MAE 575: Turbulence

Professor Werner Dahm

Final Project Report

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1. Introduction

1.1 Project Description

This project outlines the effects of various turbulence models for a nominal two-dimensional flow over a backward-facing step, as shown in Fig. 1. The flow conditions of the experiment were established from the experiments of Driver & Seegmiller (1995). The experimental results from the study will be compared to three different turbulence models: k-ε, k-ω, and the Reynolds Stress Transport Model (RSTM). Each model is simulated on ANSYS Fluent with three different meshes: coarse (6,056 cells), medium (54,504 cells), and fine (490,536 cells).

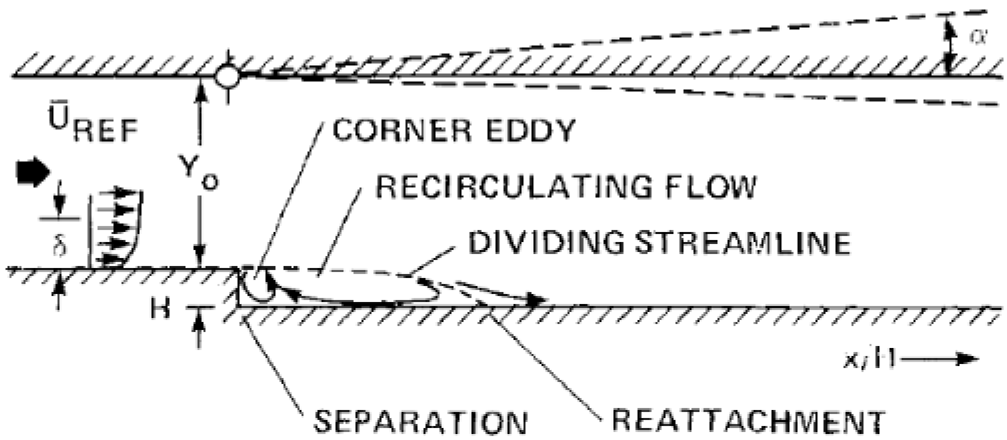

Fig. 1 Backward-Facing Step [1]

1.2 Report Outline

This final project report will be organized starting with an overview of the modeling process and equations. Then a comparison of each turbulence model with three different meshes between seven different output variables. From here, each model will be compared against each other for the fine meshes for the seven output variables. Next, the simulated data for the three models will be compared to the experimental data of Driver & Seegmiller (1995) at various (x/H) locations. Then, comparing the k-ε model's fine mesh for standard and enhanced wall functions to the experimental data of Driver & Seegmiller (1995) at various (x/H). Each section will consist of questions based on the resulting figures. Finally, the report will have a final discussion/conclusion to the overall project, references used, and the appendix with the MATLAB code.

2. Modeling

2.1 Description

The modeling of the backward-facing step was conducted in ANSYS Fluent. ANSYS is set up to create a geometry, generate a mesh, and then solve a turbulence model that outputs pre-made functions or yields the ability to generate user-defined functions.

2.2 Geometry

Beginning with opening up ANSYS, the first step was to select an analysis system. Since the project description is to model the turbulent flow over a backward-facing step, this must be a fluid flow (Fluent) analysis system. After making this selection, there are five things to complete/view, the first being geometry.

Going into the geometry, the project description yielded a strict design with the step split into 6 different zones. Each zone was given a length and height based on a value of H = 1.27 cm. The zones and sizing are shown in Fig. 2. Using Fig. 2, the geometry can be added into ANSYS, and the resulting zones can be seen in Fig. 3.

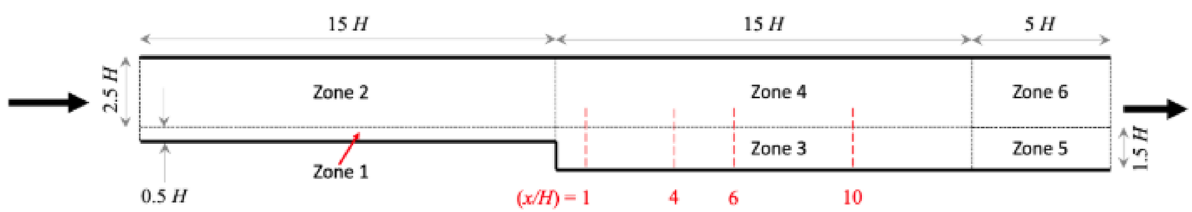

Fig. 2 Zone and Sizing of Backwards-Facing Step

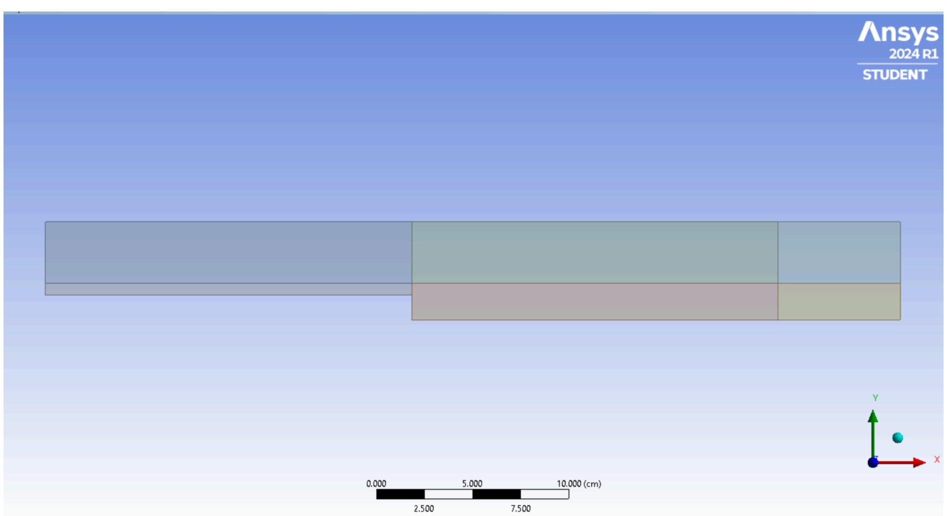

Fig. 3 ANSYS Geometry of Backwards-Facing Step

2.3 Meshing

After finishing the geometry, the next step was to complete the meshing. Table 1 shows the given meshing cells for three different meshes: coarse, medium, and fine. For each of the turbulence models, these three meshes were created.

Table 1 Zonal Meshing Specifications

Zones	Coarse	Medium	Fine
1	X = 80, Y = 10	X = 240, Y = 30	X = 720, Y = 90
2	X = 80, Y = 14	X = 240, Y = 42	X = 720, Y = 126
3	X = 80, Y = 30	X = 240, Y = 90	X = 720, Y = 270
4	X = 80, Y = 14	X = 240, Y = 42	X = 720, Y = 126
5	X = 14, Y = 30	X = 42, Y = 90	X = 126, Y = 270
6	X = 14, Y = 14	X = 42, Y = 42	X = 126, Y = 126
Total Cells	6,056	54,504	490,536

Based on Table 1, the mesh can be completed using a mixture of face meshes and edge sizing for each of the specified zones. In Fig. 4 the coarse meshing can be seen, detailing zones of higher-density cells near the walls.

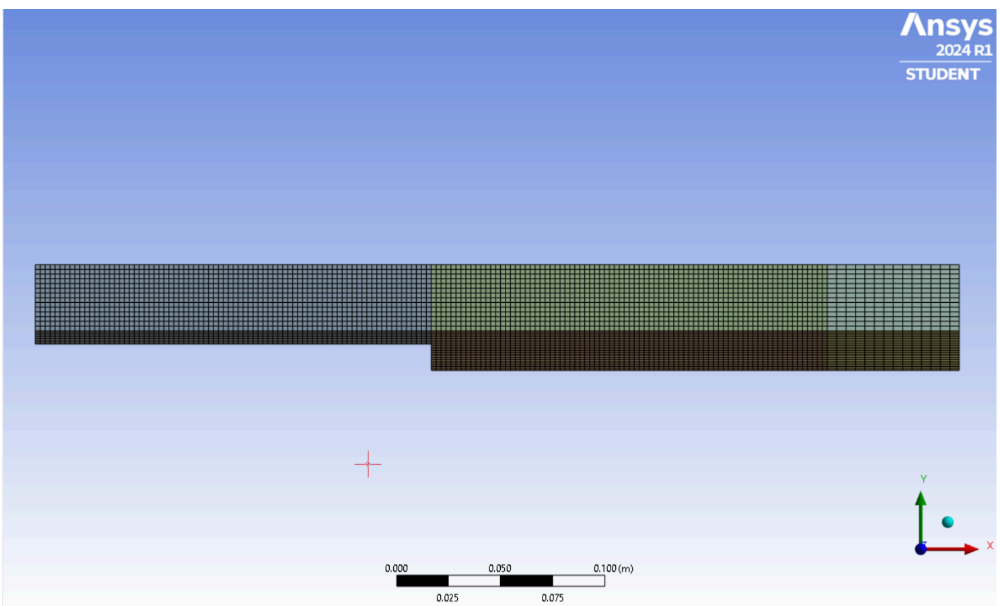

Fig. 4 ANSYS Coarse Meshing of Backwards-Facing Step

2.3 FLUENT

Once the meshing is completed, the next step is creating the turbulence simulation. Going into Fluent, the first step was selecting which model the simulation would be run using. The three specified in the problem statement were k-ε, k-ω, and the Reynolds Stress Transport Model (RSTM). The k-ε transport equation is a combined model that includes both the "standard k-equation" and the "standard ε-equation". The model in ANSYS includes the first-order single-point moment equations. The k and ε equations are expressed in Eqs. 1 through 9.

$$\frac{Dk}{Dt} = P - \varepsilon + D \tag{1}$$

Where,

$$P = -\overline{u_i' u_j'} \frac{\partial \overline{u_i}}{\partial x_j} \tag{2}$$

$$\varepsilon = \nu \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_j'}{\partial x_i}} \tag{3}$$

$$D = \frac{\partial}{\partial x_l}\left[\overline{u_l' k'} + \frac{1}{\rho}\overline{u_l' p'} - \nu \frac{\partial k}{\partial x_j}\right] \tag{4}$$

And,

$$\frac{D\varepsilon}{Dt} = P_\varepsilon - \varepsilon_\varepsilon + D_\varepsilon \tag{5}$$

Where,

$$P_\varepsilon \approx -C_{\varepsilon 1} \frac{\varepsilon}{k} \overline{u_i' u_j'} \frac{\partial \overline{u_i}}{\partial x_j} \tag{6}$$

$$\varepsilon_\varepsilon \approx C_{\varepsilon 2} \frac{\varepsilon^2}{k} \tag{7}$$

$$D_\varepsilon \approx \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] \tag{8}$$

$$\nu_T = C_\mu \frac{k^2}{\varepsilon} \tag{9}$$

Eqs. 1-9 do not detail the remaining first-order single-point moment equations such as mass conservation, momentum conservation, or the gradient transport (G.T.) approximation and local linear equilibrium (LLE). ANSYS then takes these to model the k-ε transport equation, using standard and accepted values, shown in Table 2.

Table 2 Standard Values For k-ε Transport Equation

Variable	Value
C_{μ}	.09
$C_{\varepsilon 1}$	1.44
$C_{\varepsilon 2}$	1.9
σ_{ε}	1.3

For the k-ω transport equation, it isn't a completely new set of equations but rather a slight variation that results in Eqs. 10 through 12.

$$\omega = \frac{\varepsilon}{k} \tag{10}$$

$$\frac{D\omega}{Dt} = \frac{1}{k}\frac{D\varepsilon}{Dt} - \frac{\varepsilon}{k^2}\frac{Dk}{Dt} \tag{11}$$

After subbing Eq. 11 into Eq. 1 and 5,

$$\frac{D\omega}{Dt} = -C_{\omega 1}\frac{\omega}{k}\overline{u_i' u_j'}\frac{\partial \bar{u}_i}{\partial x_j} - C_{\omega 2}\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] \tag{12}$$

The constants are different from the k-ε transport equation, and in ANSYS they use a slightly different notation where instead of C_ω , they use defined constants of α and □. Finally, the Reynolds Stress Transport Model (RSTM), is defined in Eqs. 13 through 16.

$$\begin{aligned}\frac{D}{Dt}a_{ij} = & -\left(\frac{P}{\varepsilon} - 1\right)\frac{\varepsilon}{k}a_{ij} - \frac{4}{3}\overline{S}_{ij} - \left(a_{il}\overline{S}_{lj} + \overline{S}_{il}a_{il} - \frac{2}{3}a_{nl}\overline{S}_{nl}\delta_{ij}\right) \\ & + \left(a_{il}\overline{R}_{lj} - \overline{R}_{il}a_{il}\right) + \frac{1}{k}\Pi_{ij} - \frac{1}{k}\left[\varepsilon_{ij} - \frac{2}{3}\varepsilon\delta_{ij}\right] + \frac{1}{k}\left[D_{ij} - \left(a_{ij} + \frac{2}{3}\delta_{ij}\right)D\right]\end{aligned} \tag{13}$$

Where,

$$\Pi_{ij} = \Pi_{ij}^{(r)} + \Pi_{ij}^{(s)} \tag{14}$$

$$\Pi_{ij}^{(s)} \sim C_1 \varepsilon a_{ij} \tag{15}$$

$$\Pi_{ij}^{(r)} \approx C_2 \overline{S_{ij}} + C_3\left(a_{il}\overline{S_{lj}} + \overline{S_{il}} a_{lj} - \frac{2}{3} a_{nl}\overline{S_{nl}} \delta_{ij}\right) - C_4\left(a_{il}\overline{R_{lj}} - \overline{R_{il}} a_{il}\right) \tag{16}$$

Eq. 13 details the Laurdner-Reese-Rodi (LRR) model which is the basis for the RSTM modeling in Fluent. This model also includes the k-standard equation, ε-standard equation, and the first-order single-point moment equations. Similar to the k-ε model, there are standard and accepted values for the constants, shown in Table 3.

Table 3 Standard Values For Slow and Rapid Pressure-Strain Correlation Equations

Variable	Value
C_1	1.5
C_2	.8
C_3	.875
C_4	.655

After understanding some of the math behind the three different turbulence models, in Fluent, one method at a time is selected and processed using the given standard values, as per the problem statement. Additionally, there are specifications and properties for the setup that are shown in Table 4.

Table 4 Fluent Specifications and Properties

Variable	Value
μ	$1.8(10^{-5})$ kg/(m-s)
ρ	1.2 kg/m^3
U_{REF}	44.2 m/s
Residuals	$1(10^{-5})$

Then, individual models have inlet boundary conditions that specify the values at the start of the simulation. Table 5 describes the inlet conditions for k-ε and RSTM, and Table 6 describes k-ω.

Table 5 Inlet Boundry Conditions for k-ε and RSTM

Variable	Value
k	58.6 m^2/s^2
ε	2.7(10^4) m^2/s^2

Table 6 Inlet Boundry Conditions for k-ω

Variable	Value
k	58.6 m^2/s^2
ω	460.75 s^{-1}

Before running a simulation, a few user-defined functions are added into ANSYS to evaluate the turbulent stresses for the k-ε and k-ω, as seen in Eqs. 17 and 18. RSTM has the stress functions pre-loaded, and thus Eqs. 17 and 18 won't be required.

$$\overline{u_i^{'}u_j^{'}} = \frac{2}{3}k\delta_{ij} - C_{\mu}\frac{k^2}{\varepsilon}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) \tag{17}$$

$$\overline{u_i^{'}u_j^{'}} = \frac{2}{3}k\delta_{ij} - C_{\mu}\frac{k}{\omega}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) \tag{18}$$

The problem statement requires the turbulent stresses to be evaluated in x, y, and xy, as expressed in Table 7.

Table 7 Turbulent Stress

Variable	Value
$\overline{u^{'2}}$	i = 1 j = 1
$\overline{v^{'2}}$	i = 2 j = 2
$\overline{u^{'}v^{'}}$	i = 1 j = 2

Upon adding in the user-defined functions, the simulations can be run. Transferring the data into MATLAB and doing some data manipulation to non-dimensionalize and plot certain sections of x/H, the final plots can be calculated and shown in sections 3 through 5.

