Problem 2

Double Integrals can be calculated as,

$$I = \iint_A f(x,y)\,dy\,dx$$

Domain is $a \leq x \leq b$ and $c \leq y \leq d$

$$\therefore\ I = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Let's consider the inner integral $\int_c^d f(x,y)\,dy$

$$P(x) = \int_c^d f(x,y)\,dy \Rightarrow I = \int_a^b P(x)\,dx$$

Now, we can approximate $I$ using one-dimensional trapezoidal rule.

$$I \approx h_x\left[\frac{1}{2}P(x_0) + P(x_1) + \cdots + P(x_{n-1}) + \frac{1}{2}P(x_n)\right]$$

Now, $x_i = a + ih_x$ and $h_x = (b-a)/n$

Therefore, we can now approximate $P(x_j)$

$$P(x_j) = \int_c^d f(x_j, y)\,dy$$

$$= h_y\left[\frac{1}{2}P(x_i, y_{j,0}) + P(x_i, y_{j,1}) + \cdots + P(x_i, y_{j,m-1}) + \frac{1}{2}P(x_i, y_{j,m})\right]$$

Here, $y_j = c + jh_y$ and $h_y = (d-c)/m$.

Therefore, the integral now becomes,

$$\boxed{I \approx \sum_{i=0}^{n} \sum_{j=0}^{m} f(x_i, y_j)\, h_x h_y}$$