CR 167 824

82-FM-52 JSC-18507

Jacchia - Lineberry Upper Atmosphere Density Model

(NASA-CR-167824) JACCHIA-LINEBERRY UPPER ATMOSPHERE DENSITY MODEL (Computational Mechanics Consultants) 46 p HC A03/MF A01 CSCL 04A N83-20474 Unclas G3/46 09189

Mission Planning and Analysis Division

October 1982

NASA
National Aeronautics and Space Administration

Lyndon B. Johnson Space Center
Houston, Texas

82-FM-52

JSC-18507

SHUTTLE PROGRAM

JACCHIA-LINEBERRY UPPER ATMOSPHERE DENSITY MODEL

Prepared by: Alan C. Mueller, Computational Mechanics Services

Task Monitor: Victor R. Bond

NAS 9 /16546

Approved: Elric N. McHenry
Elric N. McHenry
Software Development Branch

Approved: Ronald L. Berry
Ronald L. Berry, Chief
Mission Planning and Analysis Division

Mission Planning and Analysis Division

National Aeronautics and Space Administration
Lyndon B. Johnson Space Center
Houston, Texas

October 1982

CONTENTS

TABLES

FIGURES

ACRONYMS

EUV	extreme ultraviolet
FDS	flight design system
J/L	Jacchia/Lineberry
J/W	Jacchia/Walker
RTCC	real-time computer complex
SQRT	square root

1.0 INTRODUCTION

The upper atmosphere is, as one might expect, an extremely complex system sensitive to the diurnal changes in the temperature, seasonal and latitudinal variations, sunspot activity, and the effects of the solar winds on the geomagnetic field. Add to this the mathematical difficulties in solving the gas diffusion equilibrium equations for each constituent and the result is a modeling nightmare. Nevertheless, from the observations of satellite orbital decay, mass spectrometry data, and extreme ultraviolet (EUV) absorption data, Jacchia (refs. 1, 2, 3, and 4) has developed a series of increasingly accurate models which are a careful blend of empirical and theoretical formulae.

In the most recent Jacchia model, the exospheric temperature is assumed to be a function of:

1) the average and daily variations in the solar flux
2) the average and three hourly variations in the geomagnetic index
3) the angle between the position vector and the axis of the unsymmetric atmospheric bulge
4) the angle between the position vector and the geomagnetic pole.

The exospheric temperature is related to the density by the solution of the diffusion equilibrium equations for the different constituents of the atmosphere as a function of altitude. Other variations are modeled directly as changes in the density. They are:

1) changes due to the semiannual effect
2) changes due to the seasonal-latitudinal effect.

The causes for these variations are not exactly known but may be modeled sufficiently by empirical formulae. The Jacchia model is assumed to be valid over the altitude range of 90 to 2500 km. The residuals between the observed density from satellite drag observations and the computed densities show the mean relative error to be generally less than 10 percent with occasional peak errors near 50 percent.

Although the model recovers most of the important characteristics of the upper atmosphere, it is at a great computational expense. The inefficiency of the model cannot be reconciled by simply neglecting some of the effects. For example, the 62 standard atmosphere (ref. 5), which assumes that density is solely a function of altitude, is grossly inaccurate for altitudes above 150 km (ref. 6). The major drawback of Jacchia's model is the tabular form chosen to represent the results of numerically integrating the diffusion equations. The storage required to implement the table is prohibitive. Walker (ref. 7) has averted the storage cost by developing rather complex analytical solutions to the diffusion equations over a limited altitude range (125-700 km). The analytic expressions require several standard function evaluations which hamper the computational speed.

Lineberry, in an effort to reduce the computation time, has assumed that the log of the density may be expressed as a truncated Laurent series in temperature and altitude. The atmosphere is layered into several altitude bands and the series coefficients of each band are found by pointwise fit to Jacchia's tabular results. This layered model together with a more efficient method of computing the temperature reduces the computation time five-fold over the Walker model and yet maintains the model accuracy and reasonable storage costs. In addition, the Lineberry layered expressions are valid over the entire domain of the Jacchia model (90-2500 km) and are of such general form that differences in the Jacchia models may be easily accommodated.

This model, which we will call the Jacchia/Lineberry (J/L) model, has already been implemented into several orbit propagation software packages (refs. 8 and 9) but has never been properly documented. The purpose of this report is to show the development and concisely define the model, document the accuracy, and caution the user of its limitations.

2.0 OUTLINE OF JACCHIA/LINEBERRY MODEL

The J/L model is most closely fashioned to the Jacchia 71 (ref. 3) model. It consists of three basic steps.

1) Computation of the exospheric temperature requiring the position vector of interest, the position of the Sun, the solar flux ($F_{10.7}$), and the geomagnetic index (Kp).

2) Computation of the temperature contribution of the log of the density - $\Delta \ln \rho_{T_\infty}$ requiring the geodetic altitude and temperature.

3) Computation of the contributions due to the semiannual effect - $\Delta \ln \rho_{SA}$ and the seasonal-latitudinal effects - $\Delta \ln \rho_{SL}$ requiring the time of the year, geodetic altitude, and latitude.

The density, ρ, is then found from the sum of these contributions

$$\ln \rho = \Delta \ln \rho_{T_\infty} + \Delta \ln \rho_{SA} + \Delta \ln \rho_{SL} \tag{2.1}$$

Lineberry assumes as in Mueller (ref. 10) that the temperature contribution has the following form

$$\Delta \ln \rho_{T_\infty} = b_1 + b_2 z' + b_3/z' \tag{2.2}$$

where the coefficients $\{b_i\}$ are selected according to the altitude band of interest. The base altitude z' is assumed to be of similar form

$$z' = a_1 + a_2 z + a_3/z \quad . \tag{2.3}$$

where z is the geodetic altitude. The coefficients $\{a_i\}$ are related to the exospheric temperature T_∞ by the same function

$$a_i = a_{1i} + a_{2i} T_\infty + a_{3i}/T_\infty \qquad i = 1,2,3 \quad . \tag{2.4}$$

Again, the coefficients $\{a_{ij}\}$ are selected according to the particular altitude band of interest.

The contribution due to the semiannual effect is assumed to be of the form

$$\Delta \ln \rho_{SA} = (c_1 + c_2 z + c_3/z) g(t) \tag{2.5}$$

where the coefficients $\{c_j\}$ are selected according to the altitude band and $g(t)$ is a periodic function of time specified in Jacchia's empirical model. Lastly, the seasonal-latitudinal contribution is also assumed to have the familiar form

$$\Delta \ell n \rho_{SL} = (d_1 + d_2 z + d_3/z)p(t)f(\phi) \tag{2.6}$$

with band dependent coefficients $\{d_i\}$ and time dependent function $p(t)$ and latitude dependent function $f(\phi)$ specified by Jacchia's empirical formulae.

The coefficients $\{a_{ij}\}$ and $\{b_j\}$ are considered constants over each altitude band and have been determined by pointwise fit of the equations (2.2-2.4) to Jacchia's tables, being careful to maintain continuity over the altitude band boundaries. The width of each band is chosen to maintain three-digit accuracy of the assumed expressions and is, therefore, not uniform. The coefficients $\{c_j\}$ and $\{d_j\}$ are also considered constants over each band and are determined by pointwise fit of the altitude-dependent parts of (2.5-2.6) to the corresponding altitude-dependent parts of the empirical formulae defined by Jacchia.

An important exception to this general outline is the manner in which hydrogen contributions of the density are included. In fitting the coefficients $\{a_{ij}\}$ to the Jacchia tables, the contributions due to hydrogen have been removed to smooth out the anomalies in the contour plots of the density log versus altitude and exospheric temperature. The hydrogen contributions may be analytically computed directly from the hydrogen diffusion equation by assuming that the temperature is a constant. Since the effects due to hydrogen are only significant for altitudes greater than 500 km, where the temperature is almost radially isothermal, the assumption is a valid one.

2.1 INTERPOLATION OF THE JACCHIA TABLES AND EMPIRICAL FORMULAE

As we have seen, the J/L model uses extensively the interpolation function of the form

$$f(x) = a_1 + a_2 x + a_3/x \tag{2.7}$$

If the left-hand side $f(x)$ is given in some form, such as in Jacchia's tables, or by empirical formulae, then the coefficients $\{a\}$ may be found by inversion of (2.7). Given the value of the function $f(x_j) = f_j$ at three different points $\{x_i\}$ $i = 1,2,3$, the coefficients are then found to be

$$a_3 = \frac{\left(f_3 - f_1 + \left((x_1 - x_3)/(x_2 - x_1)\right)(f_2 - f_1)\right)}{(x_1 - x_3)\left(1/(x_1 x_3) - 1/(x_1 x_2)\right)} \tag{2.8}$$

$$a_2 = (f_2 - f_1)/(x_2 - x_1) + a_3/(x_1 x_2) \tag{2.9}$$

$$a_1 = f_1 - a_2 x_1 - a_3/x_1 \tag{2.10}$$

Equations (2.8-2.10) are to be called the "inverse" relations of (2.7) and will be used on many occasions in the following development.

2.1.1 The Base Altitude Interpolation of Density

The density at a point is dependent on the altitude and the exospheric temperature at that location. If one fixes the temperature at an arbitrary value, say $T'_\infty = 600^\circ K$, then the density becomes a function of the altitude only. This altitude is what we call the base altitude z'. Equation (2.2) is the interpolant of the functional dependence of density to the base altitude. The coefficients $\{b_i\}$ may be readily determined by the inversion relations along with Jacchia's tables evaluated at the base exospheric temperature $T'_\infty = 600^\circ K$. As stated earlier, the effects of hydrogen have been removed from the tables to give a smoother interpolant and thus reduce the number of altitude band layers in the model. The boundary altitudes of each band were selected in determining the coefficients to maintain continuity across the boundaries. The third altitude was selected so as to give a good fit. The width of each altitude band was selected so that the maximum interpolant error was generally less than a few percent. The altitude bands and the values of the coefficients $\{b_i\}$ are tabulated in table I and table II for the Jacchia 71 and 70 models, respectively.