3. Turbulence Model Effects With Mesh Resolution

3.1 Non-Dimensional Mean X-Velocity Results

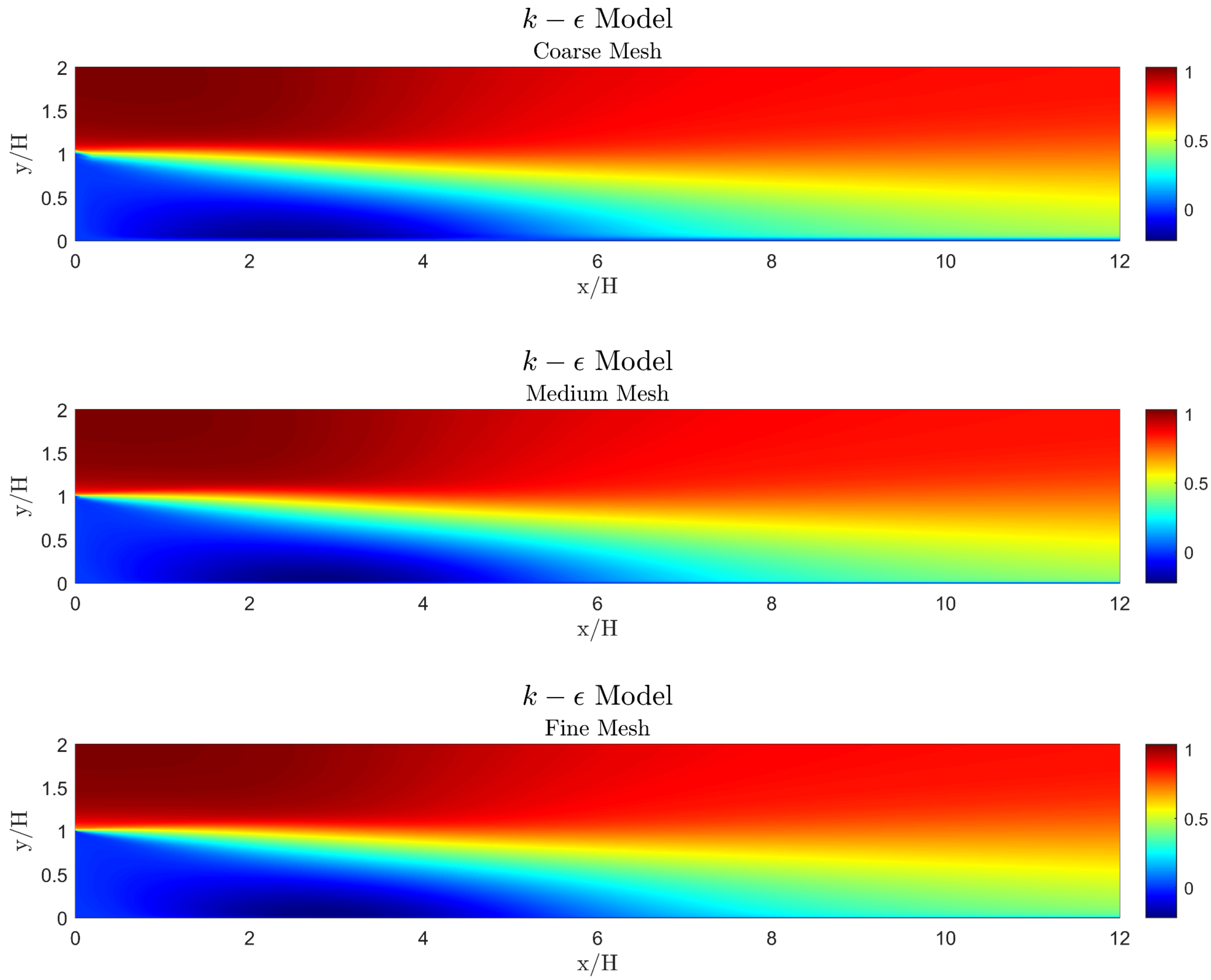

Fig. 5 Comparison of Non-Dimensional Mean X-Velocity Meshes of k-ε Model

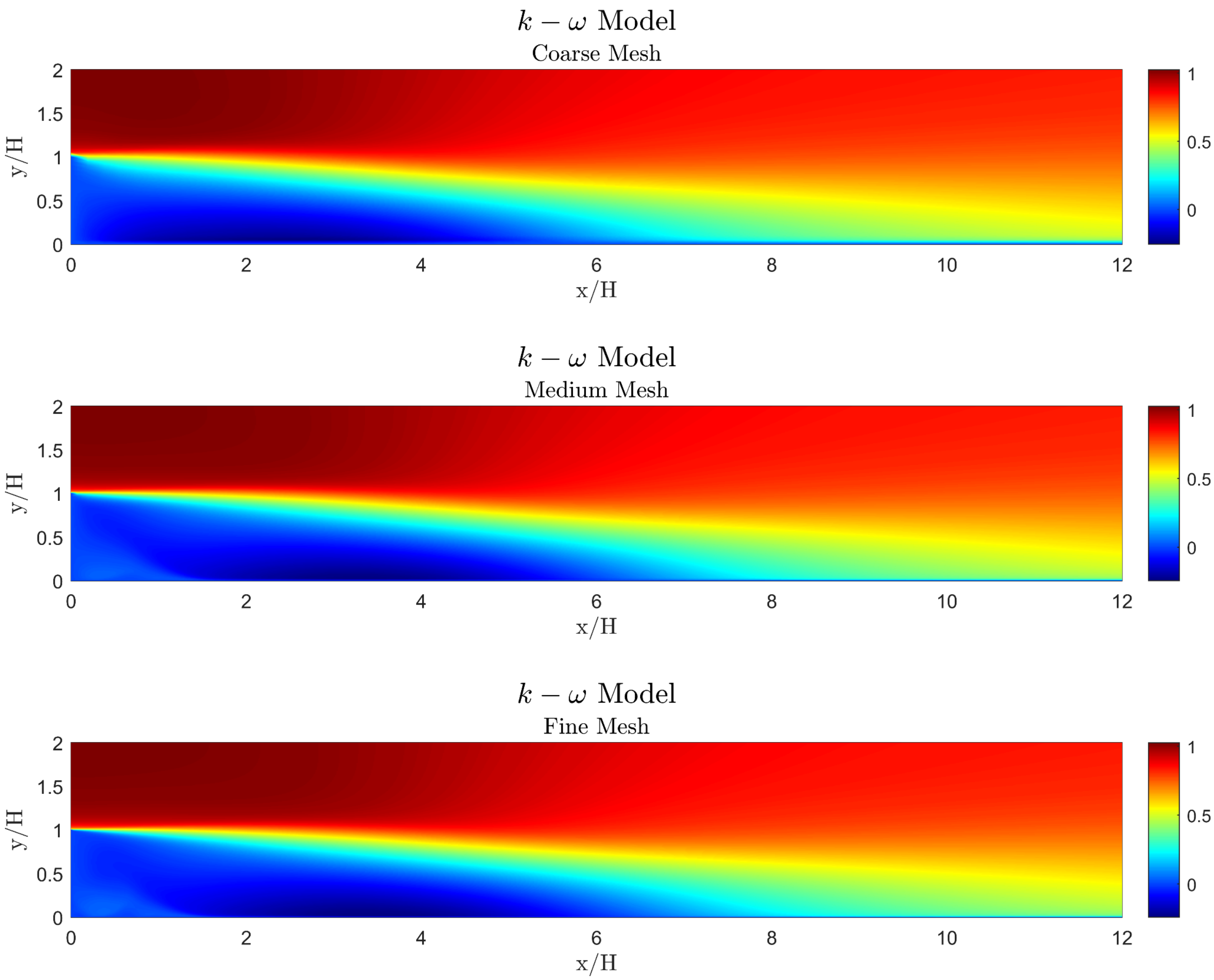

Fig. 6 Comparison of Non-Dimensional Mean X-Velocity Meshes of k-ω Model

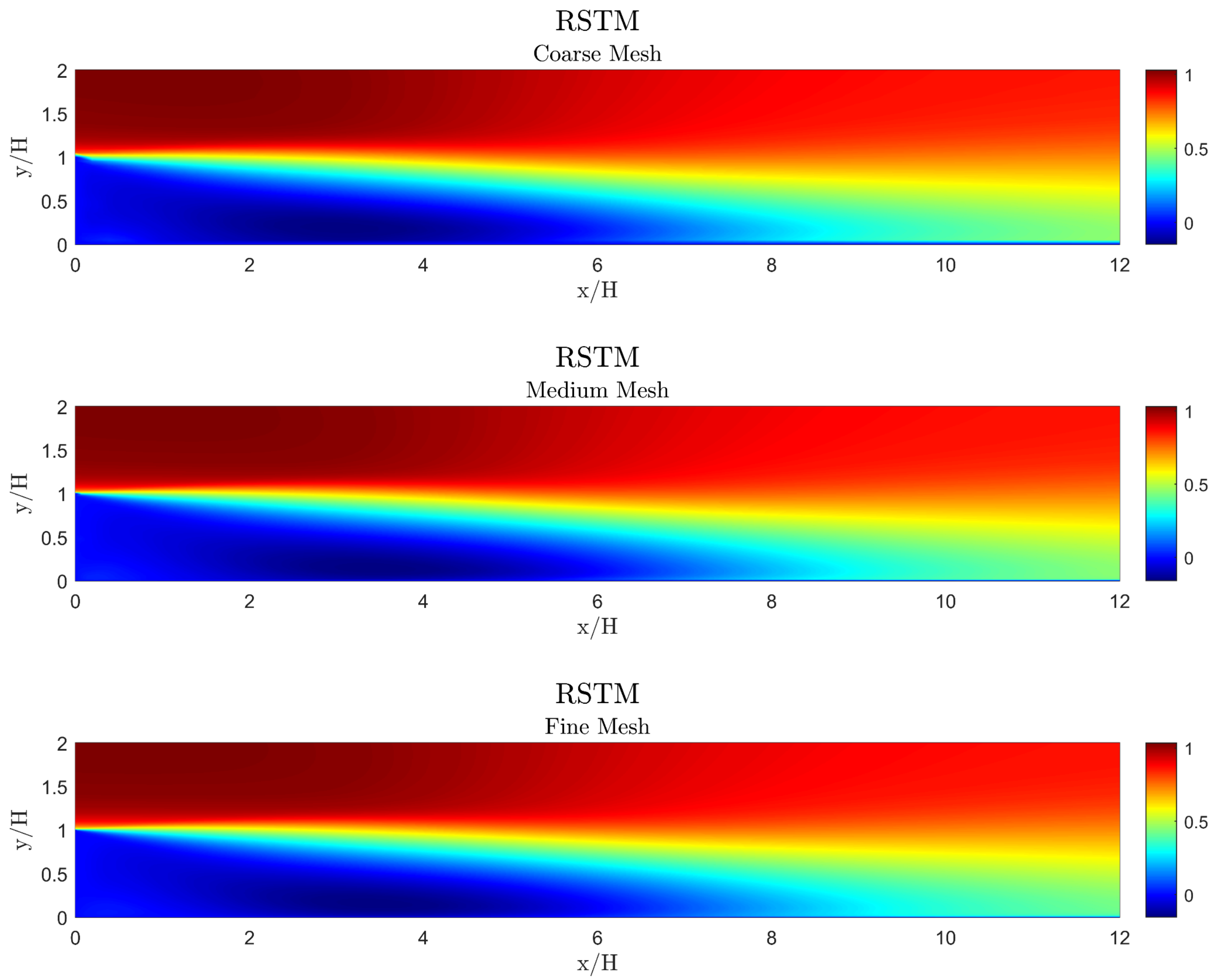

Fig. 7 Comparison of Non-Dimensional Mean X-Velocity Meshes of RSTM Model

3.2 Non-Dimensional Mean Y-Velocity Results

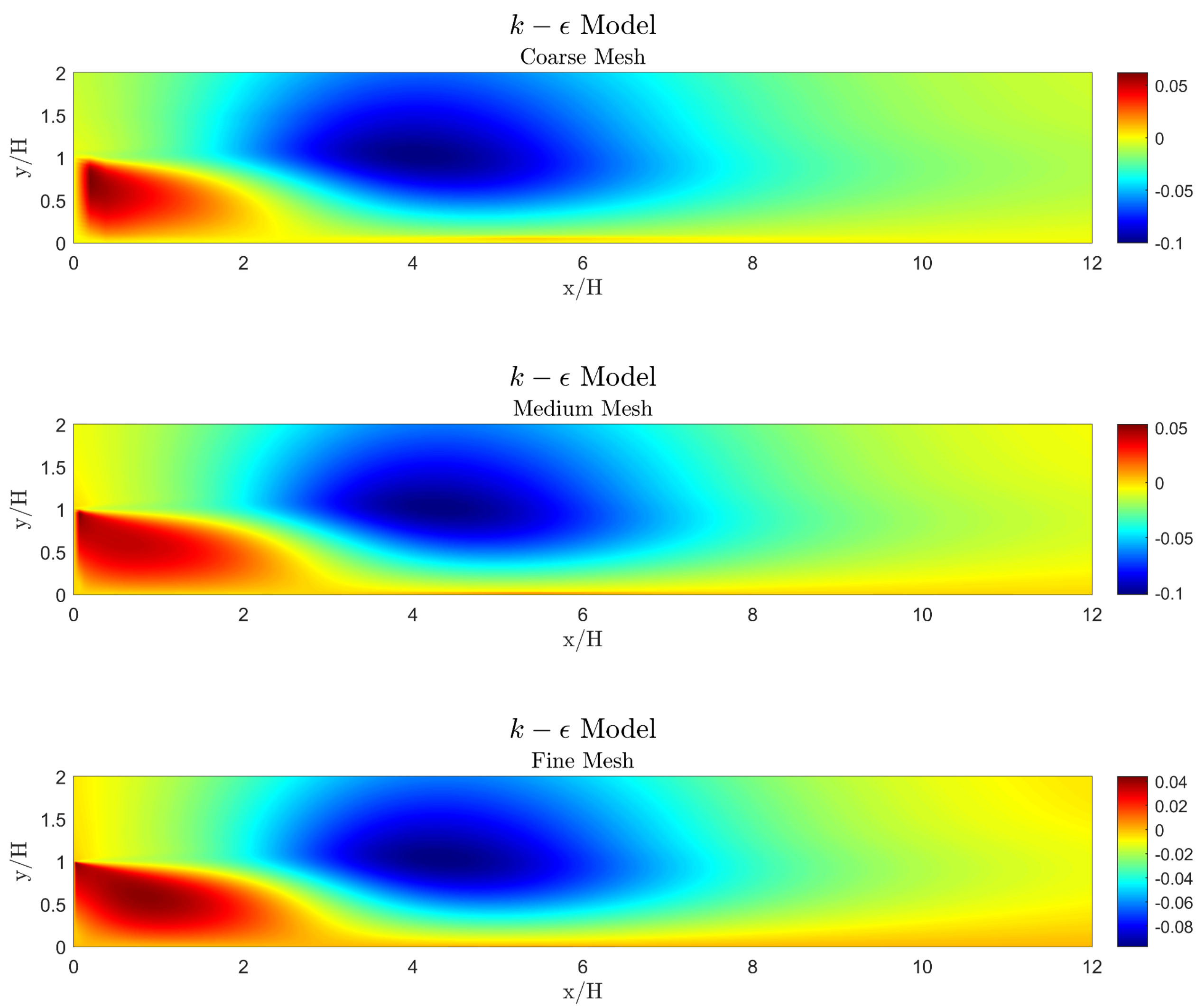

Fig. 8 Comparison of Non-Dimensional Mean Y-Velocity Meshes of k-ε Model

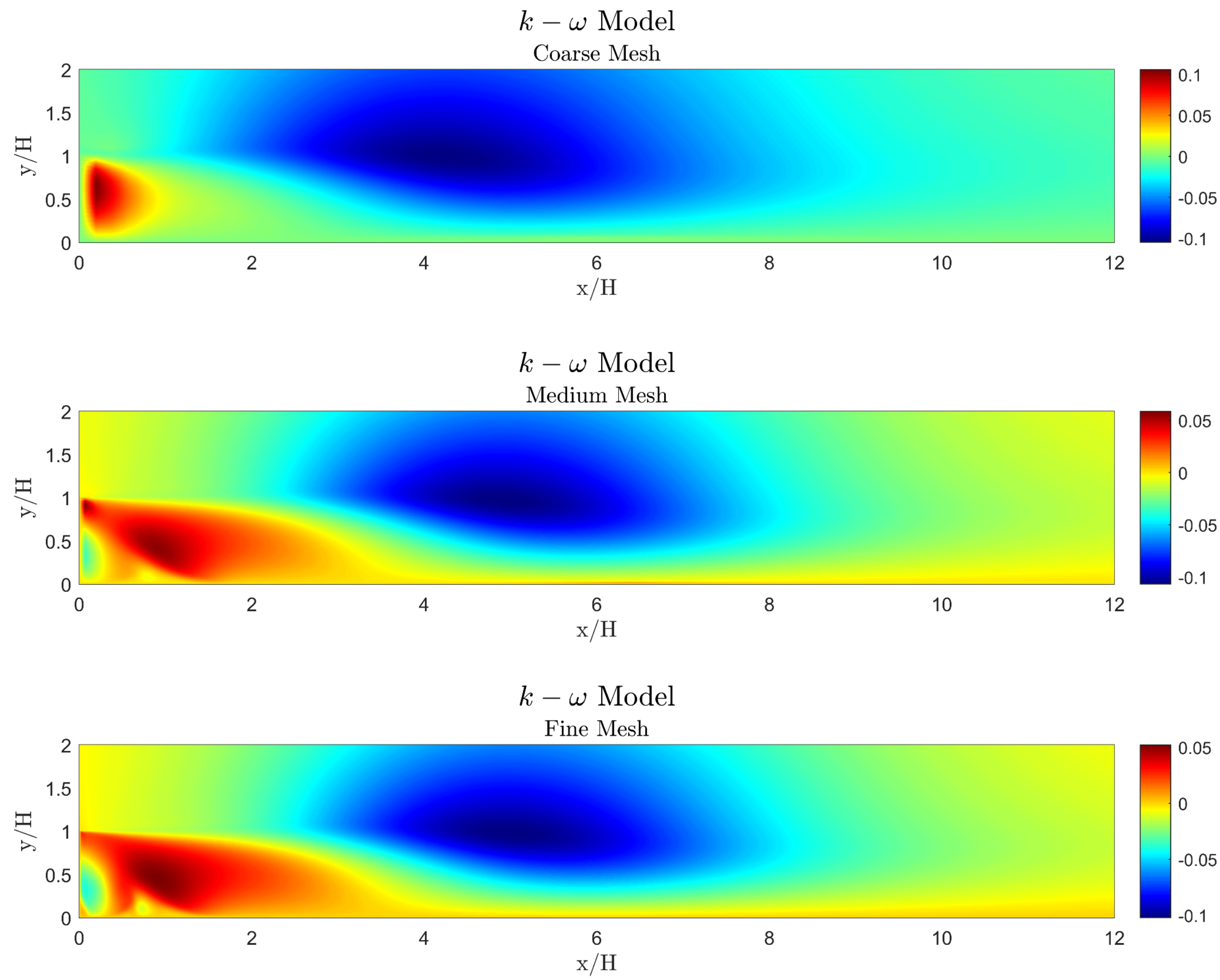

Fig. 9 Comparison of Non-Dimensional Mean Y-Velocity Meshes of k-ω Model

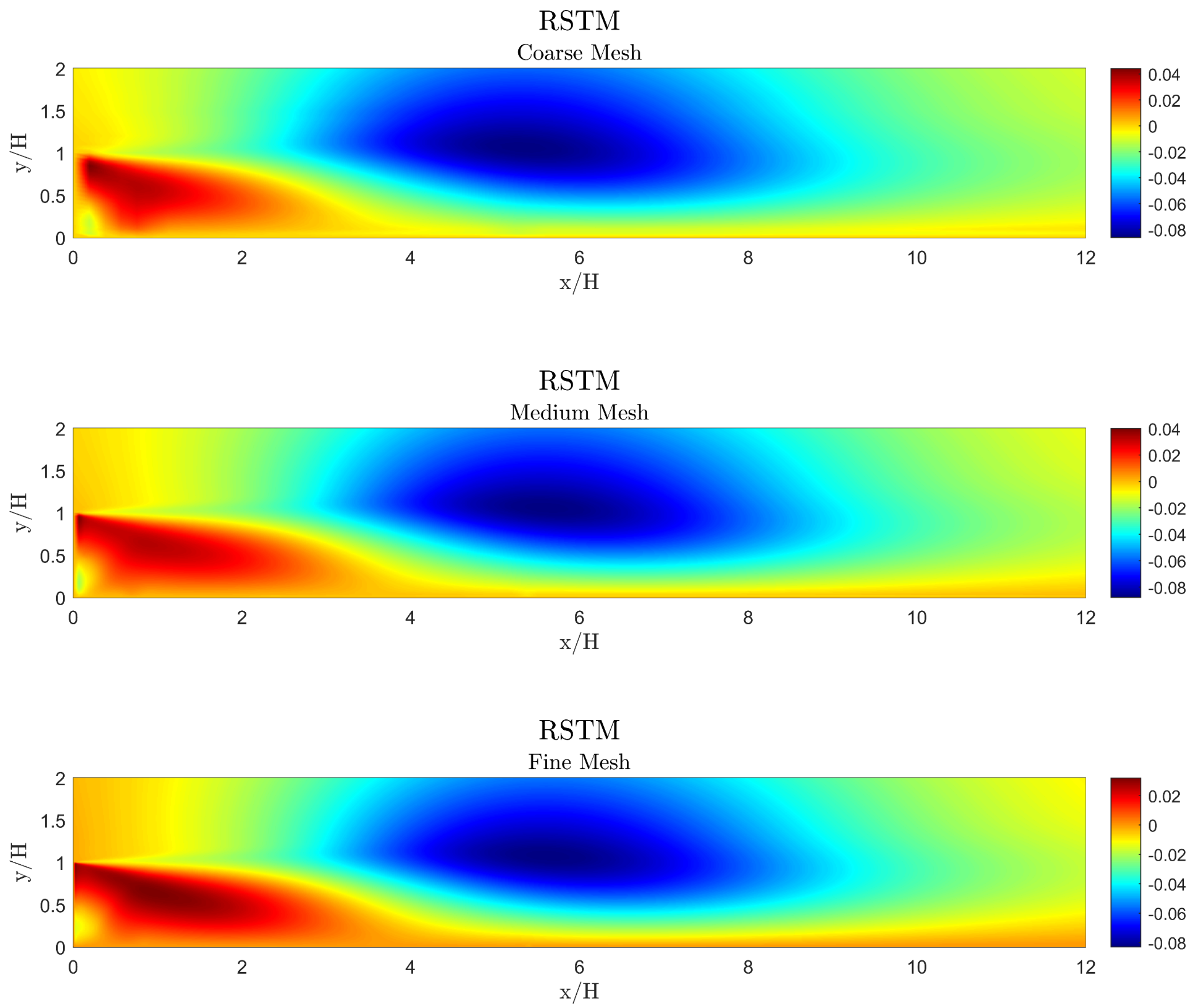

Fig. 10 Comparison of Non-Dimensional Mean Y-Velocity Meshes of RSTM Model

3.3 Non-Dimensional X-Reynolds Stress Results

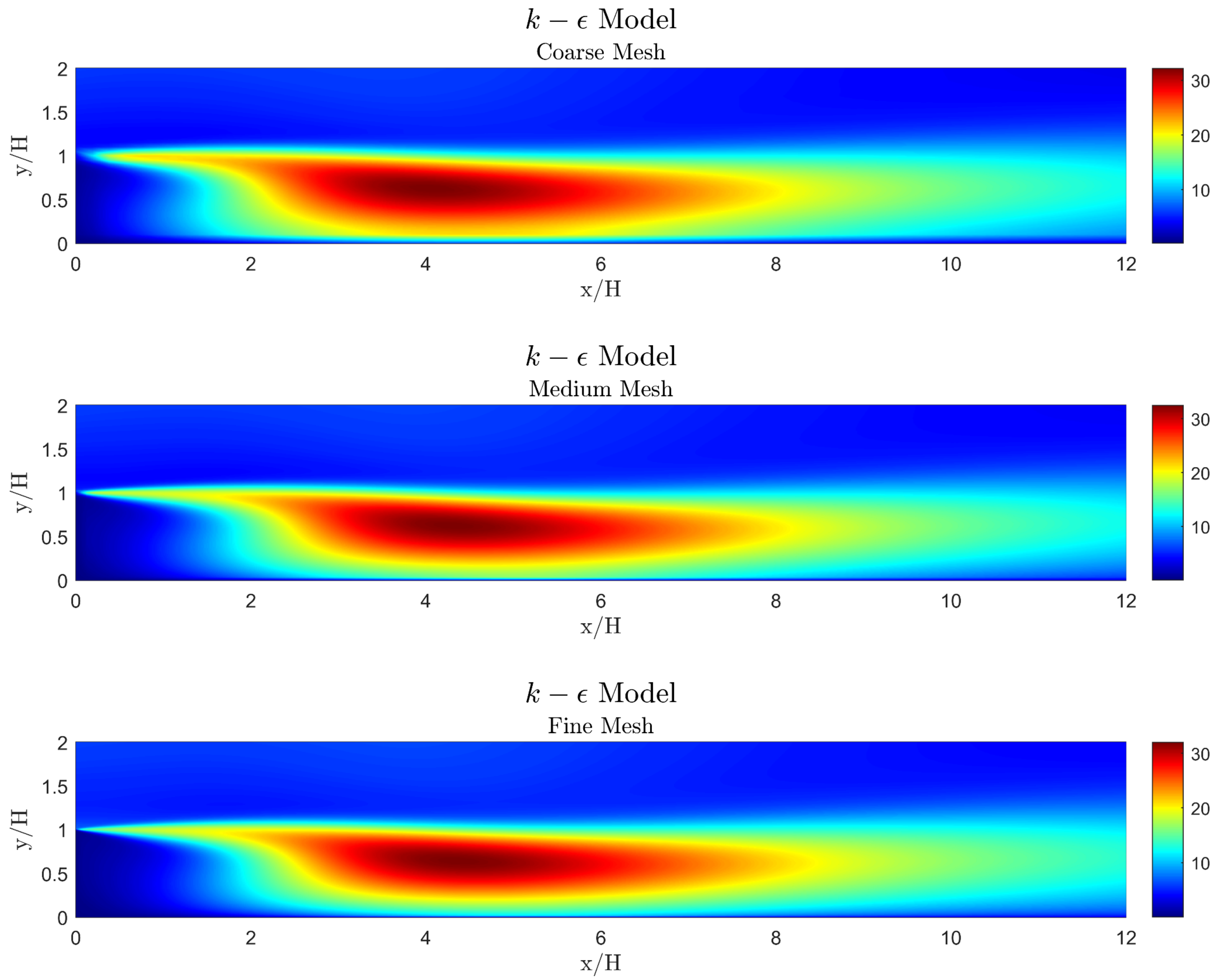

Fig. 11 Comparison of Non-Dimensional X-Reynolds Stress Meshes of k-ε Model

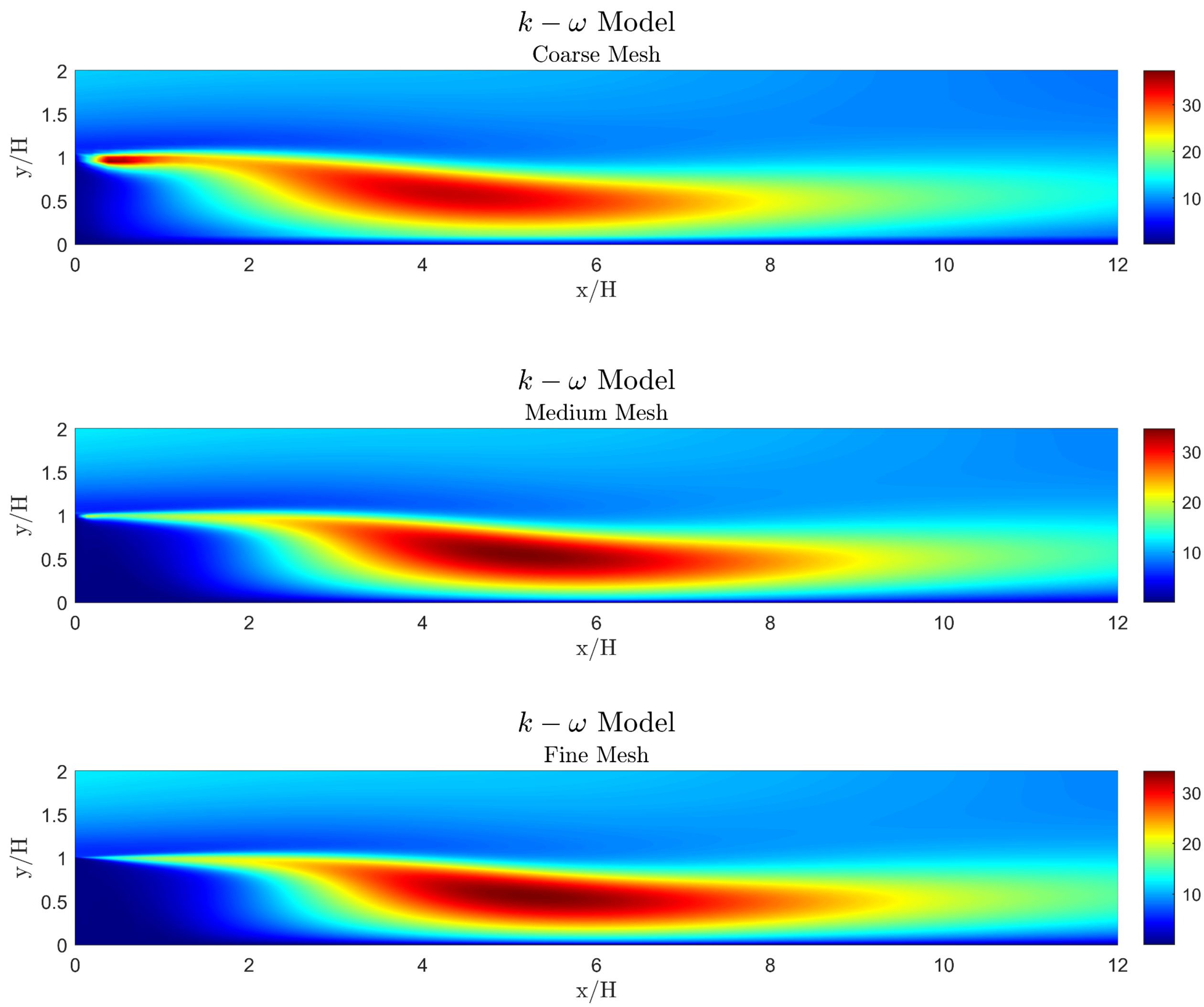

Fig. 12 Comparison of Non-Dimensional X-Reynolds Stress Meshes of k-ω Model

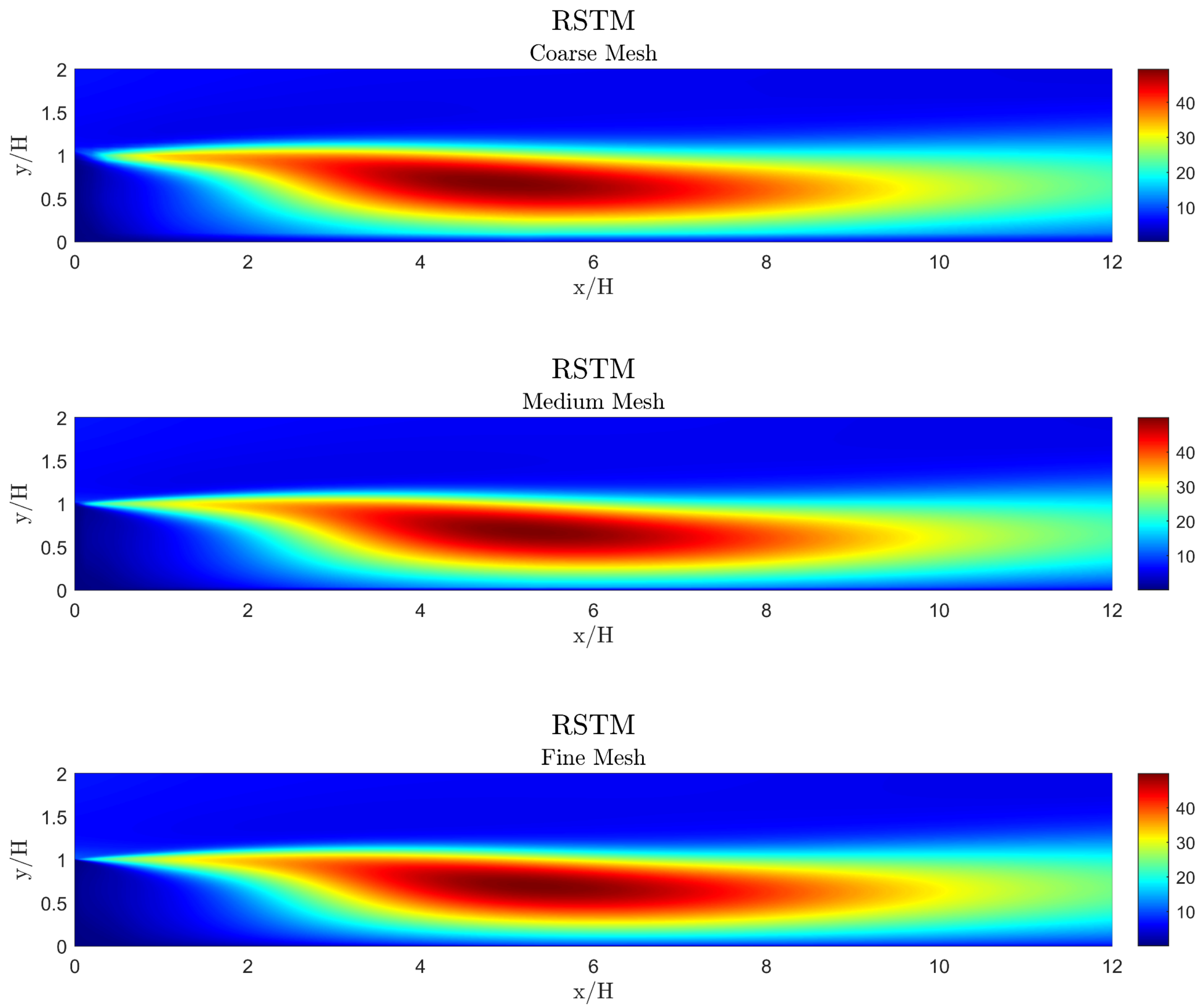

