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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES

L. G. HANSCOM FIELD, BEDFORD, MASSACHUSETTS

A FORTRAN Program That Derives Air Temperature, Density, and Composition as a Function of Height and Exospheric Temperature

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AIR FORCE SYSTEMS COMMAND

United States Air Force

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AFCRL COMPUTATION CENTER PROJECT 0001

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Abstract

Dr. L.C. Jacchia (1971) has developed a static diffusion model of the atmosphere that matches parameters at 150 km and the observed densities from satellite drag at greater heights. His report presents tables showing temperature, density and composition as a function of height and exospheric temperature.

The Jacchia model has been used in working programs by reading the tables and converting from log densities to simple densities and vice-versa, to interpolate the tables and make the necessary corrections.

This report presents a FORTRAN subroutine which gives the precise density information (along with the appropriate corrections) directly by using the analytic equations.

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Illustrations

A FORTRAN Program That Derives Air Temperature, Density and Composition as a Function of Height and Exospheric Temperature

1. INTRODUCTION

In Smithsonian Astrophysical Observatory Special Report No. 332, Dr. L. C. Jacchia has presented a static diffusion model of the atmosphere. Modifying his previous model, Dr. Jacchia has arrived at a satisfactory model that matches parameters at 150 km and the observed densities from satellite drag at greater heights. Using an appropriate set of equations, his report presents tables showing temperature, density and composition as a function of height given for exospheric temperatures ranging from 500° to 1900° at 100°K intervals, and for heights from 90 to 2500 km. There are also auxiliary tables to help in evaluating the diurnal, geomagnetic, semiannual and seasonal latitudinal effect. The Jacchia model has been used in working programs by reading the tables and converting from log densities to simple densities and vice-versa, to interpolate the tables and make the necessary corrections. This method is inefficient from the viewpoint of the amount of data storage used and from the inconvenience of interpolation.

Our report presents a FORTRAN subroutine which gives the precise density information (along with the appropriate corrections) directly by using the analytic equations. First we present the mathematical model along with the material relevant to the equations and the derivations of them. Then we present a detailed FORTRAN flow chart of the program generated in Appendix A. Finally, calling

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information to the subroutine and a printout of the program are presented in Appendix B, and examples using the model are presented in Appendix C.

2. MATHEMATICAL MODEL

This model solves the problem of finding the atmospheric density at a point given the height in kilometers, the geographical coordinates, time of the year, year involved, time in U. T., smoothed solar flux $\overline{F}_{10.7}$ for that date, actual flux $F_{10.7}$ on the day before (to account for the lag $1^{\circ}.0$), and K_p for a time 6.7 hours before the desired date.

The plan for presentation of the mathematical model is shown in Figure 1. Each block shown in Figure 1 is investigated in detail in the rest of this report.

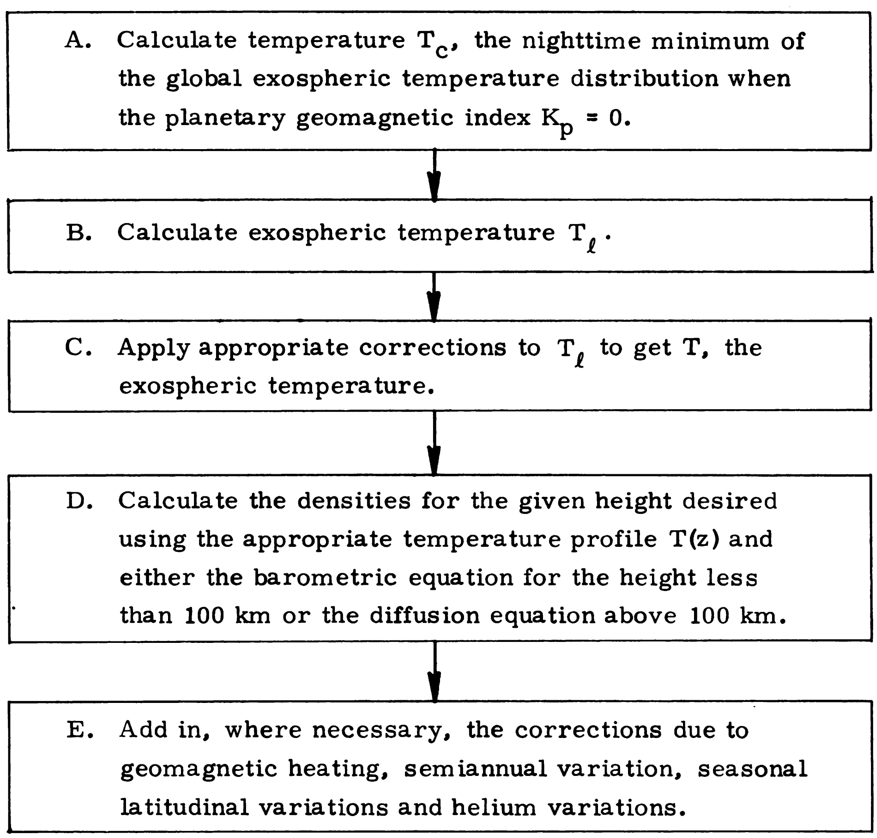


Figure 1. Chart for Presentation of Mathematical Model

2.1 Block A of Figure 1

The ultraviolet solar radiation which heats the earth's upper atmosphere consists of two components whose reactions are different: (1) the active-region component (consisting of special lines of highly ionized atoms such as FEXIV-XVI, Si IX-X, MgX); and (2) the radiation from the clear disk arising from much less ionized atoms (for example, HeI-II, O IV and helium continuum). The active regio component varies from day to day related to sun rotation and spot formation. The disk component presumably varies slowly during an eleven-year solar cycle.

$F_{10.7}$ equals the 10.7-cm solar flux which is used as an index of solar EUV radiation. Increasing the 10.7-cm flux corresponds to increases in exospheric and thermospheric temperatures.

$F_{10.7}$ equals the smoothed solar flux averaged over three solar rotations. Dr. Jacchia captures the temperature variation during the hours of the day, geographic location and geomagnetic activity in his eq. (14) for T_c:

$$T_c = 379^o + 3^o.24\ \bar{F}_{10.7} + 1^o.3\ (F_{10.7} - \bar{F}_{10.7}) \quad (K_p = 0)$$

$$\left| F_{10.7} \right| = 10^{-22} \text{ watts } m^{-2} \text{ (cycle/sec)}^{-1} \text{ bandwidth.}$$

T_c is the minimum exospheric temperature anywhere on the globe. At the desired instant for a quiet geomagnetic condition, $(K_p = 0)$.

2.2 Block B of Figure 1

We now want to calculate the exospheric temperature T_ℓ. Dr. Jacchia, in his eq. (17), presents T_ℓ as a function of the hour angle H of the sun, chosen in local solar time, counted from the upper culmination.

$$T_\ell = T_c\ [1 + R \sin^m \theta + R \times (\cos^m - \sin^m) \times \cos^n \tau/2)]$$

where

$$\tau = H + \beta + p \times \sin(H + \gamma)\ (-\pi < \tau < \pi)$$

$$m = 2.2 \qquad n = 3.0 \qquad R = 0.3$$

$$\beta = -37^o \qquad \rho = +6^o \qquad \gamma = +43^o$$

and

$$\eta = 1/2 \left| \phi - \delta_\odot \right| \qquad \theta = 1/2 \left| \phi + \delta_\odot \right|$$

where

ϕ = latitude $\qquad\qquad$ $\delta_{\odot}$ = sun's declination.

In fact, his eq. (17) gives the ratio of T_ℓ/T_c.

Combining the information from paragraph 2. 1, we get:

$$T_\ell = (T_\ell/T_c)\times T_c.$$

2.3 Block C of Figure 1

We now discuss effects of the geomagnetic effect as it deals with temperature change. K_p is the 3-hour geomagnetic planetary index with a time lag.

Dr. Jacchia considers two regions: height $z < 200$ km, and $z > 200$ km.

Case I. When $z < 200$ km, the following hybrid formula is used:

(a) $\log_{10}\rho = 0.012\ K_p + 1.2 \times 10^{-5} \exp(K_p)$

(b) $\Delta T_\infty = 14^\circ\ K_p + 0^\circ.02 \exp(K_p)$. $\qquad$ (eq. 20)

One uses eq. (a) for the final density under 200 km when using eq. (b) for ΔT_∞, from the rest value $K_p = 0$ to that corresponding to the given value of K_p at a time 6.7 hours before desired date.

Case II. When $z > 200$ km,

$$\Delta T_\infty = 28^\circ K_p + 0.03 \exp(K_p).$$

So we get

$$T_\infty = T_\ell + \Delta T_\infty \text{ (corrected exospheric temperature).}$$

In reprise, we have so far obtained exospheric temperature T via the calculations shown in Figure 2.