TABLE I.- JACCHIA 71 VERTICAL PROFILE COEFFICIENTS

Altitude Band	b_1	b_2	b_3
90-100	-6.6067	-1.6401×10^{-1}	1.6968×10^{2}
100-110	-2.2977×10	-8.2066×10^{-2}	9.8734×10^{2}
110-140	-5.4733×10	6.1437×10^{-2}	2.7441×10^{3}
140-180	-3.7147×10	4.3206×10^{-4}	1.4777×10^{3}
180-420	-2.8878×10	-2.2129×10^{-2}	7.2035×10^{2}
420-500	-3.3449×10	-1.5975×10^{-2}	1.5545×10^{3}
500-700	-5.5713×10	7.7782×10^{-3}	6.7480×10^{3}
700-1500	-3.8578×10	-4.8687×10^{-3}	9.5081×10^{2}
1500-2500	-4.1433×10	-3.8731×10^{-3}	2.9930×10^{3}

TABLE II.- JACCHIA 70 VERTICAL PROFILE COEFFICIENTS

Altitude Band	b_1	b_2	b_3
90-100	-2.26064	-1.87247×10^{-1}	-3.325619×10^{1}
100-110	-2.467081×10^{1}	-7.517851×10^{-2}	1.087119×10^{3}
110-140	-5.856595×10^{1}	7.759401×10^{-2}	2.967037×10^{3}
140-180	-3.381609×10^{1}	-9.501784×10^{-3}	1.209134×10^{3}
180-420	-2.977882×10^{1}	-2.103046×10^{-2}	8.559544×10^{2}
420-500	-3.496874×10^{1}	-1.404274×10^{-2}	1.803085×10^{3}
500-700	-5.376797×10^{1}	6.48995×10^{-3}	6.069527×10^{3}
700-1500	-3.839121×10^{1}	-4.928746×10^{-3}	9.00959×10^{2}
1500-2500	-4.214804×10^{1}	-3.607654×10^{-3}	3.571183×10^{3}

2.1.2 The Exospheric Temperature Interpolation of the Base Altitude

Now if the exospheric temperature is different from that of the base temperature $T'_\infty = 600^oK$, the base altitude used in (2.2) will be different from that of the true altitude. Suppose that at an altitude z and the exospheric temperature T_∞, the density is given by $\rho = \rho(z, T_\infty)$. The base altitude is then that particular altitude z' in which the density has the same numerical value but at the base temperature T'_∞. In other words, z' is defined implicitly by the equation

$$\rho(z', T'_\infty) = \rho(z, T_\infty) \tag{2.11}$$

Equations (2.3) and (2.4) are the explicit interpolants of the above implicit equation. The values of the coefficients $\{a_i\}$ may be determined by selecting a particular exospheric temperature T_∞^k and three different altitudes $\{z_j\}$ $j = 1,2,3$. By equation 2.11 and Jacchia's modified tables give us three different base altitudes $\{z'\}$ which are implicit functions of the selected temperature and altitude. With the values of z' and z one can invert (2.3) to give the coefficients $\{a_j(T_\infty^k)\}$ which are implicit functions of the selected exospheric temperature. But (2.4) is the explicit interpolant form of these implicit functions. By evaluating the coefficients $\{a_j(T_\infty^k)\}$ for three different values of the exospheric temperature $\{T_\infty^k\}$ $k = 1,2,3$, the inverse of (2.4) will give the desired coefficients $\{a_{ik}\}$. The altitude bands and the value of these coefficients are found in table III and table IV for Jacchia 71 and Jacchia 70, respectively.

The interpolant log density versus the exospheric temperature for several altitudes has been plotted against the Jacchia 71 tables in figure 1. The interpolant also includes the hydrogen contributions developed in section 2.2. The agreement appears to be quite good except for the high altitude, low temperature regime in which the hydrogen contribution is significant. The discrepancy can be traced to differences in the boundary conditions for hydrogen number density in the Jacchia 71 and Jacchia 77 models. The correction in the J/L model to include hydrogen assumes boundary conditions as stated in the Jacchia 77 model. A comparison to the Jacchia 77 model in figure 2 reflects the similarity in the boundary conditions.

TABLE III.- JACCHIA 71 BASE ALTITUDE COEFFICIENTS

Constant	90-110 km	110-180 km	180-2500 km
a_{11}	1.11475×10	3.39245×10^{2}	1.86895×10^{2}
a_{12}	1.36100×10^{-5}	-5.32690×10^{-2}	1.59030×10^{-2}
a_{13}	-6.69343×10^{3}	-1.84370×10^{5}	-1.17862×10^{5}
a_{21}	9.44287×10^{-1}	-5.06112×10^{-1}	-9.33360×10^{-2}
a_{22}	7.75000×10^{-7}	2.16963×10^{-4}	1.34400×10^{-5}
a_{23}	3.31488×10	8.25561×10^{2}	6.51163×10^{2}
a_{31}	-5.51954×10^{2}	-1.90923×10^{4}	-5.47081×10^{3}
a_{32}	-7.52700×10^{-3}	3.23731	-2.47382
a_{33}	3.33882×10^{5}	1.02899×10^{7}	4.17306×10^{6}

TABLE IV.- JACCHIA 70 BASE ALTITUDE COEFFICIENTS

Constant	90-110 km	110-180 km	180-2500 km
a_{11}	1.535026×10^{2}	3.86469×10^{2}	1.27264×10^{2}
a_{12}	-9.35111×10^{-3}	-7.610145×10^{-2}	4.535789×10^{-2}
a_{13}	-8.873513×10^{4}	-2.0448485×10^{5}	-9.268724×10^{4}
a_{21}	2.321941×10^{-1}	-7.287919×10^{-1}	-3.388665×10^{-2}
a_{22}	4.72682×10^{-5}	3.268459×10^{-4}	-1.339225×10^{-5}
a_{23}	4.43667×10^{2}	9.196106×10^{2}	6.251532×10^{2}
a_{31}	-7.596×10^{3}	-2.158925×10^{4}	4.176991×10^{3}
a_{32}	4.58726×10^{-1}	4.417025	-7.151575
a_{33}	4.392459×10^{6}	1.136342×10^{7}	6.83728×10^{4}

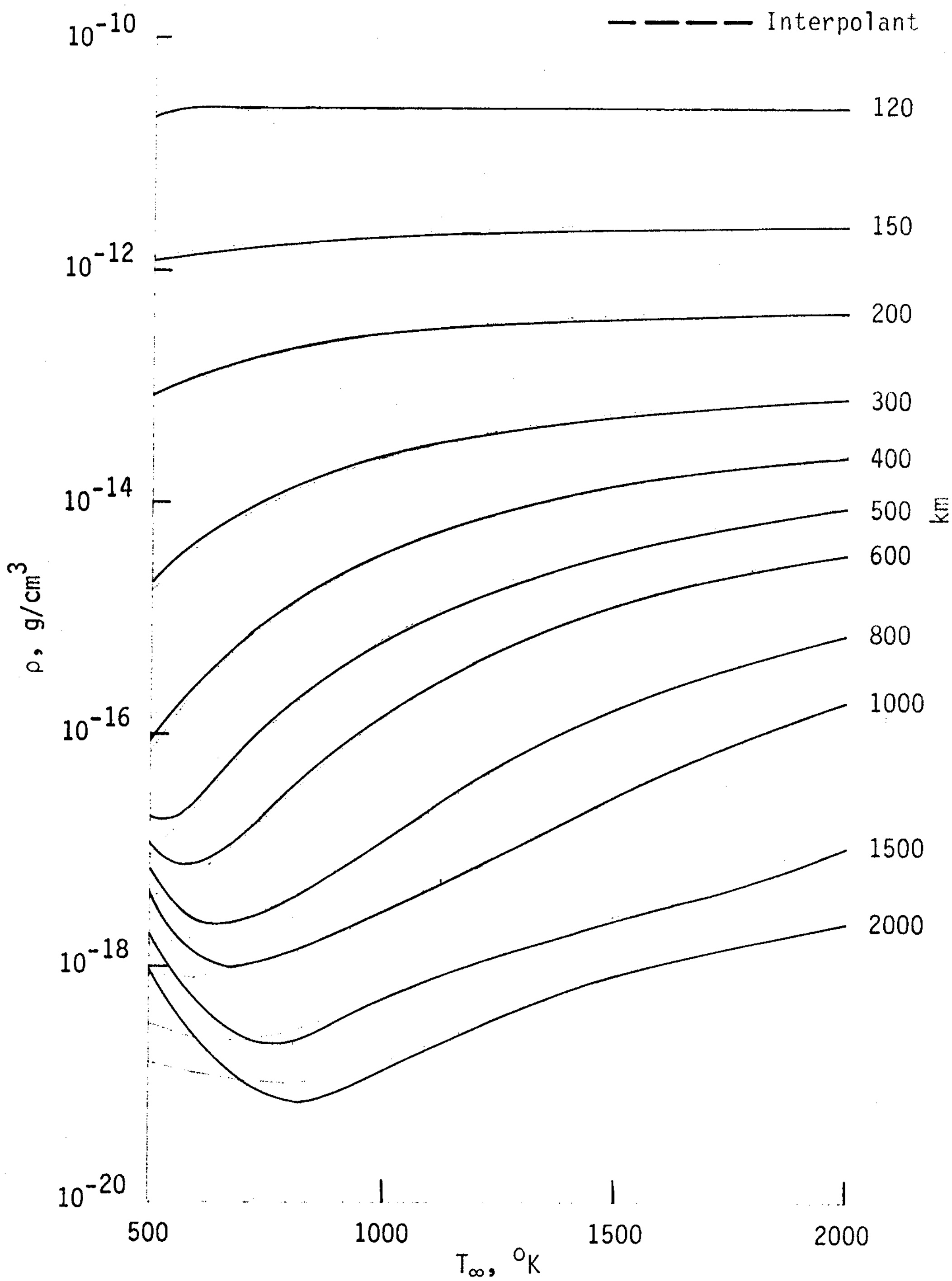

Figure 1.- Dependence of atmospheric density on exospheric temperature at different heights: Jacchia 71.