Fig. 13 Comparison of Non-Dimensional X-Reynolds Stress Meshes of RSTM Model

3.4 Non-Dimensional Y-Reynolds Stress Results

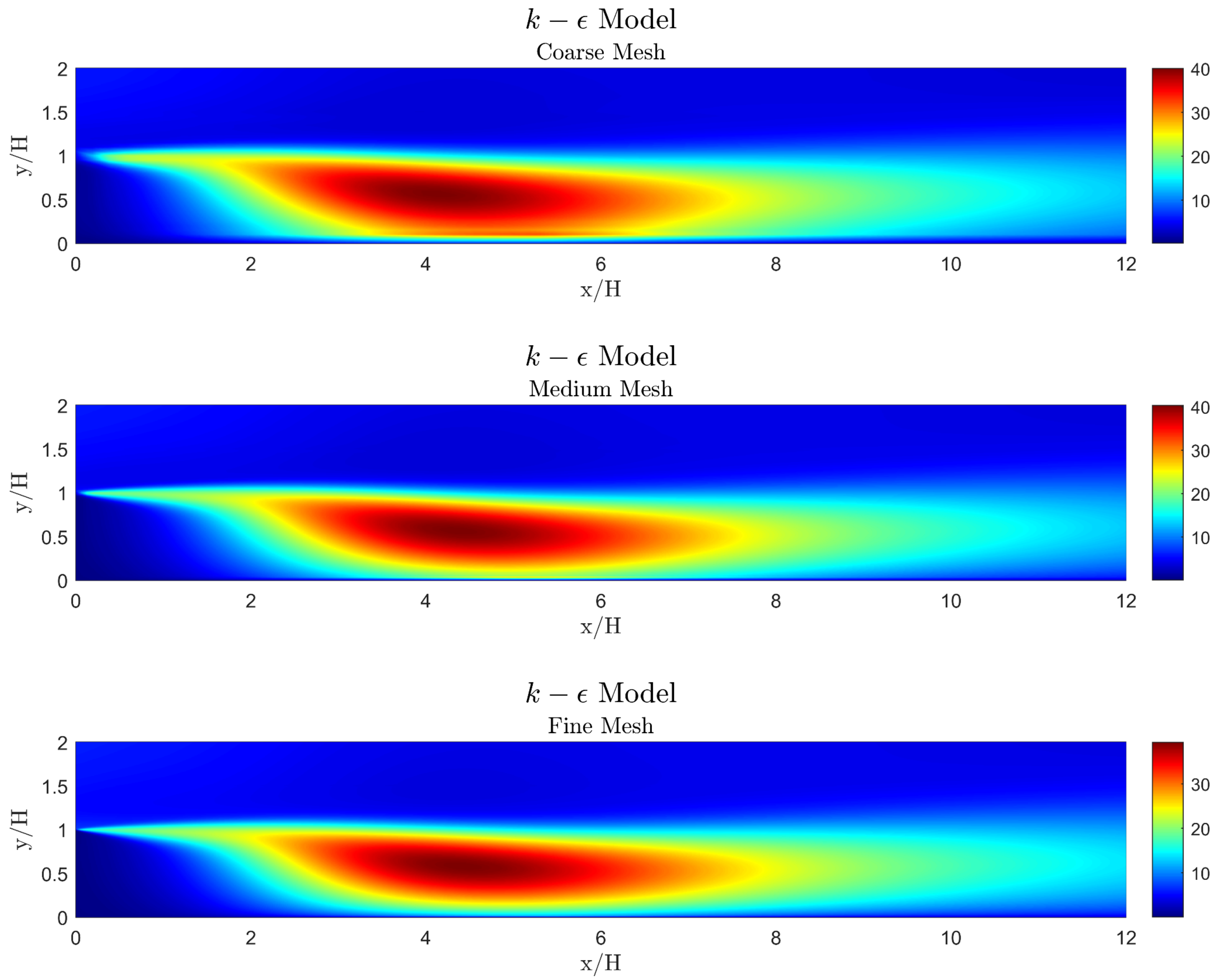

Fig. 14 Comparison of Non-Dimensional Y-Reynolds Stress Meshes of k-ε Model

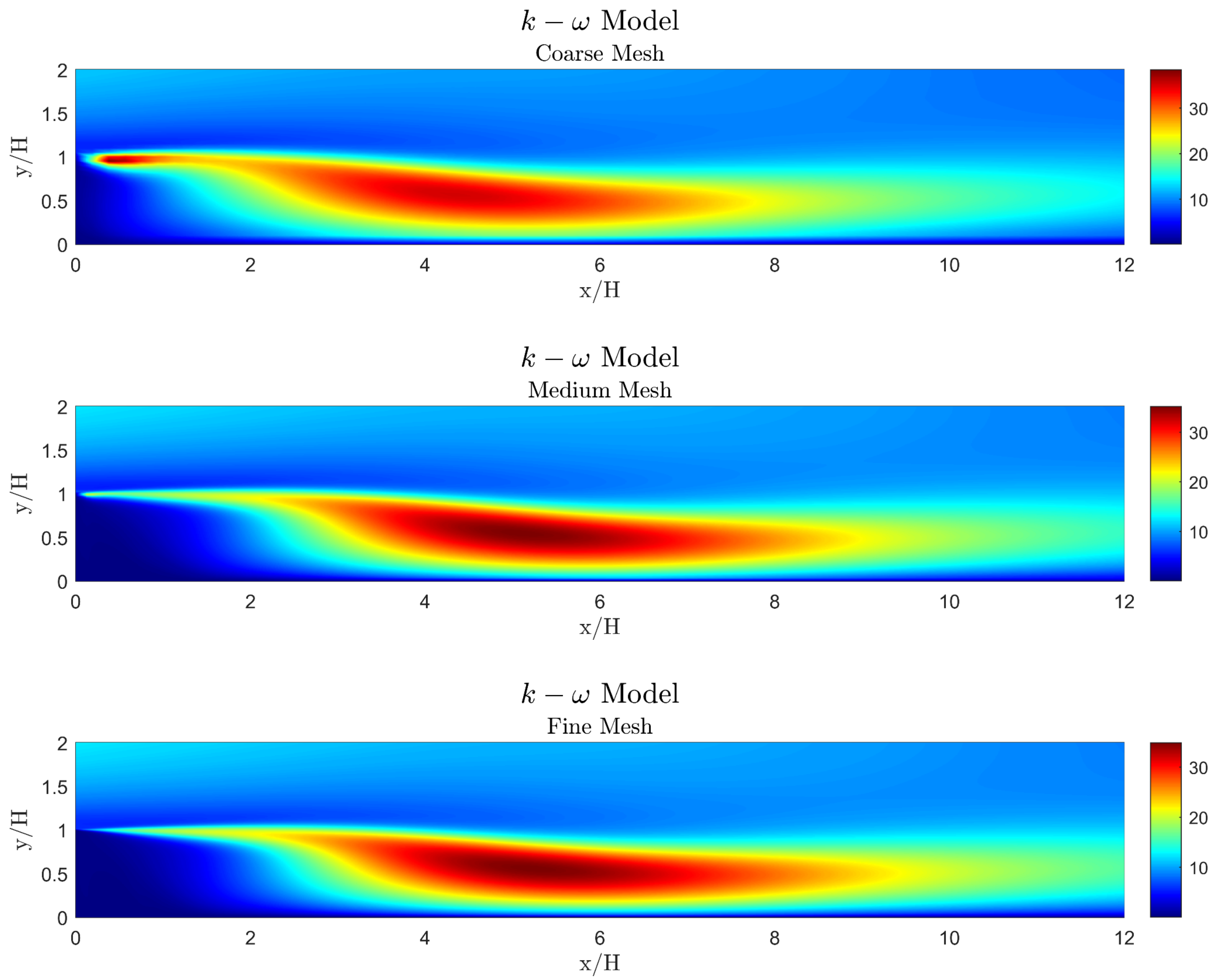

Fig. 15 Comparison of Non-Dimensional Y-Reynolds Stress Meshes of k-ω Model

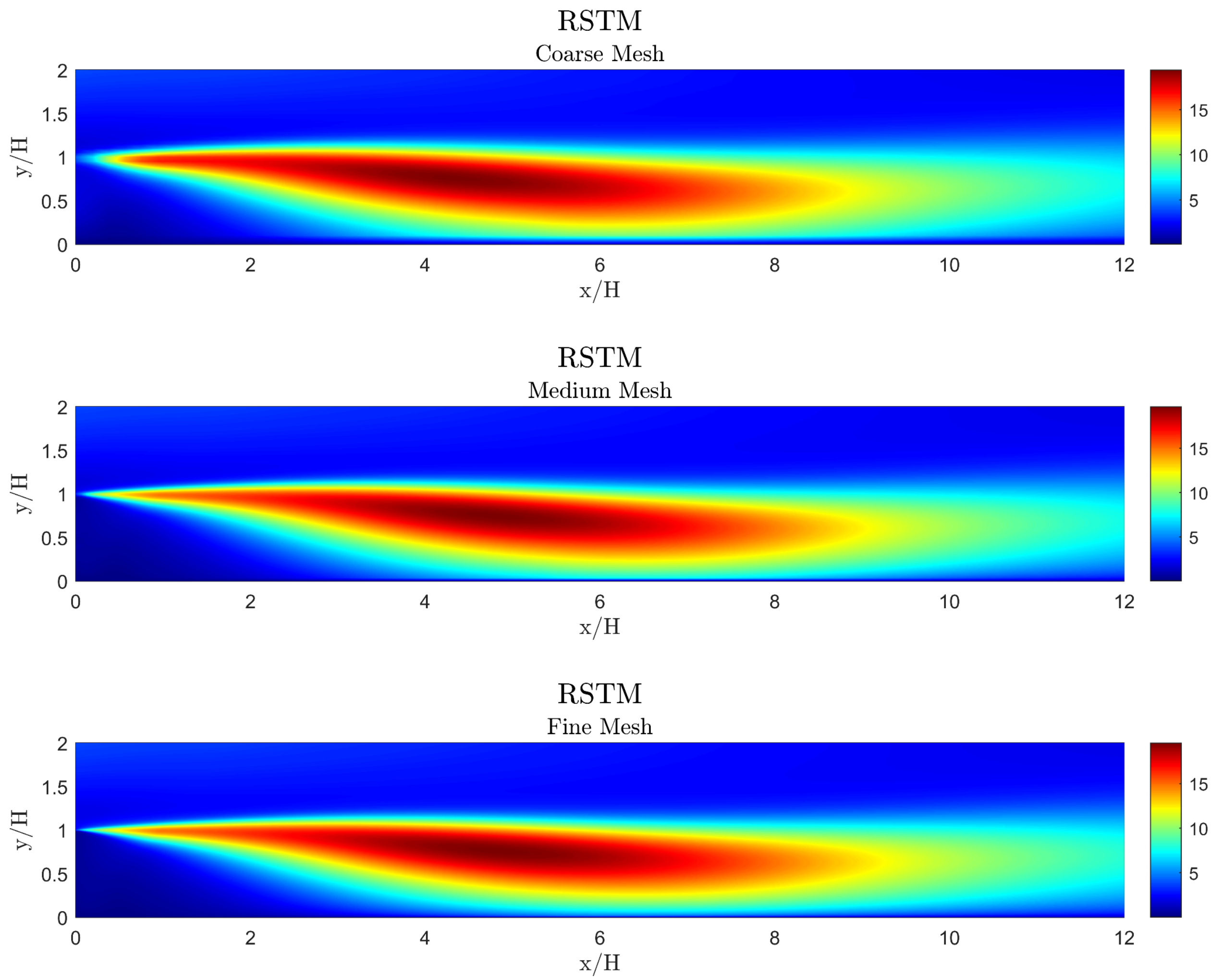

Fig. 16 Comparison of Non-Dimensional Y-Reynolds Stress Meshes of RSTM Model

3.5 Non-Dimensional XY-Reynolds Stress Results

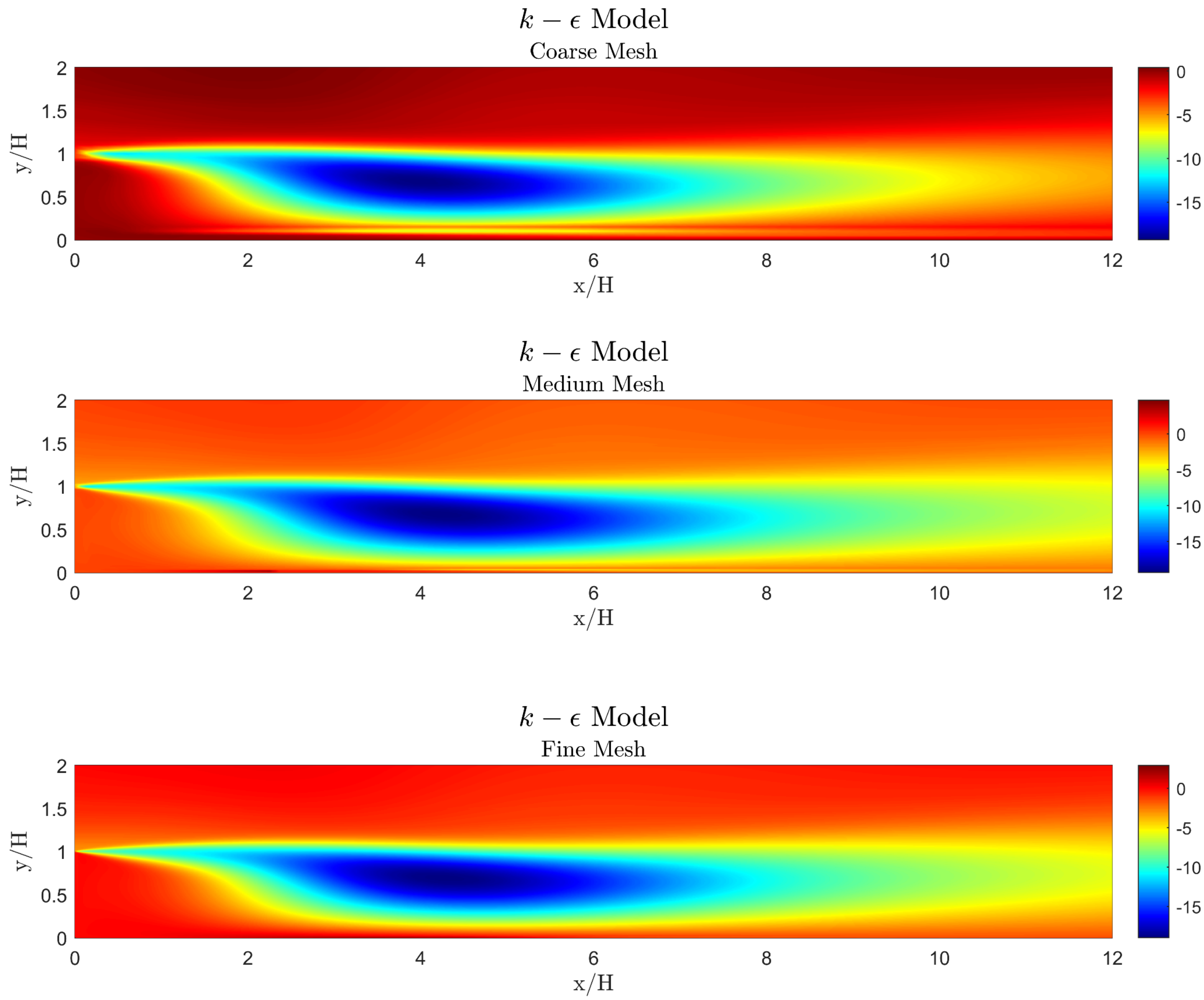

Fig. 17 Comparison of Non-Dimensional XY-Reynolds Stress Meshes of k-ε Model

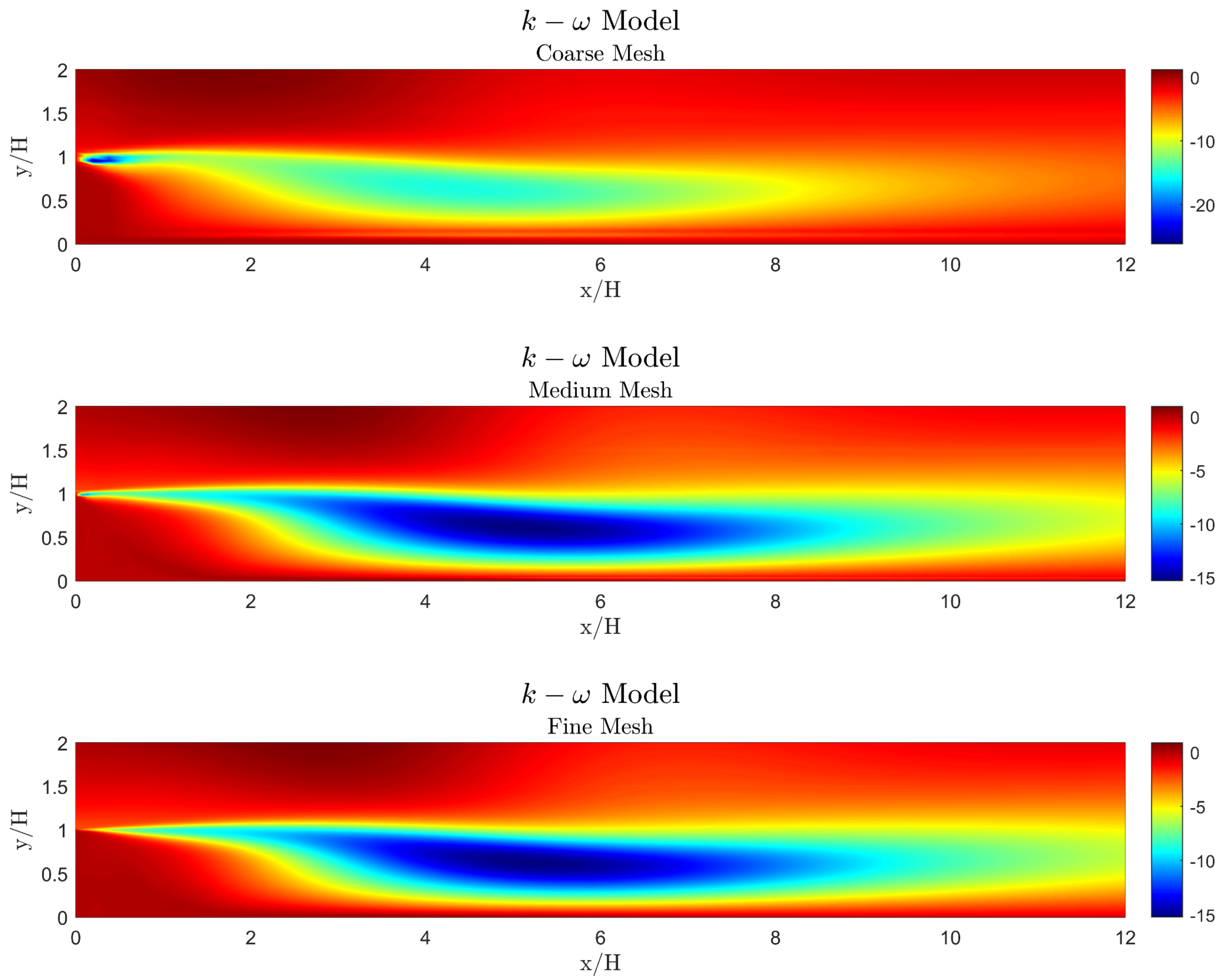

Fig. 18 Comparison of Non-Dimensional XY-Reynolds Stress Meshes of k-ω Model

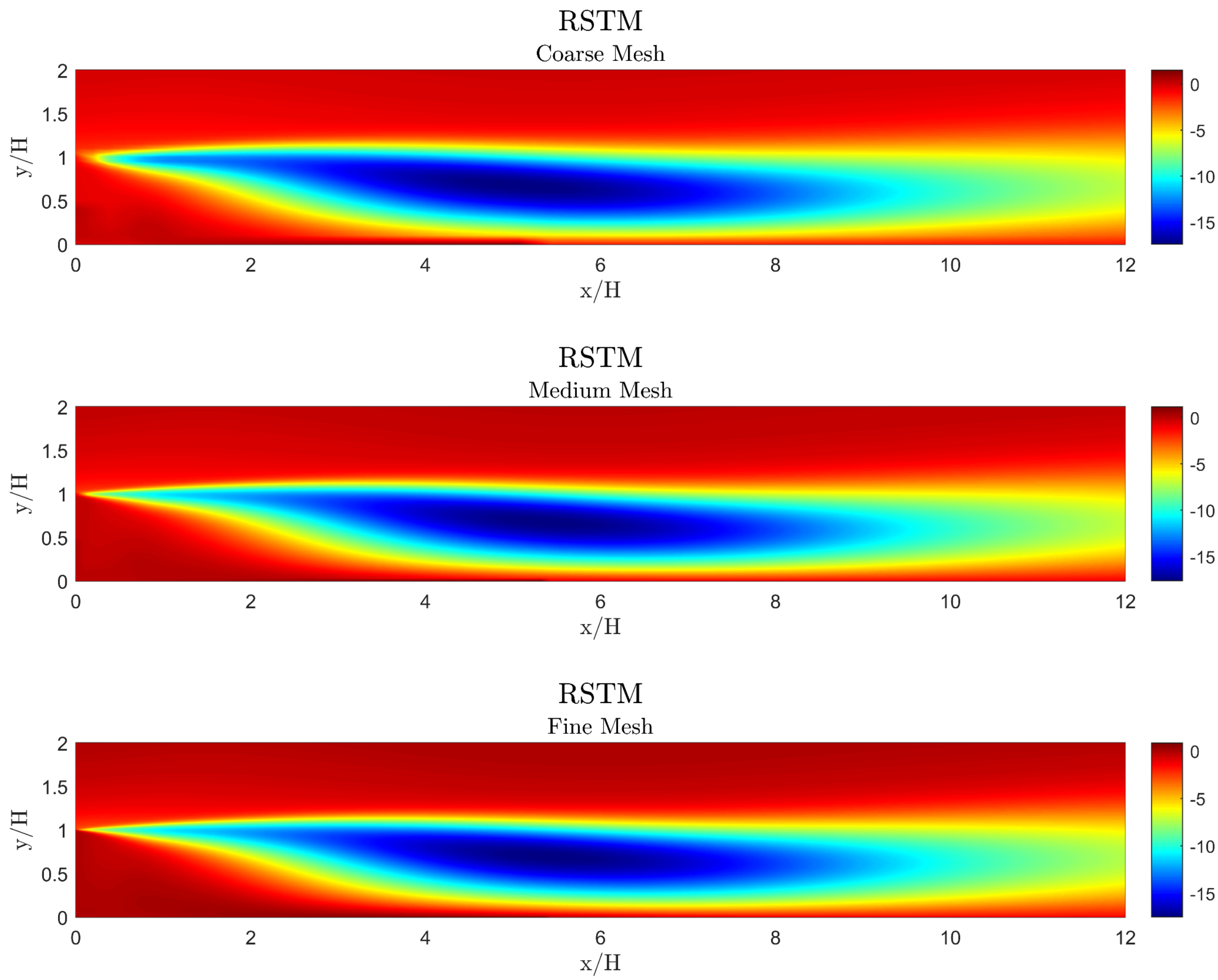

Fig. 19 Comparison of Non-Dimensional XY-Reynolds Stress Meshes of RSTM Model

3.6 Non-Dimensional Turbulent Kinetic Energy Results

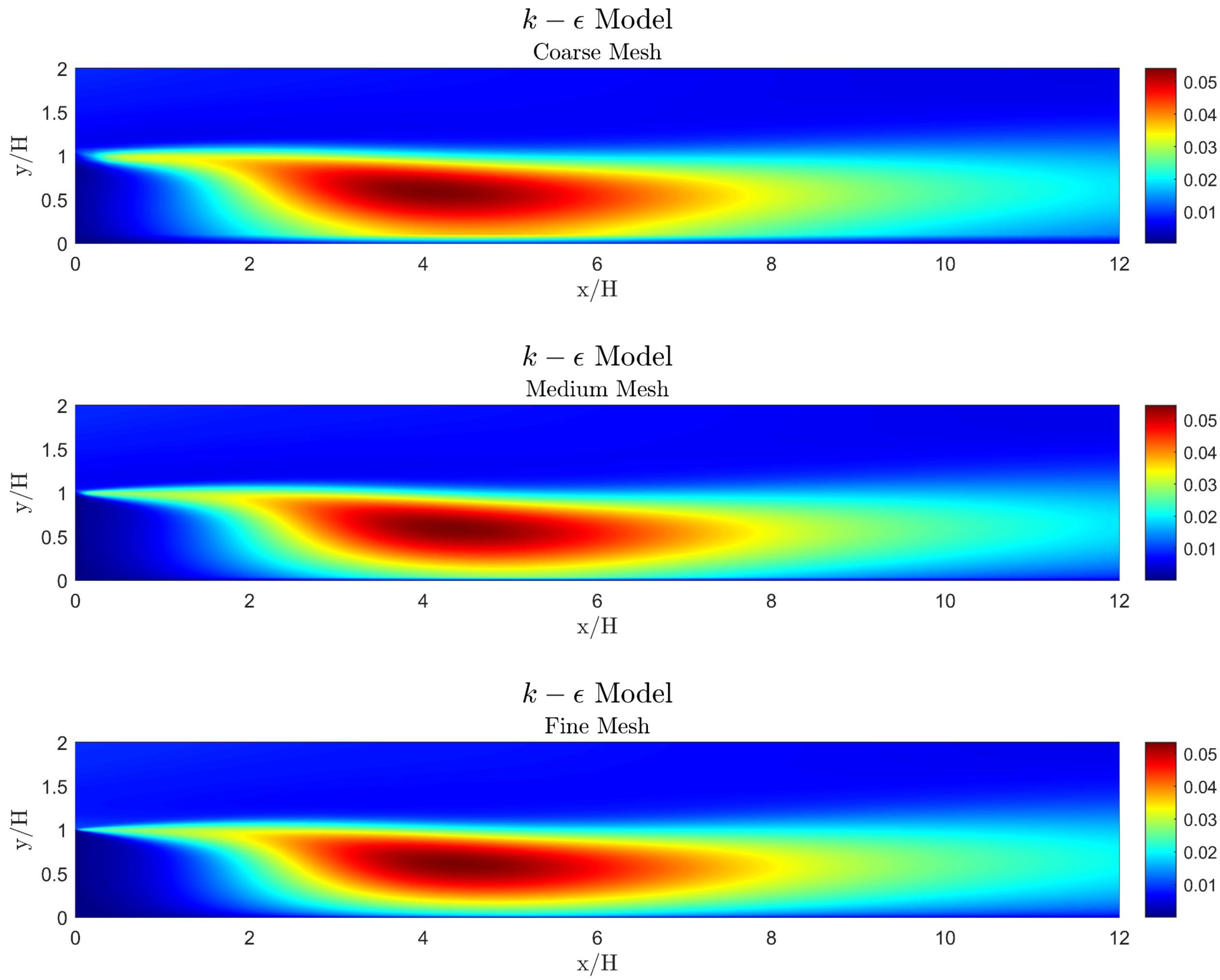

Fig. 20 Comparison of Non-Dimensional Turbulent Kinetic Energy Meshes of k-ε Model

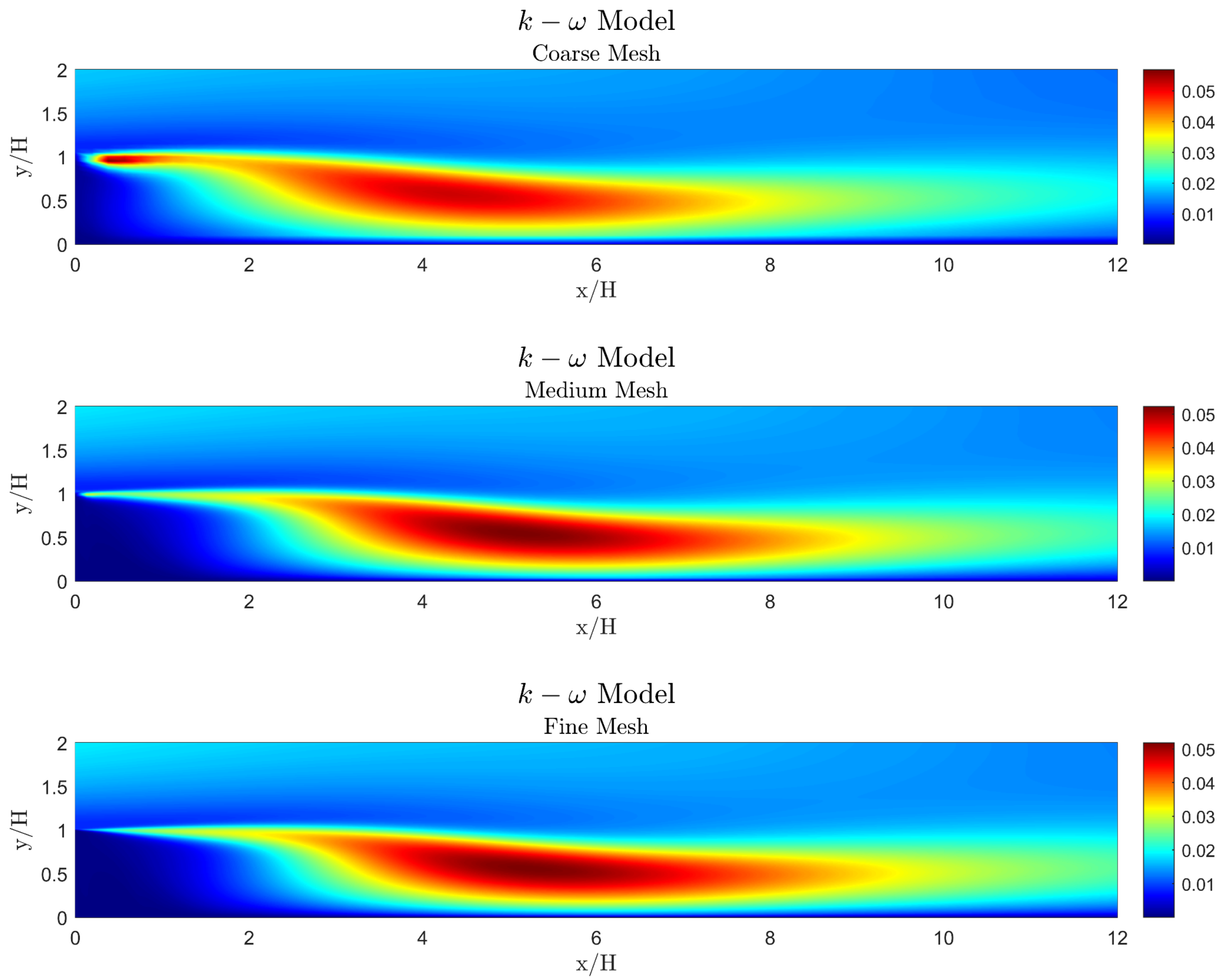

Fig. 21 Comparison of Non-Dimensional Turbulent Kinetic Energy Meshes of k-ω Model