2.4 Block D of Figure 1

The way the density is calculated requires some explanation. There are two main ingredients we need: a temperature profile $T(z)$ and equations that deal with the basic constituents of the atmosphere.

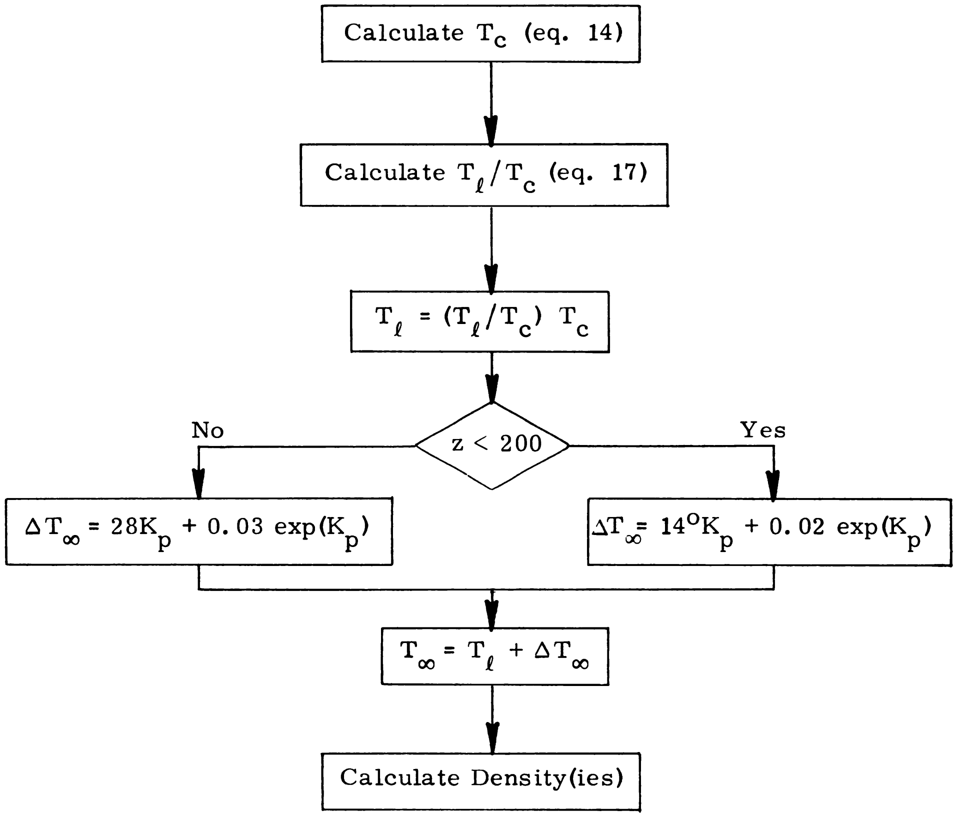


Figure 2. Chart for Obtaining Exospheric Temperature T_∞

2.4.1 TEMPERATURE PROFILES

Dr. Jacchia found that the experimentally obtained temperature profiles may be divided into two classes based upon the height ranges: 90 km < z < 125 km, and z > 125 km.

For 90 km < z < 125 km, the temperature profiles may be modeled by a 4th degree polynomial $T(z) = T_x + \sum_{n=1}^{4} C_n (z-125)^n$ (eq. 10)

$T_x = 371.6678 + 0.0518806\ T_\infty - 294.3505 \exp(-0.00216222 \times T_\infty)$

From the boundary conditions

$$z = 90 \quad \left\{ \begin{array}{l} T(z) = 183^oK \\ \\ \left(\dfrac{dT(z)}{dz}\right)_{z=90} = 0 \end{array} \right\}$$

$$z = 125 \quad \left\{ \begin{array}{ll} \left(\dfrac{dT(z)}{dz}\right)_{z=125} & = \dfrac{1.90}{35} (T_x - 183) = G_x \\ \\ \left(\dfrac{d^2T(z)}{dz^2}\right)_{z=125} & = 0 \end{array} \right\}$$

we solve to find:

$$C_1 = 5.4285714286 \times 10^{-2} \times (T_x - 183)$$

$$C_2 = 0$$

$$C_3 = -3.965014577 \times 10^{-5} \times (T_x - 183)$$

$$C_4 = -5.331112 \times 10^{-7} \times (T_x - 183)$$

For $z > 125$ km

$$T(z) = T_x + A \tan^{-1} \left\{ \frac{G_x}{A} (z-125) \left[1 + B(z-125)^{\beta} \right] \right\}$$

where:

$$A = \frac{2}{\pi} (T_\infty - T_x)$$

$$B = 4.5 \times 10^{-6}$$

$$\beta = 2.5 \, .$$

Dr. Jacchia explains that his choice of arctan from several suitable asymptotic functions was determined by its ready availability in computer libraries and in tabulated form. The corrective term $[1 + B\,(z-125)^{\beta}]$ frees the temperature profiles from strict dependence on the selected type of asymptotic functions.

2.4.2 DENSITY EQUATIONS

Dr. Jacchia has found that from the region 90 to 100 km and a given temperature profile $T(z)$, the density can be computed by integrating the barometric equation:

$$d \ln (\rho) = d \ln \left(\frac{\overline{M}}{T(z)} \right) - \left(\frac{\overline{M}g}{R^* \times T(z)} \right) dz \qquad \text{(eq. 5)}$$

where:

(a) $g = 9.80665\ (1+z/Re)^{-2} \text{cm sec}^{-2}$ $\quad Re = 6.356766 \times 10^{3}\text{km}$

(b) $R^{*} = 8.31432$ joules $(^{o}K)^{-1}$ mole^{-1}

(c) $\bar{M}(z) = \sum_{n=0}^{6} C_n (z-90)^n$ $(90 < z < 100,\ z \text{ in km})$ (eq. 1)

$C_0 = 28.82678$

$C_1 = -7.40066 \times 10^{-2}$

$C_2 = -1.19407 \times 10^{-2}$

$C_3 = 4.51103 \times 10^{-4}$

$C_4 = -8.21895 \times 10^{-6}$

$C_5 = 1.07561 \times 10^{-5}$

$C_6 = -6.97444 \times 10^{-7}$

(d) for z = 90 km, Dr. Jacchia assumed the boundary conditions

$\rho_o = 3.46 \times 10^{-9}$ g cm^{-3}

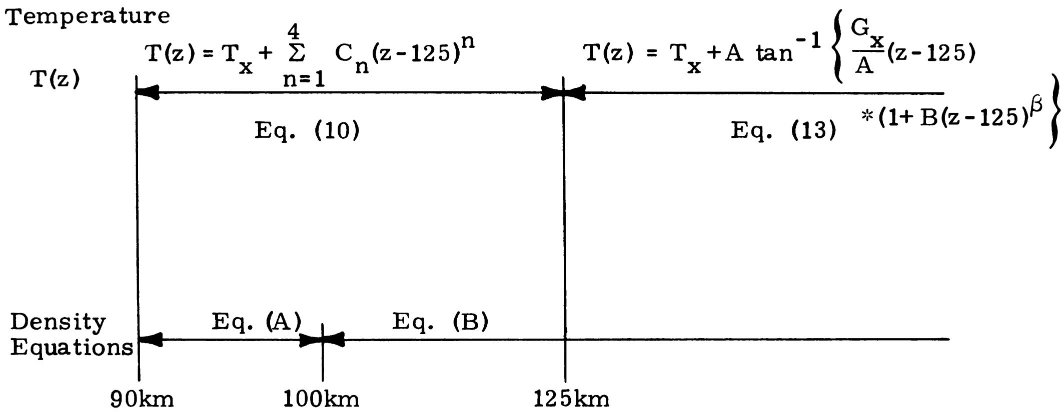
$T_o = 1830$ K

Above 100 km, one must calculate the number density of each individual species n(i) by integrating the diffusion equation.

$$\frac{dn(i)}{n(i)} = \frac{M_i g}{R^{*} \times T(z)}\, dz - \frac{dT}{T}(1 + \alpha_i)$$

$\alpha_{helium} = -0.38$

$\alpha_{n(i)} = 0$ for other species



$$\text{Eq. (A)} \quad d\,\ell n\ \rho = d\,\ell n\ \rho\left(\frac{\bar{M}(z)}{T(z)}\right) - \left(\frac{\bar{M}(z)}{R^{*}\times T(z)}\right)dz \qquad \text{(eq. 5)}$$

$$\text{Eq. (B)} \quad \frac{dn(i)}{n(i)} = -\left(\frac{M_i g}{R^{*}\times T(z)}\right)dz - \left(\frac{dT(z)}{T(z)}\right)(1+\alpha_i) \qquad \text{(eq. 6)}$$

Figure 3. Ranges of Temperature and Density Equations

$M(N_2) = 28.0134$ $\qquad$ $M(He) = 4.0026$

$M(O_2) = 31.9988$ $\qquad$ $\bar{M}_o = 28.96$ = sea-level molecules mass

$M(AR) = 39.948$

Figure 3 shows that a density in the 90 to 100 km range uses T(z) from eq. (10) and the density from eq. (5) to get $\ell n\ \rho$ by integrating from 90 to 100.

