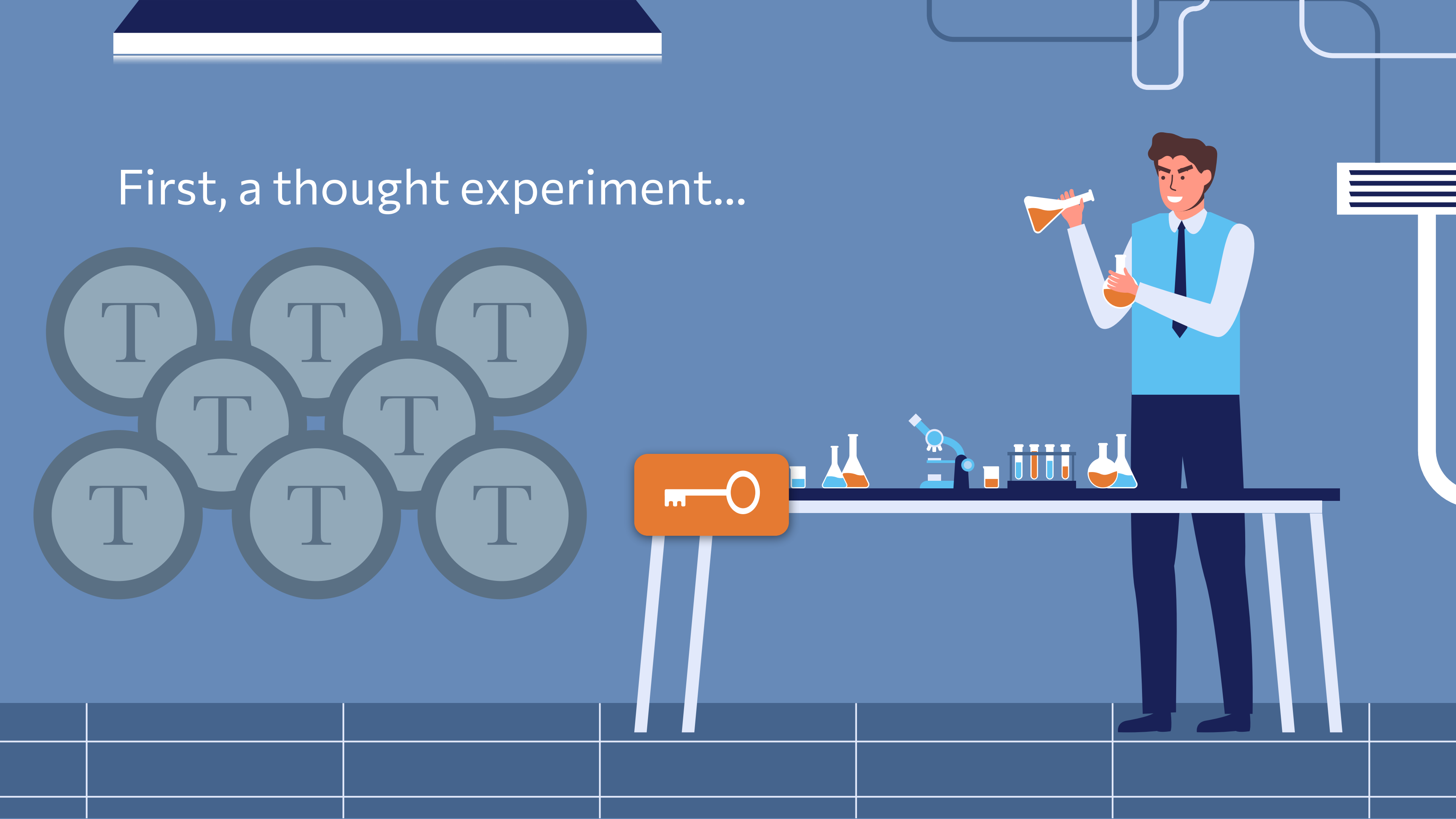
Probability as Logical Inference

Benji Metha's dramatic retelling of ET Jaynes' textbook


First, a thought experiment...

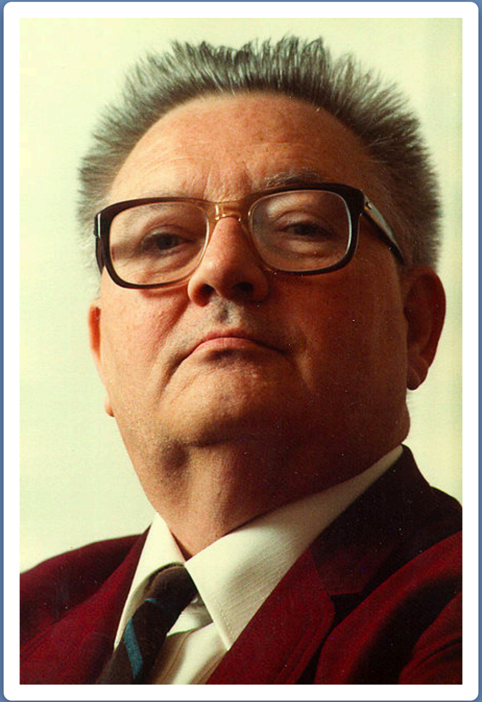
Frequentist

“If you change your mind, you’re committing the *gambler’s fallacy*, and you’re being a **bad scientist.**”

VS

Bayesian

“If you don’t change your mind, you’re *not accounting for all of the information*, and you’re being a **bad scientist.**”



Who is E.T. Jaynes?