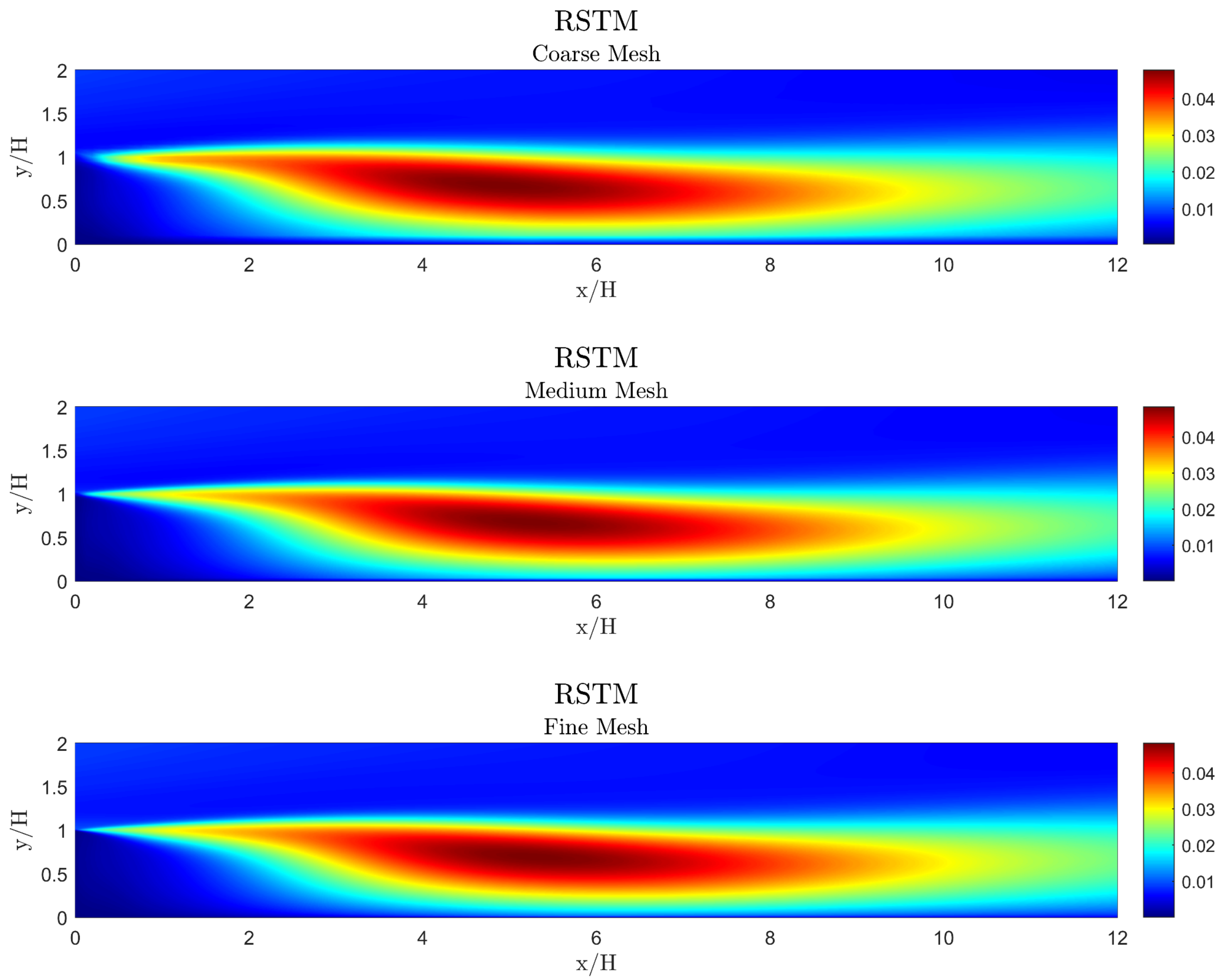

Fig. 22 Comparison of Non-Dimensional Turbulent Kinetic Energy Meshes of RSTM Model

3.7 Non-Dimensional Turbulent Dissipation Rate Results

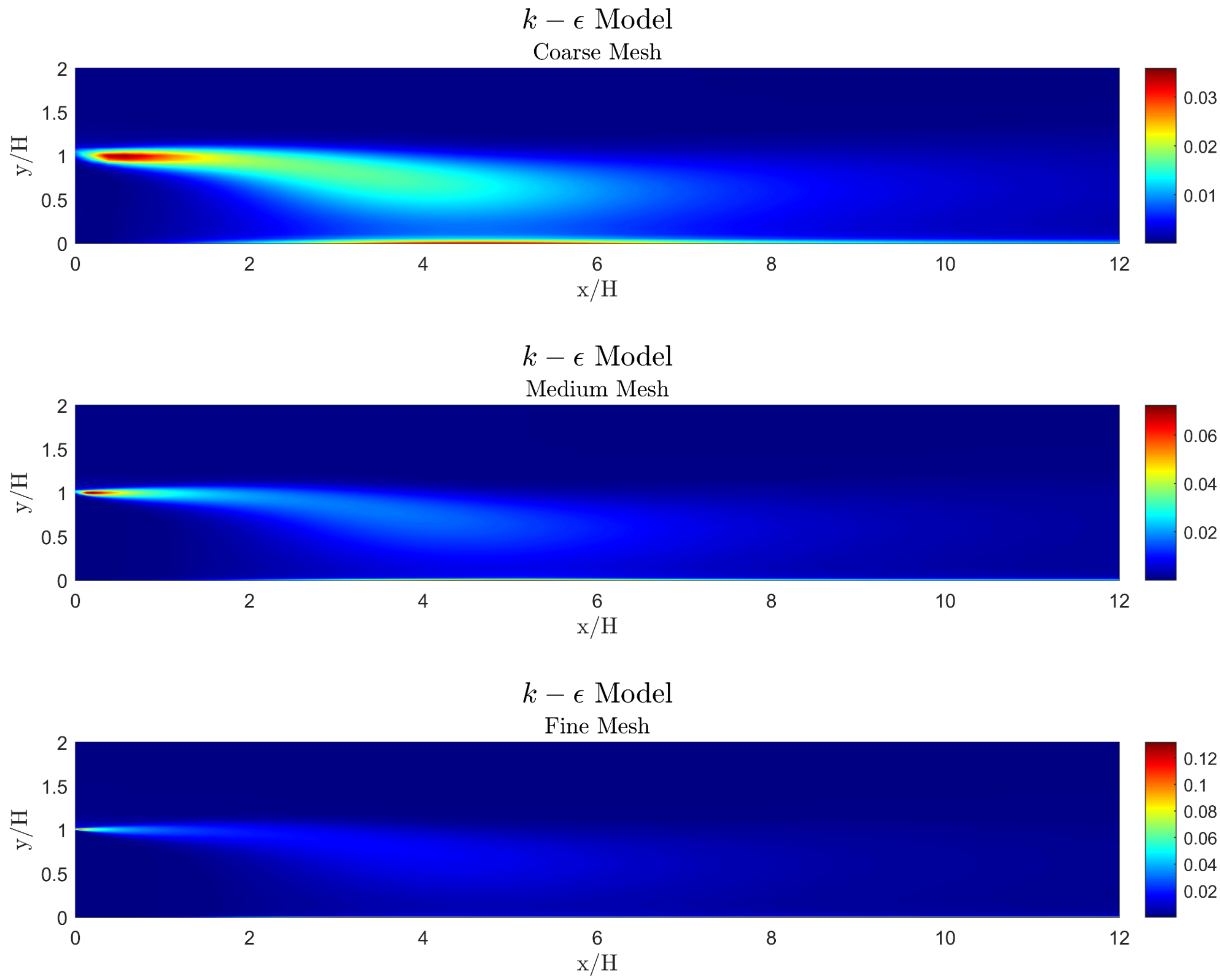

Fig. 23 Comparison of Non-Dimensional Turbulent Dissipation Rate Meshes of k-ε Model

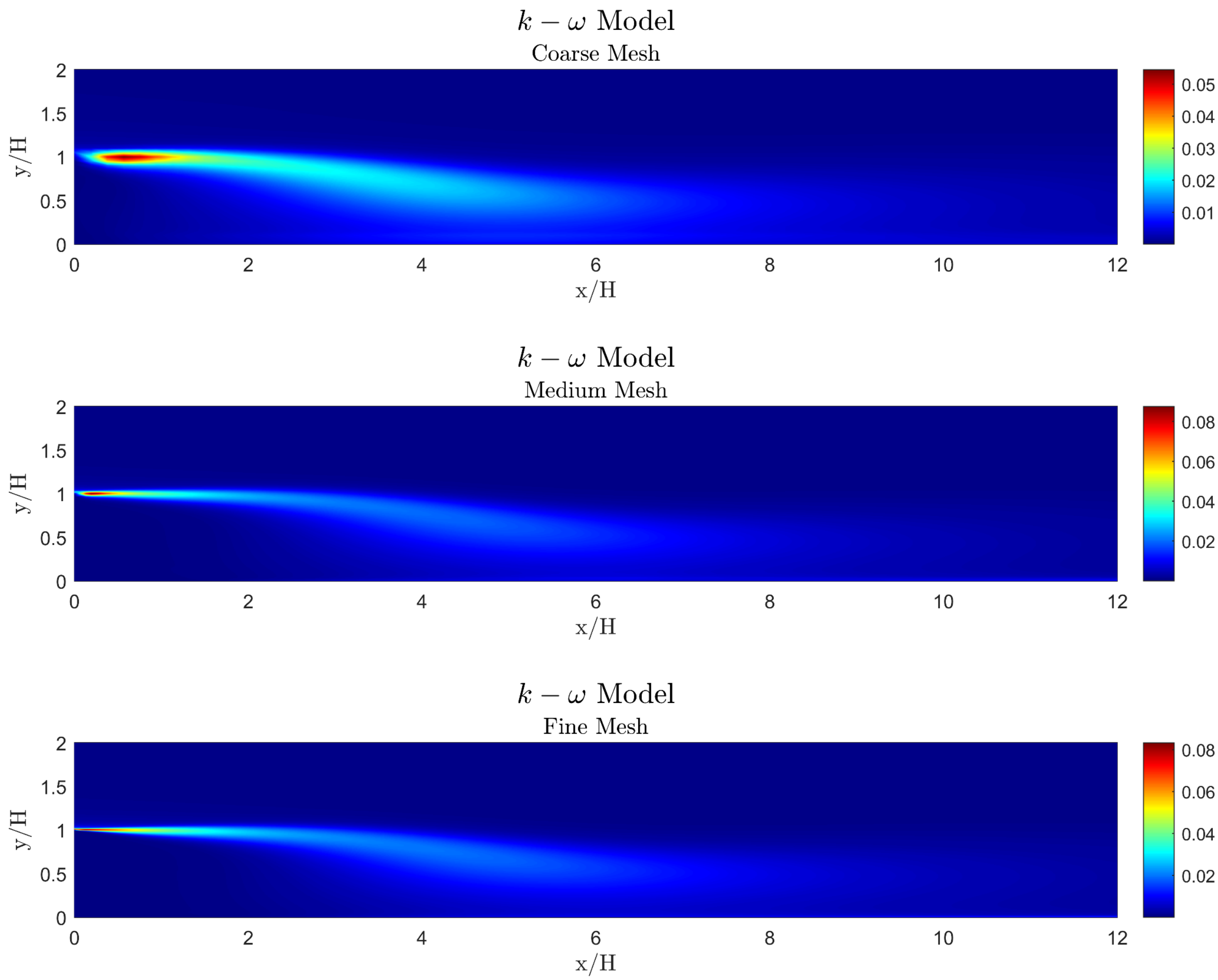

Fig. 24 Comparison of Non-Dimensional Turbulent Dissipation Rate Meshes of k-ω Model

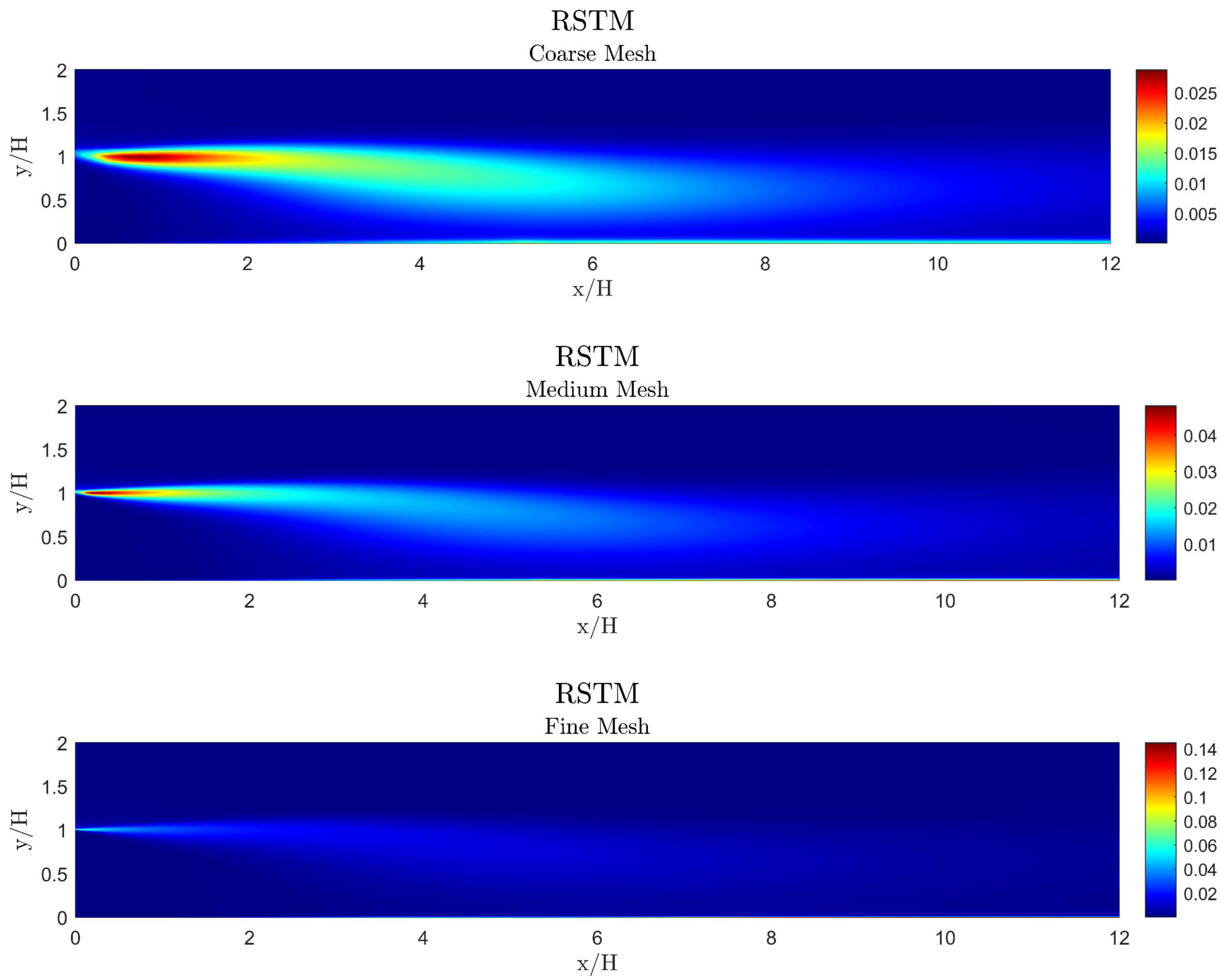

Fig. 25 Comparison of Non-Dimensional Turbulent Dissipation Rate Meshes of RSTM Model

3.8 Discussion

Question 1: Do you see significant differences in any of these color fields among the three meshes (coarse, medium, fine); if so, discuss what you believe are the most important differences you see?

Answer 1: After completing the plots from sections 3.1 through 3.7, there are a few key differences between the three meshes.

In section 3.1, for all three models, the coarse mesh starts with a sharp line down at (0,1), gets less sharp in medium, and then becomes a smooth line/curve in the fine mesh. Otherwise, increasing the mesh didn't have any drastic effects.

In Section 3.2, for the k-ε plots, there is a change in the contour structure on the lower left of the plot. The darkness of the red decreases from coarse to medium, but then increases again in the fine mesh. The overall size of the red increases from coarse to fine as well, which is interesting. This makes sense that as the mesh is greater refined, the smaller values of velocity from the flow of the step are being shown. This is expressed even more dramatically in the k-ω plots. In the coarse, it starts with a small condensed area of "higher" velocity values but then as the mesh is refined it becomes slightly lower and spread out across a much larger area of the step. What is particularly interesting is that there are small concentrations of negative velocity just below the areas of positive velocity, which appears to be boundary layer separation at the corner eddy based on Fig. 1. Finally in RSTM, similar results are shown where the area of the velocity increases as the mesh becomes more refined and the overall values of the velocity slightly decrease. For all the plots, it appears to be a center area of negative velocities, detailing the recirculating flow area in Fig. 1.

In Sections 3.3 and 3.4, there were slight changes in the area of higher values of Reynolds stress. As the mesh becomes more refined, for k-ω, the amount of higher values is increasing around x/H = 5. Additionally, the beginning of the flow has a concentrated area of high Reynolds stress values but this dissipates as the mesh becomes more refined.

In Section 3.5, the top of the contour plots show a change from zero to negative Reynolds stress. This is consistent as the mesh becomes more refined, the values that are "zero" begin to be expressed as small values of stress, in this case negative.

In Section 3.6, there are similarities between sections 3.3 and 3.4, where k-ω details high values of energy increasing as the mesh goes from coarse to fine and that there is a concentration at the beginning of the flow in the coarse but it is refined out as the mesh gets better.

In section 3.7, each model details an initial small value of dissipation rate but over a "large" area. This then changes as the mesh refines to be a much smaller area but with a slightly increased value directly at the start of the step at (0,1) of the x/H and y/H.

In summary, there are a variety of differences between the mesh sizes. It is very clear that as the mesh is refined, the resolution is increased, which is especially noticeable near the walls of the backward-facing step. Additionally, values became much more refined from starting at zero to small values, most likely due to the increase in nodes near those positions. The most noticeable is lines becoming smooth and transitions across x/H or y/H becoming less noticeable as they blend. However, as refining the mesh returns more detailed results, it does exponentially increase the time of the simulation as well.

