4 Long Questions

1. (**30 points**) M15 is a globular cluster in the constellation Pegasus. The Hertzsprung–Russell diagram (apparent visual magnitude versus color index) of the cluster is shown in fig. 1. Considering that the mass (M)– luminosity (L) relation for main sequence stars is given by $\frac{L}{M^3}$ =constant, answer the following questions. In this problem, ignore the interstellar reddening and dust extinction effects.

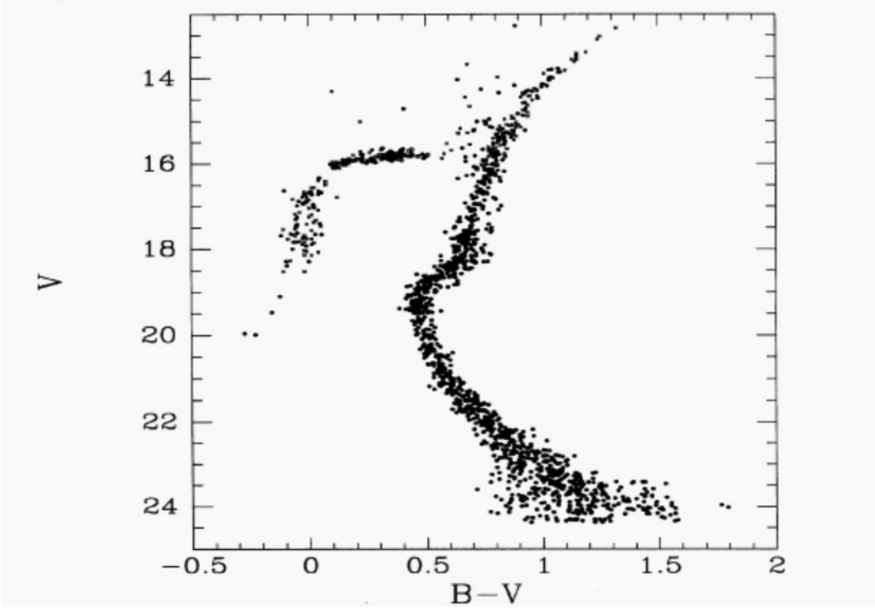


Figure 1: HR diagram for M15

(a) Given that all the stars are formed at the same time, estimate the age of the globular cluster. The color index of the sun ($(B-V)_\odot$) is 0.65 and its life time on the main sequence is 10 billion years.

(b) Estimate the distance of this globular cluster from the Earth. Give the answer in parsec. The absolute visual magnitude of the Sun is 4.83.

(c) Given that stars spend about 10% of their main-sequence life time in the post main sequence phase, find the mass of the most massive star in the post main sequence stage.

(d) The number of stars in the mass range of (M_1, M_2) can be written as:

$$N(M_1 \leqslant M \leqslant M_2) = A(M_1^{-1.35} - M_2^{-1.35}) \tag{1}$$

where A is a constant, M_1 and M_2 are in units of solar masses. Assuming that the number of stars in the post main sequence phase is 515, calculate the value of constant A in equation 1.

(e) M15 is one of the most densely packed globular clusters such that in a visual band ($\lambda \sim 5500\mathring{A}$) image of M15 taken by a telescope with diameter of 10 cm, the stars at the center of cluster cannot be resolved. Estimate the minimum number of stars in this cluster. The angular diameter of M15 is 12.3 arc minutes. Assume that the number density of stars is constant within the cluster.

(f) Use your answers from parts (d) and (e) to estimate the mass of the lowest possible mass star in this cluster. For this part, assume that the mass of the most massive star in the cluster is $20M_\odot$.