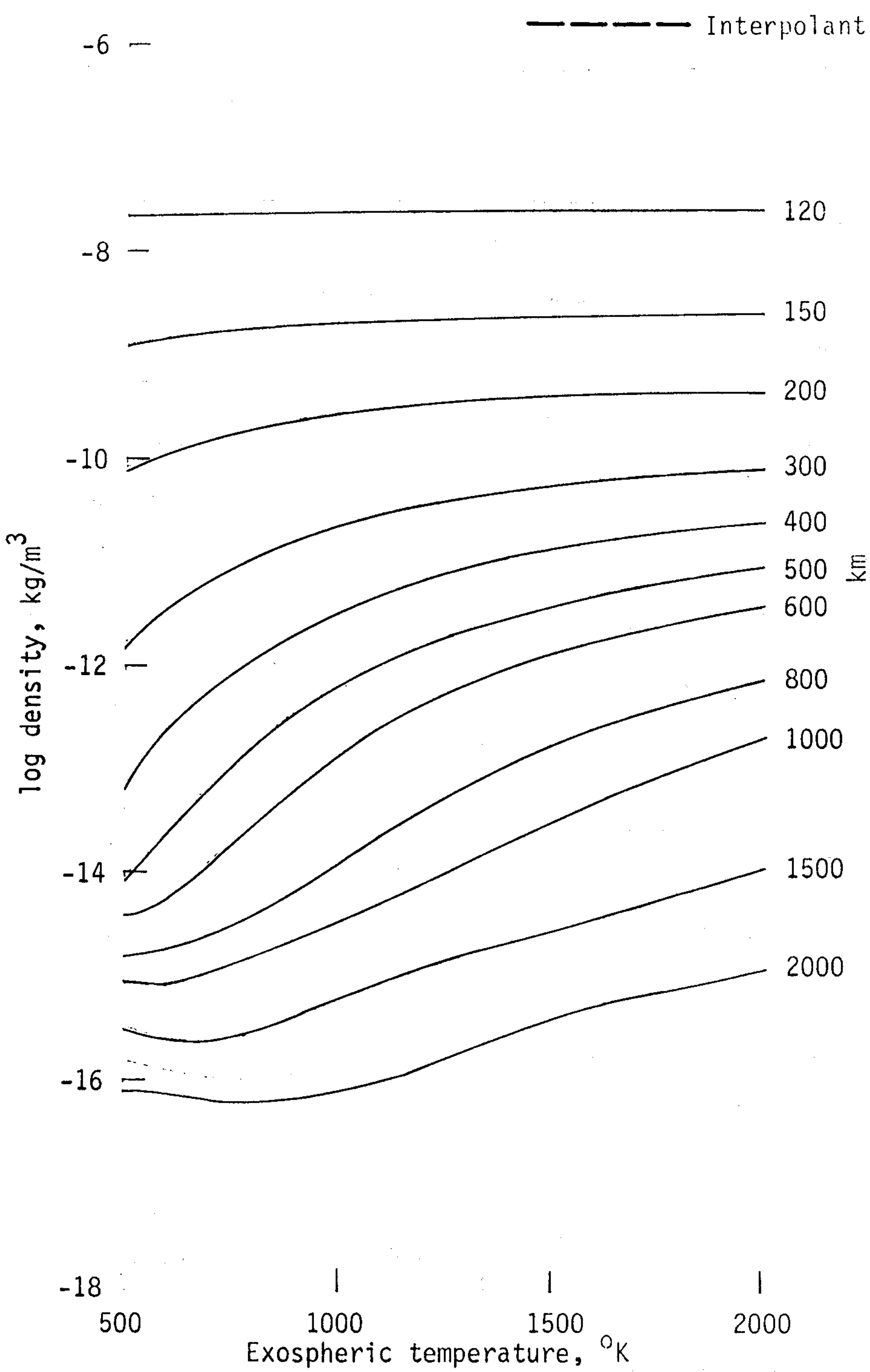

Figure 2.- Dependence of atmospheric density on exospheric temperature at different heights: Jacchia 77.

2.1.3 The Semiannual Interpolation

The variation in the semiannual effect due to changes in the altitude z is given in Jacchia (ref. 3) to be

$$f(z) = 2.302(5.876 \times 10^{-7} z^{2.331} + 0.06328)\exp(-2.868 \times 10^{-3} z) \qquad (2.12)$$

(z in km)

The factor of 2.302 is needed to convert from the base 10 log used in Jacchia's empirical formulas to the corresponding natural log used in Lineberry's interpolant (2.5). By evaluating this empirical formulae at three different altitudes for each altitude band one may determine the coefficients using the inverse relations of the interpolant. The altitude bands and the values of the coefficients $\{c_i\}$ can be found in table V. The plot of $f(z)$ and its interpolant is shown in figures 3 and 4 for altitudes from 90 to 500 km and 500 to 2500 km, respectively. The maximum difference between the two amounts to a relative error of no more than one percent.

TABLE V.- SEMIANNUAL COEFFICIENTS

Altitude Band	C_1	C_2	C_3
90-100	-6.9999×10^{-2}	1.4737×10^{-3}	7.8748
100-110	-1.2204×10^{-2}	1.1513×10^{-3}	5.3190
110-140	-4.6896×10^{-2}	1.3202×10^{-3}	7.0920
140-180	-1.3067×10^{-1}	1.6233×10^{-3}	1.2880×10
180-420	-6.5716×10^{-2}	1.4902×10^{-3}	6.1341
420-500	1.0002	1.5000×10^{-4}	-2.0940×10^{2}
500-700	1.6544	-4.3650×10^{-4}	-3.8535×10^{2}
700-1500	2.4757	-1.0458×10^{-3}	-6.6170×10^{2}
1500-2500	-8.7290×10^{-1}	9.7800×10^{-5}	1.788×10^{3}

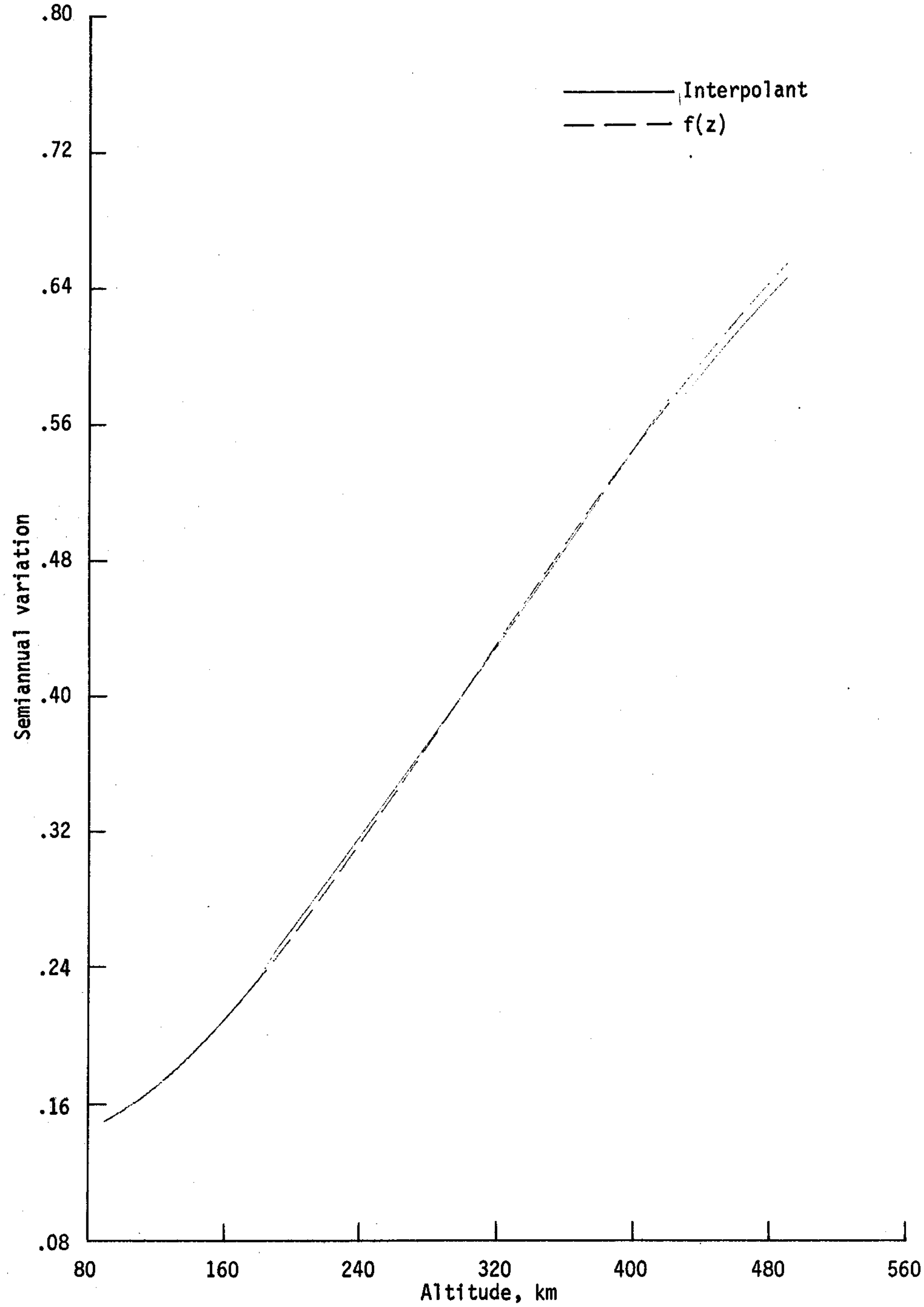

Figure 3.- Semiannual variation. (90-500 km.)

