

Physics 305 – Computational Physics, Fall 2020

Term Project

Full project submission Due Date: Tuesday December 15, 5pm

Presentation Phase: November 30 - December 11

The program in your term project can be either submitted as a python program or ipython notebook, where the latter is preferred. The program, an explanation of what the program does, along with answers to all questions asked should be uploaded to d2l.

You are expected to write a term paper (in word or Latex) on your project that discusses the problem you are trying to solve, the basic equations that govern the problem, includes plots that show the solutions, and describes the solution and the numerical method involved. In addition, you must demonstrate that your solution is correct by showing that the code converges at the expected order. If your code does not converge at the expected order you should try to identify potential reasons for why this is the case. You are expected to work on your term project by yourself.

Your term project will receive full credit **only** if: (a) the program runs successfully without errors using python 3, (b) the programs have explanatory comments and variable names that identify with the problem equations you are trying to solve, (c) give the correct output, and (d) demonstrate the validity of the solution through convergence plots. No credit will be given to late term projects.

The term paper is as important as the code (50% of the term project credit will go to the code and the other 50% to the paper). Answers to the questions and analysis requested below should be elaborated in the report. Plots should be clearly labeled and be properly described in the report, and not just shown. You will need to explain what each and every plot demonstrates. A polished paper written in word or LaTeX (preferred, e.g. please try overleaf) is expected to get full credit.

Note: Before you present results from numerical integrations that answer the questions in the project, it is critical to ***first*** perform the convergence tests for one case, and to estimate errors. This will tell you how small a step size is necessary for accurate solutions. Only after errors are estimated, does it make sense to run your code for producing results that help you learn more about the system you study.

I. FOUCAULT'S PENDULUM: THE EFFECT OF THE CORIOLIS FORCE

In 1851, the French physicist Jean Leon Foucault hung a 67-meter pendulum from the dome of the Pantheon to demonstrate the rotation of the earth for the first time. A pendulum swinging in a rotating frame exhibits deflection due to the coriolis force. You will be studying this effect.

Ignoring air resistance and losses due to imperfect pivot points etc., the equations of motion describing the tip of the pendulum on a rotating frame are given by

$$\begin{aligned}\frac{d^2x}{dt^2} &= -\frac{g}{\ell}x + 2\Omega\frac{dy}{dt} + \Omega^2x \\ \frac{d^2y}{dt^2} &= -\frac{g}{\ell}y - 2\Omega\frac{dx}{dt} + \Omega^2y\end{aligned}\tag{1}$$

Note that in these equations the terms proportional to the velocity correspond to the Coriolis acceleration due to the rotating frame with angular velocity Ω . The Ω^2 terms are the usual centrifugal acceleration terms. In addition, we have g , and ℓ being the gravitational acceleration and the length of the pendulum. We would like to cast these equations in a dimensionless form. Notice that $\sqrt{\frac{\ell}{g}}$ is related to the fundamental period of the pendulum. Introduce a new dimensionless time $\tilde{t} = t/\tau$ with $\tau = \sqrt{\frac{\ell}{g}}$ and rewrite the equations of motion. The dimensionless quantity $\frac{1}{R_0} = \Omega\frac{\ell}{g}$ will appear in your equations. Write the resulting dimensionless time equations with the parameter R_0 included. Show your work in the term paper. This parameter is called the Rossby number and is given by $R_0 = \sqrt{g/\ell}/\Omega$. Rossby number smaller than unity, means that the rotation period is shorter than swing period of the pendulum, while $R_0 > 1$ means that the rotation period is longer than swing period of the pendulum. You will be studying solutions to the dimensionless equations of motion as the Rossby number varies.

To solve the dimensionless equations numerically you will need to cast them to first-order form. Show your work in the term paper. You will need initial conditions. For all cases below assume that the pendulum at $\tilde{t} = 0$ is on the x-axis, at rest at the maximum angle. In particular choose, $x(0) = 1, y(0) = 0, \frac{dx}{dt} = 0 = \frac{dy}{dt}$.

Do the following:

1. Use RK4 to integrate numerically the first-order form of the dimensionless version of Eq. (??). You will need to evolve for at least the longest period involved in the problem. As mentioned above there are two periods involved: the pendulum swing period $T_p = 2\pi\sqrt{\ell/g}$, and the rotation period $T_\omega = 2\pi/\Omega$. However, in the non-dimensional form all times are normalized to $\tau = \sqrt{\ell/g}$, thus in the dimensionless system of equations these periods are $\tilde{T}_p = 2\pi$, and $\tilde{T}_\omega = 2\pi/\Omega\tau = 2\pi R_0$. This shows clearly now what we mentioned above, i.e., Rossby number smaller than unity, means that the rotation period is shorter than swing period of the pendulum, while $R_0 > 1$ means that the rotation period is longer than swing period of the pendulum. Run your code for the longest period in the problem in the following cases.
 - Set $R_0 = 10$. Show a plot of x vs \tilde{t} and x vs \tilde{t} . Also show x vs y to see the orbit that the tip of the pendulum traces. What do you notice happens to the pendulum? Does it stay along the x-axis on which it starts?
 - Set $R_0 = 2$. Show a plot of x vs \tilde{t} and x vs \tilde{t} . Also show x vs y to see the orbit that the tip of the pendulum traces. What do the orbits look like now? How do they compare to the $R_0 = 10$ case?
 - Set $R_0 = 1$. Show a plot of x vs \tilde{t} and x vs \tilde{t} . Also show x vs y to see the orbit that the tip of the pendulum traces. What do the orbits look like now? How do the orbits compare to the $R_0 = 2, 10$ cases? Is the pendulum swinging?

Use your judgement as to how small a step size you need to solve this system accurately. If you cannot figure this out from pure thought, experiment with different step sizes and use the solution x, y at the final \tilde{t} (for a fixed choice of R_0) and choose the step size such that x and y at that \tilde{t} does not change appreciably, say within 10^{-3} or less. Then you have decent accuracy.

2. **Self-convergence:** Use a number of step sizes and for a specific choice of R_0 , make a plot to demonstrate that the code solution for x , and y at \tilde{t} of your choosing self-converges. Does the order of convergence match your expectation? If not, try to explain why.
3. Using the order of convergence you determined, employ Richardson extrapolation to determine an error for the solution for $x(\tilde{t})$ $y(\tilde{t})$, at a sufficiently large \tilde{t} of your choosing.