If the height z lies in the 100 to 125 km range, we still use T(z) from eq. (10) but now must calculate the individual species $\ell n(n(i))$ from which density is easily obtained. Said another way, if $100 < z < 125$, we must use eqs. (10) and (5) to find out ρ at 100 km, break ρ into the species components at 100 km to have $n(i)\big|_{100}$ and then integrate from $100 \rightarrow z$.

If $z > 125$, we use T(z) from eq. (13) and the diffusion eq. (6). For $z > 125$ our integration goes from $90 \rightarrow 100$ km, $100 \rightarrow 125$ km and $125 \rightarrow z$ km.

Let us go through these integration ranges to explicate the above explanation and to demonstrate the derivation of the coefficients used in the program.

Case I: $90 < z < 100$ km

$$T(z) \underset{\Delta}{=} Tz1(z) = T_x + (T_x - 183) \times (C_1(z-125) + C_3(z-125)^3$$

$$+ C_4(z-125)^4) \quad \text{(eq. 10)}$$

$$\bar{M}(z) = \sum_{n=0}^{6} C_n(z-90)^n \quad \text{(eq. 1)}$$

We integrate eq. (5):

$$\text{S1} \quad \int_{90}^{z} d\,\ell n = \int_{90}^{z} d\,\ell n \frac{\bar{M}(z)}{T(z)} - \int_{90}^{z} \frac{\bar{M}(z)}{R^* \times Tz1(z)}\, dz$$

$$\text{S2} \quad \ell n\, \rho \Big|_z - \ell n\, \rho_{90} = \ell n \left[\frac{\bar{M}(z)}{Tz1(z)}\right]_z - \ell n \left[\frac{\bar{M}(z)}{Tz1(z)}\right]_{90} - \int_{90}^{z} \frac{\bar{M}(z)}{R^* \times Tz1(z)}\, dz$$

$$\text{S3} \quad \ell n\, \rho \Big|_z = \ell n\, \rho \Big|_{90} + \ell n\, \bar{M}(z)\Big|_z - \ell n\, (Tz1(z))\Big|_z$$

$$+ \ell n\, (\underbrace{Tz1(90)}_{T_o}) - \ell n\, \bar{M}(90) - \int_{90}^{z} \frac{\bar{M}(z)}{R^* \times Tz1(z)}\, dz$$

Initial boundary conditions are $\rho_o = 3.46 \times 10^{-9}$ g cm^{-3}

$T_o = 183^o$

$$\ell n(\rho_o) = -19.4819972$$

$$\ell n(T_o) = 5.2094862$$

$$\ell n\, \rho_o + \ell n\, T_o = -14.2725110$$

Thus S3 becomes

$$\text{S4} \quad \ell n\, \rho \Big|_z = -14.2725110 + \ell n\, \bar{M}(90) + \ell n\, \bar{M}(z)\Big|_z - \ell n\, Tz1(z)\Big|_z$$

$$- \int_{90}^{z} \frac{\bar{M}(z)}{R^* \times Tz1(z)}\, dz\ .$$

From eq. (1), $\bar{M}(90) = C_o = 28.82678$.

Evaluating S4 at height z gives the density ρ.

Case II: $\dfrac{100 < z < 125 \text{ km}}{T(z) = Tz1(z)}$

We integrate diffusion eq. (6) to get:

$$\text{T1} \quad \int_{100}^{z} \frac{d(n(i))}{n(i)} = -\frac{Mi}{R^*} \int_{100}^{z} \frac{g(z)}{Tz1(z)} - \int_{100}^{z} \frac{dT}{T}(1+\alpha_i)$$

$$\text{T2} \quad \ln n(i)\Big|_z - \ln n(i)\Big|_{100} = -(1+\alpha_i) \ln Tz1(z)\Big|_z$$

$$+ (1+\alpha_i) \ln Tz1(z)\Big|_{100} - \frac{Mi}{R^*} \int_{100}^{z} \frac{g(z)}{Tz1(z)}\, dz$$

At 100 km, $Tz1(z) = T_x$ (either eq. (10) or eq. (13)). Thus T3 is:

$$\ln n(i)_z = \ln n(i)\Big|_{100} + (1+\alpha_i) \ln (T_x) - (1+\alpha_i) \ln Tz1(z)\Big|_z$$

$$- \frac{Mi}{R^*} \int_{100}^{z} \frac{g(z)}{Tz1(z)}\, dz \, .$$

From T3 we notice the only thing we cannot directly evaluate is $\ln (n(i))\Big|_{100}$. We must use Case I with z = 100 to get $\ln\Big|_{100}$ and from this get $\ln n(i)\Big|_{100}$ for each species. Assuming we have used Case I and evaluated eq. S4 at z = 100, the procedure for obtaining each n(i) is as follows:

Use $N = \dfrac{6.02257 \times 10^{23} \rho}{\bar{M}(z)}$ (eq. 2)

For N_2, Ar and He, $n(i) = q_o(i)\left(\dfrac{\bar{M}(z)}{M_o}\right) N$ (eq. 3)

Combining eq. (2) and eq. (3) and simplifying,

$$n(i) = \frac{q_o(i)}{\bar{M}_o} \times 6.02257 \times \rho \times 10^{23}$$

where $q_o(i)$ is the fraction by volume of each specie (given below), and $\bar{M}_o$ = 28.960 = sea-level mean molecular mass.

For O and O_2 Jacchia uses eq. (4):

$$n(O_2) = 2N\left(1 - \frac{\bar{M}}{\bar{M}_o}\right) = 2 \times \frac{6.02257 \times 10^{23}}{\bar{M}(z)} \times \left(1 - \frac{\bar{M}(z)}{\bar{M}_o}\right)$$

$$n(O) = 12.04514 \times 10^{23} \times \rho \times \left(\frac{1}{\bar{M}(z)} - \frac{1}{28.96}\right)$$

For O_2:

$$n(O_2) = N\left\{\frac{\bar{M}(z)}{\bar{M}_o}[1 + q_o(O_2)] - 1\right\} = \frac{6.02257 \times 10^{23}}{\bar{M}(z)} \times$$

$$\times \left\{\frac{\bar{M}(z)}{\bar{M}_o}[1 + 0.20955] - 1\right\} = 6.02257 \times 10^{23} \times$$

$$\times \rho\left\{\frac{1.20955}{28.96} - \frac{1}{\bar{M}(z)}\right\}$$

where

	$q_o(i)$
N_2	0.78110
O_2	0.20955
Ar	0.0093432
He	0.0000061471

After this tedious computation we get, ρ_{100}

$$n(N_2) = 0.1624339 \times 10^{23} \times \rho_{100}$$

$$n(O_2) = 10^{23} \times \rho_{100} \quad (0.25154 - \frac{6.02257}{28.82678})$$

$$n(Ar) = 0.1943027 \times 10^{21} \times \rho_{100}$$

$$n(He) = 0.12783612 \times 10^{18} \times \rho_{100}$$

From eq. T3 we get $\ln n(i)\big|_z$ for each species using the above values for $n(i)\big|_{100}$ by taking the natural log and evaluating $(z < 125)$.

To get $\rho\big|_z$ we use the equation $\rho = \Sigma\ M_i \times (n(i))$.

Case III: When $z > 125$, we use eq. (13) for T(z)

$$T(z) \underset{\Delta}{=} Tz2(z) = T_x + A \tan^{-1} \left\{ \frac{G_x}{A} (z-125) \left[1+B(z-125)^{2.5} \right] \right\}$$

Integrating the diffusion eq. (6), we get

$$\ln n(i)\Big|_z = \ln n(i)\Big|_{125} + \left[\ln (T_x) + \ln Tz2(z) \right]_{125} \times (1+\alpha_i)$$

$$- \int_{125}^{z} \frac{Mi \times (z)}{R^* \times Tz2(z)}\, dz$$

To evaluate $\ln (n(i))$ at 125, 23 go back to Case II and set $z = 125$ for eq. T3. Then converting to real values, we see $\rho = \Sigma\ n(i) \times M_i$.

The flow of the above logic is shown in Figure 4.