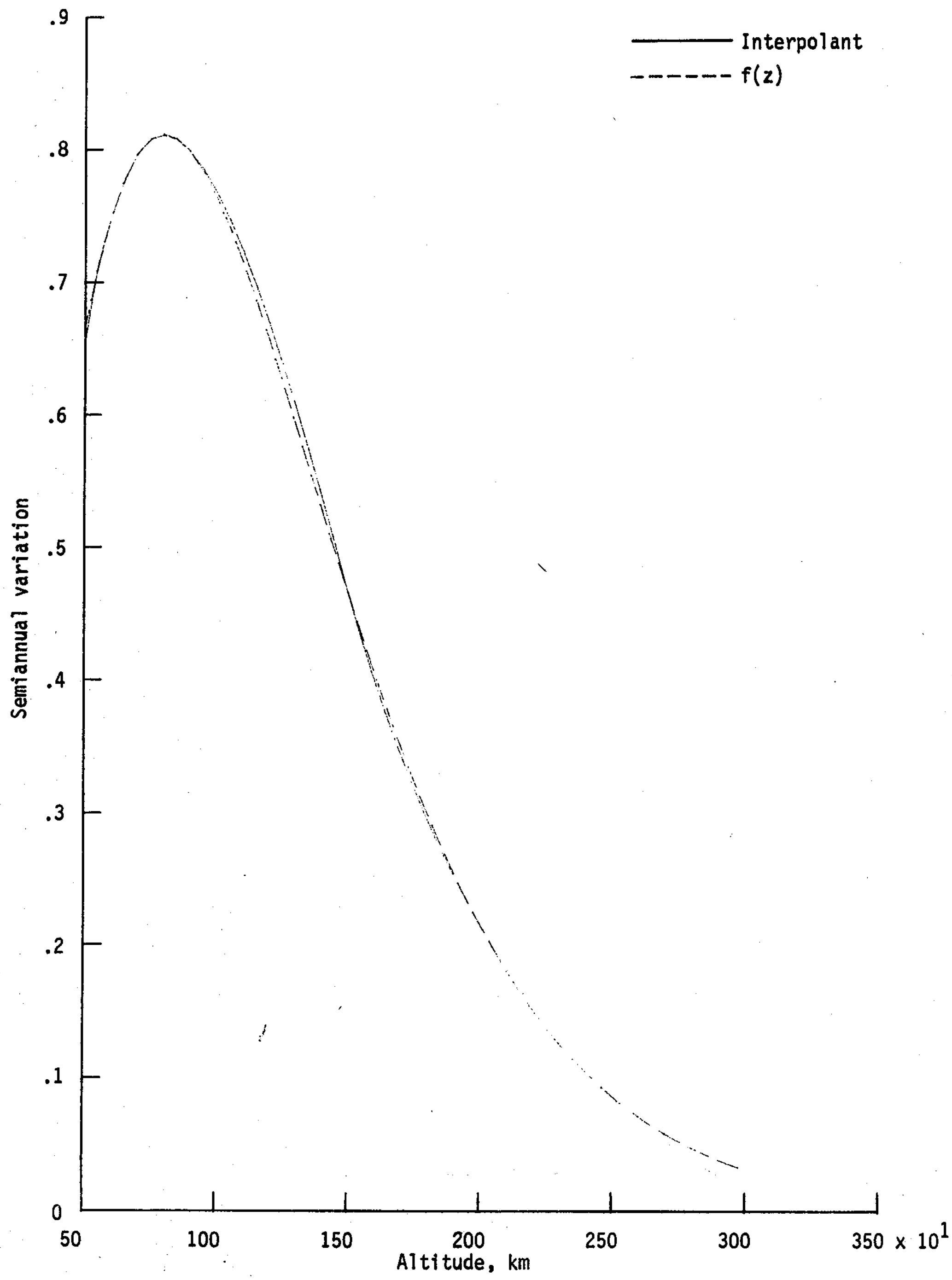

Figure 4.- Semiannual variation. (500-2500 km.)

2.1.4 The Seasonal Latitudinal Interpolation

The variation in the seasonal-latitudinal effect due to changes in the altitude z is given by Jacchia (ref. 3) to be

$$g(z) = 2.302 \cdot 0.014(z-90)\exp\left(-0.0013(z-90)^2\right) \tag{2.13a}$$

$$(z \text{ in km}) \qquad (90 < z < 180)$$

Again, the factor of 2.302 is necessary to convert bases to the corresponding Lineberry interpolant (2.6). By evaluating this empirical formula at three different altitudes for each altitude band, the coefficients of the interpolant $\{d_i\}$ may be found by the inverse relations. The altitude bands and coefficients are tabulated in table VI. The plot of $g(z)$ and its interpolant is shown figure 5 over the valid range of 90 to 180 km. The agreement is quite good below 110 km but is less accurate above this altitude because of the width of the altitude band. The maximum error results in a relative error in density of no more than three percent. An additional band layer could reduce the error. But since the error in fitting Jacchia's tables is of the same order, the added expense may not necessarily increase the total accuracy.

The Jacchia 70 model (ref. 2) assumes a slightly different form for the seasonal latitudinal variation so that

$$g(z) = 2.302 \cdot 0.02(z-90)\exp\left(-0.045(z-90)\right) \tag{2.13b}$$

The coefficients fit to this expression are shown in table VII.

TABLE VI.- JACCHIA 71 SEASONAL LATITUDINAL COEFFICIENTS

Altitude Band	D_1	D_2	D_3
90-100	8.2812	-2.8680×10^{-2}	-5.1300×10^{2}
100-110	2.4695×10	-1.1106×10^{-1}	-1.3306×10^{3}
110-140	5.1205	-2.4927×10^{-2}	-2.1960×10^{2}
140-180	-4.2401	1.2570×10^{-2}	3.5595×10^{2}

TABLE VII.- JACCHIA 70 SEASONAL LATITUDINAL COEFFICIENTS

Altitude Band	D_1	D_2	D_3
90-100	2.4107×10^{1}	-1.1142×10^{-1}	-1.2671×10^{3}
100-110	1.5097×10^{1}	-6.626×10^{-2}	-8.1774×10^{2}
110-140	4.3439	-1.8338×10^{-2}	-2.1474×10^{2}
140-180	-1.6246	3.4375×10^{-3}	1.9404×10^{2}

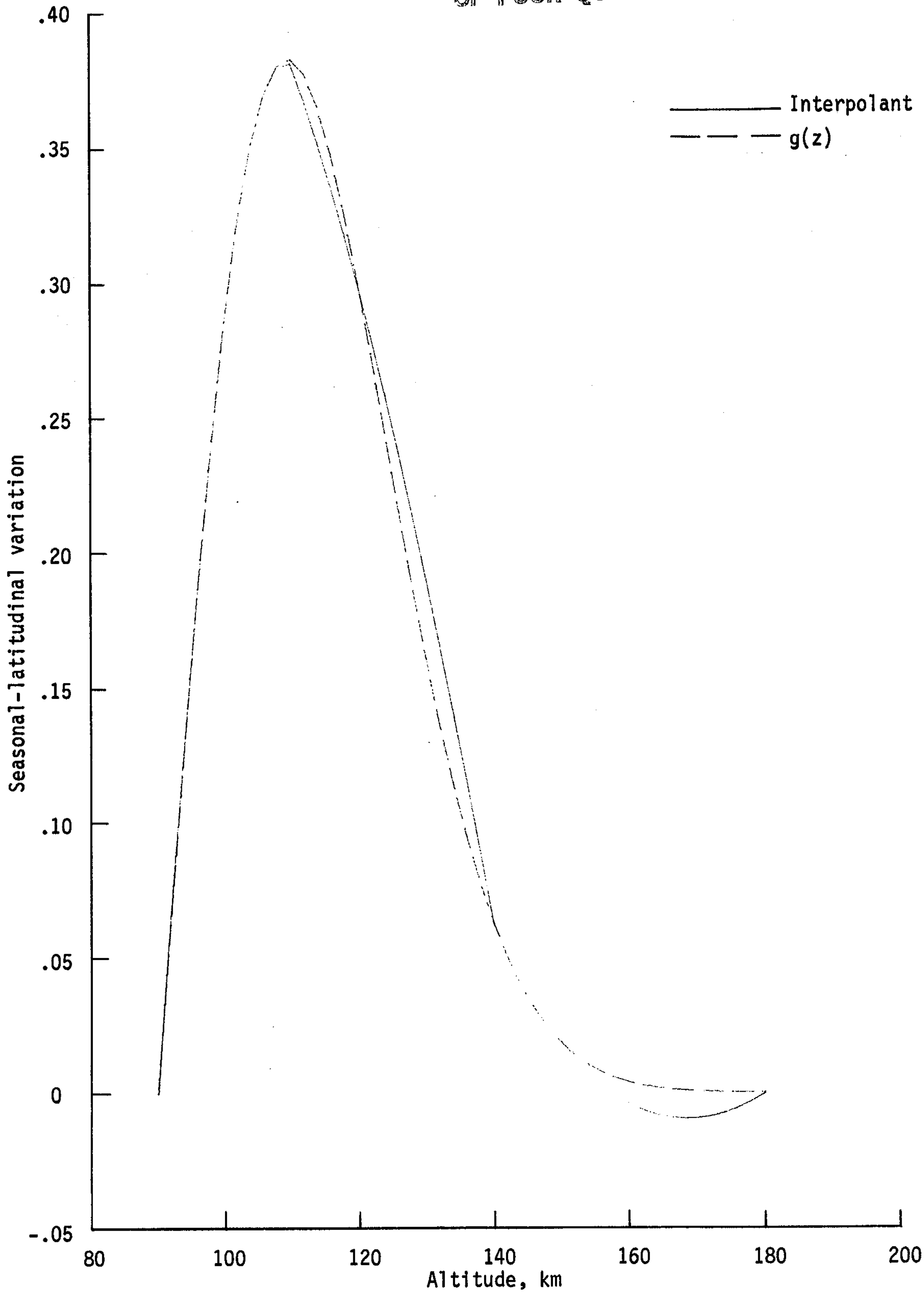

Figure 5.- Seasonal-latitudinal variation.

2.2 INCLUSION OF HYDROGEN

As we mentioned earlier, the contributions due to hydrogen have been removed to smooth out anomalies created in Jacchia's tables. In this section we will develop the analytic procedure for including the effects of hydrogen back into the J/L model. Since the hydrogen contributions are insignificant at the lower altitudes, this development will apply to altitudes of greater than 500 km.

The barometric diffusion equation governing the number density $n(H)$

$$\frac{dn(H)}{n(H)} + \frac{dT}{T}(1 + \alpha_H) + \frac{M_H g}{R^* T} dz = 0 \qquad (2.14)$$

is where

- T : temperature
- M_H : molecular weight of hydrogen
- R^* : universal gas constant
- g : local acceleration of gravity
- α_H : thermal diffusion coefficient

Since at altitudes of $z > 500$ km the temperature reaches a nearly constant value T_∞, the above equation reduces to

$$\frac{dn(H)}{n(H)} + \frac{M_H g}{R^* T_\infty} dz = 0 \qquad (2.15)$$

The acceleration g obeys closely the relation

$$g = g_o (1 + z/R_e)^{-2} \qquad (2.16)$$

where g_o is the acceleration of gravity on the Earth's surface and R_e is the mean Earth radius. Replacing (2.16) into (2.15) and integrating yields

$$\ell n\left(n(H)\right) = \frac{M_H g_o R_e^2}{R^* T_\infty (z + R_e)} + \text{const.} \qquad (2.17)$$

In ref. 4, Jacchia prescribes the hydrogen number density at an altitude of 500 km to be

$$\ln\left(n(H)\right)_{z=500} = 13.677355 + 66.544709 T_\infty^{-\frac{1}{4}} \tag{2.18}$$

which allows us to evaluate the constant of integration.

The density contribution due to hydrogen is found from

$$\rho_H = M_H / A n(H)$$

or

$$\ln\rho_H = \ln\left(n(H)\right) + \ln\left(M_H/A\right) \tag{2.19}$$

with A being Avogadro's number.

