(c) (2 points) The relationship from part (b) can be expressed as $\mu = \gamma L$. γ is usually referred to as the *classical* gyromagnetic ratio of a particle. Evaluate the classical gyromagnetic ratio for an electron and for a neutron in SI units.

(d) (7 points) For extended objects such as planets, the magnetic dipole moment is not directly accessible whereas the surface magnetic field can be measured. Assuming a magnetic dipole of magnetic moment μ located at the center of a sphere of radius r, write down the expression for the surface magnetic field B_{surf} and the surface magnetic moment defined as $\mathcal{M}_{surf} = B_{surf} r^3$. You may use the value of the angular dependence at the magnetic equator for the following parts.

(e) (3 points) Assuming a gyromagnetic relationship exists between magnetic moment μ and angular momentum L of an extended object, write down the relationship between the surface magnetic moment $\mathcal{M}_{surf}$ and angular momentum L as $\mathcal{M}_{surf} = \kappa L$. You will observe that κ depends only on fundamental constants and intrinsic properties of the extended object.

(f) (3 points) The surface magnetic moments for Mercury and Sun are 5×10^{12} T m^3 and 3×10^{23} T m^3 respectively. Assuming the bodies are perfect spheres, evaluate the constant κ for Mercury and the Sun. Comment on values obtained and if they fit into the model developed in parts (c) and (d).

(g) (5 points) The surface magnetic moments $\mathcal{M}_{surf}$ and angular momenta L of various solar system bodies are plotted in the figure 3. Justify that the data implies $\mathcal{M}_{surf} \sim L^\alpha$ and calculate the constant α. What is the expected value of α from the model developed in parts (c) and (d)?

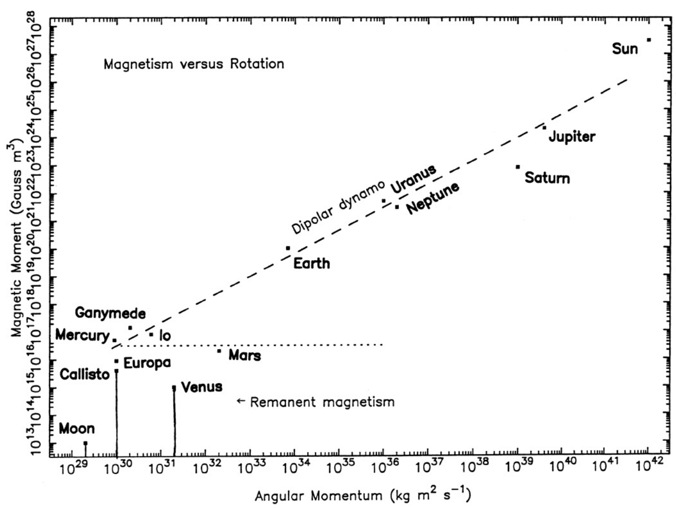

Figure 3: Surface magnetic moment vs angular momentum for solar system objects. Figure taken from Vallée, Fundamentals of Cosmic Physics, Vol. 19, pp 319-422, 1998.

(h) (2 points) Certain bodies such as Venus, Mars and the Moon are remarkably separated from the trend observed for other bodies. What can you say about magnetism in these bodies when compared to the others?

11. (25 points)

Cygnus X-1/HDE 226868 is a binary system consisting of a black hole Cygnus X-1 and blue supergiant HDE 226868. The mass of HDE 226868 is $30M_\odot$ and the period of the binary system is 5.6 days. Radial velocity data reveals that the orbital velocity of HDE 226868 is 116.68 km/s at apoapse and 123.03 km/s at periapse.

(a) (5 points) Determine the eccentricity of the orbit of HDE 226868.

(b) (5 points) Determine the length of the semimajor axis of the orbit of HDE 226868.

(c) (5 points) Determine the mass of Cygnus X-1, to at least 3 significant figures.

The peak blackbody temperature of an accretion disk occurs at a distance of r_{peak} and a temperature of T_{peak}. One can determine the peak blackbody temperature by assuming that it corresponds to the peak in the x-ray spectrum. Due to relativistic effects, the actual peak blackbody temperature T_{peak} is related to the peak color temperature T_{color} derived from observed spectral data by $T_{color} = f_{GR} f_{col} T_{peak}$, where $f_{GR} \approx 0.510$ and $f_{col} \approx 1.7$. Three x-ray spectra of Cygnus X-1 are shown in Figure 4.