- Physicist
- Most famous for connecting **thermodynamics** with **information theory**
- Died of cancer in 1998
- *Probability as logical inference* published in 2003
- Based on 40 years of lecture notes, edited by colleague Larry Bretthorst
- “MUCH MORE COMING”

Logical Reasoning

01

$A \Rightarrow B$
A is true
$\therefore$ B is true

02

$A \Rightarrow B$
B is false
$\therefore$ A is false

03

$A \Rightarrow B$
B is true
$\therefore$ A is more plausible

04

$A \Rightarrow B$
A is false
$\therefore$ B is less plausible

Goal

Formalise a **system of inference** which allows us to make these kinds of deductions

— in a **qualitative way**

— that is **consistent**

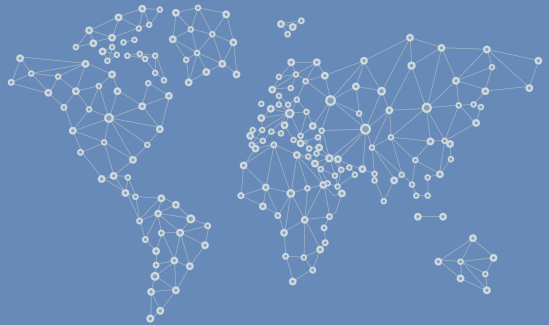
— and agrees with **common sense**

Three axioms:

Total Order

Degrees of belief should be represented by real numbers

Background



You should always take into account all relevant information

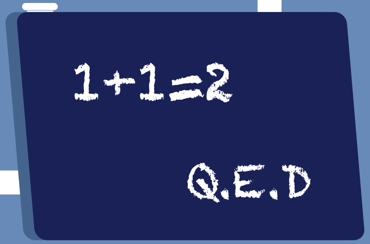
Consistency

Equally ignorant implies equally plausible

That’s it.

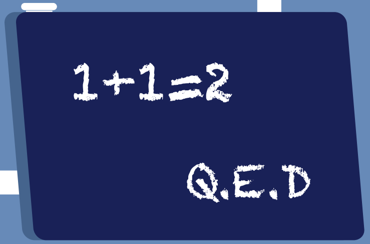
1. That which is certain has plausibility 1

Not an axiom; not a convention.



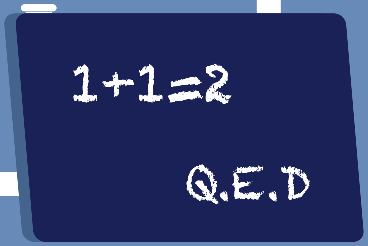
2. Something impossible must have plausibility 0, or infinity.

We choose zero (this time it is convention)



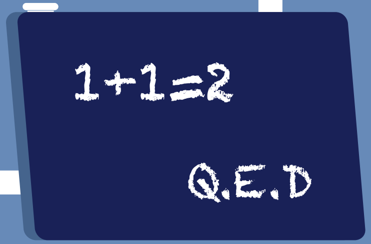
3.“Product Rule”

$$P(AB|C) = P(A|BC)P(B|C)$$



4. Negation Rule

$$P(\bar{A}|C) = 1 - P(A|C)$$



This is sufficient

All logical statements can be built out of AND or NOT; so these rules can be used to calculate the truth value of any logical statement

$A \cup B$	$\neg(\bar{A} \cap \bar{B})$
$A \Rightarrow B$	$A \cap \bar{B}$
$A \Leftrightarrow B$	$(A \cap \bar{B}) \cap (\bar{A} \cap B)$
...	...

Practical exercise

What's the *truth value* of the statement "This die will land on a 6?"

Background information:

- We have 6 different hypotheses
- We are equally ignorant about all of them
- They are *mutually exclusive and exhaustive*

$$P(H_i|B) = P(H_j|B) \ \ \forall \ \ i, j$$

$$P(H_1 + \cdots + H_6|B) = \sum_{i=1}^{6} P(H_i|B) = 1$$

Different background information implies a different truth value

Persi Diaconis' mechanical coin flipper, designed by Harvard University engineers.

All probabilities are conditional

There's no such thing as "random" in nature; just things that we don't fully understand, or have *uncertainties* about.

Probabilities represent incomplete information

Under this interpretations, statistics is the science of optimally interpreting imperfect information.

Thanks!

Do you have any questions?

Should you read Jaynes?

"It is necessary to develop a healthy disrespect for tradition and authority"

5 Queer uses for probability theory
- 5.1 Extrasensory perception
- 5.2 Mrs Stewart's telepathic powers
  - 5.2.1 Digression on the normal approximation
  - 5.2.2 Back to Mrs Stewart
- 5.3 Converging and diverging views
- 5.4 Visual perception – evolution into Bayesianity?
- 5.5 The discovery of Neptune
  - 5.5.1 Digression on alternative hypotheses
  - 5.5.2 Back to Newton

"If we humans threw away what we knew yesterday in reasoning about our problems today, we would be below the level of wild animals"

The odds are (is?)

'odds' is a grammatically slippery word.