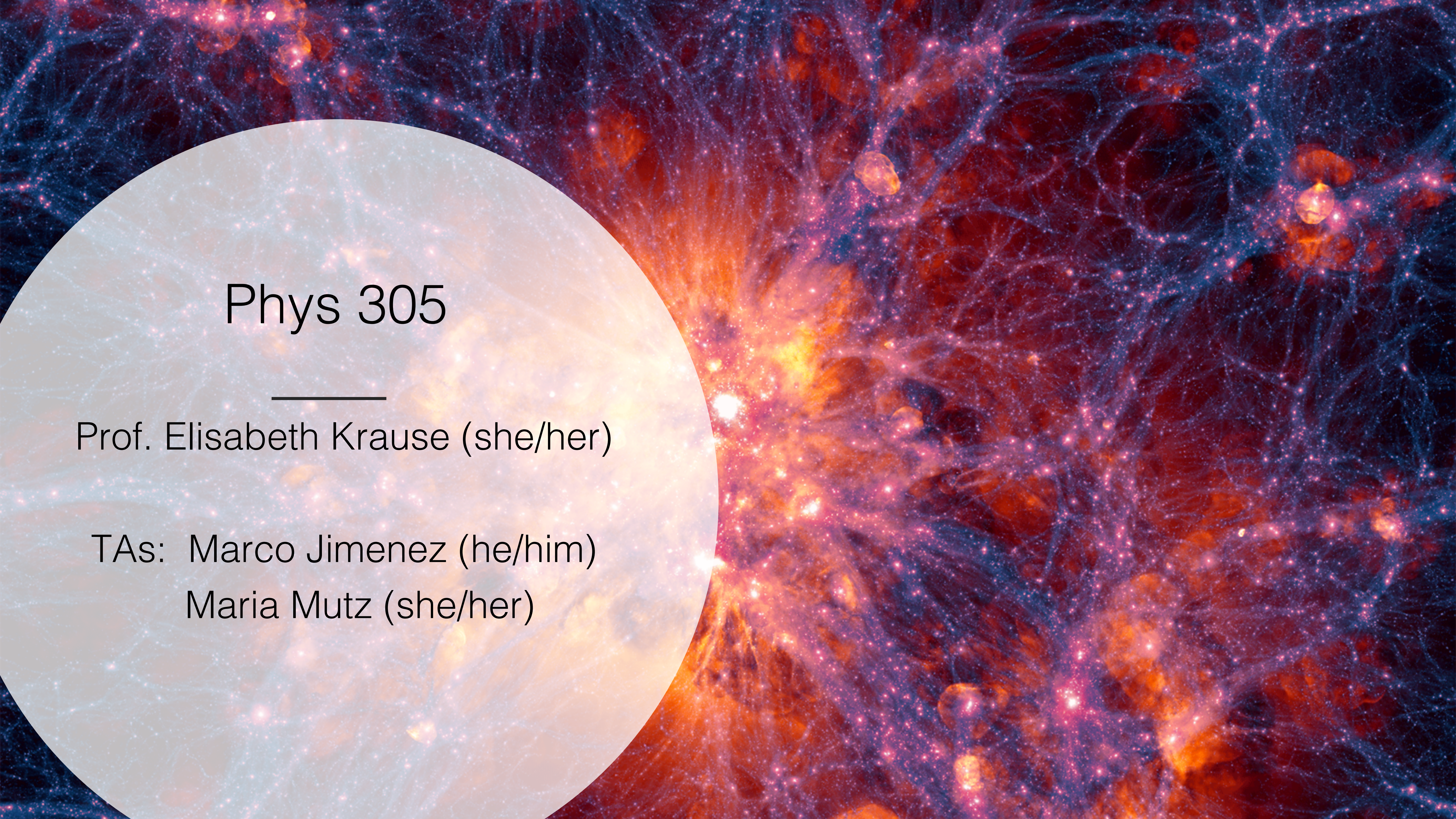

Phys 305
Prof. Elisabeth Krause (she/her)
TAs: Marco Jimenez (he/him)
Maria Mutz (she/her)