Combining (2.17) through (2.19) and replacing numerical values for the physical constants one arrives at

$$\ln\rho_H = -47.977466 + 66.544709/T_\infty^{\frac{1}{4}} - 7.00612\text{x}10^3/T_\infty + 7.5572\text{x}10^3/\left(T_\infty(1 + Z/6378.14)\right) \tag{2.20}$$

3.0 THE JACCHIA 71 AND 70 EMPIRICAL MODEL

The Jacchia 71 (ref. 3) and 70 (ref. 2) model assumes the exospheric temperature to be a function of the solar flux, geomagnetic index, and the relative position with respect to the diurnal bulge. The exospheric temperature T_∞ used in (2.4) has been found to closely obey the following empirical formula.

$$T_\infty = T_L + T_G \tag{3.1}$$

$$T_G = \Delta T_G \cdot Kp + \delta T_G \cdot \exp(Kp) \tag{3.2}$$

$$T_L = T_C(1 + R \cdot D) \tag{3.3}$$

$$T_C = T_{CO} + \Delta T_C \bar{F}_{10.7} + \delta T_C(F_{10.7} - \bar{F}_{10.7}) \tag{3.4}$$

$$R = R_o + \delta R \cdot \bar{K}p \tag{3.5}$$

$$D = \sin^m \sigma + (\cos^m \eta - \sin^m \sigma)\cos^n|\tau/2| \tag{3.6}$$

$$\begin{aligned} \sigma &= \tfrac{1}{2}|\phi + \phi_o| \\ \eta &= \tfrac{1}{2}|\phi - \phi_o| \\ \tau &= H + \beta + p\sin(H + \gamma) \\ H &= \alpha - \alpha_o \end{aligned} \tag{3.7}$$

where

- ϕ : latitude of given plot
- ϕ_o : latitude of the Sun
- α : right ascension of given plot
- α_o : right ascension of the Sun
- $F_{10.7}$: solar flux 10^4 Jansky (10^{-22} W $m^{-2}Hz^{-1}$ bandwidth)
- $\bar{F}_{10.7}$: averaged solar flux

Kp : geomagnetic planetary index

$\overline{Kp}$: averaged geomagnetic planetary index

and γ, p, β, m, n, R_o, δR, T_{co}, ΔT_c, δT_c, ΔT_G, δT_G are model parameters whose values for both models may be found in table VIII.

The semiannual term $g(t)$ found in (2.5) is given as

$$g(t) = 0.02835 + 0.3817\left(1 + 0.467\sin(\Phi + 4.14)\right)\sin(2\Phi + 4.259) \qquad (3.8)$$

with

$$\Phi = \eta t + 0.191\pi\left\{\left(\tfrac{1}{2} + \tfrac{1}{2}\sin(\eta t + 6.035)\right)^{1.650} - \tfrac{1}{2}\right\} \qquad (3.9)$$

The Jacchia 70 model represents the semiannual variation indirectly as changes in the exospheric temperature. This approach is roughly consistent with the direct method in Jacchia 71.

The seasonal-latitudinal terms $p(t)$ and $f(\phi)$ found in (2.6) are for both the Jacchia 70 and 71 models defined as

$$p(t) = \sin(\eta t + 1.72) \qquad (3.10)$$

$$f(\phi) = |\sin \phi| \sin \phi \qquad (3.11)$$

The time t is in days measured from the beginning of the year (January 1) and η is the mean motion of the Sun in radians per day.

In addition, Jacchia corrects for seasonal-latitudinal variations of helium. This contribution becomes important at high altitudes and high temperatures where helium is a dominant constituent. The variation is expressed in terms of a change in the log of the number density of helium. Consequently, the number density of helium at the desired altitude and temperature must be known to establish this variation. But the J/L model interpolates for the density considering all the constituents and not simply helium. Therefore, the J/L model neglects this correction. For altitudes $z < 500$ km the error is negligible. At high altitudes and temperatures the relative error can be as large as 100 percent.

TABLE VIII.- EXOSPHERIC TEMPERATURE MODEL PARAMETERS

Term	Units	Jacchia 71	Jacchia 70
γ	deg	+43	+43
p	deg	+6	+6
β	deg	-37	-37
m	-	2.2	2.5
n	-	3.0	3.0
R_o	-	0.3	0.134
δR	-	0.0	0.090
T_{co}	^oK	379.0	383.0
ΔT_c	$^oK/10^4$ Jansky	3.24	3.32
δT_c	$^oK/10^4$ Jansky	1.3	1.8
ΔT_G	^oK	28.0	28.0
δT_G	^oK	0.03	0.03

4.0 COMPUTATIONAL CONSIDERATIONS

The empirical formulae given by (3.1) through (3.11) require a number of costly standard function evaluations which we would like to avoid if at all possible. For example, (3.6) expresses the variation of the temperature with respect to the diurnal bulge. Assuming that the position of the point for which we desire the density is located by Cartesian coordinates, then four standard function calls are necessary to compute the right ascension and latitude used in (3.6). If the position of the Sun is also given in terms of Cartesian coordinates then we need four more calls to standard functions. Add to this the four calls to trigonometric functions found in the calculation of D for a total of 12 standard function calls. Since Lineberry's layered expressions allow for more efficient computation of the density, this diurnal term becomes proportionately a major time cruncher. The difficulty is that Jacchia expresses the diurnal terms in angular coordinates but the input in most general purpose codes is in Cartesian coordinates. The problem can be avoided by expressing D completely in terms of Cartesian coordinates without the use of the intermediate angular variables.

An important trigonometric identity

$$\cos^2\sigma = (1 + \cos 2\sigma)/2 \tag{4.1}$$

$$\sin^2\sigma = (1 - \sin 2\sigma)/2 \tag{4.2}$$

enables us to express the terms in D as follows

$$\sin^m\sigma = \left((1 - \sin(\phi + \phi_o))/2\right)^{m/2} \tag{4.3}$$

$$\cos^m\eta = \left((1 + \cos(\phi - \phi_o))/2\right)^{m/2} \tag{4.4}$$

and

$$\cos^n|\tau/2| = \left((1 + \cos\tau)/2\right)^{n/2} \tag{4.5}$$

The value of the terms inside the brackets must always be positive and for this reason we can dispense with the absolute value signs appearing in the definitions of the angular variables.

Let's examine further the trigonometric term in (4.3). By identity

$$\sin(\phi + \phi_o) = \sin\phi \cos\phi_o - \cos\phi \sin\phi_o \tag{4.6}$$

But the trigonometric terms on the right can be expressed directly in terms of the Cartesian coordinates of the point of interest and the Sun.

$$\sin\phi = z/r \qquad ; \qquad \sin\phi_o = z_o/r_o \tag{4.7}$$

$$\cos\phi = \sqrt{x^2 + y^2}/r \qquad ; \qquad \cos\phi_o = \sqrt{x^2{}_o + y^2{}_o}/r_o \tag{4.8}$$

The expressions in (4.4) and (4.5) can be treated in a similar manner. A minor assumption is necessary to place (4.5) into the desired form. By identity

$$\cos\tau = \cos(H + \beta)\cos(p\sin(H + \gamma)) - \sin(H + \beta)\sin(p\sin(H + \gamma)) \tag{4.9}$$

Now the trigonometric functions of $H+\beta$ can be easily reduced to functions of the Cartesian coordinates but trigonometric evaluation of the term $p\sin(H + \gamma)$ seems unavoidable. Actually, the magnitude of p is very small, $(p = 0.1)$ reflecting the fact that the diurnal bulge is almost symmetric. A Taylor series expansion can be used with little error so that

$$\sin(p\sin(H+\gamma)) = p\sin(H + \gamma) + O(p^3) \tag{4.10}$$

$$\cos(p\sin(H + \gamma)) = 1 - (p\sin(H + \gamma))^2/2 + O(p^4) \tag{4.11}$$

whereas before $\sin(H + \gamma)$ is reduced by identity to simple functions of the Cartesian coordinates.

$$\sin H = \sin\alpha \cos\alpha_o - \cos\alpha \sin\alpha_o \tag{4.12}$$

$$\cos H = \cos\alpha \cos\alpha_o + \sin\alpha \sin\alpha_o \tag{4.13}$$

where

$$\cos\alpha = x/\sqrt{x^2 + y^2} \qquad ; \qquad \cos\alpha_o = x_o/\sqrt{x^2{}_o + y^2{}_o} \tag{4.14}$$

$$\sin\alpha = y/\sqrt{x^2 + y^2} \qquad ; \qquad \sin\alpha_o = y_o/\sqrt{x^2{}_o + y^2{}_o} \quad . \tag{4.15}$$

Since γ and β are constants, the trigonometric functions of these terms need only be computed once and stored in the database. Also, since in most applications the density model is repeatedly evaluated with the time of the year changing little, the trigonometric functions of the position of the Sun need not be evaluated frequently. The result is that two calls to SQRT are all that should usually be required to evaluate the diurnal term.

Storing the value of complicated functions of constants or near constants for later use is applicable to other computations in the model. For instance, p(t) and g(t) found in (2.5) and (2.6), due to their long period, may be considered constants over a short period like one day. Also, in many applications, the density at some future time must be predicted. Since the short period variations in the solar flux and geomagnetic index are next to impossible to predict, these values are defaulted to the predicted average values. Here again, the average solar flux and geomagnetic index have lengthy periods related to the 11 year solar cycle and may be treated as constants. All these considerations have been made in the computational algorithm of the Jacchia/Lineberry model as programmed on the flight design system (FDS) (ref. 9).

5.0 VERIFICATION OF THE JACCHIA/LINEBERRY MODEL

The Jacchia/Lineberry model has been implemented in a stand-alone program on the UNIVAC 1108-8. Also residing on this computer is the Jacchia 70 model with the Walker analytic expressions (J/W) (ref. 11) which allows for a rather straightforward comparison. For a meaningful test we have chosen to use the Lineberry model parameters fit to the Jacchia 70 model. However, the semi-annual variation is represented the same as in the Jacchia 71 model.

5.1 NUMERICAL COMPARISONS

For the first comparison, the two models are evaluated at 20 equally spaced points, each with the same altitude and all lying in the same plane. The plane is oriented so that it is inclined to the equator by 45° and intersects the equatorial plane at a right ascension of 45°. The epoch date is December 22, 1977, which places the Sun at its greatest inclination. The average and daily solar flux and the average and three hourly geomagnetic index are set at the rather quiet conditions of $F_{10.7} = 125$ and $Kp = 2.2$.

To consolidate results, only the numerical average of the densities at the 20 points are displayed for several different altitudes. The altitudes were chosen so that they lie just above or below a boundary of the layered atmosphere. This enables one to determine if the J/L model exhibits any marked discontinuities. The results of the comparison are shown in table IX.