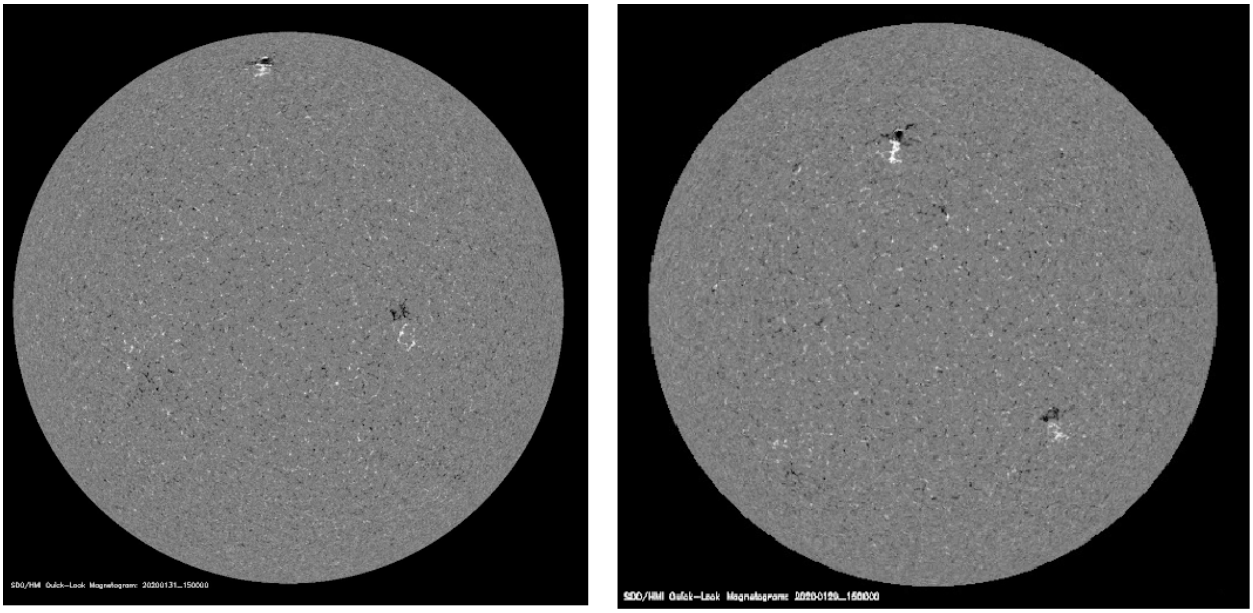
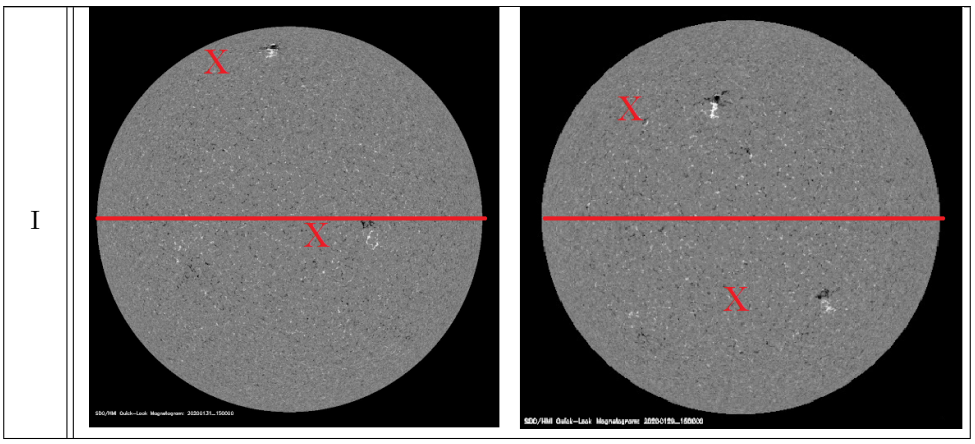
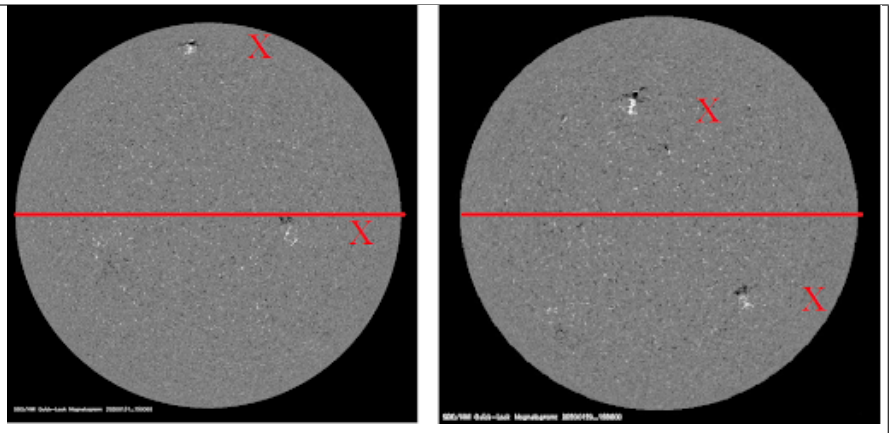
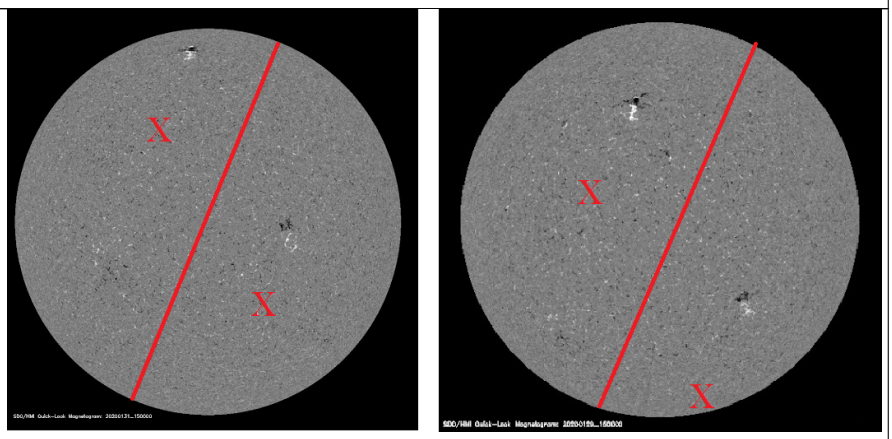
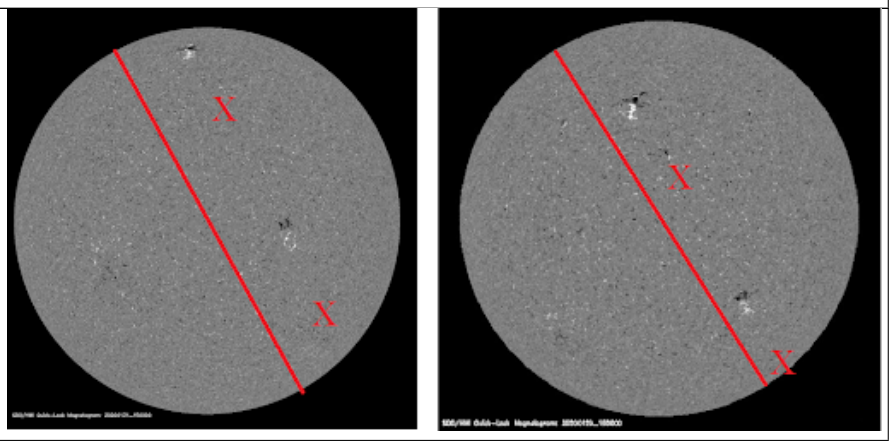