Today's Lecture

- Announcements
  - **Updated homework due date:** Mondays, at 5:00 pm (on D2L)
  - **Problem Set 1** due 9/7/2020 at 5:00 pm (on D2L)
    - Note: Submitting zip (instead of tar.gz) folder will not result any point deductions!
  - **No class next Monday (9/7/2020)** – Labour Day
  - **Consultation hours** for undergrad physics courses: https://w3.physics.arizona.edu/tutoring, Marco's slot is F 10-11am
- Today's lecture: intro to numerical integration
- Outlook: more on numerical integration (Romberg, Monte Carlo), intro to term projects next Friday

Errors in numerical computations

- **Bugs**/programming mistakes – can be avoided/corrected
- **Roundoff errors** – computer store numbers with finite digits, i.e. limited resolution
  - float in python uses *double precision*, with about 15 decimal digits; resolution varies depending on magnitude of the number
  - some roundoff errors are unavoidable, but there are strategies to minimize them
- **Truncation errors** – many numerical algorithms obtain solutions by developing finite-precision approximations to analytic results
  - often based on series (Taylor,…) expansion, which is truncated at finite order
  - e.g., finite difference derivative, which is obtained from Taylor expansion

$$f(x+h) = f(x) + hf'(x) + h^2\frac{f''(x)}{2} + h^3\frac{f'''(x)}{3!} + \dots$$

Rearranging terms, we have

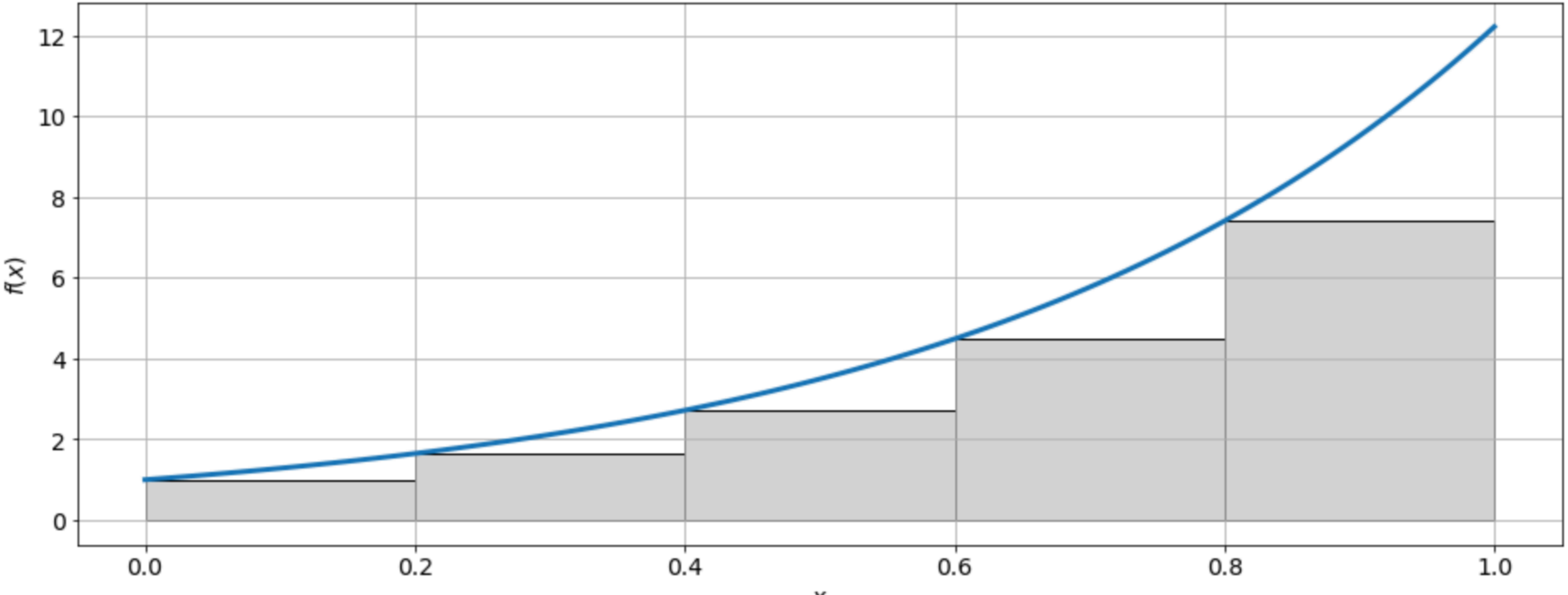
$$\frac{f(x+h) - f(x)}{h} = f'(x) + h\frac{f''(x)}{2} + \dots$$

$$\rightarrow f'(x) = \frac{f(x+h) - f(x)}{h} + \mathcal{O}(h)$$

Numerical Integration

Numerical integration: approximate area under the integrand by functions that are easy to integrate