The J/L model shows only slight discontinuities across the layer boundaries as seen from table V. The boundary at 180 km appears to be the largest, with a jump discontinuity in the second digit. Where valid, the J/L and J/W are in excellent agreement with discrepancies in the third digit. The largest difference is again near the 180 km boundary, but even so the relative error is only 2.5 percent, certainly within the Jacchia model error.

TABLE IX.- DENSITY MODEL COMPARISON

Case No.	Altitude (km)	Averaged Density (kg/m^3) J/L	J/W	Relative Error %
1	90	.344E-5	N/A	
2	100-	.524E-6	N/A	
3	100+	.524E-6	N/A	
4	110-	.967E-7	N/A	
5	110+	.965E-7	N/A	
6	125	.134E-7	.135E-7	0.7
7	140-	.384E-8	.390E-8	1.5
8	140+	.384E-8	.390E-8	1.5
9	180-	.572E-9	.558E-9	2.4

TABLE IX.- Concluded

Case No.	Altitude (km)	Averaged Density (kg/m^3) J/L	Averaged Density (kg/m^3) J/W	Relative Error %
10	180+	.546E-9	.558E-9	2.1
11	420-	.218E-11	.220E-11	0.9
12	420+	.218E-11	.220E-11	0.9
13	500-	.574E-12	.582E-12	1.3
14	500+	.575E-12	.582E-12	1.2
15	700-	.336E-13	.340E-13	1.2
16	700+	.336E-13	N/A	
17	1500-	.581E-15	N/A	
18	1500+	.573E-15	N/A	
19	2500	.650E-16	N/A	

To demonstrate the agreement between the models at each point, data from test cases 6, 12, 14 and 15 have been plotted showing density versus the angular parameter α measured in the plane marking the position of each point. The plots are shown on figures 6, 7, 8, and 9, respectively. Be careful to observe the scale of each plot. The agreement is very close for each case. The small variations in case 6 are due primarily to the seasonal-latitudinal term. Note the significant variations in the density at the higher altitudes, due to the diurnal bulge.

The last experiment is intended to show that the models agree for different orientations with respect to the diurnal bulge. In test cases 20 and 21 the Sun is placed at the vernal equinox. In case 20 the plane is oriented so that it coincides with the plane of the maximum diurnal variation. In contrast, for case 21 the plane is chosen to be perpendicular to the maximum diurnal bulge plane so we should see little variation. In each case, the altitude is 500 km with the $F_{10.7} = 78$ and $Kp = 2.2$. Figures 10 and 11 show the density versus angular position for each of the points. The agreement between models is within the Jacchia model error.

ORIGINAL PAGE IS
OF POOR QUALITY

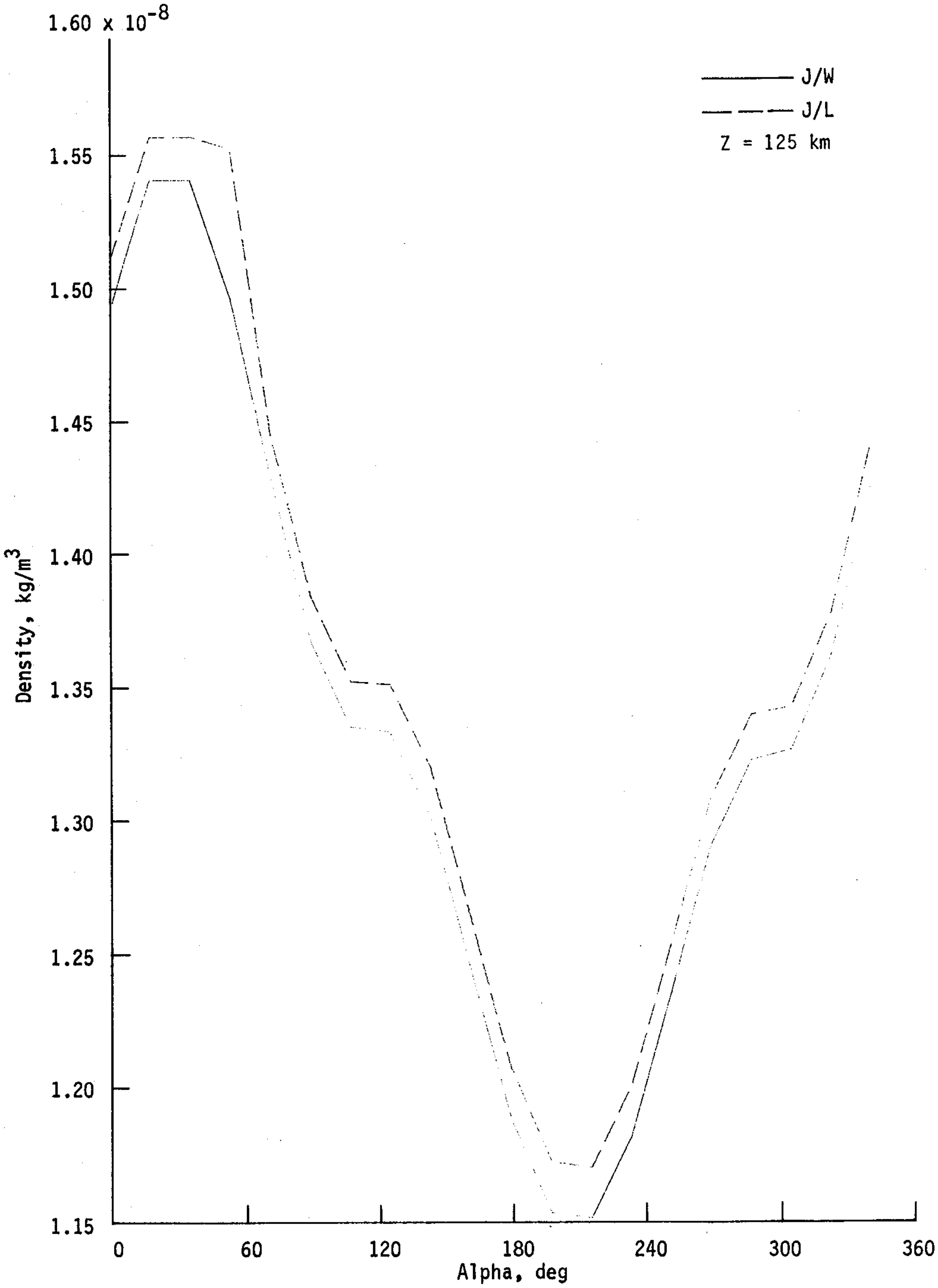

Figure 6.- Case 6 density comparison.

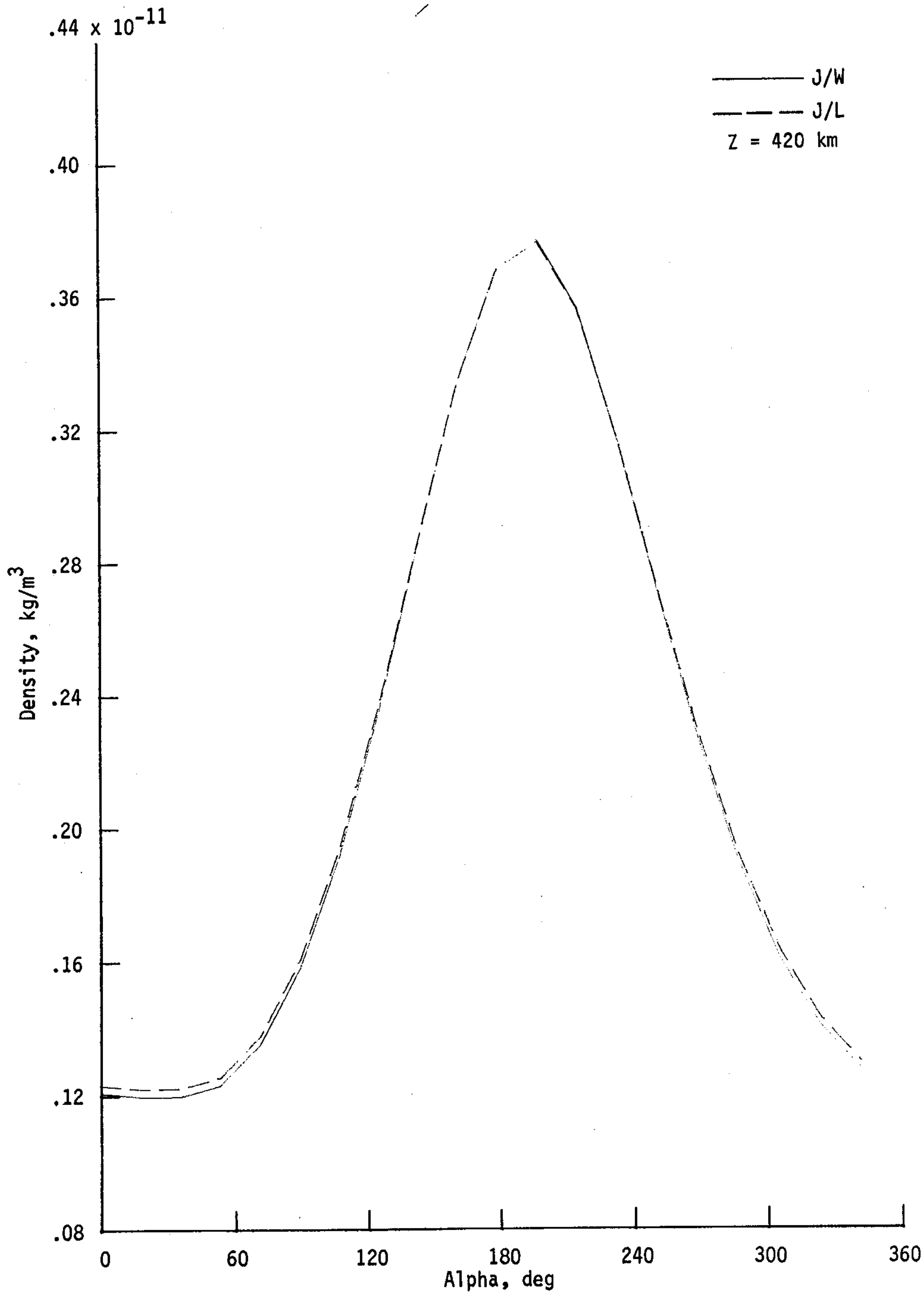

Figure 7.- Case 12 density comparison.

