CSCE 222 [501, 502] Discrete Structures for Computing
Spring 2015 – Hyunyoung Lee

Problem Set 10

Due dates: Electronic submission of hw10.pdf file of this homework is due on **Wednesday, 4/29/2015 before 23:59** on `http://ecampus.tamu.edu`. Please do not archive or compress the file. A signed paper copy of the PDF file is due on **Thursday, 4/30/2015** at the beginning of class.

Name: Natalie Cluck **Section: 501**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____

In this problem set, you will earn total $100 + 20$ (extra credit) points.

Problem 1. (9 points) Section 13.1, Exercise 4, page 856

Solution.
a.) S → 1S → 11S → 111S → 11100A → 111000. Hence, 111000 belongs to the language generated by G.
b.) Any sentence of the language generated by G has to end with 0, because the sentence always ends with A, which is 0. Therefore, 11001 does not belong to the language generated by G.
c.) L(G) = $\{1^m 0^n | m \geq 0, n \geq 3\}$.

Problem 2. (15 points) Section 13.1, Exercise 6, page 856

Solution.
a.) $\{aabb\}$
b.) $\{aba, aa\}$
c.) $\{abb, abab\}$
d.) $\{a^n | n \geq 4\} \bigcup \{b^m | m \geq 1\}$
e.) $\{a^n b^{n+m} a^m | m \geq 0, n \geq 0\}$

Problem 3. (16 points) Section 13.1, Exercise 14, page 856

Solution.
a.) Let L = $\{10, 01, 101\}$

. Then, G = (V, T, S, P) is a phrase structure grammar which generates the language L if V = $\{0, 1, S\}$, T = $\{0, 1\}$, S is the starting symbol and the productions are S $\rightarrow$ 10, S $\rightarrow$ 01, S $\rightarrow$ 101.

b.) Let L = $\{a|$ a is a bit string that starts with 00 and ends with one or more 1s $\}$.

Then, $G = (V, T, S, P)$ is a phrase structure grammar which generates the language L if V = $\{0, 1, S, A, B\}$, T = $\{0, 1\}$, S is the starting symbol and the productions are S $\rightarrow$ 00AB, A $\rightarrow$ AA, A $\rightarrow$ 0, A $\rightarrow$ 1, B $\rightarrow$ BB, B $\rightarrow$ 1.

c.) $L = \{a|$ a is a bit string consisting of an even number of 1s followed by a final 0 $\}$.

Then, $G = \{V, T, S, P\}$ is a phrase structure grammar which generates the language L if $V = \{0, 1, S, A, B\}$,T = $\{0, 1\}$, S is the starting symbol and the productions are $S \rightarrow 11SB, S \rightarrow A, A \rightarrow 11A, A \rightarrow 11, A \rightarrow \lambda, B \rightarrow 0$

d.) Let L = {a | a is a bit string that has neither two consecutive 0s or two consecutive 1s}.

Then G = (V,T,S,P) is a phrase structure grammar which generates the language L if V = {0,1,S,A,B}, T = $\{0, 1\}$, S is the starting symbol and the productions are S $\rightarrow$ A, A $\rightarrow$ $AA, A \rightarrow A0, A \rightarrow 01, A \rightarrow \lambda$
$S \rightarrow B, B \rightarrow BB, B \rightarrow B1, B \rightarrow 10, B \rightarrow \lambda$

Problem 4. (12 points) Section 13.1, Exercise 18, page 856

Solution.

a.) L = $\{01^{2n}|n \geq 0\}$.

Then G = (V,T,S,P) is a phrase structure grammar which generates the language L if V = $\{0, 1, S, A\}, T = \{0, 1\}$, S is the starting symbol and the productions are $S \rightarrow 0A, A \rightarrow \lambda, A \rightarrow AA, A \rightarrow 11$

b.) L = $\{0^n 1^{2n}|n \geq 0\}$.

Then G = (V,T,S,P) is a phrase structure grammar which generates the language L if V = $\{0, 1, S, A\}, T = \{0, 1\}$, S is the starting symbol and the productions are $S \rightarrow 0AB, A \rightarrow 0A, A \rightarrow 0, A \rightarrow \lambda, B \rightarrow BB, B \rightarrow 11, B \rightarrow \lambda$

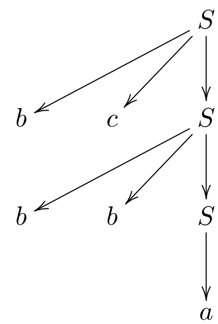
c.) L = $\{0^n 1^m 0^n|m \geq 0, n \geq 0\}$.