- Riemann sum: piece-wise constant approximation



Riemann Integration

- Divide integration interval into N steps, perform Riemann sum

$$x_i = a + ih, \quad i = 1, \dots, N+1, \quad h = \frac{b-a}{N} \qquad \int_a^b f(x)dx \sim \sum_{i=0}^{N} f(x_i)(x_{i+1} - x_i) = h \sum_{i=0}^{N} f_i$$

- Now on to the approximation error

$$f(x) = f(x_i) + (x - x_i)f'(x_i) + \frac{(x - x_i)^2}{2} f''(x_i) + \dots$$

$$\int_{x_i}^{x_{i+1}} f(x)dx = (x_{i+1} - x_i)f(x_i) + \frac{(x_{i+1} - x_i)^2}{2} f'(x_i) + \frac{(x_{i+1} - x_i)^3}{3!} f''(x_i) + \dots$$

$$= \boxed{hf_i} + \boxed{h^2 \frac{1}{2} f_i'} + h^3 \frac{1}{3!} f_i'' + \dots$$

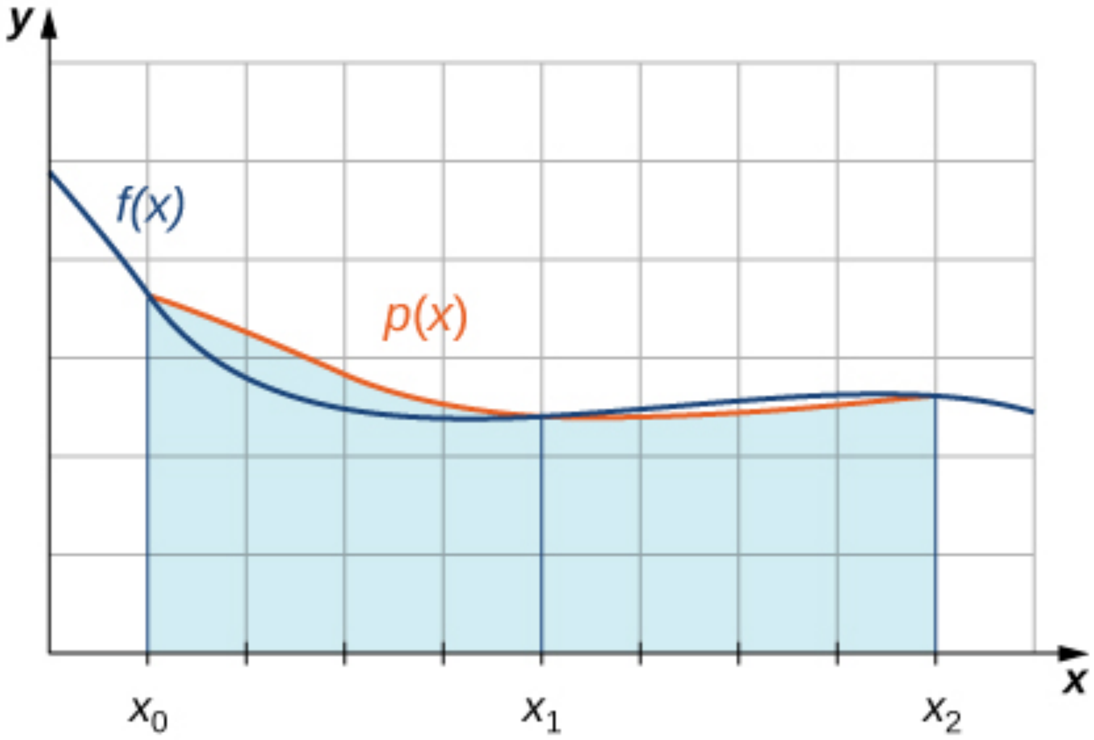
Riemann sum + leading error term

Now sum up the error over whole interval

$$\epsilon = N f'(\xi) \frac{1}{2} h^2 = N f'(\xi) \frac{1}{2} h \frac{(b-a)}{n} = h(b-a) \frac{1}{2} f'(\xi) = \mathcal{O}(h)$$

Beyond Riemann Sums

Trapezoid (linear interpolation)



Simpson rule (quadratic interpolation)