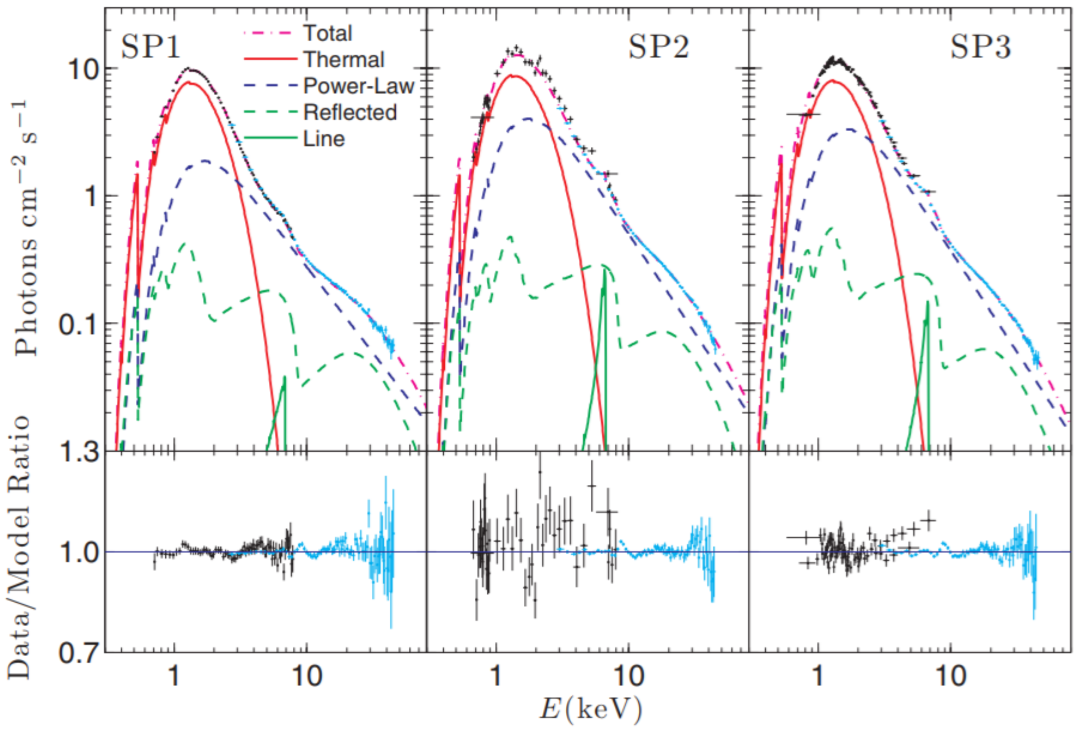

Figure 4: Three x-ray spectra from Cygnus X-1. From Gou et al. (2011).

(d) (4 points) Using spectrum SP2, determine the peak blackbody temperature T_{peak} of the accretion disk around Cygnus X-1.

The total luminosity of the blackbody component of the accretion disk can be estimated by $L_{disk} \approx 4\pi\sigma r_{peak}^2 T_{peak}^4$ (Makishima et al. 1986). The radius r_{last} of the innermost edge of the accretion disk is related to the radius r_{peak} of the peak blackbody temperature by $r_{peak} = \eta r_{last}$, where $\eta \approx 0.63$. In 1996, the blackbody luminosity of the accretion disk around Cygnus X-1 was estimated to be 2.2×10^{37} ergs/s.

(e) (4 points) Determine the radius r_{last} of the innermost edge of the accretion disk around Cygnus X-1.

Assume that the innermost edge of the accretion disk is located at the innermost stable circular orbit (ISCO), whose radius r_{isco} is a function of the spin of the black hole. The relationship between r_{isco} and a_*, the spin parameter of the black hole, can be estimated by:

$$r_{isco} = \frac{GM}{c^2}\left(\sqrt{8.354 \cdot [(2 - a_*)^2 - 1]} + 1\right)$$

(f) (2 points) Determine the spin parameter a_* of Cygnus X-1.