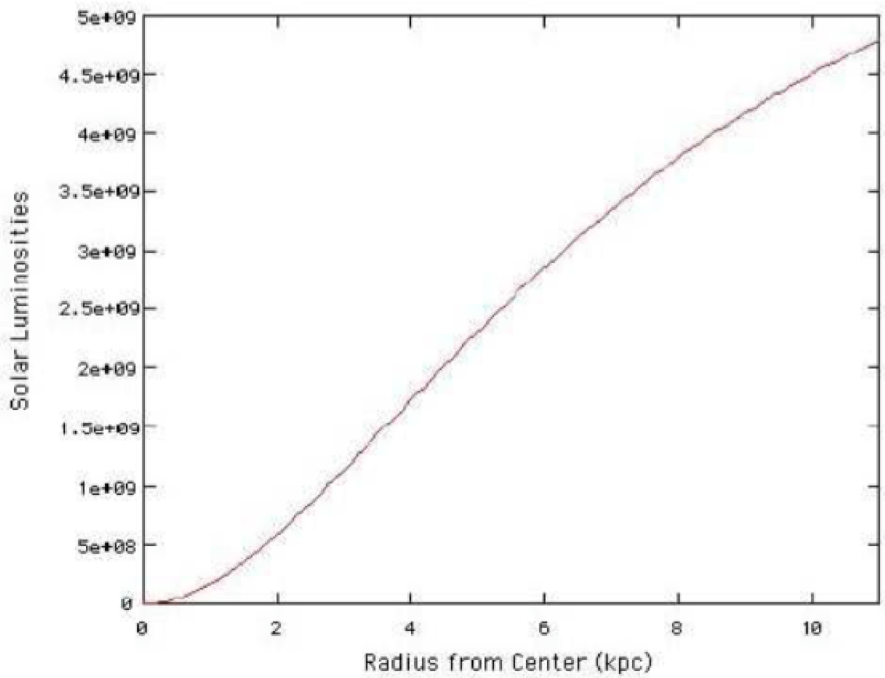

Figure 2: The amount of light emitted from inside a given radius in NGC 2742.

5. At the radii that you have chosen, determine a value for the luminosity of the galaxy and record it in the table. (The "e"s in the luminosity values are powers of ten: "2e+09" means 2×10^9).

6. Now calculate how much mass must be present in the galaxy based on how much light we see. Assume that it takes roughly two solar masses of matter to produce one solar luminosity. (This is based on how many low mass and high mass stars we think are typically in a galaxy.) Record the values in the table under *Luminous mass*.

7. Plot each of the masses (gravitational and luminous) versus radius, remembering to label your axes.

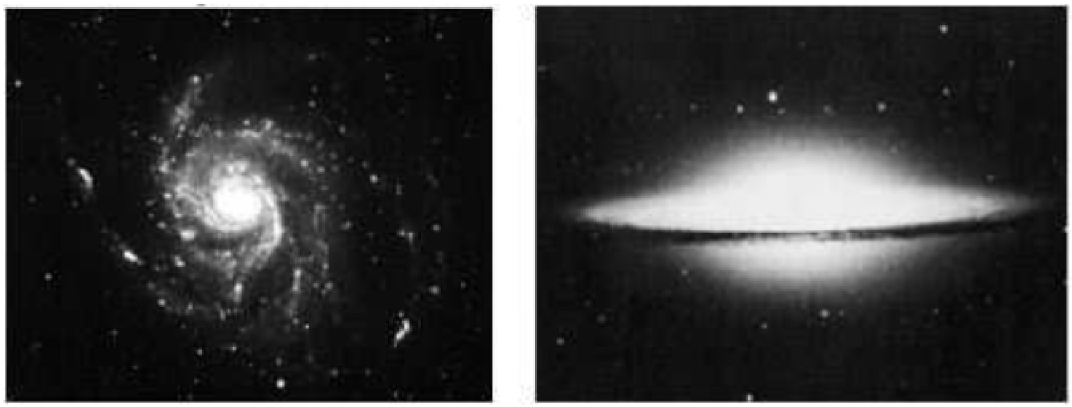

Figure 3: Left: M101, the Pinwheel Galaxy. Right: M104, the Sombrero Galaxy.

8. Determine the *mass-to-light ratio* for NGC 2742 at your largest radius (within the entire galaxy) by dividing your gravitational mass by your luminous mass.

9. What can you conclude about the matter in NGC 2742? In other words, what percentage of the total mass is luminous? What percent cannot be accounted for by the light that we see? Why is this so-called "dark matter" termed "dark?"

4 Conclusions

1. Based on the above, are you convinced dark matter exists? Briefly explain your reasoning.

2. Figure 3 shows two galaxies that we cannot use to search for dark matter. Why not?

3. If the lab was perfectly clear to you, what did you like or dislike? If not, what confused you? Any other feedback?