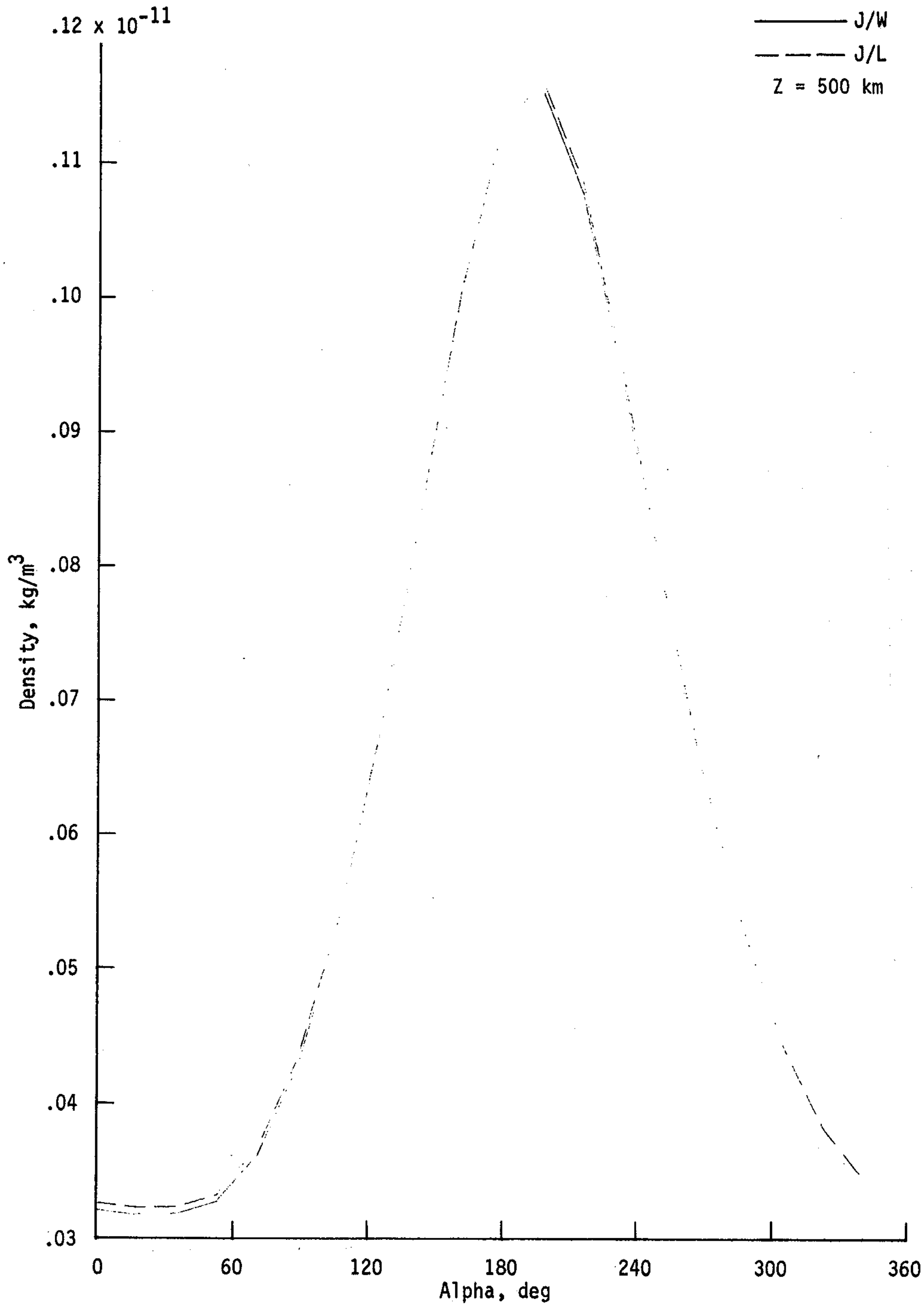

Figure 8.- Case 14 density comparison.

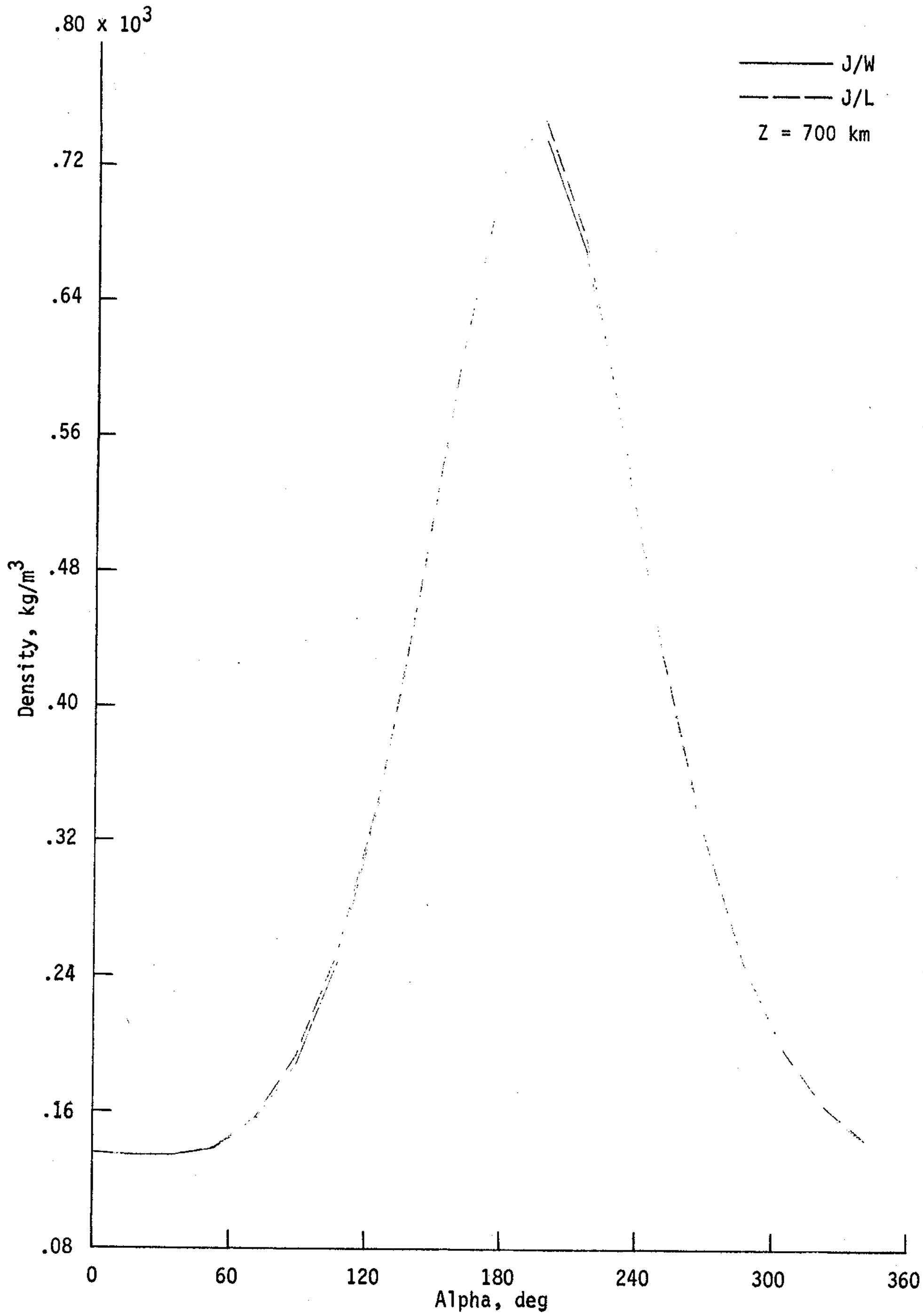

Figure 9.- Case 15 density comparison.

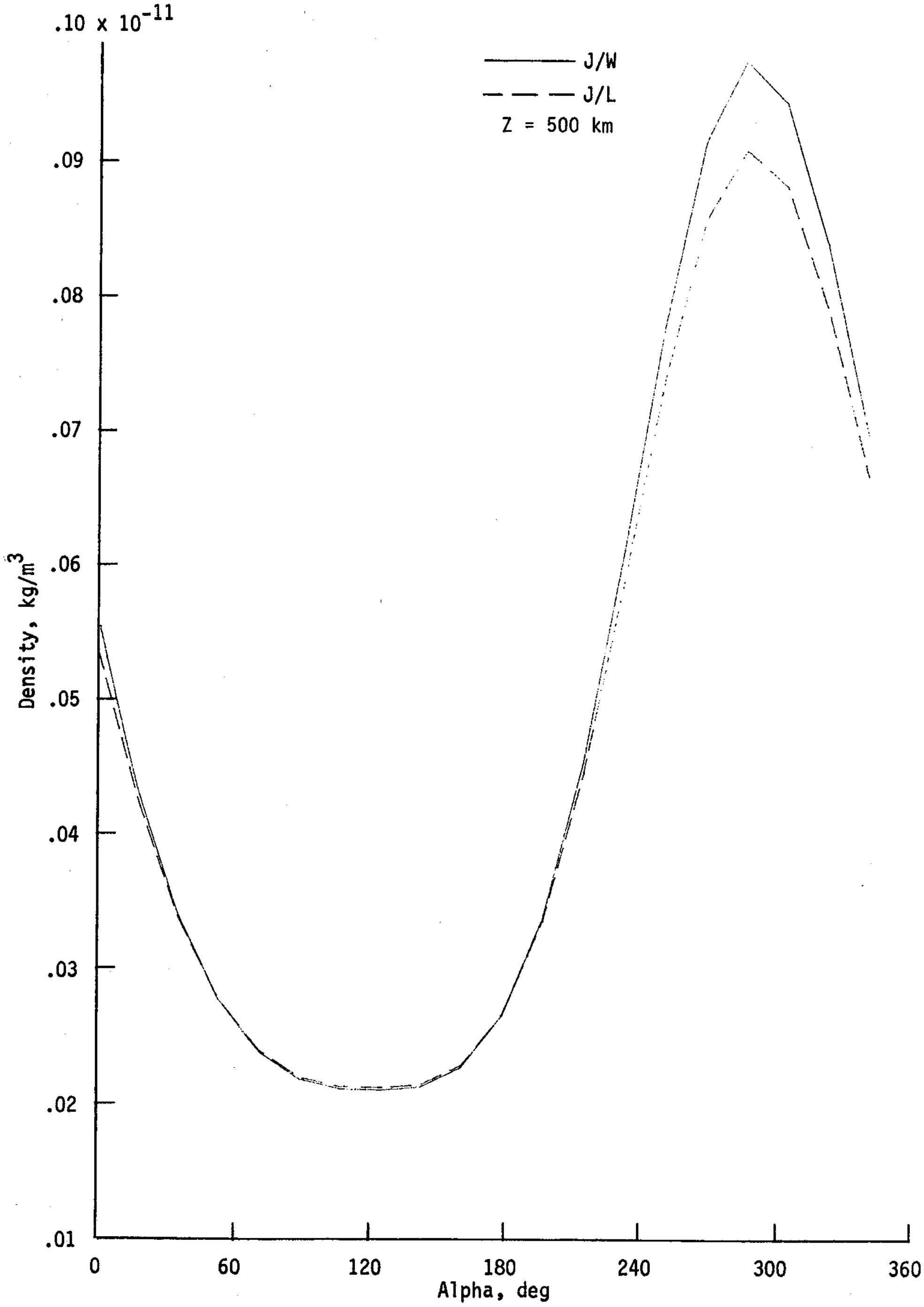

Figure 10.- Case 20 density comparison.