Then G = (V,T,S,P) is a phrase structure grammar which generates the language L if V = $\{0, 1, S, A\}, T = \{0, 1\}$, S is the starting symbol and the productions are $S \rightarrow ABA, A \rightarrow 0A, A \rightarrow 0, A \rightarrow \lambda, B \rightarrow \lambda, B \rightarrow 1B, B \rightarrow 1$

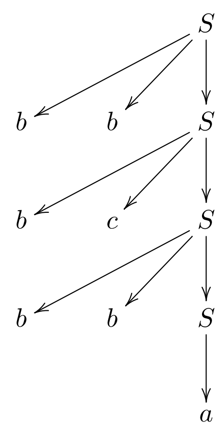
Problem 5. (15 points) Section 13.1, Exercise 24, page 857

Solution.

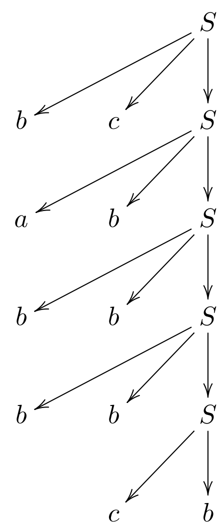
a.)



b.)



c.)



Problem 6. (5 points) Section 13.2, Exercise 2 a), page 864

Solution.

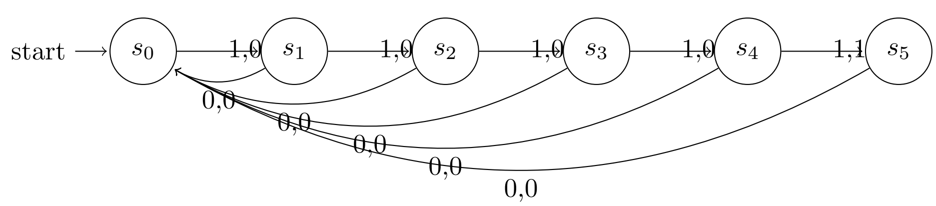
	f		g	
	Input		Input	
State	0	1	0	1
s_0	s_1	s_2	1	0
s_1	s_0	s_3	1	0
s_2	s_3	s_0	0	0
s_3	s_1	s_2	1	1

Problem 7. (5 points) Section 13.2, Exercise 4 a), page 864

Solution.

a.) 0 0 1 1 0

Problem 8. (10 points) Section 13.2, Exercise 18, page 865 (explain your FSM)



Solution.

In this FSM, if a 1 is in the string, it is one step closer to 5 consecutive ones. Same with 11, 111, 1111. Anytime a 0 is entered, it returns to the original state. Once 5 1s are detected, the string is valid as long as the rest of the string consists of 1s only. If a 0 is detected, the machine returns back to the original state again.

Problem 9. (12 points) Section 13.3, Exercise 8 a), b), e) and f), page 875

Solution.

a.) If A contains v, v $\in$ V, A = $\{v\}$. The set A^2 is $\{vv\}$. Hence, by counterexample, A is not contained in A^2.
b.) $\lambda \notin A$, so the conclusion is false.
e.) If the statement equals $A^0 A = A^0$, the equality would not hold. Therefore the statement is false.
f.) If $A = \{1, 11\}$, A^2 would be $\{11, 111, 1111\}$, with cardinality 3. If A is 2 and n is 2, the statement would be $3 = 4$. The statement is proven false.

Problem 10. (6 points) Section 13.3, Exercise 10 b), d) and f), page 875

Solution.
b.) It is not present.
d.) It is present
f.) It is not present.

Problem 11. (5 points) Section 13.3, Exercise 16, page 876

Solution.

L = $\{\lambda\} \bigcup \{1\}\{0,1\}^* \bigcup \{0\}\{1\}^*\{0\}\{0,1\}^*$

Problem 12. (5 points) Section 13.3, Exercise 18, page 876

Solution.

L = $\{\lambda\} \bigcup \{0\}\{1\}^*$

Problem 13. (5 points) Section 13.3, Exercise 28, page 876

Solution.

Checklist:
☐ Did you add your name and section?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the PDF file of your homework on eCampus?
☐ Did you submit a signed hardcopy of the PDF file in class?