

Define the mantissa $m \in [0, 1)$ as a B_m bit number. The high order $B_m/2$ bits fit in a B_m bit register h , and the lower order $B_m/2$ bits fit in a B_m bit register l . The mantissa is

$$(1) \quad m = 2^{(\frac{B_m}{2})}h + l.$$

The mantissa is assumed to be normal (i.e. the high order bit is set).

Because the mantissa is normal, we can drop the leading bit and assume

$$(2) \quad \hat{m} = 1 + 2m = 1 + 2^{(\frac{B_m}{2}+1)}h + 2l,$$

where $\hat{m} \in [2, 1)$.

The square is

$$(3) \quad (\hat{m})^2 = 1 + 2^{(\frac{B_m}{2}+2)}h + 2^2l + 2^{(\frac{B_m}{2}+3)}hl + 2^{(B_m+2)}h^2 + 2^2l^2.$$

The normal representation is

$$(4) \quad \text{Norm} \left((\hat{m})^2 \right) = 2^{(\frac{B_m}{2}+1)}h + 2l + 2^{(\frac{B_m}{2}+2)}hl + 2^{(B_m+1)}h^2 + 2l^2.$$

If the 2nd highest order bit of h is set (i.e. $h \wedge (2^{(B_m/2)-1}) \neq 0$, where \wedge represents bitwise logical and) then the square will result in number greater than 1, and thus requires division by 2 to reduce it to the range $[0, 1)$,

$$(5) \quad \text{Norm} \left((\hat{m})^2 \right) = 2^{(\frac{B_m}{2}+1)}h + 2l + 2^{(\frac{B_m}{2}+2)}hl + 2^{(B_m+1)}h^2 + 2l^2.$$

$$(6) \quad \text{Adj} \left((\hat{m})^2 \right) = (\hat{m})^2 2^{-1} = 2^{-1} + 2^{(\frac{B_m}{2}+1)}h + 2^1l + 2^{(\frac{B_m}{2}+2)}hl + 2^{(B_m+1)}h^2 + 2^1l^2.$$

We can rewrite this

$$(7) \quad \text{Adj} \left((\hat{m})^2 \right) = 2^{(\frac{B_m}{2}+2)}(2^{-\frac{B_m}{2}-3} + 2^{-1}h + 2^{-\frac{B_m}{2}-2}l + hl + 2^{(B_m/2-1)}h^2 + 2^{-\frac{B_m}{2}-1}l^2).$$