# I: Short Questions

1. [5 pt] In order to detect an Earth-twin, we need significant advances in the precision of spectrographs to detect the periodic Doppler shift of nearby stars. Estimate the radial velocity semi-amplitude, in m/s, that a planet with the mass, radius, and semi-major axis of Earth would cause in the motion of a star with the mass of the Sun. Assume that the Earth-twin has zero eccentricity. Note that the mass of Earth is $5.97 \times 10^{24}$ kg, and the distance from the Earth to the Sun is $1.5 \times 10^{8}$ m.

2. [5 pt] Planet Nine is a hypothesized planet in the outer Solar System that may explain the clustering of orbital elements of distant trans-Neptunian objects. The hypothesized periapse of Planet Nine is 200 AU, and the apoapse is expected to be at approximately 1200 AU. What would the eccentricity of Planet Nine be? How does this eccentricity compare to that of the 8 major planets in the Solar System?

3. [5 pt] What is the main-sequence lifetime of a star with a mass of 0.1 solar masses, and a star with a mass of 10 Solar masses? Assume that stellar luminosity, $L \propto M^{3.5}$, where $M$ is stellar mass, and that the main-sequence lifetime of the Sun is 10 billion years.

4. [5 pt] The Very Large Array radio interferometer ($\lambda$=1 m) has maximum baseline of D = 36.4 km. How large will an optical telescope have to be to achieve a similar angular resolution in visible light ($\lambda$=5500 A$^\circ$)?

5. [5 pt] An amateur astronomer observes the Moon with 20-cm telescope, and accomplishes 160x magnification with an eyepiece with focal length 10 mm. What is the f-number of the telescope?

6. [5 pt] The average person has 1.4 m$^2$ of skin. What is the energy per second radiated by the average person in the form of blackbody radiation? What is the peak wavelength of emitted radiation? Why cant we see it with our eyes?

7. [5 pt] On March 21st at true noon, length of the shadow of a vertical rod was equal to its height. On which geographic latitude did this happen?

8. [5 pt] In stars like the Sun, helium nuclei are formed by fusing hydrogen nuclei together in a process known as the proton-proton chain. One step of the proton-proton chain consists of a deuterium nucleus ($m_d = 2.01410$ u) fusing together with a hydrogen nucleus ($m_H = 1.00783$ u) to form a helium-3 nucleus ($m_{He} = 3.01603$ u), where $u = 1.6605 \times 10^{-27}$ kg. How much energy is released during this fusion reaction?

9. [5 pt] Solar wind consists of protons that fly with the speed of 300 km/s and they fill the space of interplanetary matter around Earth with 10 particles/cm$^3$. With what force is this "wind" hitting the Moon? Recall that mass of a proton is $m_p = 1.610^{-24}$ g. Radius of the Moon is $R_m$=1737 km.

10. [5 pt] Mars orbits the Sun at an average distance of $2.28 \times 10^{11}$ m and has a radius of $3.39 \times 10^{6}$ m. The Sun has a luminosity of $3.828 \times 10^{26}$ W. How much solar energy falls on the surface of Mars each second? Ignore any effects of Mars' thin atmosphere.

11. [5 pt] When a gravitationally bound system (such as a galaxy) forms, it transitions from a just bound state ($E_{\rm kin} = |E_{\rm pot}|$) to a virialized state ($E_{\rm kin} = 0.5|E_{\rm pot}|$) and the excess binding energy has to be radiated away. Consider an idealized disk galaxy with an exactly flat rotation curve with a rotation speed of $v_{\rm circ} = 220$ km/s (you can neglect the kinetic energy in random motions). Its density profile cuts off abruptly at a radius of $R_{\rm max} = 50$ kpc. Assume that it took 500 million years for this galaxy to collapse to its present state. What was its mean luminosity (in units of solar luminosity) due to the release of the binding energy during that period?