4. Effects of Different Turbulence Models (Fine Mesh)

4.1 Non-Dimensional Mean X-Velocity Results

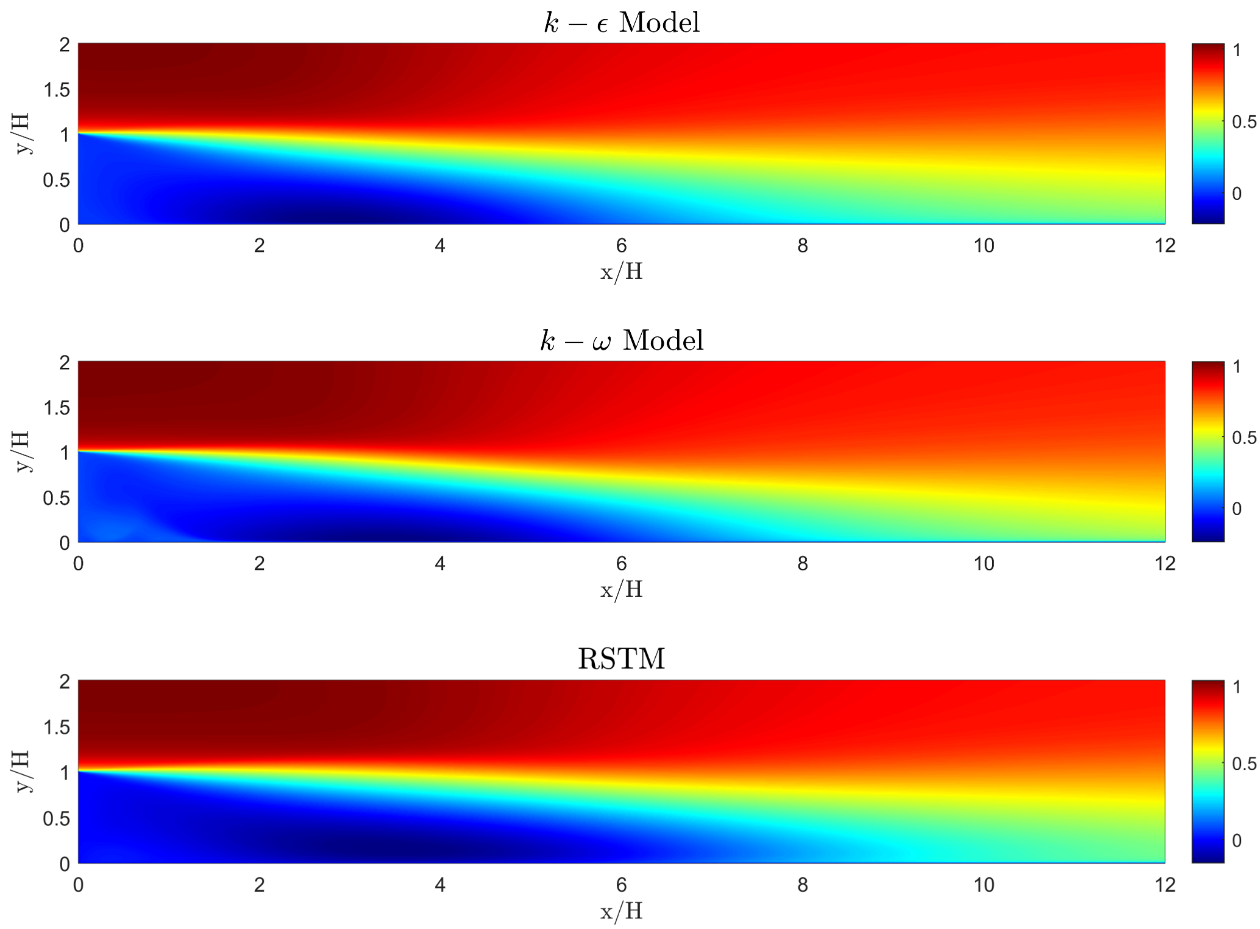

Fig. 26 Comparison of Turbulence Models for Non-Dimensional Mean X-Velocity

4.2 Non-Dimensional Mean Y-Velocity Results

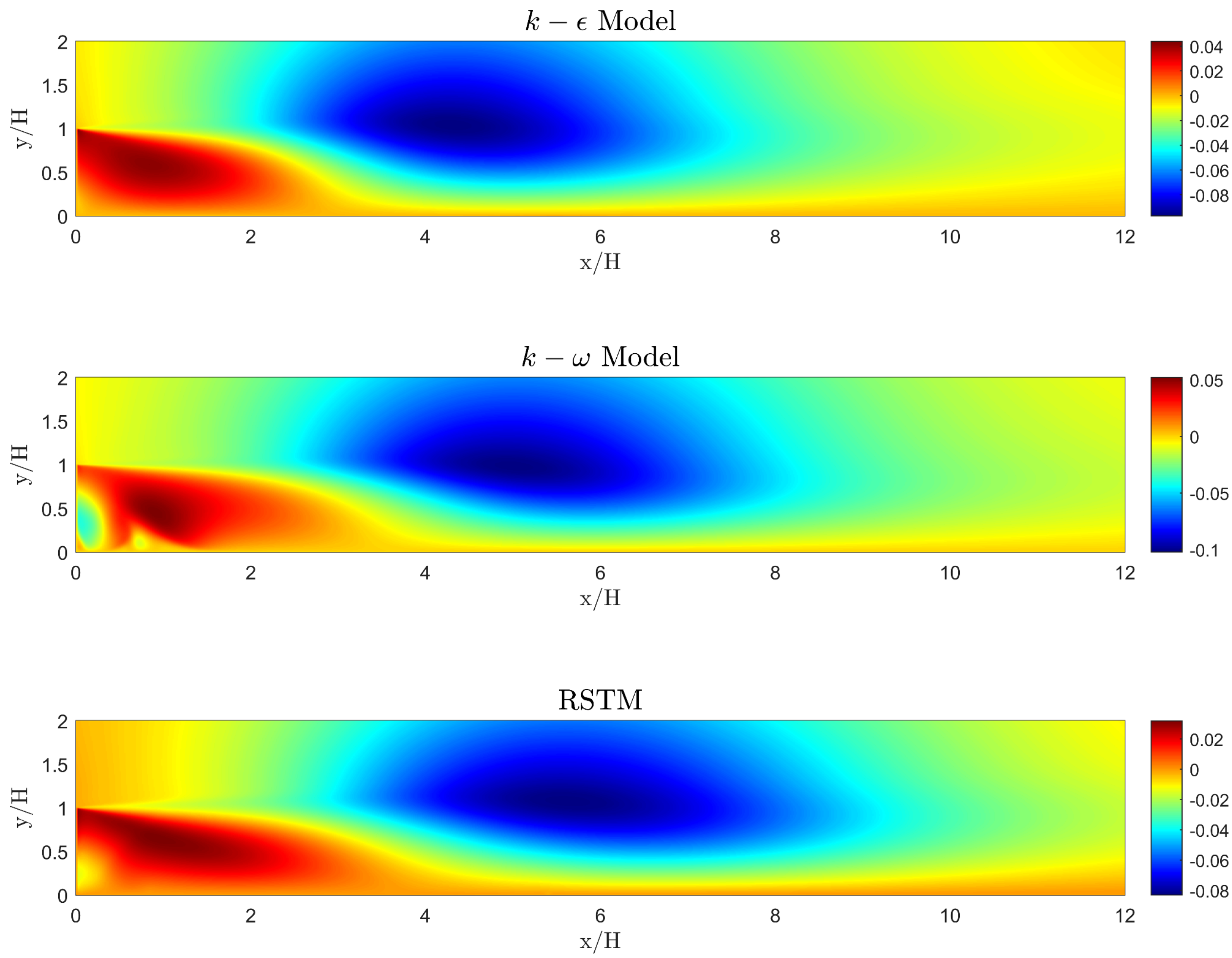

Fig. 27 Comparison of Turbulence Models for Non-Dimensional Mean Y-Velocity

4.3 Non-Dimensional X-Reynolds Stress Results

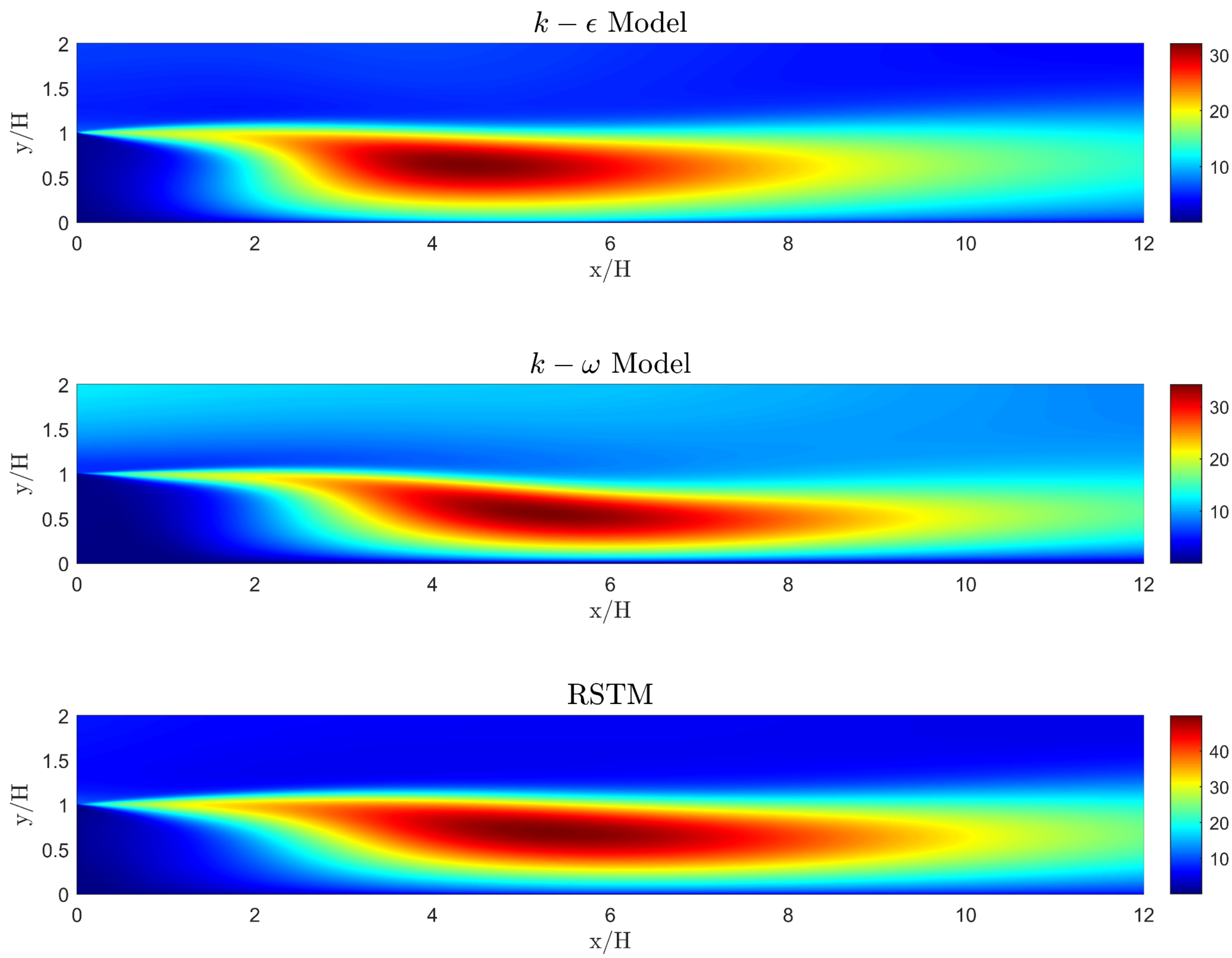

Fig. 28 Comparison of Turbulence Models for Non-Dimensional X-Reynolds Stress

4.4 Non-Dimensional Y-Reynolds Stress Results

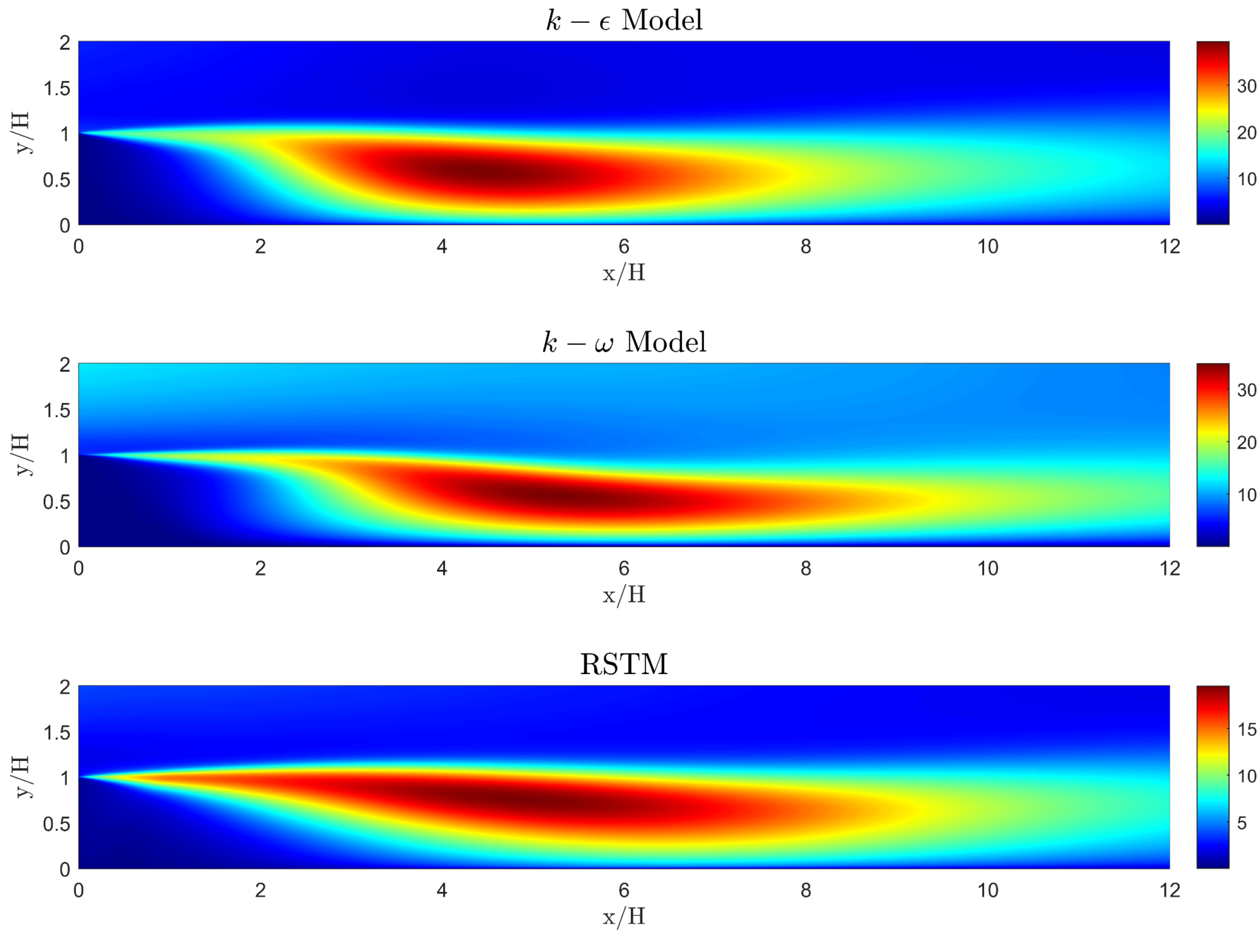

Fig. 29 Comparison of Turbulence Models for Non-Dimensional Y-Reynolds Stress

4.5 Non-Dimensional XY-Reynolds Stress Results

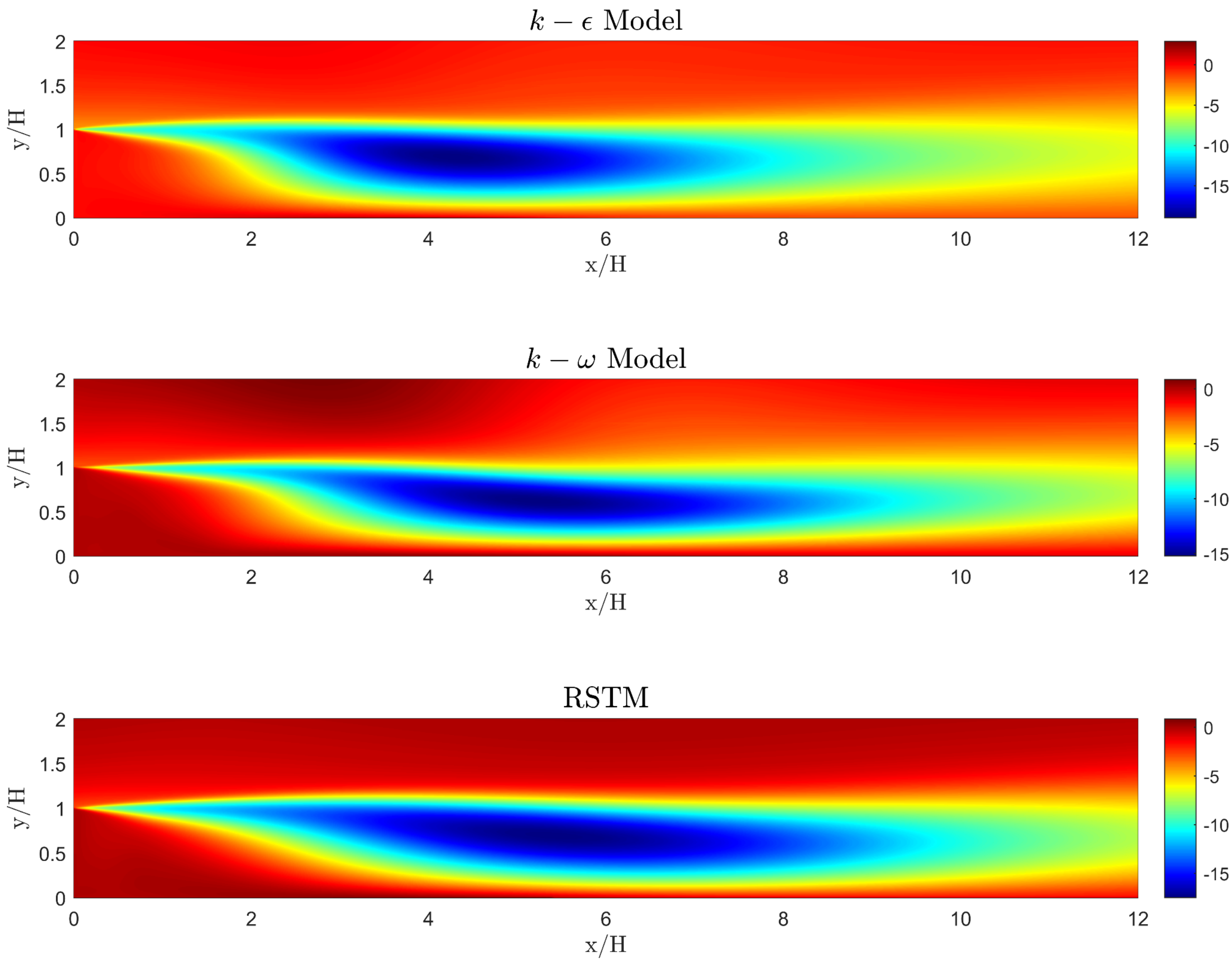

Fig. 30 Comparison of Turbulence Models for Non-Dimensional XY-Reynolds Stress

4.6 Non-Dimensional Turbulent Kinetic Energy Results

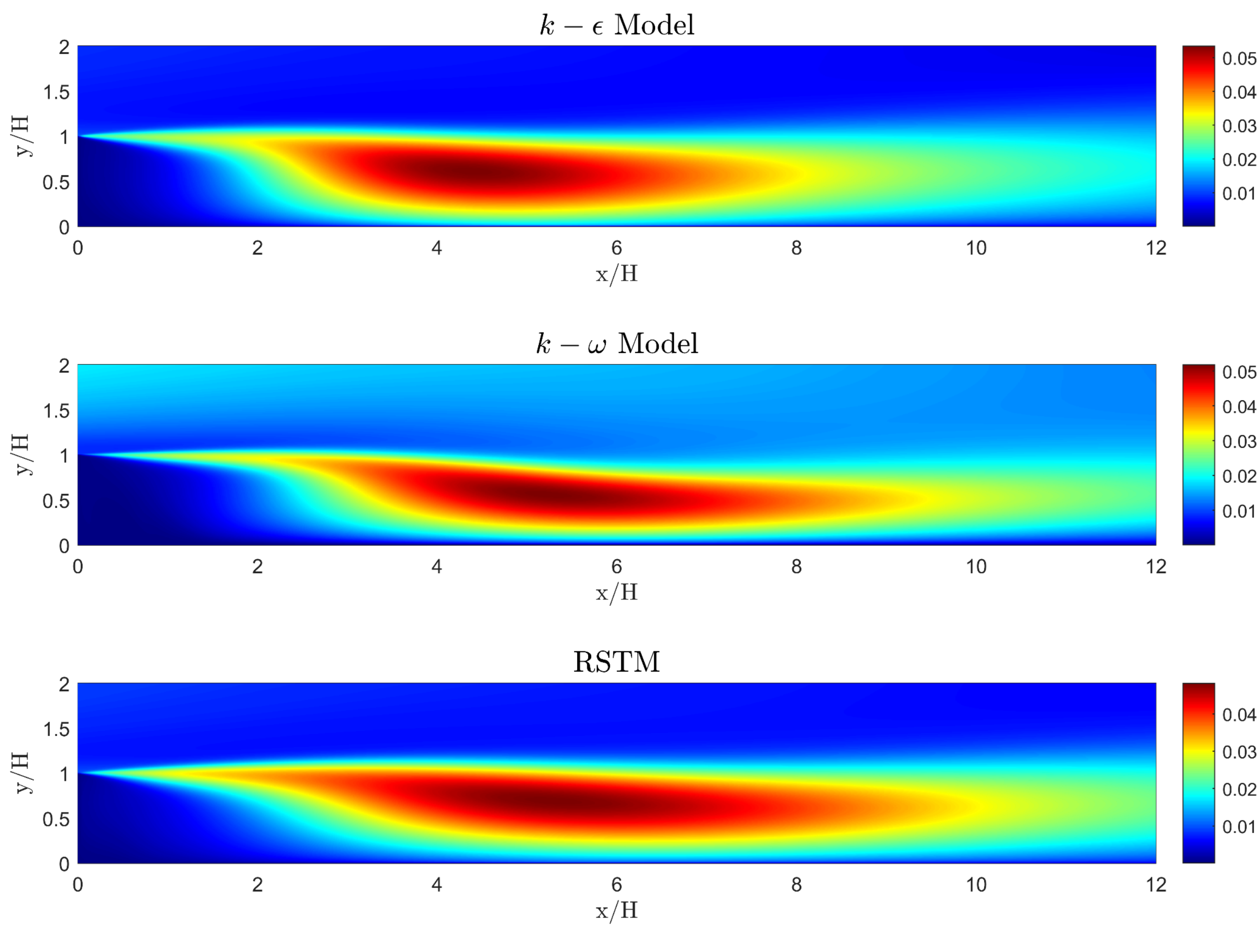

Fig. 31 Comparison of Turbulence Models for Non-Dimensional Turbulent Kinetic Energy

4.7 Non-Dimensional Turbulent Dissipation Rate Results

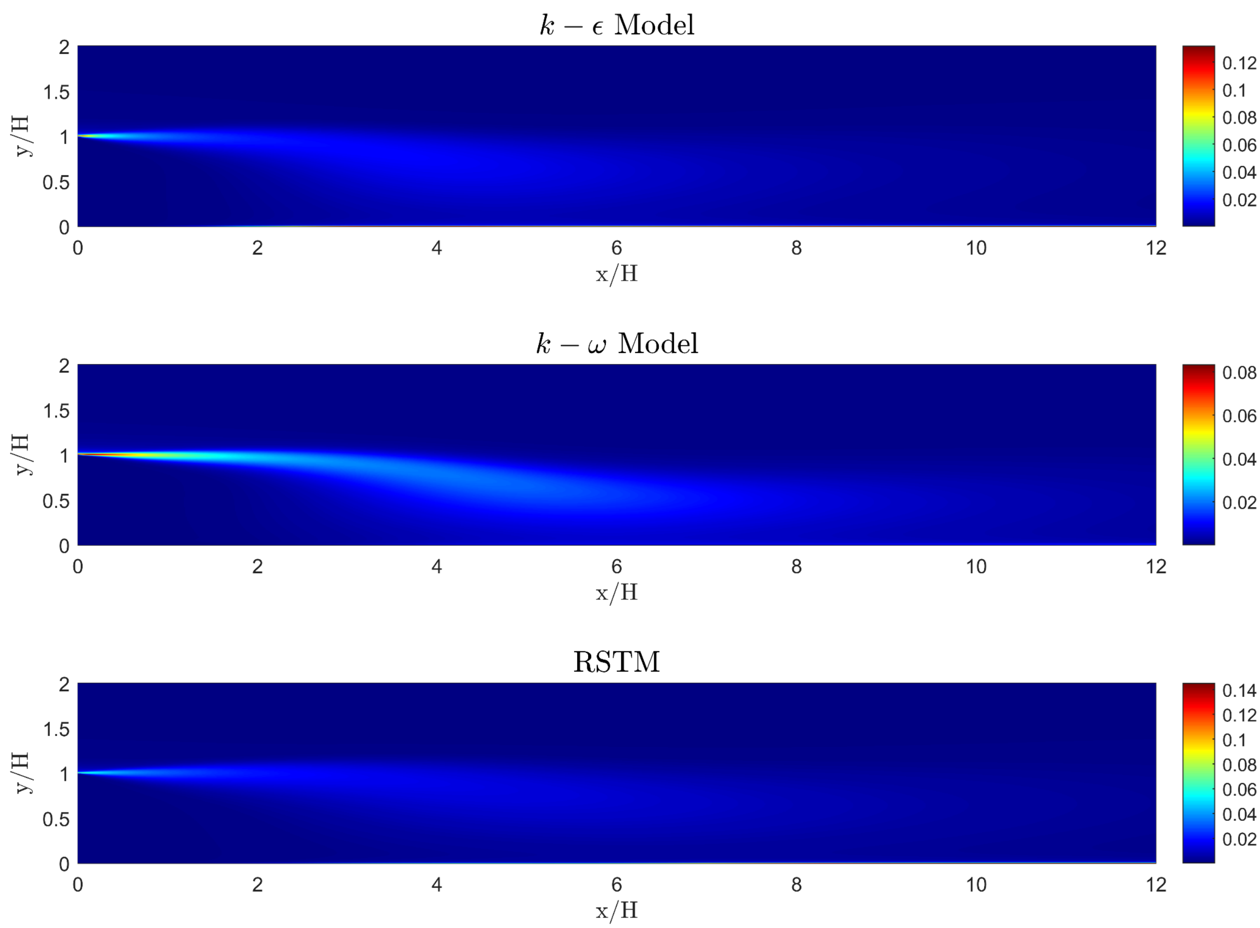

Fig. 32 Comparison of Turbulence Models for Non-Dimensional Turbulent Dissipation Rate

4.8 DISCUSSION

Question 2: Do you see significant differences in any of the color fields among the three turbulence models (k-ε, k-ω, RSTM); if so, discuss what you believe are the most important differences you see?

Answer 2: There are fairly significant differences between the contour plots between the three models. As noted in the lectures, the k-ϵ model performs better away from the walls, and k-ω performs better near the walls (more on this in Answer 7), which is important to note as discussing differences below.

In section 4.2, there is some variation between the models near the bottom wall of the contour plot. In the left corner, the k-ϵ model details some small positive velocities and a general small value along the entire bottom wall. However, k-ω shows a bubble of negative velocity directly at the left wall which then turns positive, similar to k-ε. This continues to the bottom wall but appears to have a smaller area than k-ε. For RSTM, it is a mixture of each of the previous models, where there are positive velocities but also at the left wall a small bubble of zero or slightly negative values of velocity. These velocity changes could be due to the order of the equations being solved at the wall with a potential boundary layer separation at the eddy corner (more on this in Answer 7). Additionally, for RSTM, the bottom wall is slightly larger and continues through the entire x/H with larger values of velocity.

In sections 4.3, 4.4, and 4.5 there is a slight difference in the overall structure of the contours with k-ω being a little compressed in the recirculation zone compared to k-ε and RSTM. Additionally, RSTM has a smoother transition around x/H = 2 than both k-ε and k-ω. Finally, k-ω has a larger value of Reynolds stress near the top of the contour plot, which could be a result of the model being slightly worse away from the walls.

In section 4.7, the models have vastly different plots in terms of concentration area and flow path. The k-ε and RSTM models have a small concentration at the step transition, where a larger amount of dissipation occurs. After this area, it gets quite small and the path behind it fades into zero. For k-ω, the area of concentration is a little longer and narrower, and the trail of positive dissipation continues downward near the bottom wall from x/H = 4 to x/H = 8. Additionally, the highest value of dissipation rate is smaller than both k-ε and RSTM.

In conclusion, there are differences between each of the models with the majority of the differences occurring in the vertical velocity and Reynolds stresses. The results confirmed there is a difference near and away from the wall for both k-ε and k-ω models. As well as significant downstream activity (x/H > 8) for RSTM that is not detailed in either k-ε or k-ω.

5. Simulation and Experimental Data Comparison

5.1 Non-Dimensional Mean X-Velocity Results

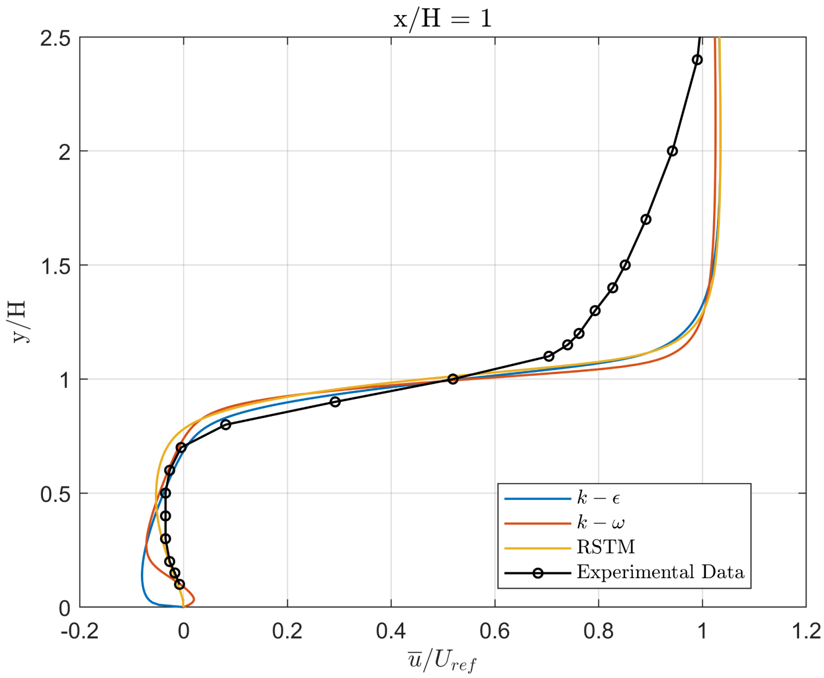

Fig. 33 Comparison of Turbulence Models for Non-Dimensional Mean X-Velocity at x/H = 1

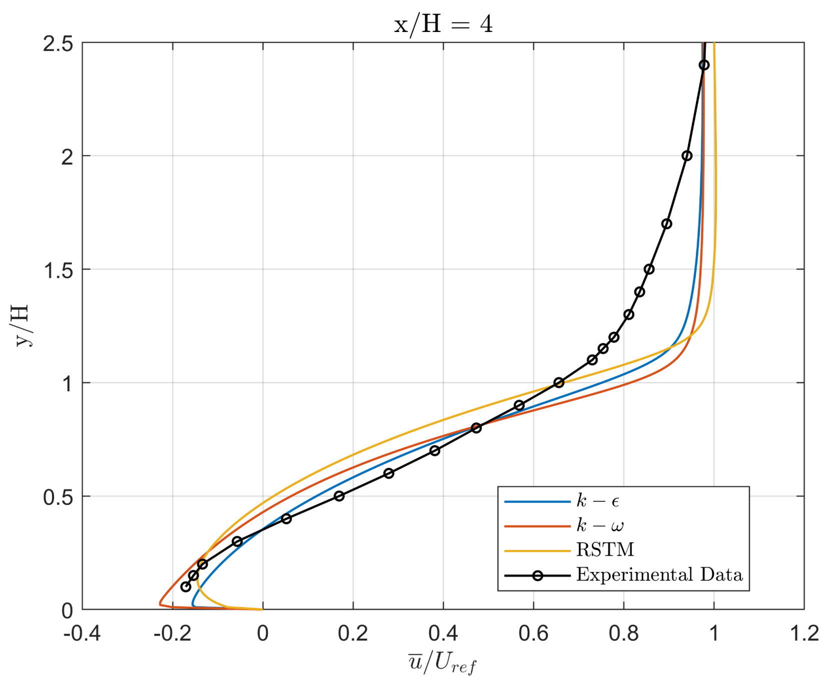

Fig. 34 Comparison of Turbulence Models for Non-Dimensional Mean X-Velocity at x/H = 4

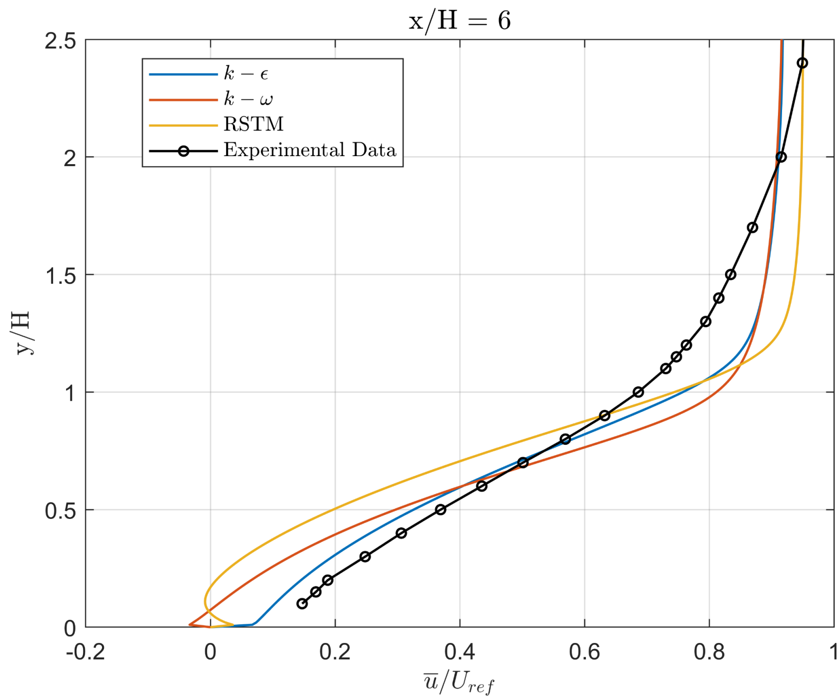

Fig. 35 Comparison of Turbulence Models for Non-Dimensional Mean X-Velocity at x/H = 6

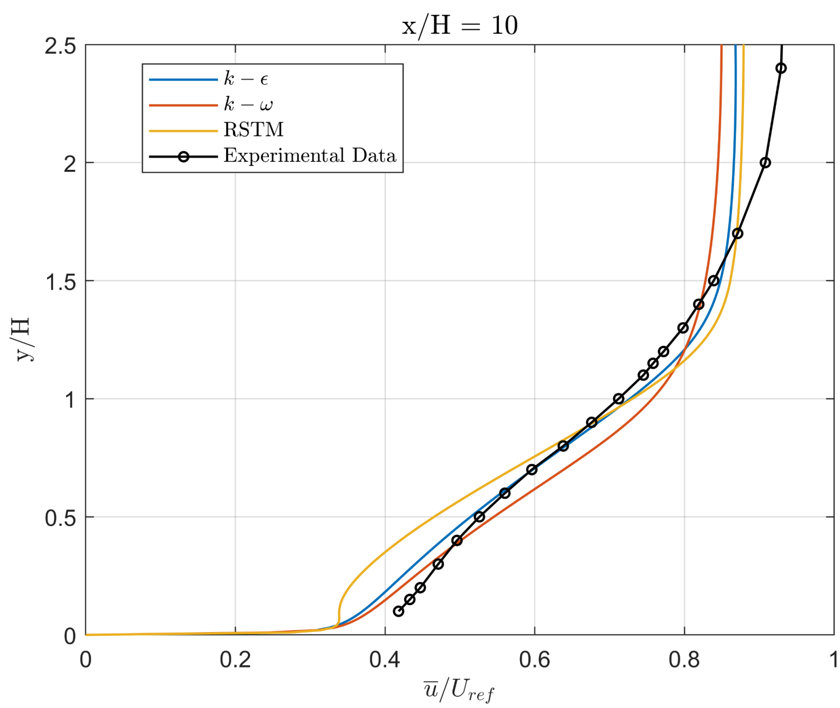

Fig. 36 Comparison of Turbulence Models for Non-Dimensional Mean X-Velocity at x/H = 10