2.5 Block E of Figure 1

The first correction applied to the density obtained above is the one associated with the semiannual variation observed in the thermosphere and lower exosphere. Dr. Jacchia indicates that there is no satisfactory explanation for this phenomenon. Historically, he linked the semiannual variation to temperature variations which are the immediate cause of the solar activity effect, diurnal variation and the geomagnetic effect. The justification for associating the semiannual density variations to temperature was the apparent dependency of the amplitude of the temperature variation on solar activity. Unfortunately, satellite drag information showed there was a discrepancy between the semiannual density variations, the model predicted, and the semiannual density variations encountered at 100 km (based on Echo 2). Other discrepancies also arose. In 1971, Dr. Jacchia discovered that the amplitude of the semiannual density variation, although strongly height dependent and variable from year to year, does not seem to be related to the solar activity. He has been led to represent the semiannual density variation in the form

$$\Delta \log_{10} \rho \text{ semi-an} = f(z) \times g(t) \qquad \text{(eq. 21)}$$

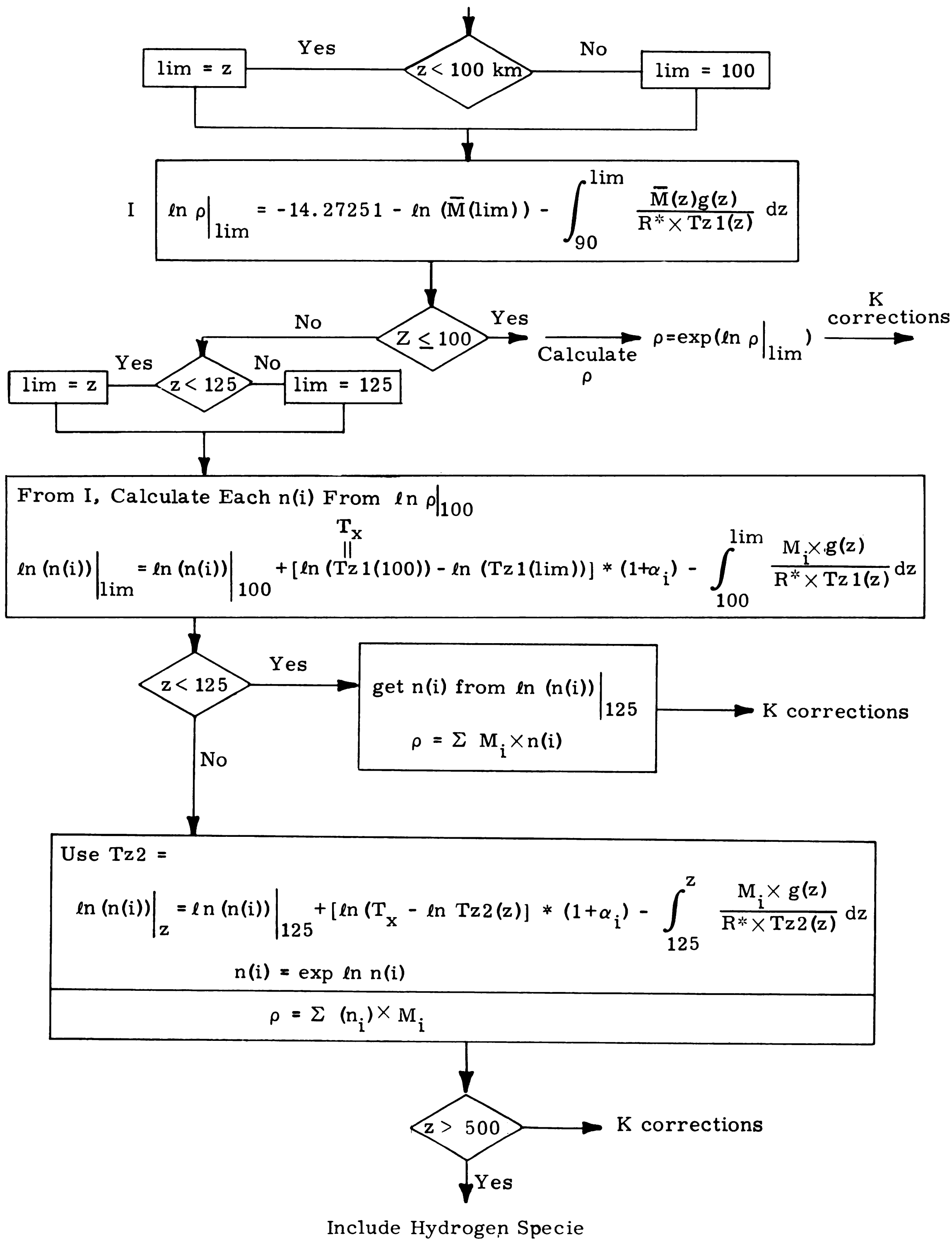


Figure 4. Chart for Obtaining Temperature Profiles and Density Equations

where g(t) represents the average density variation as a function of time in which amplitude (that is, the difference in log ρ between the principal minimum in July and the principal maximum in October) is normalized to 1, and f(z) is the relation between amplitude and the height z.

Dr. Jacchia found the following equations best fit the data obtained:

$$f(z) = (5.876 \times 10^{-7} \times z^{2.331} + 0.06328) \exp(-2.868 \times 10^{-3} z) \text{ (z in km)} \rightarrow$$

$$g(t) = 0.02835 + 0.3817 [1 + 0.4671 \sin(2\pi\tau + 4.137)] \sin(4\pi\tau + 4.259) \quad \text{(eq. 22)}$$

where

$$\tau = \Phi + 0.09544 \left\{ [1/2 + 1/2 \sin(2\pi\Phi + 6.035)^{1.650} - 1/2 \right\}$$

$$\Phi = (t - 36204)/365.2422 \qquad \text{(eq. 23)}$$

is the phase of the semiannual variation (approximately the number of days elapsed since January 1 divided by the duration of the tropical year in days).

In eq. (23), t is the time in Modified Jacchia Days (M.J.D. = Julian Day minus 2,400,000.5). M.J.D. 36204 corresponds to Jan. 1, 1958. Dr. Jacchia gives the following caveat: "It should be understood that even though temperature variations are apparently not the primary cause of the semiannual density variations, these must be accompanied by some temperature changes, however small. The determination of such temperature changes must wait, however, for better observations or, more likely, for dynamical models of the phenomenon."

The new density becomes log ρ new = log ρ + $\Delta\log_{10}\rho$ semiannual.

Here we branch. If $170 < z < 500$ km, we are done. If $z < 170$ km, we must include a correction for the seasonal-latitudinal variations of the lower thermosphere. If $z > 500$ km we must include a correction for helium variation.

Case I: Seasonal-Latitudinal Variations of the Lower Thermosphere

Due to the complexity of the seasonal-latitudinal variations, some simplifying assumptions had to be made to obtain a compromise with the various data segments observed. The models were limited to a fixed, intermediate latitude and to three seasons (summer, winter and spring/fall). The amplitude of the seasonal-latitudinal density variations increases very rapidly between 90 and 100 km, reaching an apparent maximum between 105 to 120 km. From 120 km, there is a sharp decrease because above 160 km there seems to be no appreciable seasonal latitudinal variations, other than those involved in the global pattern of the diurnal variation. Thus temperature variations are in phase at 100 km with the density variations, undergo a phase inversion at 110/cm, and reach a maximum amplitude around 150 km in opposite phase. Representing temperatures is prohibitive.

Thus Dr. Jacchia makes a simplifying assumption that the standard - latitudinal variation occurred in density (ignoring the influence of temperature variation).

Dr. Jacchia uses the Champion (1967) and Groves (1970) formulas.

$$\Delta \log_{10} \rho = S \times P \frac{\phi}{|\phi|} \sin 2\phi \qquad \text{(eq. 24)}$$

$$S = 0.014\ (z - 90) \exp\left(-0.0013\ (z - 90)^2\right)$$

$$P = \sin\ (2\pi\Phi + 1.72)$$

We add $\Delta\log_{10}\rho$ to $\log \rho$ so far obtained to get a correction $\log\ \rho$.

Case II. Helium Variations

From the drag of Explorer 10 and Explorer 24, the helium variation which is observed above 500 km can best be expressed by eq. (25).

$$\Delta \log_{10} \Delta(\mathrm{He}) = 0.65 \ \frac{\delta_\odot}{\epsilon} \ \left(\sin^3 \left(\frac{\pi}{4} - \frac{\phi}{2} \frac{\delta_\odot}{|\delta_\odot|}\right) - \sin \frac{3\pi}{4}\right)$$

where ϵ is the obliquity of the ecliptic, $23^\circ 44$, and $\delta_\odot$ is the declination of the sun.