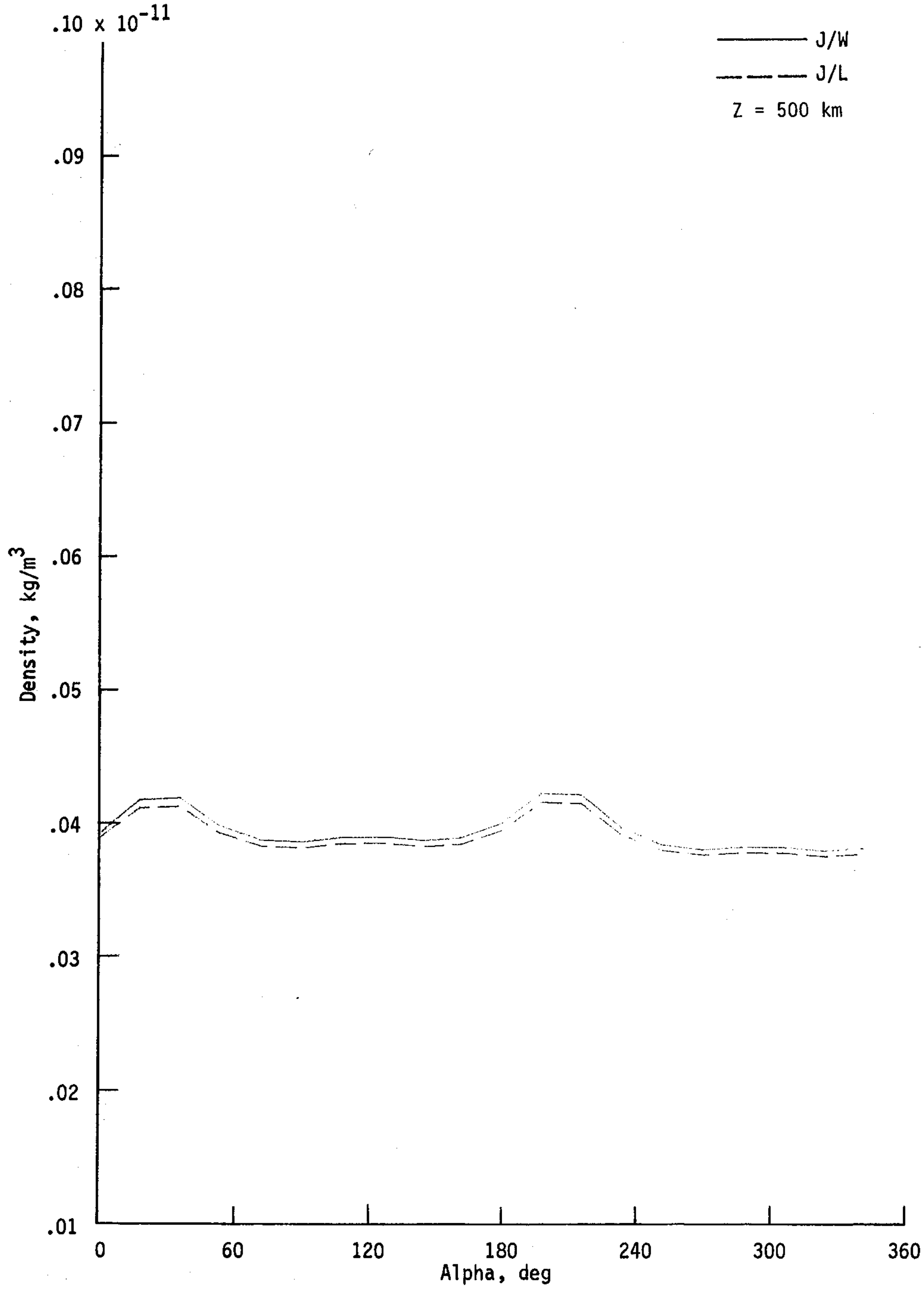

Figure 11.- Case 21 density comparison.

5.2 TIMING COMPARISONS

To make an accurate estimate of the computation cycle times, both models have been clocked over 100 calls to the algorithm. The cycle time then is the time required to evaluate the model 100 times divided by 100. In table X the cycle times on the UNIVAC 1110 are shown for both models. The times vary somewhat depending on altitude but generally fall within the ranges shown.

TABLE X.- COMPUTATION TIMES

Model	Computation Cycle Time (ms)
J/W	3.4 - 3.6
J/L	0.5 - 0.7

6.0 DENSITY MODEL DIFFERENCES

The J/L model has been fashioned so that it closely follows the Jacchia 71 model. The J/L and Jacchia 71 differ in two respects:

1) The J/L model assumes the hydrogen diffusion boundary condition defined in the Jacchia 77 model which is different from that used in the 71 model. This difference becomes significant only at high altitudes ($z > 500$ km) and low temperatures ($T_\infty < 1000^\circ$ K).

2) The J/L model does not correct for seasonal-latitudinal helium variations. Only at high altitudes ($z > 500$ km) and high temperatures ($T_\infty > 500^\circ$ K) does the difference become significant.

The J/L model differs from the Jacchia 70 model primarily in the manner in which the semiannual variation is computed. Since the direct form used in the J/L and Jacchia 71 model is roughly equivalent to the indirect form in the Jacchia 70 model, the quantitative differences are small.

The primary quantitative difference between the Jacchia 71 and 77 models is due to the form of the geomagnetic effect on temperature. In the 77 model the position with respect to the geomagnetic field is an important feature in describing geomagnetic variations in temperature. The difference between models is negligible for "quiet" conditions but becomes very large during solar storms (large Kp). The Jacchia 77 diurnal variation takes a simpler form than in the 71 model. The 77 model also suggests methods for determining diurnal variations in temperature for each of the constituents. The number density for each component is computed according to its temperature. Algorithmically, this represents a large departure from the 71 model, although numerically the differences may be only slight. In such a case, the interpolants in the J/L model must be for each constituent instead of the total density.

7.0 CONCLUSIONS

The Jacchia/Lineberry density model is an efficient yet accurate method for computation of the upper atmospheric density. Relative errors between the J/L model and Jacchia 71 model are generally less than a few percent. Relative errors between the models may become large at high altitudes where the density is extremely small. For orbit calculations, such an error is not critical except possibly for long lifetime studies. The J/L model represents a five-fold increase in efficiency with comparable accuracy compared to the J/W model (ref. 11) used on the real-time computer complex (RTCC). The model is of such general form that differences in Jacchia models can be readily accounted for.

8.0 REFERENCES

1. Jacchia, L. G.: "Static Diffusion Models of the Upper Atmosphere with Empirical Temperature Profiles," Smithsonian Contr. Astrophys., vol 8, no. 9, pp. 215-217, 1965.

2. Jacchia, L. G.: "New Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles," Smithsonian Astrophysical Observatory Special Report No. 313, 1970.

3. Jacchia, L. G.: "Revised Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles," Smithsonian Astrophysical Observatory Special Report No. 332, 1971.

4. Jacchia, L. G.: "Thermospheric Temperature, Density, and Composition: New Models," Smithsonian Astrophysical Observatory Special Report No. 375, 1977.

5. U.S. Standard Atmosphere 1962, U.S. Government Printing Office, Washington, DC, 1962.

6. Lewis, J. R.: "Final Atmosphere Model Analysis," TRW Memo 5522, 7-70-73, April 30, 1970.

7. Walker, J. C. G.: "Analytic Representation of Upper Atmospheric Densities Based on Jacchia's Static Diffusion Models," Journal of the Atmospheric Sciences, vol. 22, pp. 462-3, July 1965.

8. Mueller A. C.: "The Decay of the Low Earth Satellite," Orbital Debris Study J.O. 69-067, Lockheed Engineering and Management Services Company, LEMSO - 17520, December 1981.

9. Flight Design System - Utility Density Model.

10. Mueller, A. C.: "Atmospheric Density Models," ACM Technical Report No. 106, June 1977.

11. Lewis, J. R., et al.: "RTCC and MOPS Requirements for: Modified 1970 Jacchia Atmosphere Model," MSC Internal Note No. 70-FM-202, MSC-03768, January 26, 1971.

☆ U.S. GOVERNMENT PRINTING OFFICE: 1983—669-013/2344

DISTRIBUTION FOR JSC IN 82-FM-52

NASA - JSC
FM/R. Berry
E. Lineberry (25)
C. Graves
FM14/Editing
Reports Control (50)
Word Processing
FM2/K. Young
J. P. Basu
R. Becker
A. Menchaca
G. Hunt
J. Bell
E. Mann
R. Moore
L. Davis
C. Osgood
M. Jenness
J. Kirkpatrick
FM4/M. Jenkins
F. Buzzard
FM5/E. E. Smith
J. Harpold
P. Rennert
FM6/E. McHenry
A. Nolting
J. Yencharis
E. Fridge
W. Reini
R. Davis
H. Sullivan
G. Weisskopf
V. Bond (10)
S. Newman
I. Johnson
FDS Project Library
FM8/E. Schiesser
W. Lear (TRW)
P. Pixley
R. Savely
O. Olszewski
R. Osburn
J. deVezin
J. Currie
J. Weaver
JM2/Technical Library (2)
JM23/R. Jones
SN/D. Kessler

Barrios - Houston
D. Fabian
J. Yglesias (2)
H. Flinn (2)
C. Mallini
A. Smith

CSC - Houston
O. Dial (2)
L. Duncan
J. King

MDTSCO - Houston
B. Brown
M. McRae
L. Murphy

IBM - Houston
L. Burnett

University of Texas at Austin
Dept. of Aerospace Engineering
and Engineering Mechanics
WRW, 308D
Austin, Texas 78712
Mr. Alan C. Mueller (10)