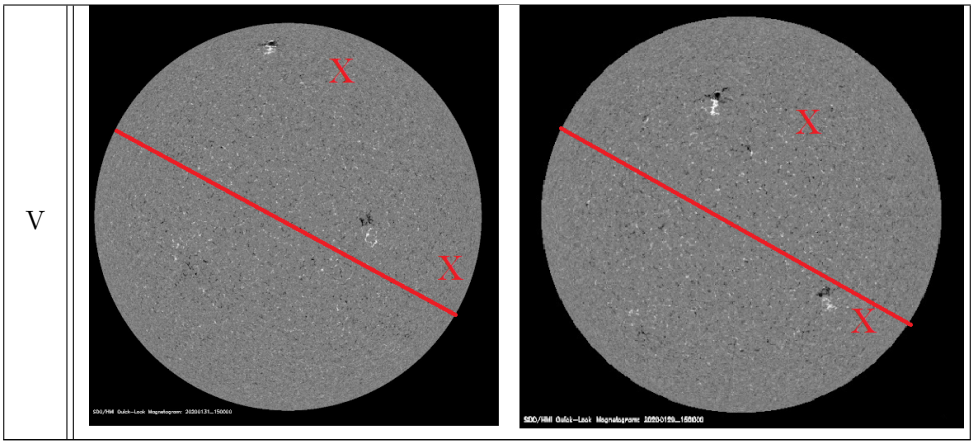
Figure 2: Solar magnetograms taken at the end of Jan 2020

