Numerical Integration & Mapping

1. Calculate the integral $\int_0^{10} x^2 dx$ using the Trapezoidal Rule. Calculate the results for 5 and 10 equal divisions. Also compare the obtained result with the analytical Results
2. Using polynomial exactness, derive the Gauss–Legendre integration points and weights for the cases of (a) one-point and (b) two-point Gauss quadrature on the interval -1 to +1.
3. Compute the integral $\int_{-1}^{+1} cosx\, dx$ using Gauss quadrature with 1-point, 2-point and 3-point rules. Compare the numerical results with the analytical result.
4. Map the integral $\int_{-0}^{+10} cosx\, dx$ Map the function to natural coordinates (i.e. between - 1 and +1). Provide the transformed integrand and show the Jacobian explicitly.
5. Integrate $\int_{-0}^{+10} cosx\, dx$ using Gauss quadrature integration
6. Explain how the Jacobian is used to map a function from one independent variable to another. Provide a worked example (explicit mapping and Jacobian) that illustrates the procedure.
7. Make a pseudocode to integrate any function using the Gauss Quadrature integration using 3-point Gauss quadrature points. The code must implement the mapping to the natural coordinated in the code itself.

Stiffness Modelling of Link Elements

8. With a neat sketch, explain how a 1D link element is mapped to natural coordinates in the isoperimetric formulation. Show the coordinate mapping and define the Jacobian.
9. Derive the stiffness matrix of a 1D link element (linear two-node element) in matrix form in the natural coordinate system, starting from the weak form.
10. Repeat the same process for a heat transfer problem.
11. Write pseudocode to compute the element stiffness matrix using the derived natural-coordinate formulation. Use 2-point Gauss quadrature for numerical integration.
12. Calculate the stiffness matrix for a 3-node link element (with quadratic shape functions) using the above-mentioned method.
13. What are the essential requirements that a shape function must satisfy in the finite element method?
14. Explain the continuity requirement of shape functions across element boundaries. How does this requirement vary between problems involving displacement, temperature, and stress fields?
15. Explain what is H Method and P method.
16. Explain how displacements inside an element can be computed if the nodal displacements are known. Provide the interpolation formula and explain continuity and convergence aspects.
17. Compare the stiffness matrix and displacement approximation of a single 3-node quadratic element with the assembly of two linear (2-node) elements occupying the same domain. Discuss whether the displacement field inside the element is identical and explain why.

Nonlinear FEM

18. Explain the types of non-linearity encountered in finite element analysis (material nonlinearity, geometric nonlinearity, boundary/contact nonlinearity).
19. With a clear sketch, explain why a single-step (direct) solution is generally not possible for nonlinear systems and why iterative solution procedures are required.
20. On the same figure, illustrate how the Newton–Raphson iterative scheme converges to the nonlinear solution (show residual vs iteration and tangent updates).
21. Write a pseudocode to solve a nonlinear one-dimensional link element problem using Newton–Raphson iteration.

Modal Analysis using FEM

22. Why is the mass matrix required in modal analysis, and how does the choice between a lumped and consistent mass matrix affect the computed natural frequencies?
23. Explain how boundary conditions influence the natural frequencies and mode shapes obtained from a modal analysis? Why might

constraining or releasing a single degree of freedom significantly alter the modal results?

24. Explain how the mass matrix is formulated. Derive both (a) the consistent mass matrix and (b) the lumped mass matrix for a 1D element.
25. For a simple system of link elements, formulate the eigenvalue problem for modal analysis and describe how natural frequencies and mode shapes are obtained.
26. Consider a tapered rod of total length 1 m divided into 3 equal elements. Model the rod using link elements, assemble mass and stiffness matrices, and compute the natural frequencies and mode shapes (numerical calculation).

Transient and Dynamic Analysis

27. Explain the difference between transient analysis and dynamic analysis
28. Explain how finite difference approximations are used to formulate acceleration and time-integration in dynamics.
29. Explain explicit and implicit time integration methods.
30. Derive the matrix equations in case of a transient heat conduction equation governed by $\rho C \frac{\partial T}{\partial t} + \frac{\partial}{\partial x}\left[k \frac{\partial T}{\partial x}\right] = 0$, where ρ is the density, C is specific heat, and K is thermal conductivity.
31. Obtain the matrix equations to simulate the forced vibration analysis using explicit analysis. The system consists of 2 link elements. You may use lumped mass formulation for the mass matrix.
32. Explain the relationship between minimum element size and maximum permissible time step for explicit schemes.
33. Under what circumstances can a dynamic analysis be used to approximate a quasi-static response? Discuss time scaling and mass scaling and explain why they are used to accelerate computations (with limitations).

2D & 3D Elements

34. Explain the governing equations for the heat transfer in 2D.
35. Present the governing equations for 2D elasticity (plane stress/strain) and their weak form.
36. In the matrix FE formulation for a 4-noded quadrilateral element, define the strain–displacement matrix [B], material constitutive matrix [D], and the Jacobian [J]. Explain how they are assembled into the element stiffness.
37. Derive the stiffness matrix for a 4-noded plane-stress quadrilateral element.
38. Explain shear locking. Why reduce integration is preferred to overcome
39. Explain the constant strain triangle (CST) element. Explain the shape functions in terms of aera coordinates
40. Derive the shape functions for a 3-node CST element.
41. Explain Lagrangian and serendipity elements in 2D and 3D. Sketch typical 8-node and 9-node quadrilaterals, and derive their shape functions.
42. On computational grounds, discuss why serendipity elements may be superior to full Lagrangian higher-order elements (tradeoffs between DOFs and interpolation completeness
43. A rectangular area is meshed with 4 elements as shown in the figure below. Create element connectivity matrix and nodal coordinate matrix
44. Write a pseudo code to calculate solve the D problem using FEM. The programme has to output the stress in the elements.

Advanced Topics (Basic understanding only)

45. Explain gap elements: formulation and how they model separation and closing between non-conforming surfaces.
46. Explain contact elements and typical methods to enforce contact constraints (penalty, Lagrange multipliers, augmented Lagrangian) at a conceptual level.
47. Explain cohesive zone elements and how they simulate material degradation and progressive failure.
48. List a few methods in FEM to simulate frack propagation
49. List one application each of FEM in electromagnetics, Acoustics and biomedical engineering.

Closing Remarks

50. In the first assignment you might have identified 5 job positions and the skills mentioned in the job description. Now you may find 5 job position which you may be able to apply with the skills you earned from this course.