5.2 Non-Dimensional Mean Y-Velocity Results

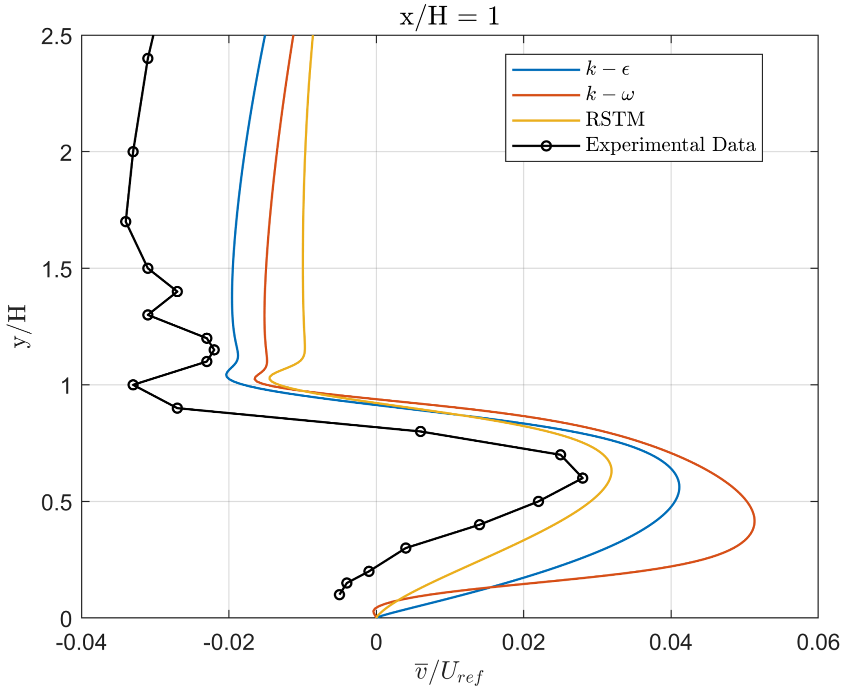

Fig. 37 Comparison of Turbulence Models for Non-Dimensional Mean Y-Velocity at x/H = 1

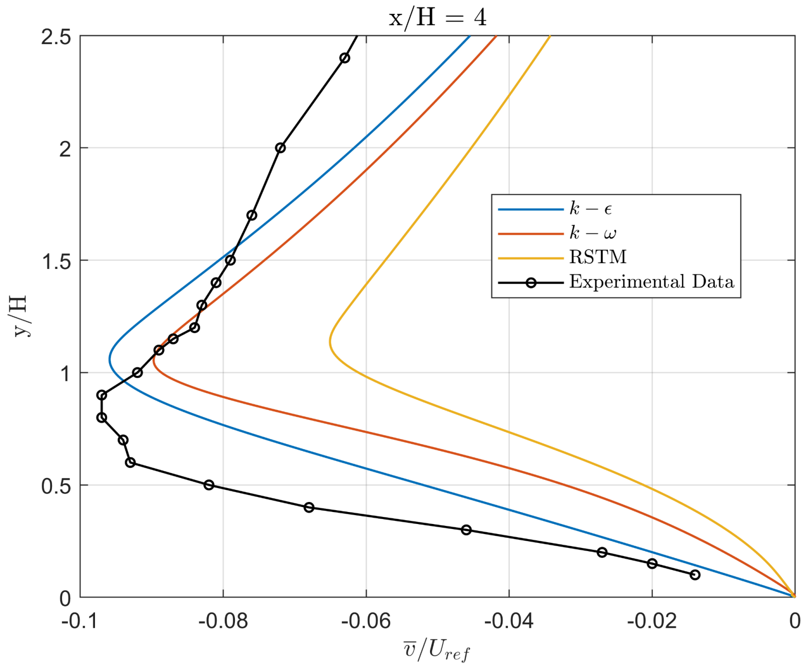

Fig. 38 Comparison of Turbulence Models for Non-Dimensional Mean Y-Velocity at x/H = 4

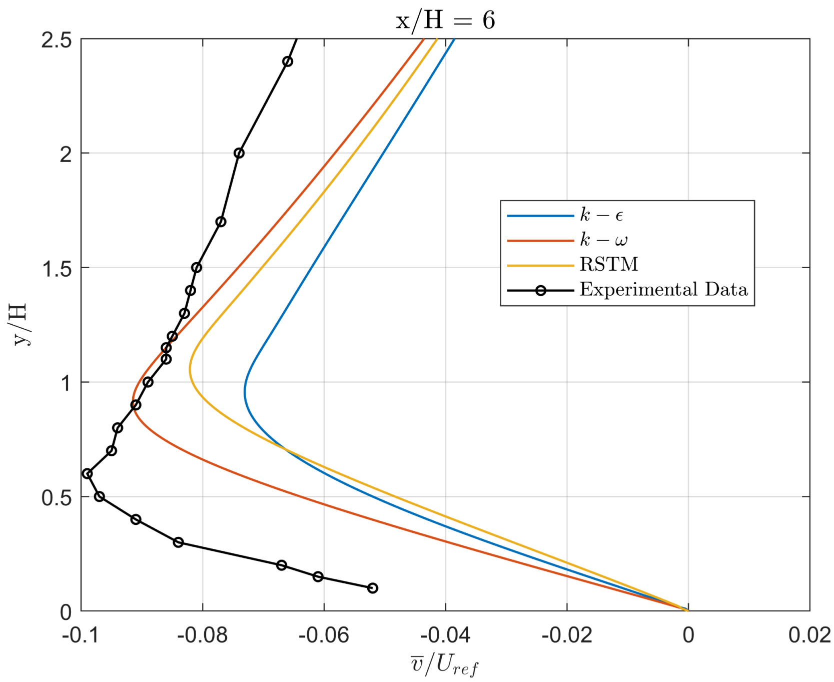

Fig. 39 Comparison of Turbulence Models for Non-Dimensional Mean Y-Velocity at x/H = 6

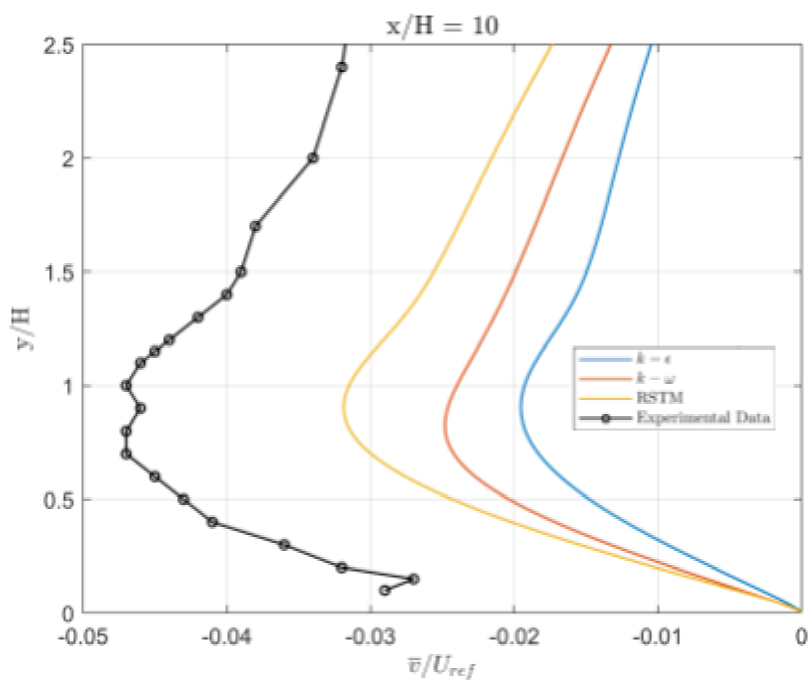

Fig. 40 Comparison of Turbulence Models for Non-Dimensional Mean Y-Velocity at x/H = 10

5.3 Non-Dimensional X-Reynolds Stress Results

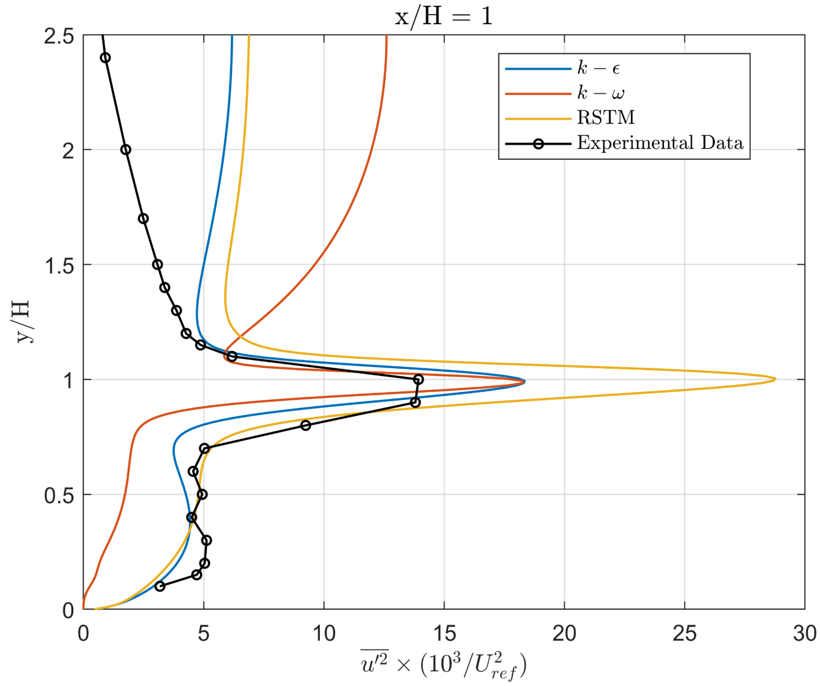

Fig. 41 Comparison of Turbulence Models for Non-Dimensional X-Reynolds Stress at x/H = 1

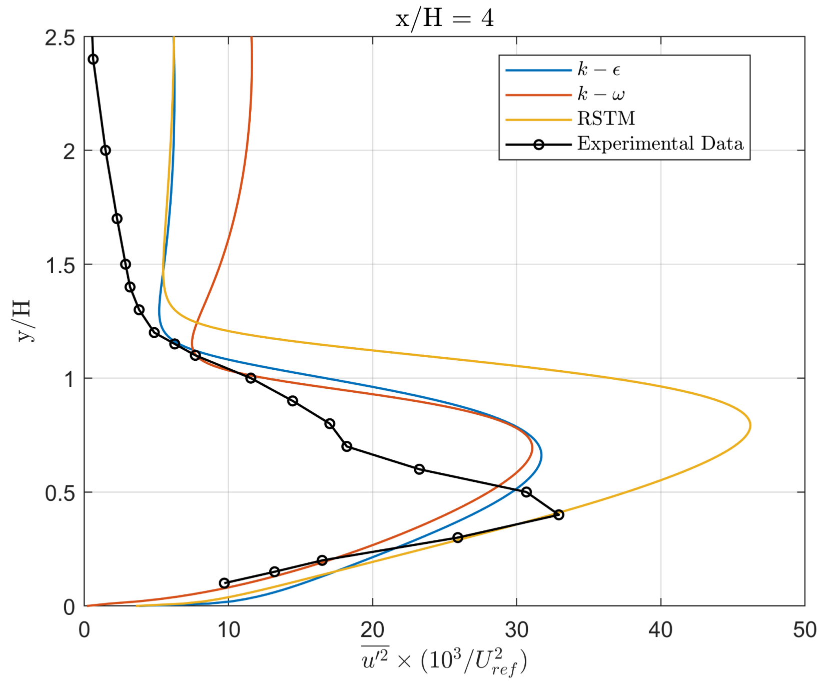

Fig. 42 Comparison of Turbulence Models for Non-Dimensional X-Reynolds Stress at x/H = 4

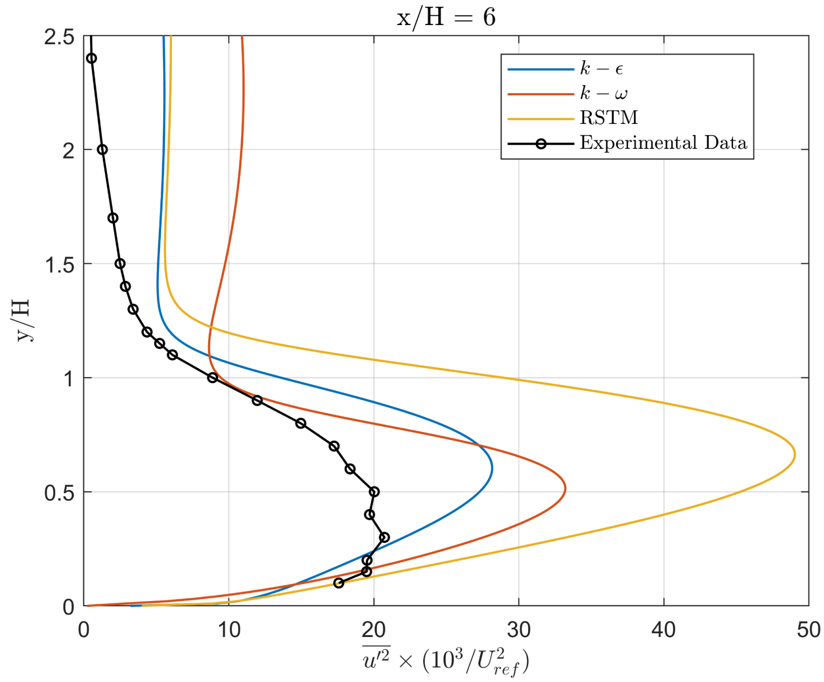

Fig. 43 Comparison of Turbulence Models for Non-Dimensional X-Reynolds Stress at x/H = 6

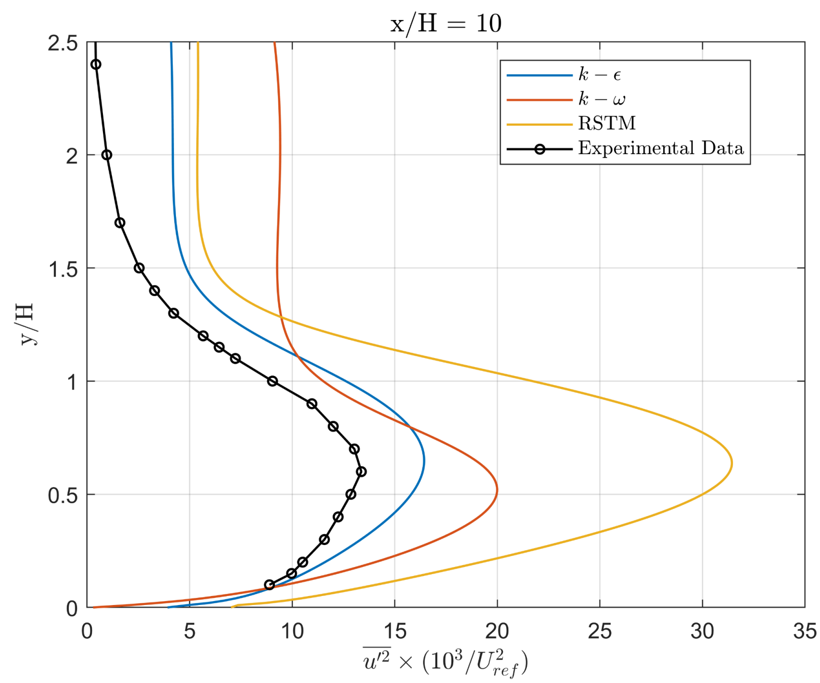

Fig. 44 Comparison of Turbulence Models for Non-Dimensional X-Reynolds Stress at x/H = 10

5.4 Non-Dimensional Y-Reynolds Stress Results

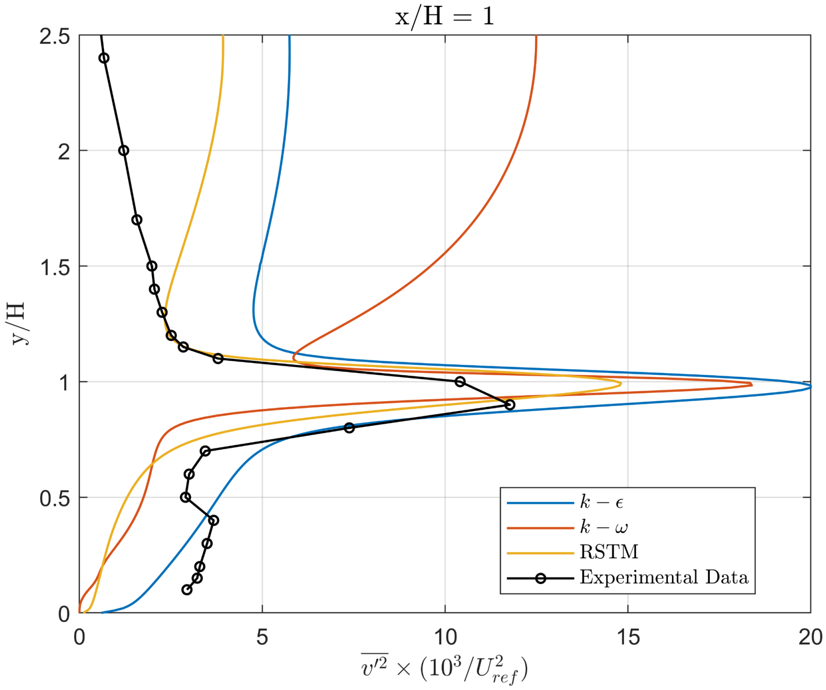

Fig. 45 Comparison of Turbulence Models for Non-Dimensional Y-Reynolds Stress at x/H = 1

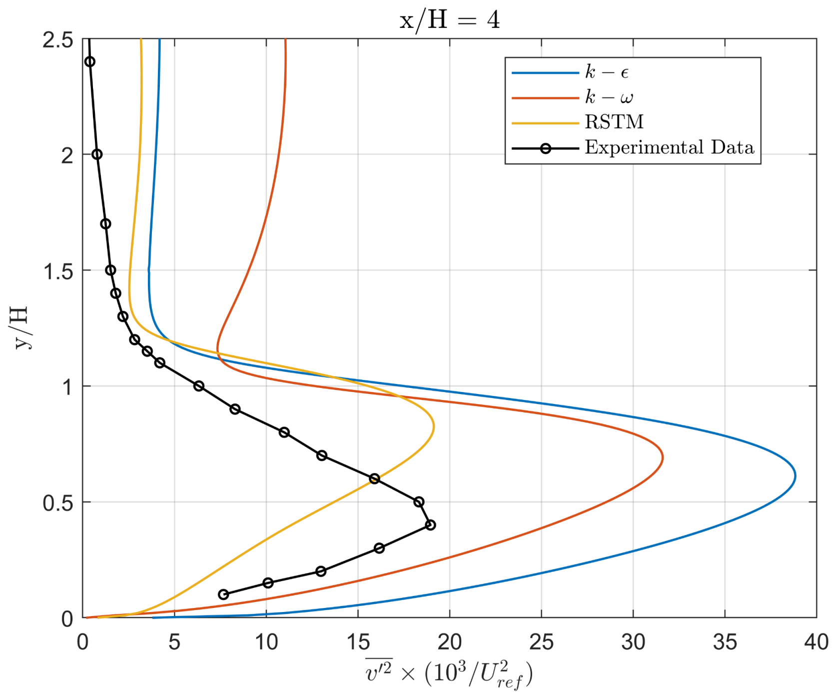

Fig. 46 Comparison of Turbulence Models for Non-Dimensional Y-Reynolds Stress at x/H = 4

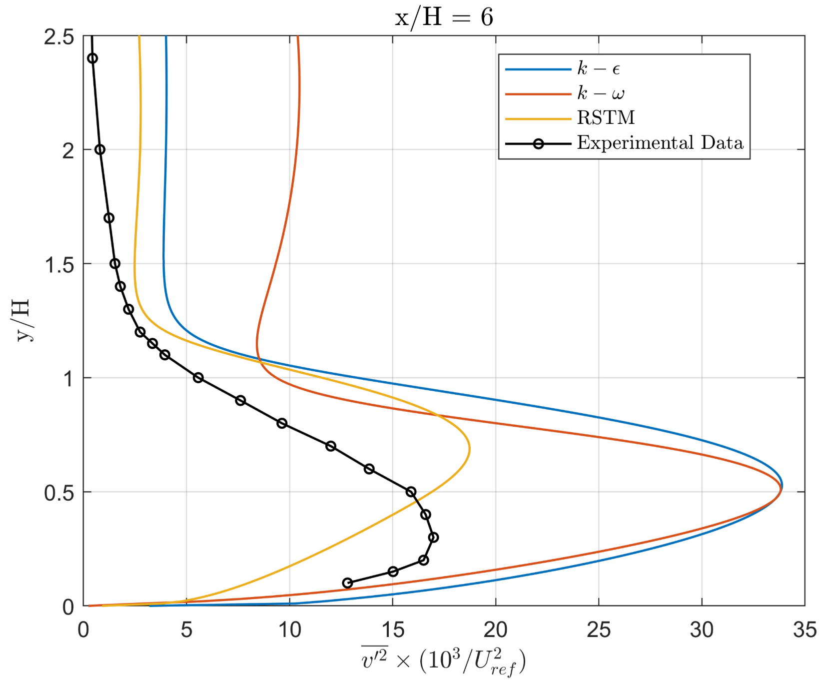

Fig. 47 Comparison of Turbulence Models for Non-Dimensional Y-Reynolds Stress at x/H = 6

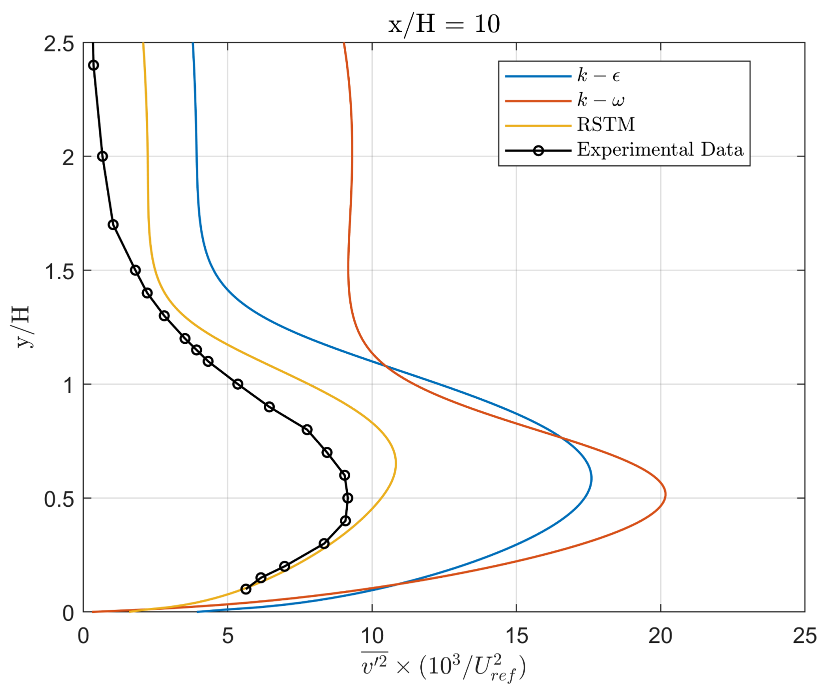

Fig. 48 Comparison of Turbulence Models for Non-Dimensional Y-Reynolds Stress at x/H = 10

5.5 Non-Dimensional XY-Reynolds Stress Results

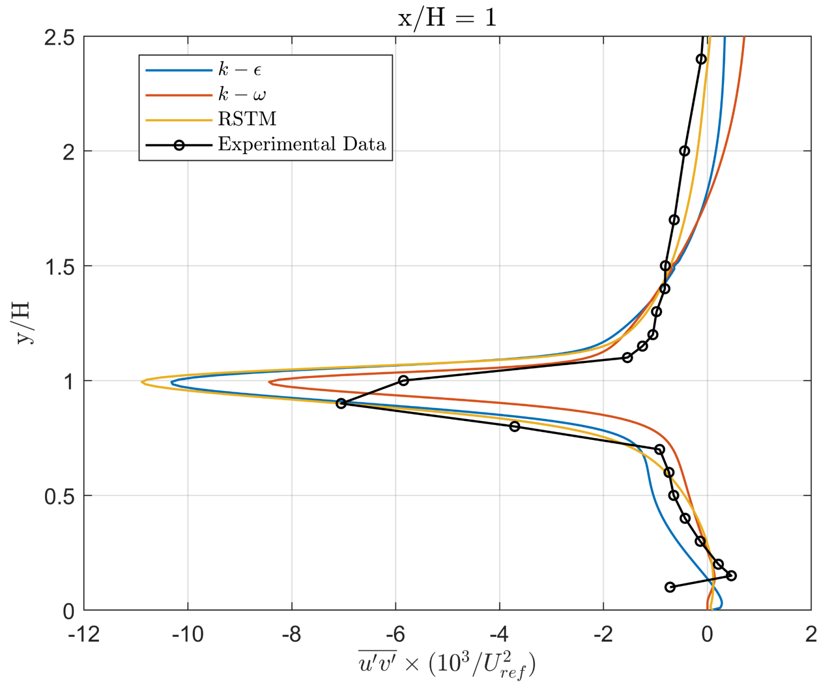

Fig. 49 Comparison of Turbulence Models for Non-Dimensional XY-Reynolds Stress at x/H = 1

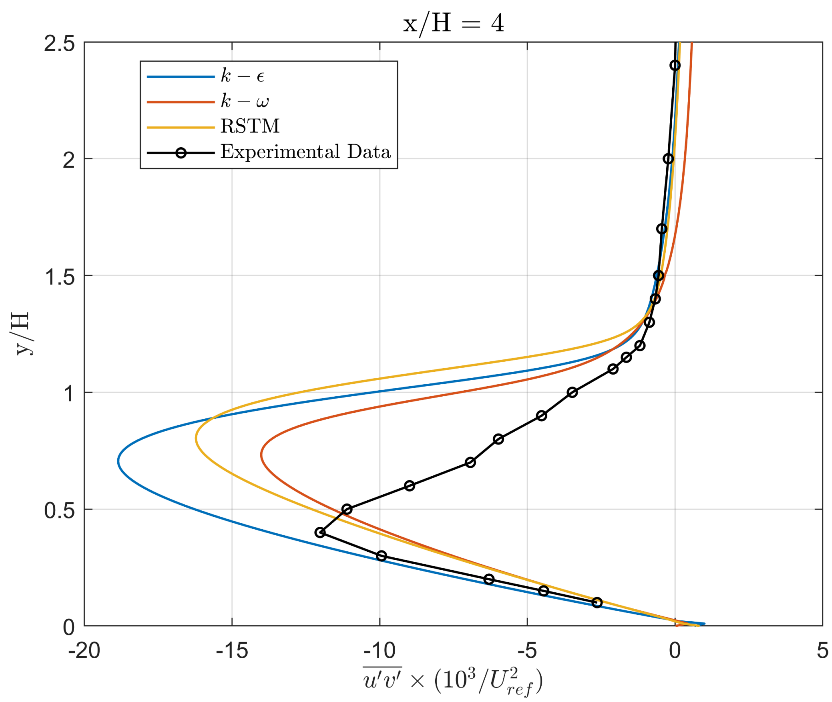

Fig. 50 Comparison of Turbulence Models for Non-Dimensional XY-Reynolds Stress at x/H = 4