We use the correction $\Delta\log_{10} n(\mathrm{He})$ by adding it to $\log_{10} n_o(\mathrm{He})$. Then taking the anti-log base 10, we get the corrected n(He). We define $\Delta n = n(\mathrm{He}) - n_o(\mathrm{He})$ and use it to get $\Delta\rho = 6.64599^{963} \times 10^{-24} \times \Delta n$. Our final ρ is found to be:

$$\rho = \rho + \Delta\rho\ .$$

The flow diagram of the corrections can be seen in the chart of Appendix A.

Appendix A

Major Flow Chart Diagram

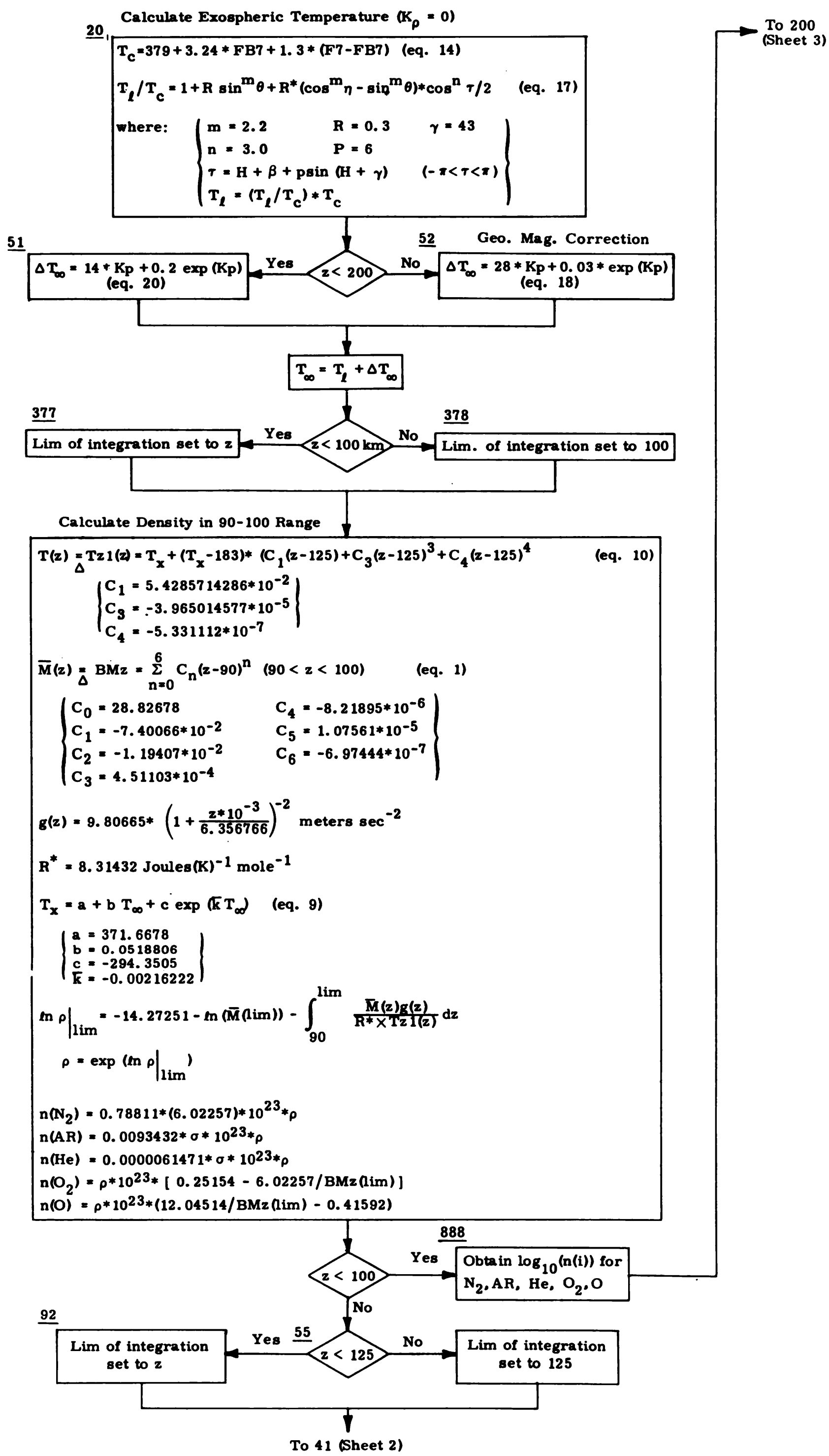


Figure A1. Flow Diagram of Corrections (Sheet 1 of 3)

From 92 (Sheet 1)

To 200 (Sheet 3)

<u>41</u> Calculate Density in 100-125 Range

For each space:

$$\ln(n(i))\Big|_{\lim} = \ln(n(i))\Big|_{100} + \{\ln Tz1(100) - \ln Tz1(\lim)\} \times (1+\alpha_i) - \int_{100}^{\lim} \frac{M_i g(z)}{R^* \times Tz1(z)}\, dz$$

$\alpha_i = -0.38$ for He

$\alpha_i = 0$ for other species

$M(N_2) = 28.0134$ $\quad M(AR) = 39.948$

$M(O_2) = 31.9988$ $\quad M(He) = 4.0026$

$M(O) = 15.9994$

<u>569</u> $z < 125$

Yes:

$n(i) = \exp(\ln(n(i))$ for each specie

$\rho = (\Sigma\, n(i) \times M_i) \times 0.166042071 7 \times 10^{-23}$

No:

Calculate Density in Range Greater Than 125 km.

$$T(z) \triangleq Tz2(z) = T_x + A \tan^{-1}\left\{\frac{G_x}{A}(z-125)(1+B(z-125)^{\beta})\right\} \quad \text{(eq. 13)}$$

$A = 2/\pi\,(T_\infty - T_x)$ $\quad B = 4.5 \times 10^{-6}$

$\beta = 2.5$ $\quad G_x = 1.90\left(\frac{T_x - 183}{35}\right)$

$$\ln(n(i))\Big|_z = \ln(n(i))\Big|_{125} + \{\ln T_x - \ln Tz2(z)\} * (1+\alpha_i) - \int_{125}^{z} \frac{M_i \times g(z)}{R^* \times Tz2(z)}\, dz$$

$n(i) = \exp \ln n(i)\Big|_z$ for each specie

density $\rho = (\Sigma\, n(i) \times M_i) \times 0.166042071 7 \times 10^{-23}$

<u>569</u> $z \geq 500$

No: To 200

Yes:

Include Hydrogen

$$\mathrm{Log}_{10}\, n(H)_{500} = 73.13 - 39.4 \times \log_{10}(Tz2(500)) + 5.5 \times (\log_{10} Tz2(500))^2$$

$$n(H)_{500} = 10^{\log_{10} n(H)_{500}}$$

$$\ln n(H)_z = \ln n(H)_{500} + \{\ln Tz2(500) - \ln Tz2(z)\} - \int_{500}^{z} \frac{g(z)}{R^* \times Tz2(z)}\, dz$$

$$n(H)_z = \exp(n(H)_z)$$

$$\rho_{new} = \rho + (n(H)_z \times 0.166042071 7 \times 10^{-23})$$

Figure A1 (Contd). Flow Diagram of Corrections (Sheet 2 of 3)

