# Exercise 1

```
type("rredf.m")
```

```
function R=rredf(A)
format
[m,n]=size(A);
rankA=rank(A);
A=sym(A);
R=A;
%Forward Phase
for k=1:m
    X=R(k:m,:);
    i=find(any(X),1);
    [~,j]=max(abs(X(:,i)));
    if j~=1
        X([1 j],:)=X([j 1],:);
    end
    if k<m
        for p=2:m-k+1
            if X(p,i)~=0
                r=-X(p,i)/X(1,i);
                X(p,:)=X(p,:)+(r*X(1,:));
            end
        end
    end
    R(k:m,:)=X;
    R=closetozeroroundoff(R,7);
end
%Backward Phase
for k=rankA:-1:1
    j=find(any(R(k,:),1),1);
    h=1/R(k,j);
    R(k,:)=h*R(k,:);
    if k>1
        for i=1:k-1
            if R(i,j)~=0
                r=R(i,j);
                R(i,:)=R(i,:)-(r*R(k,:));
            end
        end
    end
    R=closetozeroroundoff(R,7);
end
disp('the constructed matrix R is')
disp(double(R))
rf=rref(A)
if closetozeroroundoff(R-rf,7)==0
    disp('R is the reduced echelon form of A')
    R=double(R);
else
    disp('Something went wrong!')
    R=[]
end
end
```

```
type("closetozeroroundoff.m")
```

```
function B=closetozeroroundoff(A,p)
A(abs(A)<10^-p)=0;
B=A;
end
```

```
%(a)
```

```
A=[2 0 1 3]
```

```
A = 1×4
     2     0     1     3
```

```
R=rredf(A);
```

```
the constructed matrix R is
    1.0000         0    0.5000    1.5000
rf =
```

$$\begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

```
R is the reduced echelon form of A
```

```
%(b)
A=[2 4 1 ; 1 2 3 ; 1 2 1]
```

```
A = 3×3
     2     4     1
     1     2     3
     1     2     1
```

```
R=rredf(A);
```

```
the constructed matrix R is
     1     2     0
     0     0     1
     0     0     0
rf =
```

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

```
R is the reduced echelon form of A
```

```
%(c)
A=[zeros(4),magic(4)]
```

```
A = 4×8
     0     0     0     0    16     2     3    13
     0     0     0     0     5    11    10     8
     0     0     0     0     9     7     6    12
     0     0     0     0     4    14    15     1
```

```
R=rredf(A);
```

```
the constructed matrix R is
     0     0     0     0     1     0     0     1
     0     0     0     0     0     1     0     3
     0     0     0     0     0     0     1    -3
     0     0     0     0     0     0     0     0
rf =
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
R is the reduced echelon form of A
```

```
%(d)
A=pascal(3)
```

```
A = 3×3
     1     1     1
     1     2     3
     1     3     6
```

```
R=rredf(A);
```

```
the constructed matrix R is
     1     0     0
     0     1     0
     0     0     1
rf =
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R is the reduced echelon form of A
```

```
%(e)
A=ones(3,6); A(:,1:2:5)=magic(3)
```

```
A = 3×6
     8     1     1     1     6     1
     3     1     5     1     7     1
     4     1     9     1     2     1
```

```
R=rredf(A);
```

```
the constructed matrix R is
     1     0     0     0    -1     0
     0     1     0     1    15     1
     0     0     1     0    -1     0
rf =
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 15 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{pmatrix}$$

```
R is the reduced echelon form of A
```

```
%(f)
A=zeros(3,6); A(:,2:2:6)=magic(3)
```

```
A = 3×6
     0     8     0     1     0     6
     0     3     0     5     0     7
     0     4     0     9     0     2
```

```
R=rredf(A);
```

```
the constructed matrix R is
     0     1     0     0     0     0
     0     0     0     1     0     0
     0     0     0     0     0     1
rf =
```

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

R is the reduced echelon form of A

```
%(g)
A=[magic(4);hilb(4)]
```

```
A = 8×4
   16.0000    2.0000    3.0000   13.0000
    5.0000   11.0000   10.0000    8.0000
    9.0000    7.0000    6.0000   12.0000
    4.0000   14.0000   15.0000    1.0000
    1.0000    0.5000    0.3333    0.2500
    0.5000    0.3333    0.2500    0.2000
    0.3333    0.2500    0.2000    0.1667
    0.2500    0.2000    0.1667    0.1429
```

```
R=rredf(A);
```

```
the constructed matrix R is
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
rf =
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

R is the reduced echelon form of A

```
%(h)
A=[magic(4),hilb(4)]
```

```
A = 4×8
   16.0000    2.0000    3.0000   13.0000    1.0000    0.5000    0.3333    0.2500
    5.0000   11.0000   10.0000    8.0000    0.5000    0.3333    0.2500    0.2000
    9.0000    7.0000    6.0000   12.0000    0.3333    0.2500    0.2000    0.1667
    4.0000   14.0000   15.0000    1.0000    0.2500    0.2000    0.1667    0.1429
```

```
R=rredf(A);
```

```
the constructed matrix R is
    1.0000         0         0    1.0000         0    0.0041    0.0052    0.0054
         0    1.0000         0    3.0000         0    0.0262    0.0253    0.0222
         0         0    1.0000   -3.0000         0   -0.0196   -0.0182   -0.0154
         0         0         0         0    1.0000    0.4400    0.2533    0.1657
```

rf =

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & \frac{169}{40800} & \frac{641}{122400} & \frac{16}{2975} \\ 0 & 1 & 0 & 3 & 0 & \frac{1069}{40800} & \frac{3101}{122400} & \frac{66}{2975} \\ 0 & 0 & 1 & -3 & 0 & -\frac{133}{6800} & -\frac{1111}{61200} & -\frac{183}{11900} \\ 0 & 0 & 0 & 0 & 1 & \frac{11}{25} & \frac{19}{75} & \frac{29}{175} \end{pmatrix}$$

R is the reduced echelon form of A

```
%(i)
A=randi(10,5,3);A=[A, sum(A,2)]
```

```
A = 5×4
     3     2     4     9
     4     3    10    17
     5     2     5    12
     3     3     2     8
     9     5    10    24
```

```
R=rredf(A);
```

```
the constructed matrix R is
     1     0     0     1
     0     1     0     1
     0     0     1     1
     0     0     0     0
     0     0     0     0
rf =
```

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

R is the reduced echelon form of A