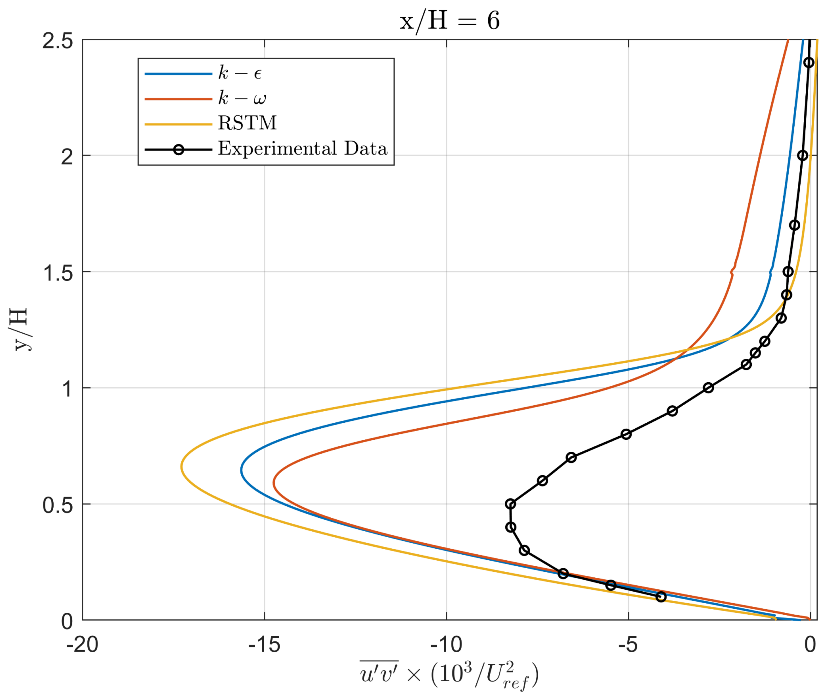

Fig. 51 Comparison of Turbulence Models for Non-Dimensional XY-Reynolds Stress at x/H = 6

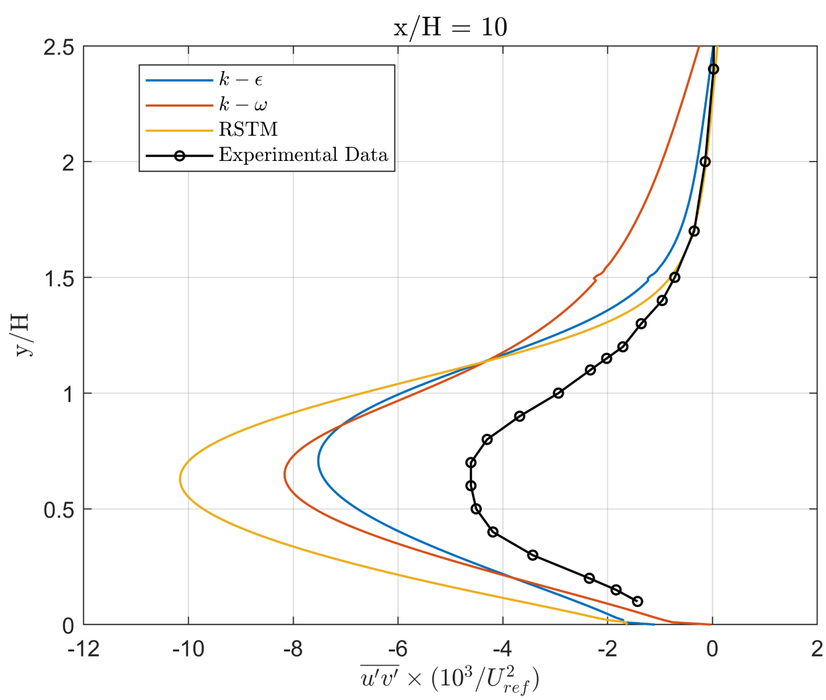

Fig. 52 Comparison of Turbulence Models for Non-Dimensional XY-Reynolds Stress at x/H = 10

5.6 Non-Dimensional Turbulent Kinetic Energy Results

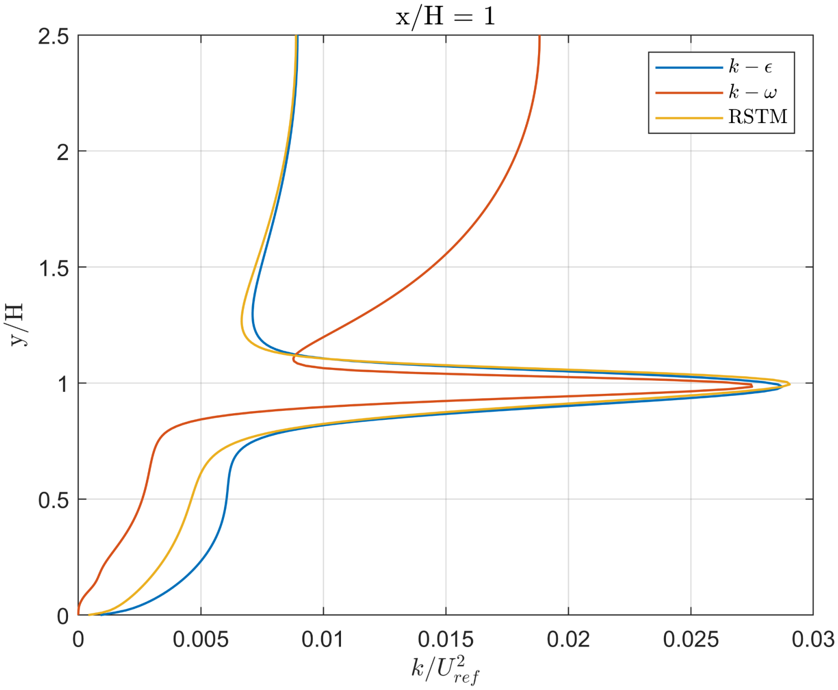

Fig. 53 Comparison of Turbulence Models for Non-Dimensional Turbulent Kinetic Energy at x/H = 1

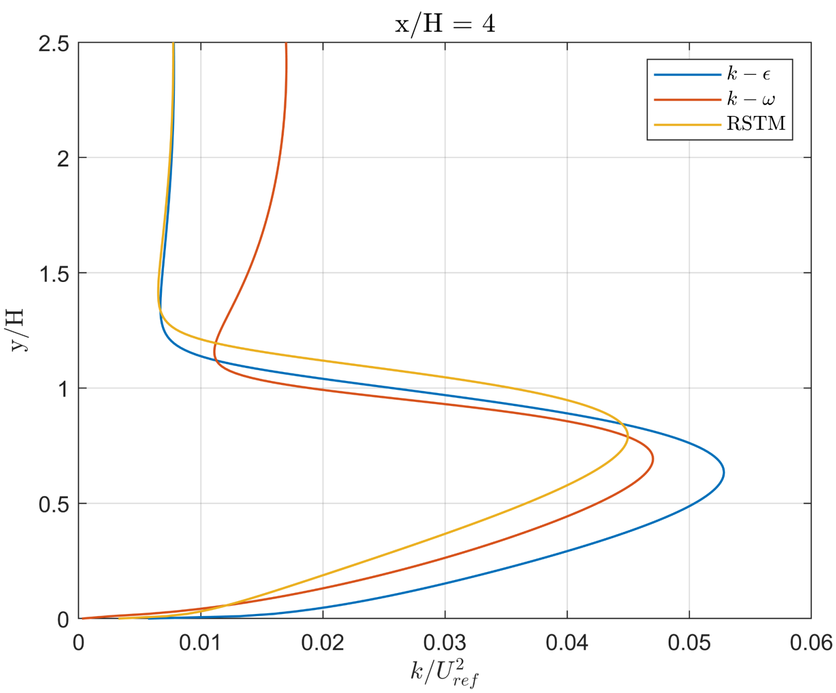

Fig. 54 Comparison of Turbulence Models for Non-Dimensional Turbulent Kinetic Energy at x/H = 4

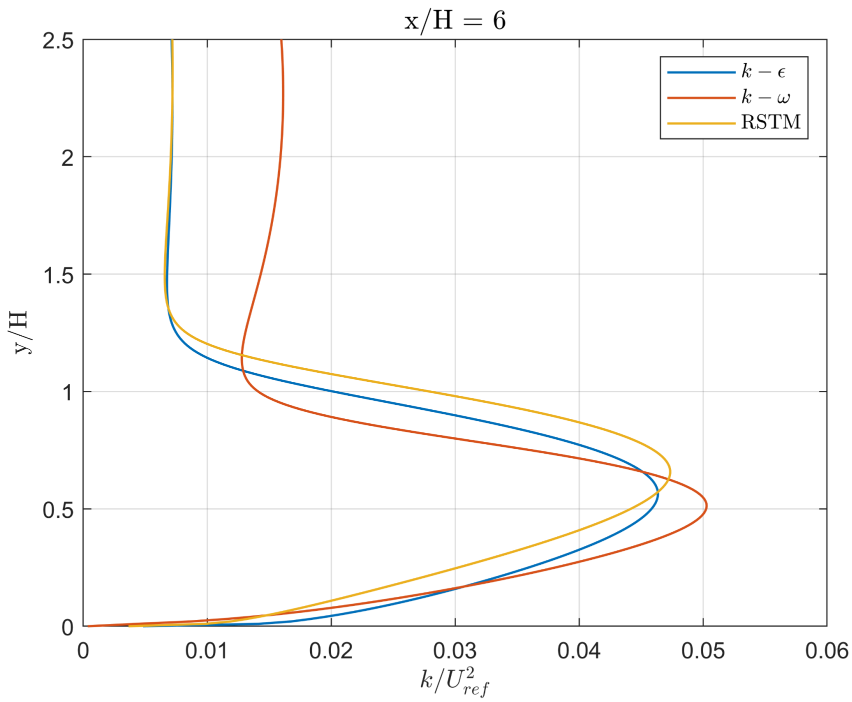

Fig. 55 Comparison of Turbulence Models for Non-Dimensional Turbulent Kinetic Energy at x/H = 6

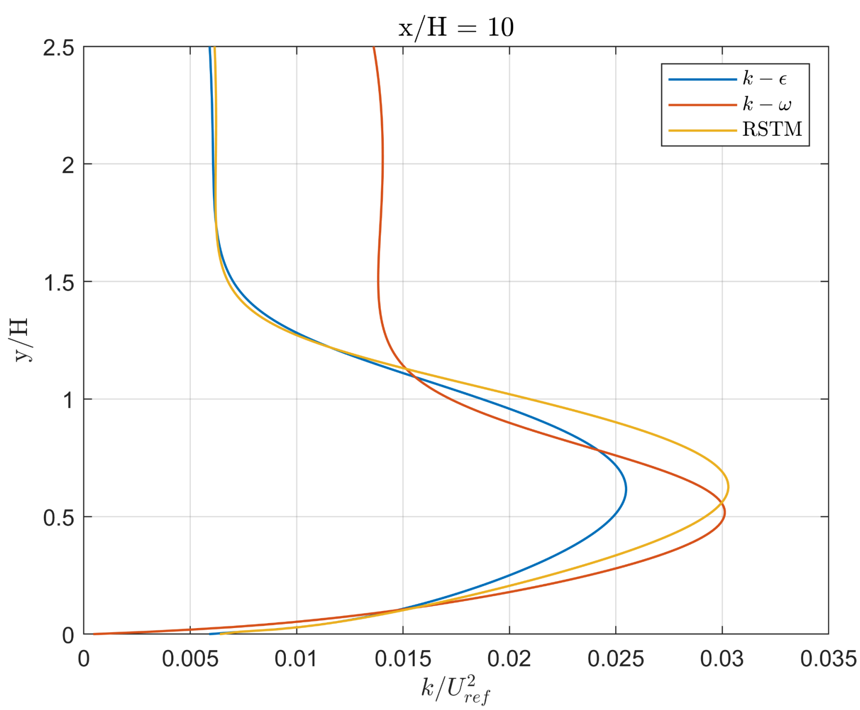

Fig. 56 Comparison of Turbulence Models for Non-Dimensional Turbulent Kinetic Energy at x/H = 10

5.7 Non-Dimensional Turbulent Dissipation Rate Results

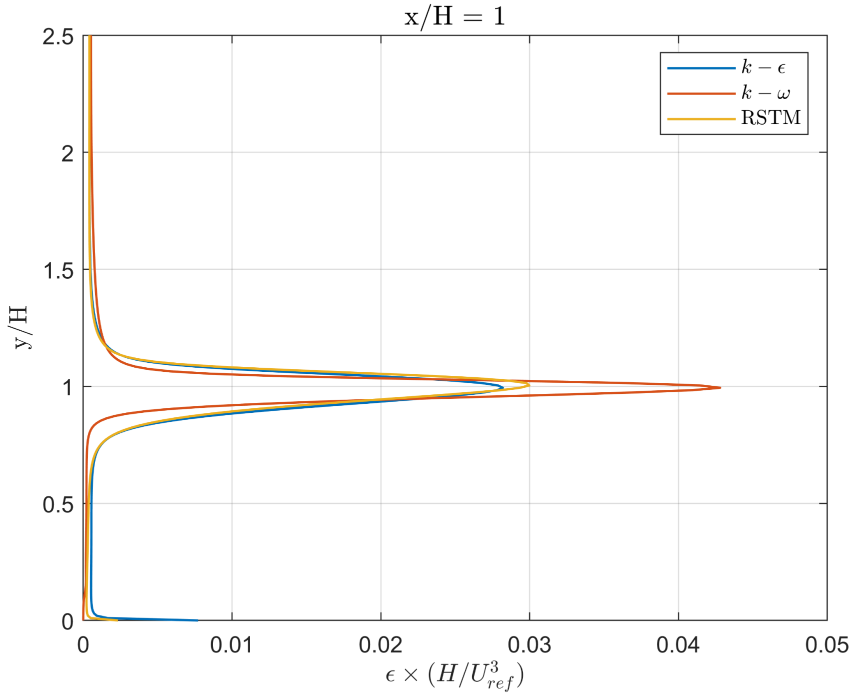

Fig. 57 Comparison of Turbulence Models for Non-Dimensional Turbulent Dissipation Rate at x/H = 1

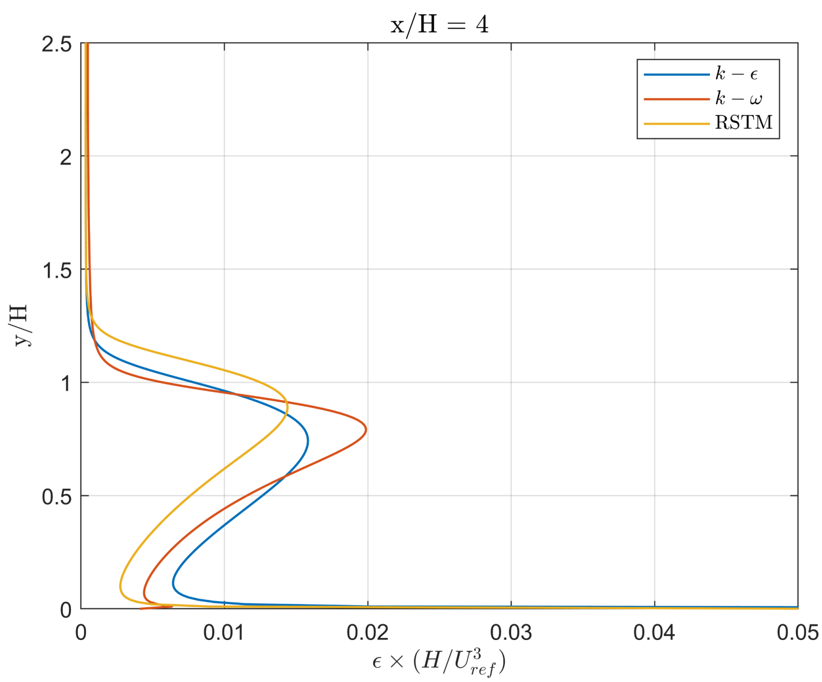

Fig. 58 Comparison of Turbulence Models for Non-Dimensional Turbulent Dissipation Rate at x/H = 4

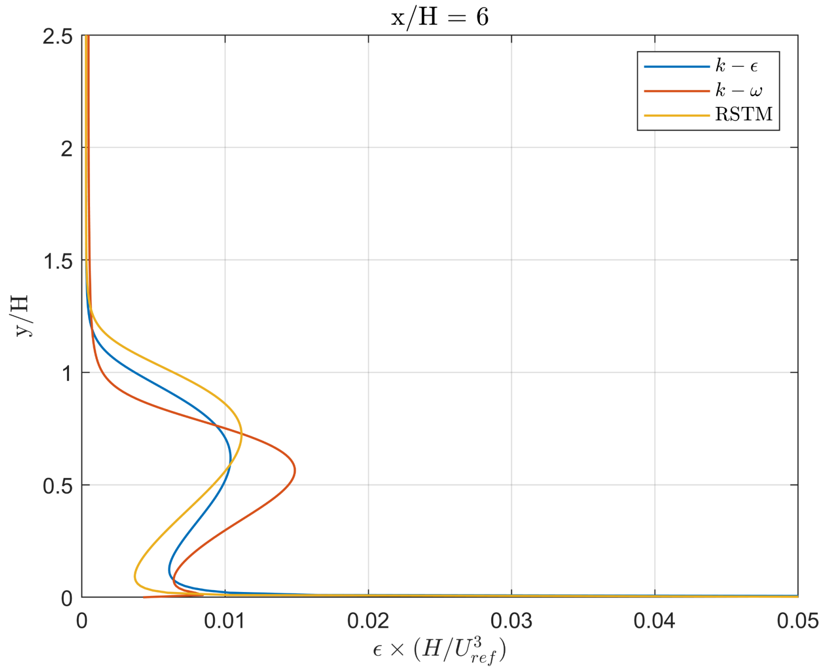

Fig. 59 Comparison of Turbulence Models for Non-Dimensional Turbulent Dissipation Rate at x/H = 6

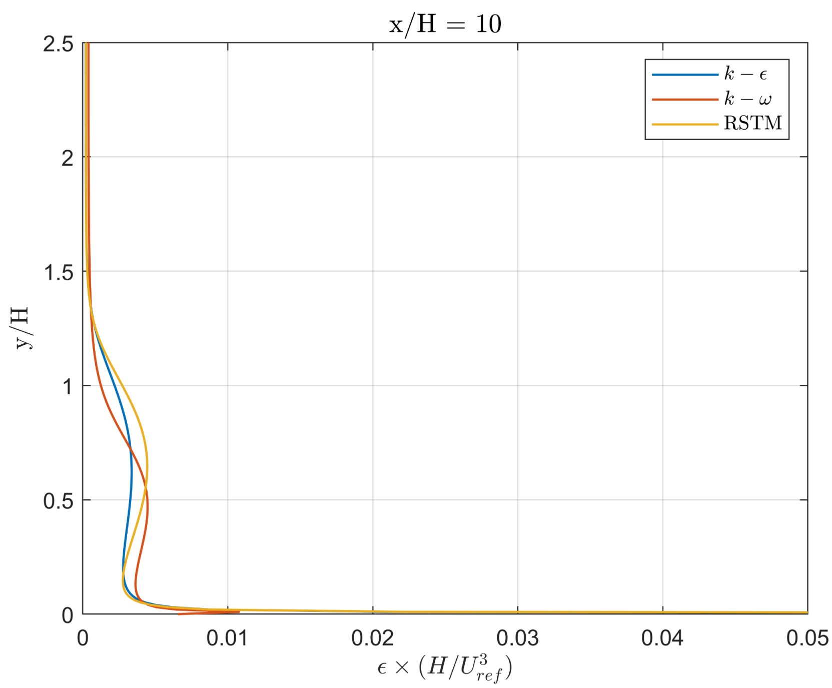

Fig. 60 Comparison of Turbulence Models for Non-Dimensional Turbulent Dissipation Rate at x/H = 10

5.8 Discussion

Question 3: Discuss briefly what you think is most important in the $\overline{u}$ and $\overline{v}$ graphs, and comment on how accurately or inaccurately these models predict these quantities.

Answer 3: After reviewing the results of the $\overline{u}$ graphs, it appears that all three models are rather inaccurate at the beginning of the flow near the wall (x/H = 1), but become closer to the experimental results as the x/H increases. By x/H = 10, the three models are more closely repressing the curve. It appears that the $\overline{u}$ is typically predicted to be larger than what is seen in the experimental results. The values of $\overline{u}$ are especially inaccurate with y/H values larger than 1, showing a large overprediction. This overprediction is corrected when y/H is below 1, for x/H = 1 and 4, where the values for $\overline{u}$ are much more accurate. As the overall accuracy increases as x/H increases, the values for $\overline{u}$ begin to be lower than the experimental.

For the $\overline{v}$ graphs, it appears the opposite to $\overline{u}$, where x/H = 1 is the most accurate for the three models and then becomes further from the true value by x/H = 10. The curves themselves become more accurate and similar to the experimental results but they are increasingly getting further from the experimental results. However, similar to $\overline{u}$, the values are mostly predicted higher than the experimental values show.

Question 4: Does any of these three turbulence models consistently predict $\overline{u}$ and $\overline{v}$ more accurately than the others, and if so which one?

Answer 4: When comparing the models against each other, it doesn't appear to be a clear favorite in terms of being the most accurate for $\overline{u}$ and $\overline{v}$. For $\overline{u}$, the results show that the k-ε preforms potentially slightly better than k-ω or RSTM but this isn't true for every x/H. This is because it is the closest to the experimental curve for x/H = 4, 6, and 10.

Then for $\overline{v}$, there is no true favorite as k-ε may begin to be the closest for y/H = 2, but RSTM or k-ω could end up being the favorite when y/H = 0. There is significant overlap and change between each of the models showing there is no true leader model for $\overline{v}$.

Question 5: Discuss briefly what you think is most important in the $\overline{u_i'u_j'}$ graphs, and comment on how accurately or inaccurately these turbulence models predict these quantities.

Answer 5: For $\overline{u'^2}$, it is clear that at the beginning of the flow, the Reynolds stress is centered in an almost Gaussian curve with the largest amount of stress at y/H = 1. As x/H increases, the larger values of stress shift towards y/H = .5, moving the overall distribution towards y/H = 0. Again, the models are rather inaccurate with large overpredictions of the stress values. For x/H = 1, the values are larger for y/H greater than 1, then at y/H = 1, the values for RSTM are significantly larger, and finally for y/H less than 1, the values are more accurate for RSTM but less for the other two models.

For $\overline{v'^2}$, the general curves are similar to $\overline{u'^2}$ with the x/H = 1 having the largest stress values at y/H = 1, and then it shifting towards y/H = .5 for the remaining x/H. Additionally, the curves are even more inaccurate than seen for $\overline{u'^2}$, mostly due to the overprediction of the stress values for nearly the entire length of y/H for each x/H. There are some instances where the values are underpredicted, especially at y/H less than .5, for x/H = 1.

Finally for $\overline{u'v'}$, the stresses are no longer positive and the models can portray or reflect similar curves to the experimental values. In contrast to $\overline{u'^2}$ and $\overline{v'^2}$, the curves are undershooting, showing the Reynolds stress as more negative. Again, the same pattern is seen where x/H = 1 has the largest stress values at y/H =1, and then shifts towards y/H = .5 for the remaining x/H.

Question 6: Does any of these three turbulence models consistently predict the $\overline{u_i'u_j'}$ components more accurately than the others, and if so which one?

Answer 6: Similarly to question 4, there doesn't appear to be a particular model that is better than another. For $\overline{u'^2}$, RSTM is not a good representation of the Reynolds stress due to the extreme over-prediction at the maximum stress values. However, it is quite accurate at y/H less than .5 for x/H = 1 and 4. The k-ε model is the most accurate, with k-ω not being too far off. They swap back and forth for the closest but generally, k-ε has the right curve and doesn't overshoot the stress values.

In contrast for $\overline{v'^2}$, RSTM is much more accurate than k-ε and k-ω. This is clear with the curve shapes, but also the values of Reynolds stress being fairly accurate, with only a slight amount of overprediction. Both k-ε and k-ω are quite inaccurate, portraying similar curves but are overshooting in value.

Finally for $\overline{u'v'}$, k-ω most closely portrays the experimental results. Although all the models are not very close in values, they are rather accurate in curve shape. Similarly to $\overline{u'^2}$, the RSTM is very clearly the worst model at the highest stress points but is the most accurate at y/H greater than 1.5.

Question 7: Based on what you have learned in the lectures, what do you think are the greatest sources of error in each of these three turbulence models?

Answer 7: The three models, k-ε, k-ω, and Reynolds Stress Transport Model (RSTM) are all considered to be Reynolds-Averaged Navier-Strokes (RANS) simulations or turbulence models. The greatest sources of error stem from various limitations inherent to the equations and assumptions of each model.

In the k-ε model, significant errors can arise near solid boundaries due to challenges in accurately predicting near-wall flows, especially in cases of separation. This originates from the wall boundary condition, which is the log-law for the k-equation and ε-equation. These log-laws move the boundary condition at the wall up to a specific height (y_p > 30) so that the first row of the mesh is not directly against the wall. The k-ω model exhibits limitations in transition and separated flows, along with complexities in predicting flow behavior in rather complex geometries. Additionally, for both of these models, the first-order moment equations have wall boundary conditions, known as Dirichlet and Neumann. However, for the near-wall treatment, high values of shear generate extremely high anisotropy and the Reynolds turbulence goes to zero. These are not particularly accurate and to counter this, there are constants (f_μ, f_1, and f_2) that are used when going below the log-law layer. These constants are entirely made up to dampen the effects and are different values between the k-ε and k-ω models.

Both of these models are run on the gradient transport (G.T.) approximation based on the idea that turbulent transport is the same as molecular transport. As discussed in the lecture, since 1877 when this was published, it has been proven incorrect due to the transport being due to a small mean free path and is entirely random. Additionally, these models use local linear equilibrium (LLE) dynamics as an assumption saying that the anisotropy is always in equilibrium with the strain rate tensor at every time position. Essentially, this implies that the strain rate tensor is varying slowly! Now, this assumption ignores the effects of the non-equilibrium response to compulsively applied shear and period shear, which ignores finite time response causing a phase lag between the strain rate tensor and the anisotropy aswell as the magnitude of the phase lag.

The Reynolds Stress Transport Model (RSTM) is mostly based on the Laurnder-Reese-Rodi (LRR) model. This model's biggest approximation is modeling of the rapid pressure-strain correlation solely in the local and linear strain rate tensor and rotation rate tensor. This means that only the first-order velocity gradients are used for the pressure-strain correlation, and subsequently, the anisotropy.

Additionally, for all the models, the constants are mostly generated by "tuning" from sample flows. These are generally accepted standard values, however, they may not be perfect for this specific model of the backward-facing step. This is less probable to cause major inaccuracies but with the amount of these constants, it could add up to a variety of small inaccuracies equaling a much more significant issue. This might be optimized by changing the values of the constants and running the simulations for a variety of small changes to determine potential differences and inaccuracies for this specific flow.