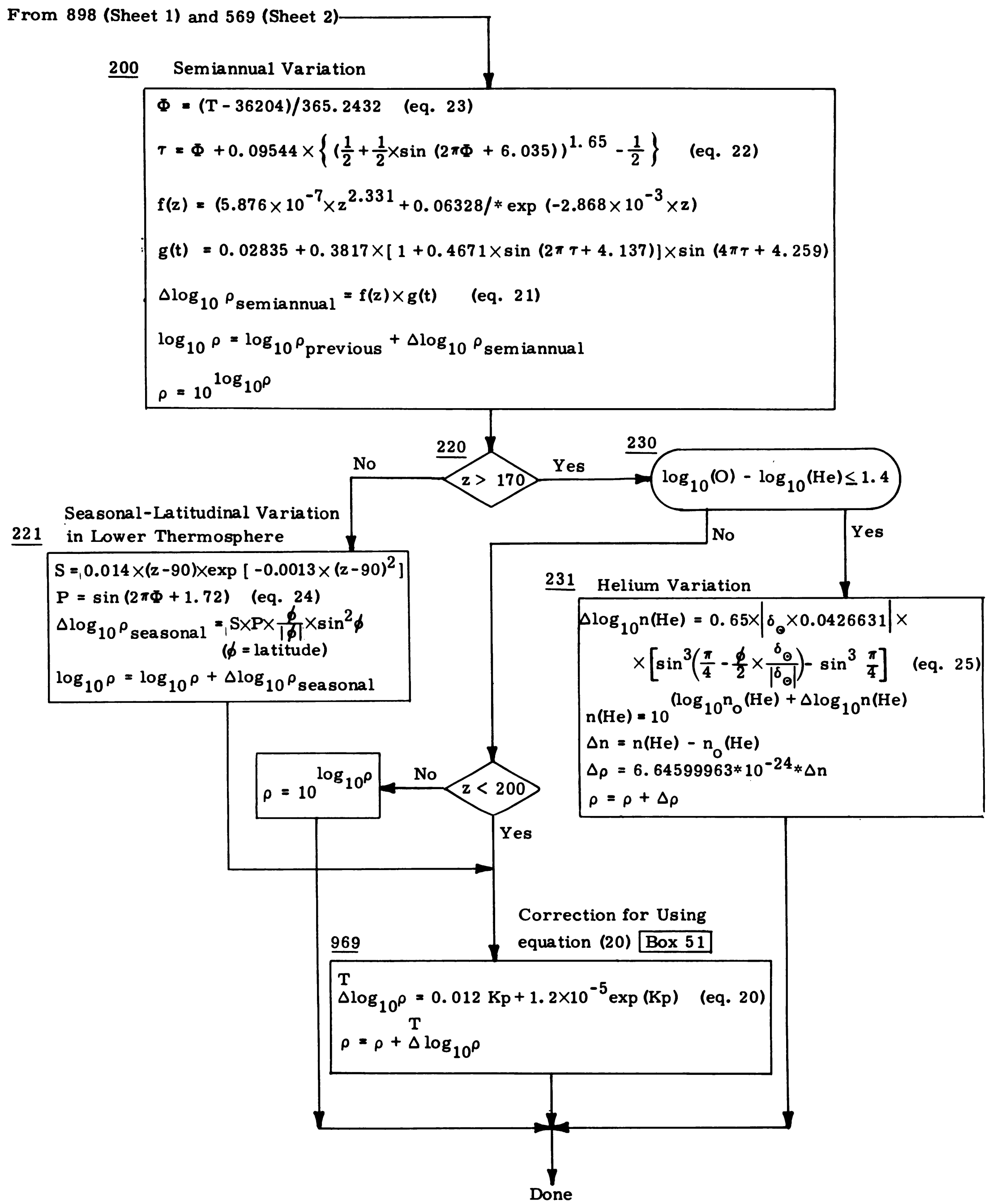


Figure A1 (Contd). Flow Diagram of Corrections (Sheet 3 of 3)

Appendix B

Calling Sequency and Printout of the Subroutine

When the subroutine named CCIA (z, H, F7, FB7, KP, T, DEC, PHI, RHO, QLN2, QLN02, QLN00, QLNHE, QLNAR, QLNHYD, ALTEMP, TNF) is supplied with:

z = height in kilometers
H = Hour angle of the sun
F7 = 10.7 Solar Flux
FB7 = Averaged Solar Flux
KP = K_p
T = Time expressed in modified Julian Days (M.J.D. = Julian Day Minus 2,400, 000.5)*
DEC = declination of the sun
PHI = latitude

it will return with:

(a) log base 10 of the individual species (for example, QLNO2 $\longleftrightarrow$ $\mathrm{Log}_{10}(O_2)$)

(b) ALTEMP temperature at height z
TNF = the exospheric temperature.

It may be used with any FTN compiler.

---

*M.J.D. 36204 corresponds to Jan. 1, 1958

SUBROUTINE CCIA TRACE

```
      SUBROUTINE CCIA(Z,H,F7,FB7,KP,T,DEC,PHI,RHO,QLN2,QLNO2,QLNOO,
     1   QLNHE,QLNAR,QLNHYD,ALTEMP,TNF)
      REAL LNN2,LNO2,LNAR,LNHE,NOHE,NHE,LGHE,MU,KP,LNOO
         EXTERNAL     IGSN,TGRN,TGPN
         QLN2=QLNO2=QLNOO=QLNHE=QLNAR=QLNHYD=0.
       DATA PI/3.14159265358979/
      DATA   RAD/.0174532925199431/
C  CALCULATE-EXOSPHERIC TEMPERATURE
   20  XTAU=RAD*(H-37.+6.0*SIN(RAD*(H+43.)))
C      MAKE XTAU FALL IN RANGE PLUSS-MINUS PI
        IF(XTAU.GT.PI)    XTAU=XTAU-2.*PI
      TC = 379.+3.24*FB7+1.3*(F7-FB7)
      ZAP=10+.3*SIN(.5*RAD*ABS(PHI+DEC))**2.2
      ZCP=SIN(.5*RAD*ABS(PHI+DEC))**2.2
       ZBP=COS(.5*RAD*ABS(PHI-DEC))**2.2
      ZFP=COS(.5*XTAU)
      ZGP=ZFF**3.
       ZDP=ZBP-ZCP
      TLC=ZAP+.3*(ZDP)*ZGP
        TL=TLC*TC
   50 IF(Z.GT.200.) GO TO 52
C  CORRECTION TO TL
   51 DTNF = 14.0*KP + .02*EXP(KP)
        GO TO 53
   52 DTNF = 28.0*KP+.03*EXP(KP)
   53 TNF = TL+DTNF
      TX = 371.6678+.0518806*TNF-294.35*EXP(-2.16222E-3*TNF)
      MU=TX-183.0
   60  IF(Z.GT.100.)    GO TO  377
  378   SIGMA=Z
        GO TO 61
  377   SIGMA=100.
   61     VZ=BMZ(90.)
       BETA1=ALOG(VZ)
       VZ2=BMZ(SIGMA)
        VZ3=TZ1(SIGMA,TX,MU)
          ALTEMP=VZ3
       BETA2=ALOG(VZ2/VZ3)
        SCND=S2MPER(90.,SIGMA,TGRN,2,5,TX,MU)
```

SUBROUTINE CCIA TRACE

```
      RHSGLN=-14.27251-BETA1+BETA2-SCND
      RHOSIG=EXP(RHSGLN)
         RHO=RHOSIG
        RHLOG=ALOG10(RHO)
      FNN2=.162438861E23*RHOSIG
           FO2=1.E23*RHOSIG*(.25154-6.02257/BMZ(SIGMA))
      FHE=.1278361189468E18*RHOSIG
      FAR=.1943027487E21*RHOSIG
      FOO=1.E23*RHOSIG*(12.04514/BMZ(SIGMA)-.4159233425)
     IF(Z.LE.100.)      GO TO    888
      GNN2=ALOG(FNN2)
      GO2=ALOG(FO2)
      GAR=ALOG(FAR)
      GHE=ALOG(FHE)
      GOO=ALOG(FOO)
      IF(Z.LE.125.)      GO TO 92
      SAGMA=125
     GO TO 41
  92  SAGMA=Z
  41 VQ=TZ1(100.,TX,MU)
     DETA2=ALOG(VQ)
     VQ2=TZ1(SAGMA,TX,MU)
           ALTEMP= VQ2
      DETA3=ALOG(VQ2)
     DDETA=DETA2-DETA3
     SCND=S2MPER(100.,SAGMA,TGPN,2,5,TX,MU)
      LNN2=GNN2+DDETA-3.36929538*SCND
     LNO2=GO2+DDETA-3.84863705*SCND
       LNAR=GAR+DDETA-4.80472245 4*SCND
      LNHE=GHE+.62*DDETA-.481410859*SCND
          LNOO=GOO+DDETA-1.923669043*SCND
           IF(Z.LE.125.)     GO TO 301
      WV=TZ2(Z,TX,MU,TNF)
           ALTEMP=WV
        VW=ALOG(WV)
      VV=ALOG(TX)
      DDETA=VV-VW
       SCND=S1MPER(125.,Z,TGSN,2,5,TX,MU,TNF)
       LNN2=LNN2+DDETA-3.36929538*SCND
      LNO2=LNO2+DDETA-3.848637 05*SCND
```

```
SUBROUTINE  CCIA      TRACE

          LNHE=LNHE+.62*DDETA-.481410859*SCND
          LNAR=LNAR+DDETA-4.804722454*SCND
          LNOO=LNOO+DDETA-1.923669043*SCND
      301 BOX1=EXP(LNN2)
          BOX2=EXP(LNC2)
          BOX3=EXP(LNHE)
          BOX4=EXP(LNAR)
          BOX5=EXP(LNOO)
          QLN2=ALOG10(BOX1)
          QLNO2=ALOG10(BOX2)
          QLNOO=ALOG10(BOX5)
          QLNHE=ALOG10(BOX3)
          QLNAR= ALOG10(BOX4)
          RHO=(BOX1*28.0134+BOX2*31.9988+BOX4*39.948+BOX3*4.0026
         1          +BOX5*15.9994)*.1660420717E-23
          RHLOG=ALOG10(RHO)
          GO TO 569
      888 QLN2=ALOG10(FNN2)
          QLNO2=ALOG10(FO2)
          QLNOO=ALOG10(FOC)
          QLNAR= ALOG10(FAR)
          QLNHE=ALOG10(FHE)
      569 IF(Z.LT.500)    GO TO 200
C...INCLUDE HYDROGEN SPECIES AS THE HEIGHT IS >500 KM
          T500=TZ2(500.,TX,MU,TNF)
          BTZ2=TZ2(Z,TX,MU,TNF)
          QLGH10=73.13-39.4*ALOG10(T500)+5.5*(ALOG10(T500))**2
          BQLG10=10.**QLGH10
          BLNHE=ALOG(BQLG10)+ALOG(T500)-ALOG(BTZ2)-.120274418*
         1 S1MPER(500.,Z,IGSN,2,5,TX,MU,TNF)
          XBLN=EXP(BLNHE)
          QLNHYC=ALOG10(XBLN)
          RHO=RHO+XBLN*.1660420717E-23
          RHLOG=ALOG10(RHO)
      200 BGPHI = (T-36204.)/365.2422
          TAU=BGPHI+0.09544*((.5+.5*SIN(2.*PI*BGPHI+6.035))**1.65-.5)
          FZ = (5.876E-7*Z**2.331+0.06328)*EXP(-2.868E-3*Z)
          S4TAU=SIN(4.*PI*TAU+4.259)
          S1TAU=1.+0.4671*SIN(2.*PI*TAU+4.137)
```

