2019 National Astronomy Olympiad (NAO)- Solutions

1 Short Questions

1. (7 points) Assuming that the present density of baryonic matter is $\rho_{b0} = 4.17 \times 10^{-28}$ kg m^{-3}, what was the density of baryonic matter at the time of Big Bang nucelosynthesis (when T $\sim 10^{10}$ K)? Assume the present temperature, T_0 to be 2.7 K.

   Solution: Answer: $\rho_{BBN} = 21$ kg m^{-3}.
   $\rho_{b,BBN} = \rho_{b0} a^{-3}$, where a is the scale factor.
   We also know that $T_0 = aT(a)$, giving us:
   $\rho_{b,BBN} = \rho_{b0}(\frac{T_{BBN}}{T_0})^3$.
   Using $\rho_{b0} = 4.17 \times 10^{-28}$ kg m^{-3}, $T_{BBN} = 10^{10}$ K and $T_0 = 2.7$ K, we get $\rho_{BBN} = 21$ kg m^{-3}.

2. **Note: constants required are the semimajor axis of the moon's orbit (384399 km), the semimajor axis of the earth's orbit (1.4960×10^8 km), the radius of the sun (695700 km), the radius of the earth (6371 km), the radius of the moon (1737 km), the mass of the earth (5.972×10^{24} kg), the mass of the sun (1.989×10^{30} kg), and G.**

   (7 points) On the night of January 21st, 2019, there was a total lunar eclipse during a supermoon. At the time, the moon was close to perigee, at a distance of 351837 km from the earth, which was 1.4721×10^8 km from the sun. The gamma (γ) of a lunar eclipse refers to the closest distance between the center of the moon and the center of the shadow, expressed as a fraction of the earth's radius. For this eclipse, $\gamma = 0.3684$. Given this information, find the closest estimate for the duration of totality of the eclipse.

   Solution: First, calculate the radius of the shadow cast by earth. The size of the umbra at a distance of 351837 km is $r_{umbra} = 6371 - \frac{695700-6371}{1.4721\times10^8} \times 351837 = 4723$ km. However, the distance travelled by the moon through the umbra is far less than twice the umbral radius. Firstly, the moon does not pass through the center of the shadow; secondly, totality occurs when the moon is completely within the shadow, not when the center of the moon is in the shadow. So, the distance travelled by the moon during totality is $d = 2 \times \sqrt{(r_{umbra} - 1737)^2 - (0.3684 \times 6371)^2} = 3693$ km. The velocity of the moon relative to the earth can be calculated using the vis-viva equation: $v_{moon} = \sqrt{G \times 5.972 \times 10^{24} \times \left(\frac{2}{351837000} - \frac{1}{384399000}\right)} = 1.108$ km/s. However, the earth is also moving around the sun, so the umbra is also moving relative to the earth. The velocity of the earth is $v_{earth} = \sqrt{G \times 1.989 \times 10^{30} \times \left(\frac{2}{1.4721\times10^{11}} - \frac{1}{1.4960\times10^{11}}\right)} = 30.259$ km/s. Therefore, the velocity of the umbra is $v_{umbra} = \frac{v_{earth}}{1.4721\times10^8} \times 351837 = 0.072$ km/s. The earth and the moon orbit in the same direction, so the velocity of the moon relative to the umbra is $v = v_{moon} - v_{umbra} = 1.036$ km/s. Finally, the duration of totality is $t_{totality} = \frac{d}{v} = 59.426$ minutes.

3. (7 points) You are in the northern hemisphere and are observing rise of star A with declination $\delta = -8^o$, and at the same time a star B with declination $\delta = +16^o$ is setting. What will happen first: next setting of the star A or rising of the star B?

Solution: Star B will rise first. At the same time as the star A rise, a point on the opposite side of the sky with declination $+8^o$ is setting. Star B sets at the same time as that point, but having a higher declination, and being in the north hemisphere, will spend less time under the horizon.

4. (7 points) Consider a star with mass M and radius R. The star's density varies as a function of radius r according to the equation $\rho(r) = \rho_{center}(1 - \sqrt{r/R})$, where ρ_{center} is the density at the center of the star. Derive an expression for dP/dr in terms of G, M, R, and r, where P is the pressure at a given radius r

Solution: The mass inside a given radius r is $m(r) = \int_0^r 4\pi r^2 \rho(r) dr$. Thus, we have

$$m(r) = \frac{4}{21}\pi\rho_{center}r^3(7 - 6\sqrt{\frac{r}{R}})$$

Note that $M = m(R)$, so we can solve for $\rho_{center} = \frac{21M}{4\pi R^3}$. Hydrostatic equilibrium requires that $dP/dr = -\rho(r)\frac{Gm(r)}{r^2}$. Thus, substituting, we have

$$dP/dr = -\frac{21GM^2r}{4\pi R^6}(7 - 13\sqrt{\frac{r}{R}} + 6\frac{r}{R})$$