



$$B_z = \frac{\mu_0 \pi I}{2} [\cos \phi_1(z) - \cos \phi_2(z)]$$

$$\cos \phi_1 = \frac{z}{\sqrt{z^2 + R^2}}, \quad \cos \phi_2 = \frac{z - z_2}{\sqrt{(z - z_2)^2 + R^2}}$$

$$B_r(r, z) = -\frac{r}{2} \frac{dB_z}{dz}$$

$$\cos \phi_1 = z (z^2 + R^2)^{-1/2}$$

$$\frac{d(\cos \phi_1)}{dz} = z \left(-\frac{1}{2}\right) (z^2 + R^2)^{-3/2} \cdot 2z + 1 \cdot (z^2 + R^2)^{-1/2}$$

$$= -\frac{z^2}{z^2 + R^2} \frac{1}{\sqrt{z^2 + R^2}} + \frac{1}{\sqrt{z^2 + R^2}}$$

$$= \frac{1}{\sqrt{z^2 + R^2}} \left(1 - \frac{z^2}{z^2 + R^2} \right)$$

$$= \frac{1}{\sqrt{z^2 + R^2}} \left(\frac{z^2 + R^2 - z^2}{z^2 + R^2} \right)$$

$$\frac{d}{dz} (\cos \phi_1) = \frac{R^2}{(z^2 + R^2)^{3/2}}$$

Second $\cos \phi_2(z) = (z - z_2) [(z - z_2)^2 + R^2]^{-1/2}$

$$\frac{d}{dz} (\cos \phi_2) = (z - z_2) \left(-\frac{1}{2}\right) [(z - z_2)^2 + R^2]^{-3/2} \cdot 2(z - z_2) + [(z - z_2)^2 + R^2]^{-1/2}$$

$$= \frac{1}{\sqrt{(z - z_2)^2 + R^2}} \left[1 - \frac{(z - z_2)^2}{(z - z_2)^2 + R^2} \right]$$

$$= \frac{1}{\sqrt{(z - z_2)^2 + R^2}} \left[\frac{(z - z_2)^2 + R^2 - (z - z_2)^2}{(z - z_2)^2 + R^2} \right]$$

$$\frac{d}{dz} (\cos \phi_2) = \frac{R^2}{[(z - z_2)^2 + R^2]^{3/2}}$$

$$B_r = -\frac{r}{2} \frac{\mu_0 \eta I}{2} R^2 \left[\frac{1}{(z^2 + R^2)^{3/2}} - \frac{1}{[(z-z_2)^2 + R^2]^{3/2}} \right]$$

$$= \frac{-\mu_0 \eta I R^2}{4} r \left[\frac{1}{(z^2 + R^2)^{3/2}} - \frac{1}{[(z-z_2)^2 + R^2]^{3/2}} \right]$$

$$B_x = \frac{-\mu_0 \eta I R^2}{4} x \left[\frac{1}{(z^2 + R^2)^{3/2}} - \frac{1}{[(z-z_2)^2 + R^2]^{3/2}} \right] \frac{x}{\sqrt{x^2 + y^2}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$B_y = \frac{-\mu_0 \eta I R^2}{4} y \left[\frac{1}{(z^2 + R^2)^{3/2}} - \frac{1}{[(z-z_2)^2 + R^2]^{3/2}} \right] \frac{y}{\sqrt{x^2 + y^2}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$B_z = \frac{\mu_0 \eta I}{2} \left[\frac{z}{\sqrt{z^2 + R^2}} - \frac{z-z_2}{\sqrt{(z-z_2)^2 + R^2}} \right]$$

Task 1

Define B-func(x,y,z) that spits Bx, By, Bz vector.

use

$$B_x = -10^{-3} \left(\frac{0.01}{4} \right) x \left[\frac{1}{(z^2 + 0.01)^{3/2}} - \frac{1}{[(z-0.2)^2 + 0.01]^{3/2}} \right] \frac{x}{\sqrt{x^2 + y^2}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

this raises infinite error at x=y=0, so has to return zero that case

$$B_y = -10^{-3} \left(\frac{0.01}{4} \right) y \left[\frac{1}{(z^2 + 0.01)^{3/2}} - \frac{1}{[(z-0.2)^2 + 0.01]^{3/2}} \right] \frac{y}{\sqrt{x^2 + y^2}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$B_z = \frac{10^{-3}}{2} \left[\frac{z}{(z^2 + 0.01)^{1/2}} - \frac{z-0.2}{[(z-0.2)^2 + 0.01]^{1/2}} \right]$$