SUBROUTINE CCIA TRACE

```
          GT=.02835+0.3817*S1TAU*S4TAU
          DLSEM=FZ*GT
          RHLOG=RHLOG+DLSEM
      220 IF(Z.GT.170.)GO TO 230
C  INCLUDE SEASONAL VARIATION
      221 S=0.014*(Z-90.)*EXP(-0.0013*(Z-90.)**2)
          P = SIN(2.*PI*BGPHI+1.72)
            DRR=S*P*SIN(RAD*PHI)**2*(PHI/ABS(PHI))
          RHLOG=RHLOG+DRR
      969   DR10=0.012*KP+1.2E-5*EXP(KP)
              RHLOG=RHLOG+DR10
          RHO=10.**RHLOG
              GO TO 302
      230 IF(QLNOO-QLNHE-1.4)   231,231,967
C  HELIUM VARIATION
      231 DELHE=.65*ABS(DEC*.04266)*(SIN(.78539-.5*PHI*RAD*
        1   (DEC/ABS(DEC)))**3.-SIN(.78539)**3.)
          NOHE=BOX3
            QLNHE=ALOG10(BOX3)
            XLNHE=QLNHE+DELHE
          NHE=10.**XLNHE
            DN=NHE-NOHE
            DLRHC=6.645999963E-24*DN
              RHO=10.**RHLOG
            RHO=RHO+DLRHC
          GO TO 302
      967   IF(Z.LE.200.)   GO TO 969
                RHO=10.**RHLOG
            GO TO 302
      302     RETURN
              END
```

```
FUNCTION     BMZ        TRACE

            FUNCTION   BMZ(Y)
      Q=Y-90.
      BMZ = 28.82678 - 7.40066E-2*Q - 1.19407E-2*Q**2 + 4.51103E-4*Q**3
     1 - 8.21895E-6*Q**4 + 1.07561E-5*Q**5 - 6.97444E-7*Q**6
 . 5        RETURN
              END

FUNCTION     GRAV       TRACE

           FUNCTION  GRAV(Y)
      GRAV=9.80665   /(1.+(.157312696E-3*Y))**2
           RETURN
              END

FUNCTION     TZ1        TRACE

             FUNCTION  TZ1(Y,Q,X)
        V=Y-125.
       TZ1=Q+X*(5.4285714286E-2*V-3.9650145777E-5*V**3
     1  -5.331112E-7*V**4)
 5           RETURN
              END

FUNCTION     TZ2        TRACE

      FUNCTION TZ2(Y,Q,X,W)
        DATA PI/3.14159265358979323846/
        A=2./PI*(W-Q)
           B=4.5E-6
 5      GX=.0542857*X
       TZ2=Q+A*ATAN(GX/A*(Y-125.)*(1.+B*(Y-125.)**2.5))
          RETURN
            END
```

FUNCTION TGPN TRACE

```
      FUNCTION TGPN(WZ,WTX,WMU)
      XGRAV=GRAV(WZ)
      XT=TZ1(WZ,WTX,WMU)
      TGPN=XGRAV/XT
      RETURN
      END
```

FUNCTION TGSN TRACE

```
      FUNCTION  TGSN(VZ,VTX,VMU,VTNF)
      QGR=GRAV(VZ)
      QTZ2=TZ2(VZ,VTX,VMU,VTNF)
      TGSN=QGR/QTZ2
      RETURN
      END
```

FUNCTION TGRN TRACE

```
      FUNCTION  TGRN(SZ,STX,SMU)
      SGRV=GRAV(SZ)
      SBM=BNZ(SZ)
      ST=TZ1(SZ,STX,SMU)
      TGRN=(.120274418*SGRV*SBM)/ST
      RETURN
      END
```

FUNCTION SIMPER TRACE

```
      FUNCTION    SIMPER(A,B,FUN,N,NSF,QTX,QMU,QTNF)
C     ADAPTIVE EVALUATION OF DEFINITE INTEGRAL USING SIMPSONS RULE AND
C     HALVING STEP SIZE
C     A=LOWER LIMIT
C     B=UPPER LIMIT
C     FUN=EXTERNAL FUNCTION DEFINING INTEGRAND
C     N=INITIAL NUMBER OF DIVISIONS
C     NSF=NUMBER OF SIGNIFICANT FIGURES DESIRED
        H=.5*(B-A)/FLOAT(N)
      NM1=N-1
      NUP=N
       SEND=FUN(A,QTX,QMU,QTNF)  + FUN(B,QTX,QMU,QTNF)
        SOD=FUN(A+H,QTX,QMU,QTNF)
      SEVEN=0.
      DO 10 J=1,NM1
       SOD=SOD+FUN(A+H*FLOAT(2*J+1),QTX,QMU,QTNF)
   10  SEVEN=SEVEN+FUN(A+H*FLOAT(2*J),QTX,QMU,QTNF)
      PREVAL=H*(SEND+4.*SOD+2.*SEVEN)/3.
C     SETS TOLERANCE USING NSF AND VALUE CALCULATED WITH N DIVISIONS
          TOL=.1**NSF*ABS(PREVAL)
C     EXISTS FROM DO LOOP IF TOLERANCE MET
         IF(TOL.LT..01**NSF) GO TO 40
         DO 33 NC=1,6
   20  H=.5*H
       SEVEN=SEVEN+SOD
         SOD=0.
         NUP=2*NUP
       DO 30 J=1,NUP
   30    SOD=SOD+FUN(A+H*FLOAT(2*J-1),QTX,QMU,QTNF)
        VAL=H*(SEND+4.*SOD+2.*SEVEN)/3.
        CORR=(VAL-PREVAL)/15.
        ACOR=ABS(CORR)
        IF(ACOR.LT.TOL) GO TO 35
        PREVAL=VAL
   33   CONTINUE
C     REPORTS TOLERANCE WAS   NOT MET
   50 WRITE(6,51)
   51 FORMAT(2X,37HTOLERANCE NOT MET WITH 64*N DIVISIONS/)
C     ADDS CORRECTION TERM WHICH MAKES RESULTS HIGHER ORDER
   35     SIMPER=VAL+CORR
```

FUNCTION S1MPER TRACE

```
      RETURN
C     EXIT FOR NEAR ZERO RESULT
   40     S1MPER=PREVAL
          WRITE (6,100)
  100 FORMAT(1X,19HINTEGRAL MAY VANISH/)
      RETURN
      END
```

