Phys 305

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Today's Lecture

- Announcements
  - **Updated homework due date:** Mondays, at 5:00 pm (on D2L)
  - **Problem Set 1** posted to D2L, due 9/7/2020 at 5:00 pm (on D2L)
  - **No class next Monday (9/7/2020)** – Labour Day
  - **Consultation hours** for undergrad physics courses: https://w3.physics.arizona.edu/tutoring, Marco's slot is F 10-11am
- Recap: Taylor Approximation
- Algorithmic Thinking Breakout
- Homework submission questions
- Outlook: the next two lectures will be on numerical integration, intro of term projects next Friday

The Taylor Approximation

Approximate a function f(x) by its first n derivatives

very important tool for many STEM problems

more often than not, n=1

Suppose that some function $f(x)$ has a continuous n-th derivative $f^{(n)}(x)$ on the interval $[a, b]$. Integrating this derivative we have

$$\begin{aligned} \int_a^x f^{(n)}(x)dx &= f^{(n-1)}(x)\Big|_a^x \\ &= f^{(n-1)}(x) - f^{(n-1)}(a). \qquad (1) \end{aligned}$$

Integrating again, we have

$$\int_a^x \left\{ f^{(n-1)}(x) - f^{(n-1)}(a) \right\} dx = f^{(n-2)}(x) - f^{(n-2)}(a) - (x-a)f^{(n-1)}(a). \qquad (2)$$

The Taylor Approximation

Approximate a function f(x) by its first n derivatives

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more often than not, n=1

$$\boxed{f(x) = f(a) + (x-a)f'(a)} + \frac{(x-a)^2}{2!}f''(a) + ... + \frac{(x-a)^{n-1}}{(n-1)!}f^{(n-1)}(a) + R_n$$

R_n is the remainder term, quantifying the error of this approximation:

$$R_n = \int_a^x ... \int_a^x f^{(n)}(\xi)(dx)^n = \frac{(x-a)^n}{n!}f^{(n)}(\xi).$$

Here ξ is some point in the interval [x,a], as can be shown by the mean value theorem. Hence we can bracket the approximation error as

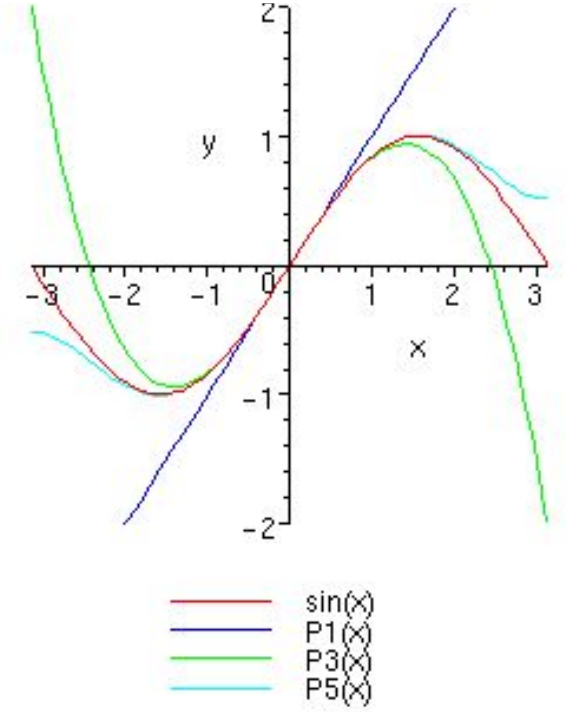
$$R_n \leq \frac{(x-a)^n}{n!} \max_{\zeta\in[x,a]} \left(|f^{(n)}(\zeta)|\right)$$

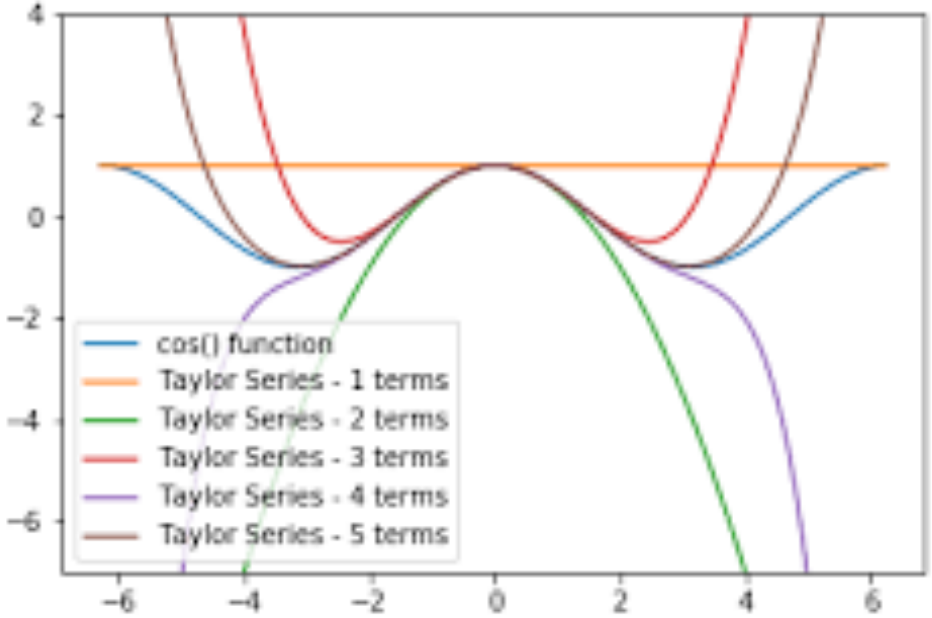
The Taylor Approximation

Two of the most common Taylor approximations in physics:

$$\sin(x) \approx x - \frac{x^3}{3!}$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$





Algorithmic Thinking: Simulating a Competition

Sketch out an algorithm that simulates a sports competition or league of 8 players/teams and ***creates a table at the end of the competition/season****.*

- We'll develop this outline in groups for 20 mins, then I'll call on 3-4 of those groups to summarize their thought process and algorithms to the class
- Everyone is welcome to ask questions!
- If you're in **breakout room 3**, please develop your algorithm in the **document for Group 3**, etc., from this folder https://drive.google.com/drive/folders/1SxENYHdyupr8BLT6ZBuHBKdw5g_GpUOX

Homework Submission

- Volunteer for demonstrating folder compression on Windows systems?
- Ask if you'd like a Mac/UNIX demo