2. (**25 points**) Fig. 2 shows two magnetograms of the Sun taken with the Heliseismic and Magnetic Imager (HMI) at the Solar Dynamics Observatory (SDO) towards the end of January 2020. The picture on the left was taken three days after the image on the right.

   (a) Select the pair of images (numbered from I to V) in which the lines are drawn at the Sun's Equator and the Xs correspond to the position of each sunspot 4 days before the pictures were taken.



II	
III	
IV	



(b) Estimate the absolute value of the latitude of both Sunspots in Figure 2.

(c) Fig. 3 is a magnetogram of the Sun in normal activity. It is possible to notice that the sunspots have different orientations in different hemispheres. In one of the hemispheres, each spot has the white part on the left and the black one on the right, and vice-versa. However, this is not the case for the images presented in fig. 2. Suggest an explanation for the anomaly on the images in fig. 2.

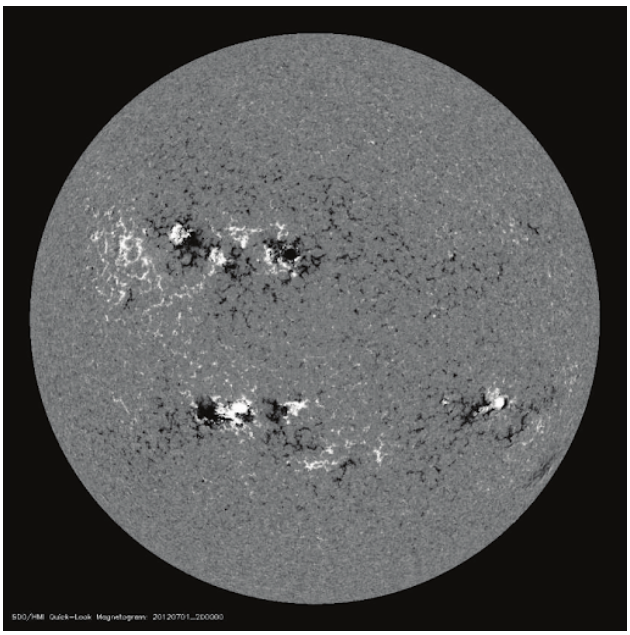


Figure 3: Sun in normal activity

(d) Assume for the sake of simplicity that a specific sunspot has a shape very similar to that of a spherical triangle. The sides of the triangle are equal to $0.176°$, $0.0981°$, and $0.201°$. Calculate the value of the three internal angles in degrees.

For the following parts, assume that this sunspot is centered at $7.89°$ South and $51.74°$ East of the center of the Solar disk for an observer on Earth.

(e) For an observer on Earth, what is the ratio between the area of the Solar disk and the observed area of the sunspot? Note that the required ratio is between the areas observed by someone on Earth,

not the ratio between the actual areas. The area of a spherical triangle is equal to $\pi R^2 E/180^\circ$, in which the spherical excess (E) in (deg) is equal to the sum of the internal angles minus 180° and R is the radius of the sphere on which the spherical triangle lies.

(f) If an observer on Earth uses a huge f/5 telescope with a focal length of 13 m to look at this sunspot, will it be possible to resolve it? Visible light is centered at 550 nm.

(g) The Sun generates its luminosity by converting Hydrogen into Helium in the proton-proton chain. In the most energetic branch of the chain, 4 protons fuse into a helium nucleus. Considering that only 10% of the solar mass can be converted into energy, calculate the time that the Sun spends in the Main Sequence.