FUNCTION S2MPER TRACE

```
        FUNCTION    S2MPER(A,B,FUN,N,NSF,QTX,QMU)
C     ADAPTIVE EVALUATION OF DEFINITE INTEGRAL USING SIMPSONS RULE AND
C     HALVING STEP SIZE
C     A=LOWER LIMIT
C     B=UPPER LIMIT
C     FUN=EXTERNAL FUNCTION DEFINING INTEGRAND
C     N=INITIAL NUMBER OF DIVISIONS
C     NSF=NUMBER OF SIGNIFICANT FIGURES DESIRED
        H=.5*(B-A)/FLOAT(N)
      NM1=N-1
      NUP=N
       SEND=FUN(A,QTX,QMU)       + FUN(B,QTX,QMU)
        SOD=FUN(A+H,QTX,QMU)
      SEVEN=0.
      DO 10 J=1,NM1
       SOD=SOD+FUN(A+H*FLOAT(2*J+1),QTX,QMU)
   10  SEVEN=SEVEN+FUN(A+H*FLOAT(2*J),QTX,QMU)
      PREVAL=H*(SEND+4.*SOD+2.*SEVEN)/3.
```

```
      FUNCTION    S2MPER    TRACE
C     SETS TOLERANCE USING NSF AND VALUE CALCULATED WITH N DIVISIONS
          TOL=.1**NSF*ABS(PREVAL)
C     EXISTS FROM DO LOOP IF TOLERANCE MET
         IF(TOL.LT..01**NSF) GO TO 40
         DO 33 NC=1,6
   20  H=.5*H
       SEVEN=SEVEN+SOD
         SOD=0.
         NUP=2*NUP
       DO 30 J=1,NUP
   30    SOD=SOD+FUN(A+H*FLOAT(2*J-1),QTX,QMU)
        VAL=H*(SEND+4.*SOD+2.*SEVEN)/3.
        CORR=(VAL-PREVAL)/15.
        ACOR=ABS(CORR)
        IF(ACOR.LT.TOL) GO TO 35
        PREVAL=VAL
   33   CONTINUE
C     REPORTS TOLERANCE WAS   NOT MET
   50 WRITE(6,51)
   51 FORMAT(2X,37HTOLERANCE NOT MET WITH 64*N DIVISIONS/)
C     ADDS CORRECTION TERM WHICH MAKES RESULTS HIGHER ORDER
   35    S2MPER=VAL+CORR
        RETURN
C     EXIT FOR NEAR ZERO RESULT
   40      S2MPER=PREVAL
           WRITE (6,100)
  100 FORMAT(1X,19HINTEGRAL MAY VANISH/)
      RETURN
      END
```

Appendix C

Test Runs Using Subroutines

```
INTEGRAL MAY VANISH

Z=   91.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1830000E+03  AND T-INFINITY    EQUALS    .1019657E+04

QLN2=  .1374977E+02  QLNO2=  .1316866E+02  QLNHE=  .8645730E+01  QLNAR=  .1182756E+ 2  QLNCO=  .1182286E+02  QLNHYD= 0.

RHO=  .3559671E-08



Z=   92.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1832616E+03  AND T-INFINITY    EQUALS    .1019657E+04

QLN2=  .1358934E+02  QLNO2=  .1299432E+02  QLNHE=  .8485944E+01  QLNAR=  .1166770E+ 2  QLNCO=  .1205555E+02  QLNHYD= 0.

RHO=  .2532989E-08



Z=   93.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1851250E+03  AND T-INFINITY    EQUALS    .1019657E+04

QLN2=  .1334872E+02  QLNO2=  .1272029E+02  QLNHE=  .8244660E+01  QLNAR=  .1142651E+ 2  QLNCO=  .1216831E+02  QLNHYD= 0.

RHO=  .1512380E-08
```

```
Z=   99.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMP=  .1917497E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1342991E+02  QLNO2=  .1236549E+02  QLNHE=  .7925878E+01  QLNAR=  .1110770E+02  QLNCO=  .1208411E+02  QLNHYD= 0.

RHO=  .7395470E-09


Z=  100.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1943219E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1295143E+02  QLNO2=  .1226766E+02  QLNHE=  .7847394E+01  QLNAR=  .1102922E+02  QLNOO=  .1203889E+02  QLNHYD= 0.

RHO=  .6399882E-09


Z=  105.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .2135692E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1256789E+02  QLNO2=  .1183538E+02  QLNHE=  .7773023E+01  QLNAR=  .1049975E+02  QLNOO=  .1180231E+02  QLNHYD= 0.

RHO=  .2732063E-09
```

```
Z= 124.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .3807637E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1137133E+02  QLNO2=  .1050432E+02  QLNHE=  .7482245E+01  QLNAR=  .8900405E+ 1  QLNCO=  .1111140E+02  QLNHYD= 0.

RHO=  .1756057E-10

Z=  125.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .3921070E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1132286E+02  QLNO2=  .1045076E+02  QLNHE=  .7469236E+01  QLNAR=  .8836712E+ 1  QLNOO=  .1097625E+02  QLNHYD= 0.

RHO=  .1568298E-10

Z=  126.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .4034555E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1127577E+02  QLNO2=  .1039874E+02  QLNHE=  .7456596E+01  QLNAR=  .8774847E+01  QLNOO=  .1094605E+02  QLNHYD= 0.

RHO=  .1405260E-10
```

```
Z=  140.0   H=-15.00   F7= 136.0   FB7= 155.0   KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .5536304E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1073379E+02  QLNO2=  .9799209E+01  QLNHE=  .7313590E+01  QLNAR=  .8060514E+01  QLNCO=  .1057765E+02  QLNHYD= 0.

RHO=  .4059708E-11

Z=  150.0   H=-15.00   F7= 136.0   FB7= 155.0   KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .6414972E+03  AND T-INFINITY     EQUALS    .1019657E+04

QLN2=  .1044046E+02  QLNO2=  .9473241E+01  QLNHE=  .7241154E+01  QLNAR=  .7669461E+01  QLNOO=  .1038273E+02  QLNHYD= 0.

RHO=  .2142937E-11

Z=  356.0   H=-15.00   F7= 136.0   FB7= 155.0   KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1032697E+04  AND T-INFINITY     EQUALS    .1047731E+04

QLN2=  .7381680E+01  QLNO2=  .6008696E+01  QLNHE=  .6705475E+01  QLNAR=  .3395565E+01  QLNCO=  .8547717E+01  QLNHYD= 0.

RHO=  .9714649E-14
```

```
Z= 450.J  H=-15.00  F7= 136.J  FB7= 155.J  KP=2.J00J000
 T=40242.7993  DEC= -2J.00  PHI= 45.0

TEMPERATJRE AT Z ALTEMF=  .1042925E+04  AND T-INFINITY     EQUALS     .1047731E+04

QLN2=  .5154855E+01  QLNO2=  .4607935E+01  QLNHE=  .0528141E+01  QLNAR=  .1647887E+ 1  QLNCO=  .7845432E+J1  QLNHYD= 0.

RHO=  .1765529E-14

INTEGRAL MAY VANISH

Z=  500.J  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.J00J000
 T=40242.7993  DEC= -2J.00  PHI= 45.0

TEMPERATJRE AT Z ALTEMF=  .1044724E+04  AND T-INFINITY     EQUALS     .1047731E+04

QLN2=  .5559042E+01  QLNO2=  .3927464E+01  QLNHE=  .0442653E+01  QLNAR=  .7985582E+ 0  QLNOO=  .7504937E+01  QLNHYD=  .43..377E+01

RHO=  .7389984E-15

Z=  600.J  H=-15.00  F7= 136.J  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -26.00  PHI= 45.0

TEMPERATJRE AT Z ALTEMF=  .1046370E+04  AND T-INFINITY     EQUALS     .1047731E+04

QLN2=  .+395800E+01  QLNO2=  .2598895E+01  QLNHE=  .0276129E+01  QLNAR= -.8558856E+.0  QLNCO=  .684.535E+01  QLNHYD=   4268195E+01

RHO=  .1920749E-15
```

```
Z=  800.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1047323E+04  AND T-INFINITY     EQUALS    .1047731E+04

QLN2=  .2170605E+01  QLNO2=  .5718329E-01  QLNHE=  .5958001E+01  QLNAR= -.4132916E+01  QLNOO=  .5549910E+01  QLNHYD=  .4188380E+01
RHO=  .1324400E-16


Z=  900.0  H=-15.00  F7= 136.0  FB7= 155.0  KP=2.0000000
 T=40242.7993  DEC= -20.00  PHI= 45.0

TEMPERATURE AT Z ALTEMF=  .1047478E+04  AND T-INFINITY     EQUALS    .1047701E+04

QLN2=  .1104640E+01  QLNO2= -.1160493E+01  QLNHE=  .5805655E+01  QLNAR= -.5553073E+01  QLNCO=  .4961245E+01  QLNHYD=  .4150264E+01
RHO=  .8958400E-17
```

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13. ABSTRACT

Dr. L.C. Jacchia (1971) has developed a static diffusion model of the atmosphere that matches parameters at 150 km and the observed densities from satellite drag at greater heights. His report presents tables showing temperature, density and composition as a function of height and exospheric temperature.

The Jacchia model has been used in working programs by reading the tables and converting from log densities to simple densities and vice-versa, to interpolate the tables and make the necessary corrections.

This report presents a FORTRAN subroutine which gives the precise density information (along with the appropriate corrections) directly by using the analytic equations.

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