Question 8: How certain are you that the Driver & Seegmiller data are actually the "right answer"? Discuss some possible sources of error in the experimental data.

Answer 8: The experiment performed by Driver & Seegmiller (1995) [1] offers valuable insights into the flow behavior for a backward-facing step. They utilized a variety of measurement techniques including wall static pressure orifices, oil-flow laser interferometry for skin friction, and thermal-tuft probes for flow direction detection. Although we are comparing three different simulations to this experiment, there are potential sources of error. These could be uncertainties in instrumentation such as the given +/- 0.009 uncertainty in wall static pressure measurements and the +/- 8% uncertainty in skin friction measurements. Additionally, factors such as data processing techniques, boundary conditions, and possible human error in experimental setup and execution should be considered when interpreting the results.

Despite the given uncertainties and potential human errors, the experiment provides valuable data that is significantly more accurate than the simulations. Using the experimental data as validation for a variety of simulation models progresses our understanding of turbulent flows and limitations in a variety of scenarios. Therefore, this data may or may not be considered "right", but it is particularly useful for validating our simulations.

Question 9: How does the magnitude of the differences between the $\overline{u_i' u_j'}$ results from the three turbulence models compare with the magnitude of the differences between the turbulence model results and the experimental data?

Answer 9: Looking at section 5.3, it appears that the models are fairly different, and as noted in answer 5, the models are not particularly close to the experiment results. Therefore, this would appear the magnitude of the difference is comparable for $\overline{u'^2}$. Now, the same can be said for $\overline{v'^2}$ in the way that the models are about as different as they are all from the experimental data. Finally, for $\overline{u'v'}$, the models are quite similar, varying only slightly. However, they are not particularly close to the experimental data.

Question 10: Based on your answer to Question 9, discuss what the implications of this are for choosing a turbulence model for a CFD simulation, at least in this flow but also in general.

Answer 10: Based on the idea that the results between each of the three models are comparably different to the models being different from the experimental data, it implies that the selection of a singular model is not particularly going to provide better results. Throughout sections 5.1 to 5.7, it has become clear that for this problem and in general, each model has an advantage and disadvantage in modeling particular areas (such as near the wall or recirculation zone) or particular values (such as velocity or Reynolds stress). There is a multitude of swaps between the models for being the most accurate and least accurate, as the advantages and disadvantages of the approximations and boundary conditions of each model are used. In general, it may be beneficial to not only compare the different meshes and different models but also compare a variety of constants being used. This will further add to the complexity of selecting a singular option, but it should provide significantly more data to select for particular areas or values of interest.

6. Effects of Turbulence Wall Functions

6.1 Non-Dimensional Mean X-Velocity Results

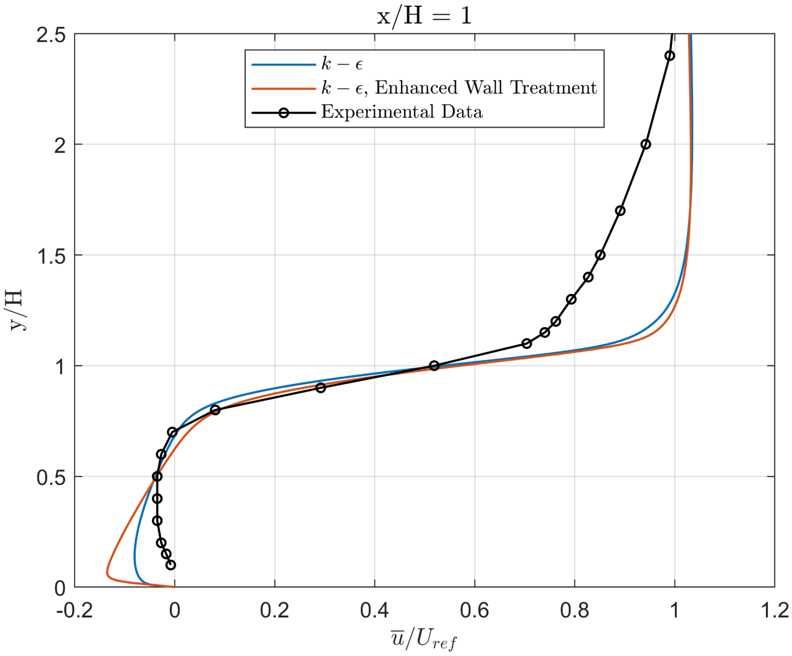

Fig. 61 Comparison of Wall Treatment for Non-Dimensional Mean X-Velocity at x/H = 1

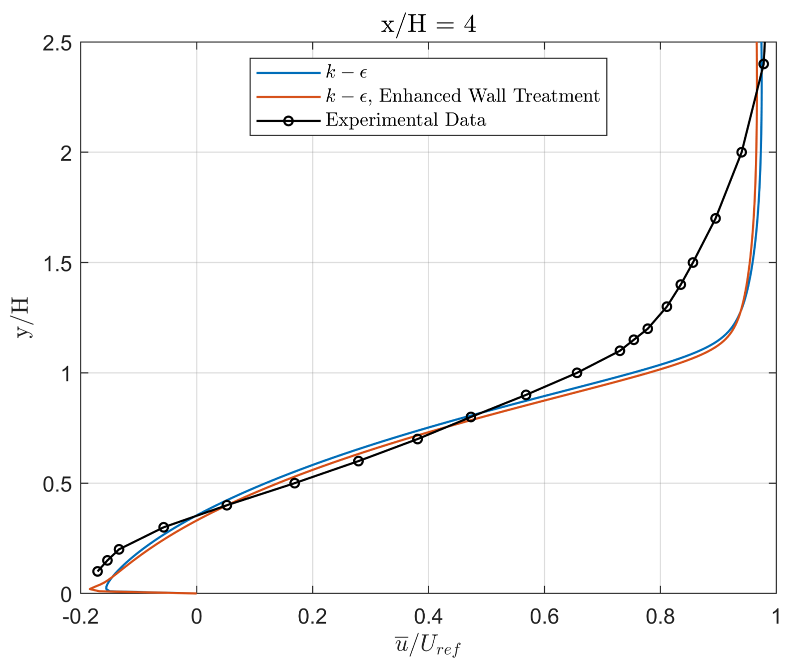

Fig. 62 Comparison of Wall Treatment for Non-Dimensional Mean X-Velocity at x/H = 4

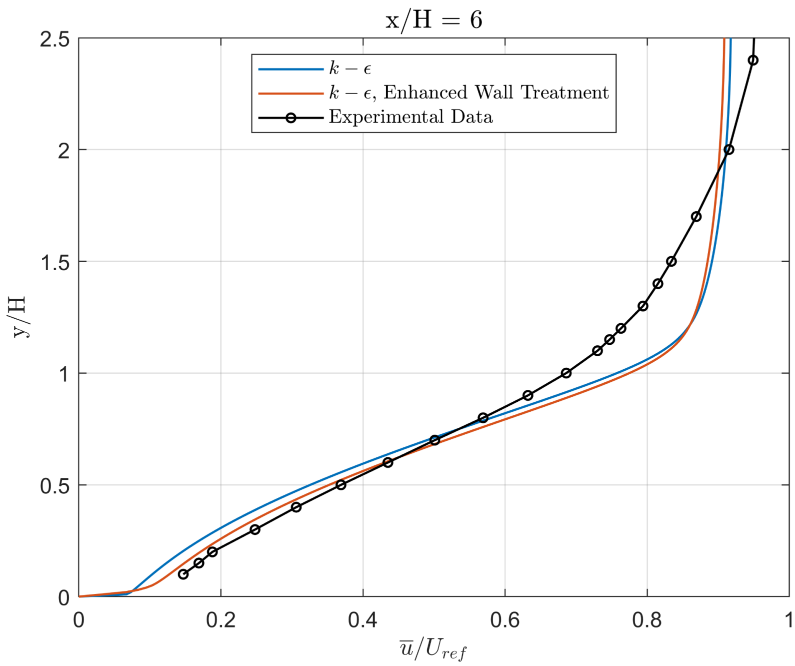

Fig. 63 Comparison of Wall Treatment for Non-Dimensional Mean X-Velocity at x/H = 6

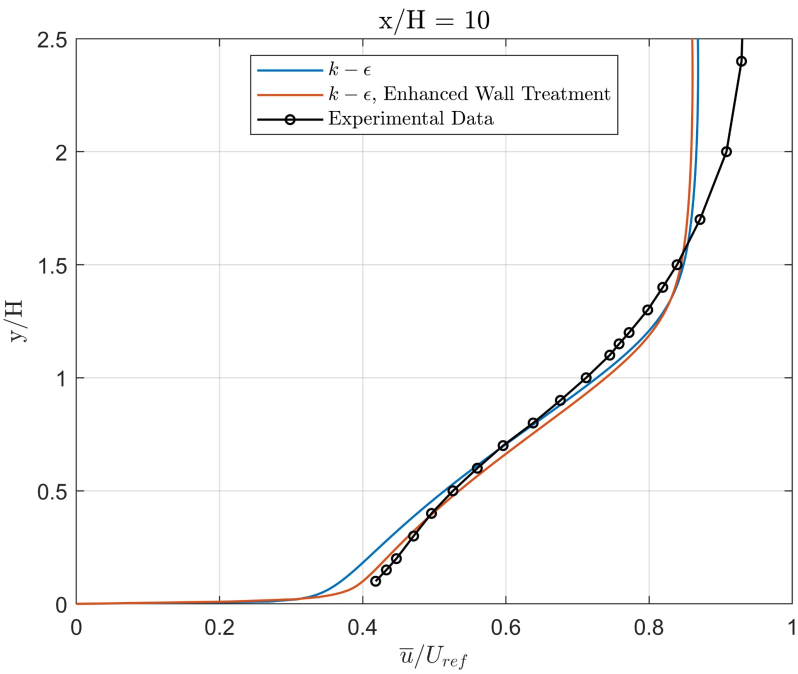

Fig. 64 Comparison of Wall Treatment for Non-Dimensional Mean X-Velocity at x/H = 10

6.2 Non-Dimensional Mean Y-Velocity Results

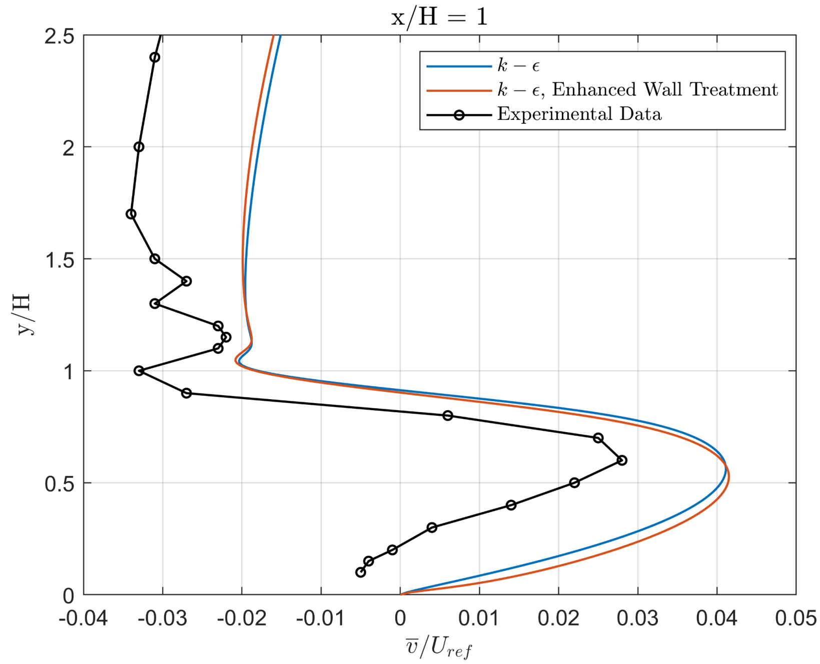

Fig. 65 Comparison of Wall Treatment for Non-Dimensional Mean Y-Velocity at x/H = 1

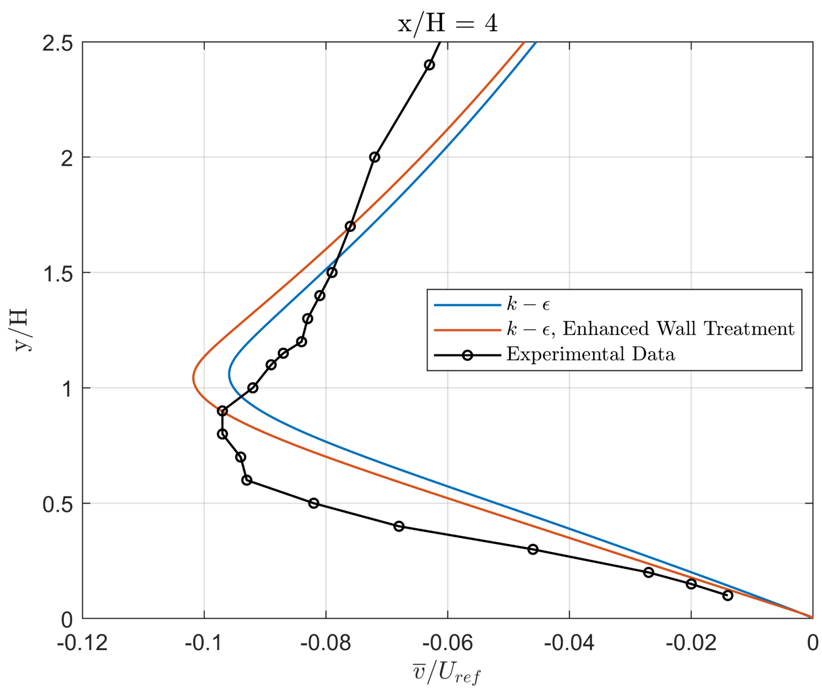

Fig. 66 Comparison of Wall Treatment for Non-Dimensional Mean Y-Velocity at x/H = 4

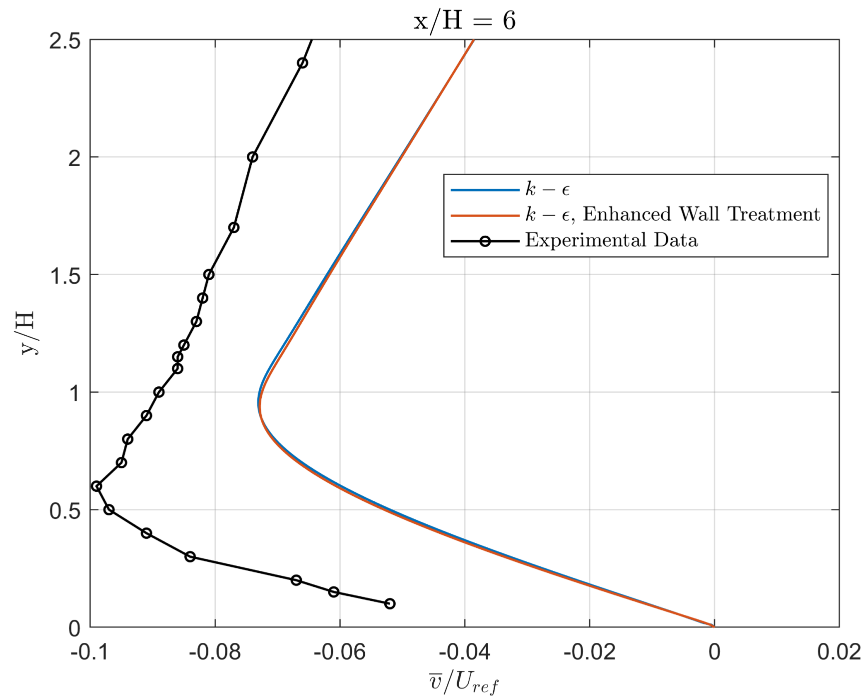

Fig. 67 Comparison of Wall Treatment for Non-Dimensional Mean Y-Velocity at x/H = 6

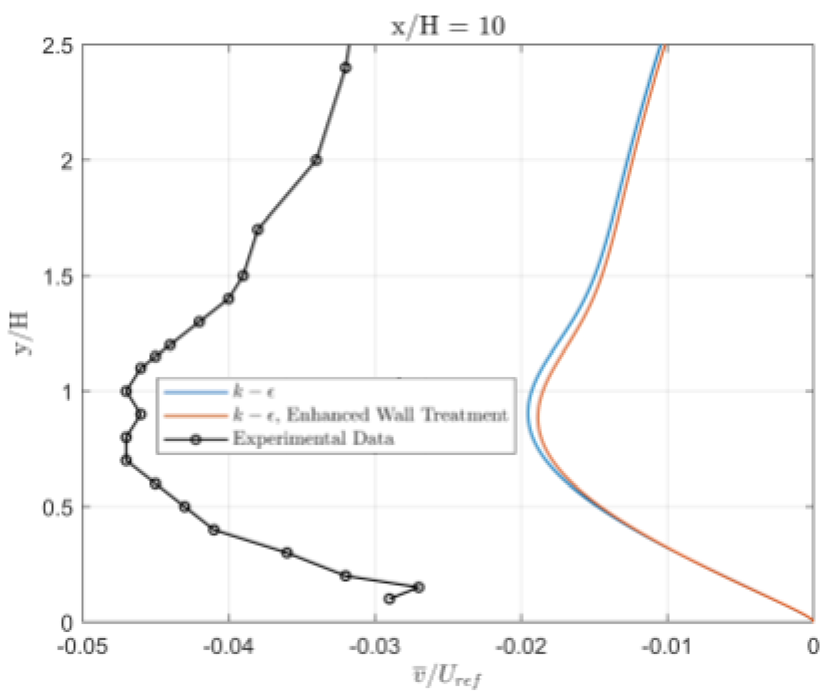

Fig. 68 Comparison of Wall Treatment for Non-Dimensional Mean Y-Velocity at x/H = 10

6.3 Non-Dimensional X-Reynolds Stress Results

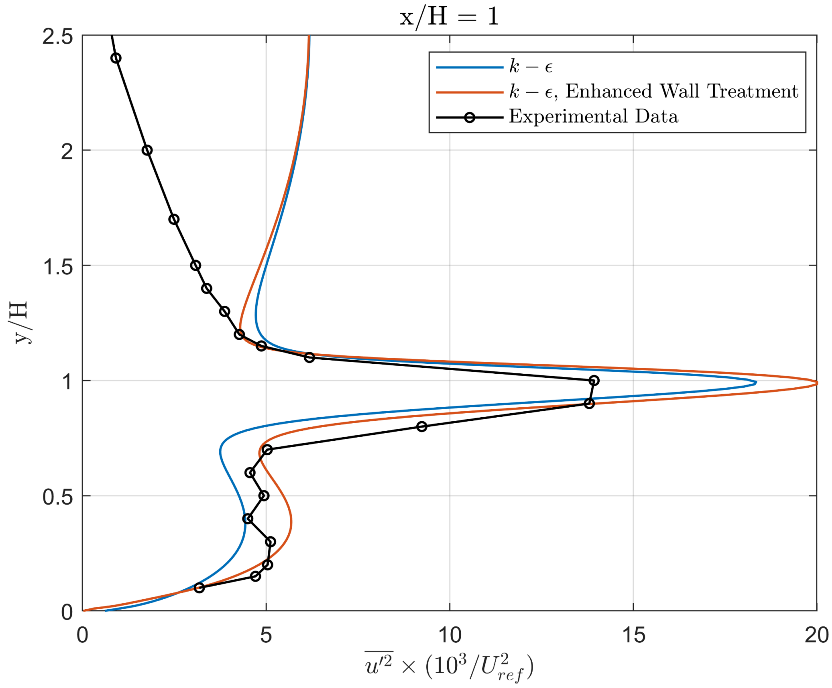

Fig. 69 Comparison of Wall Treatment for Non-Dimensional X-Reynolds Stess at x/H = 1

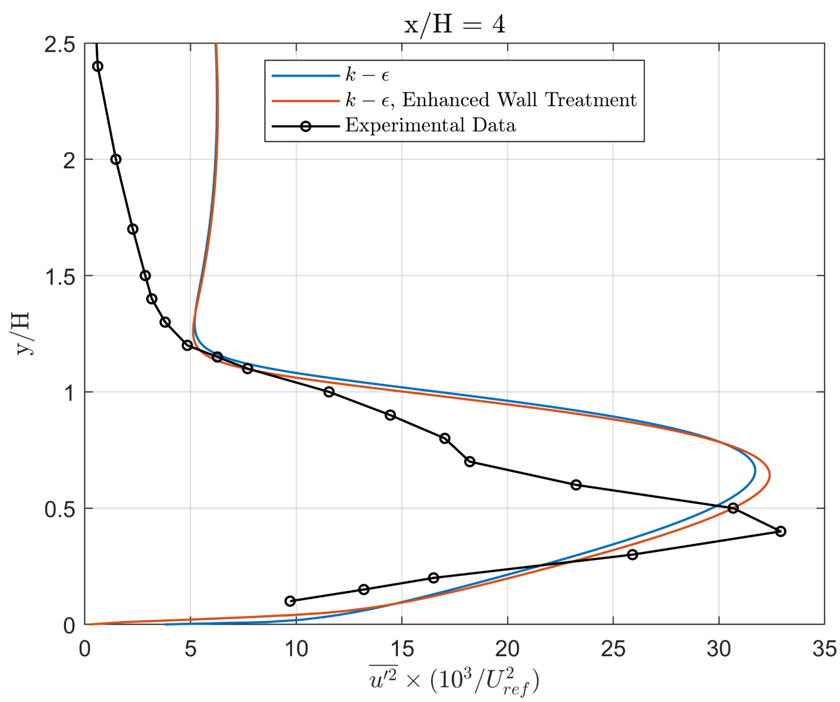

Fig. 70 Comparison of Wall Treatment for Non-Dimensional X-Reynolds Stess at x/H = 4

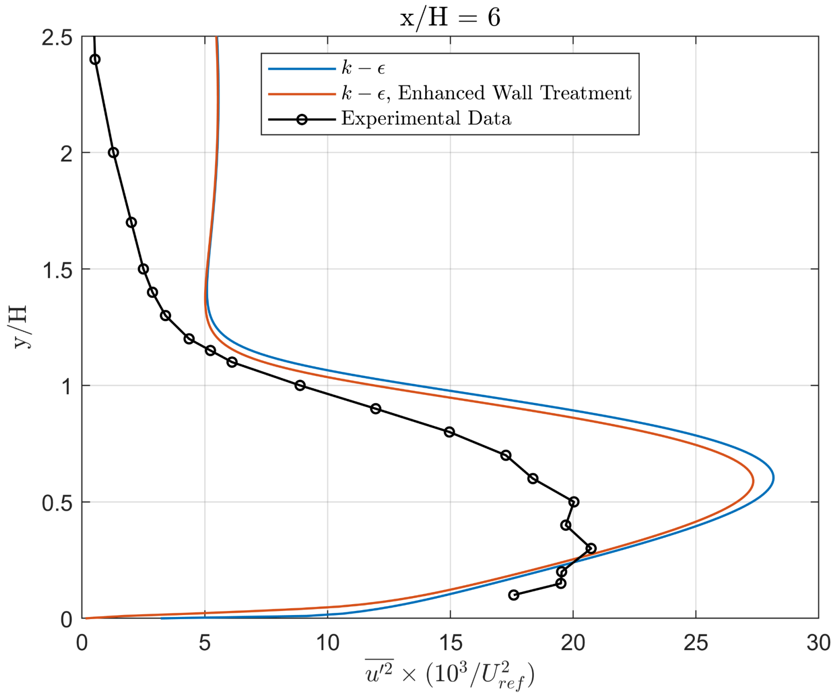

Fig. 71 Comparison of Wall Treatment for Non-Dimensional X-Reynolds Stess at x/H = 6

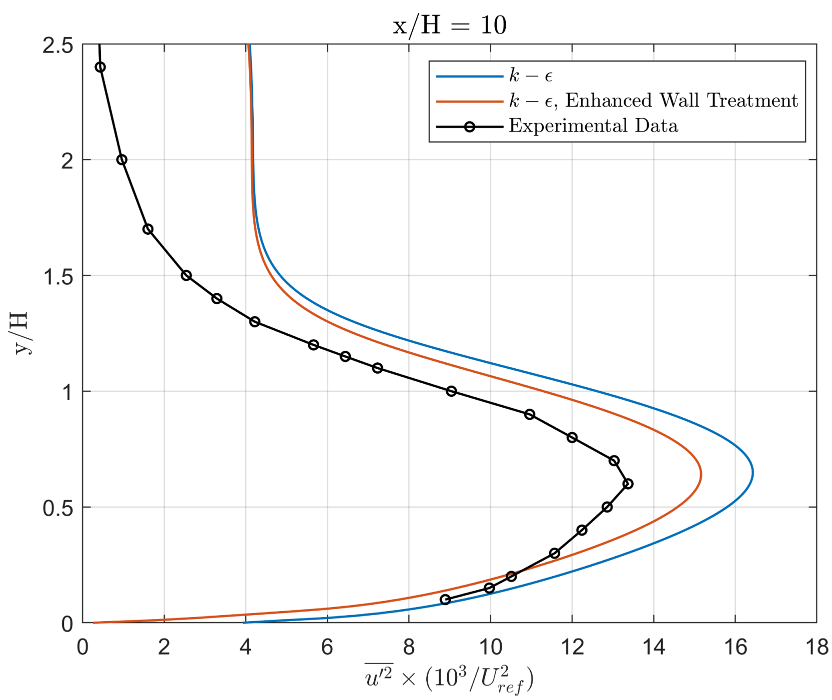

Fig. 72 Comparison of Wall Treatment for Non-Dimensional X-Reynolds Stess at x/H = 10

6.4 Non-Dimensional Y-Reynolds Stress Results

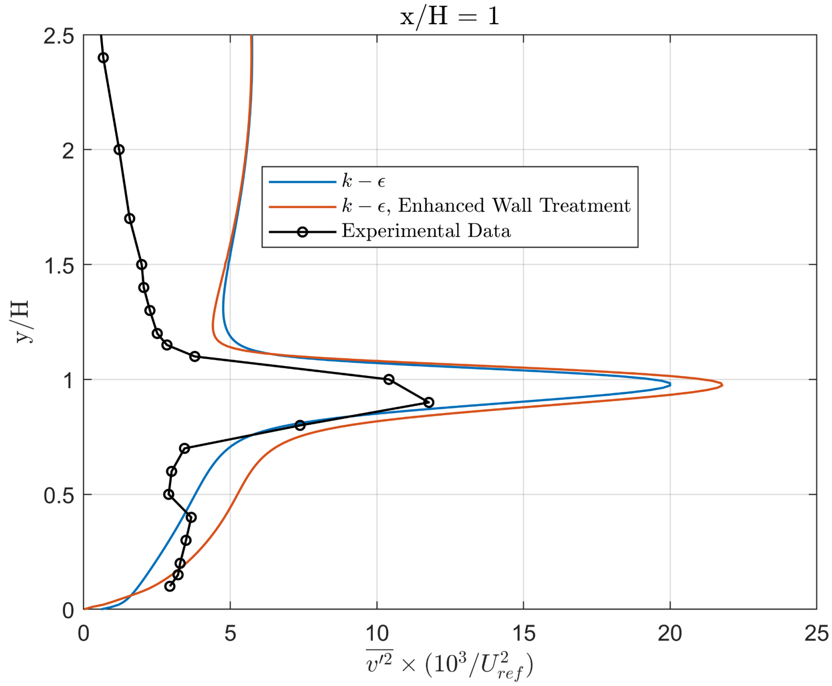

Fig. 73 Comparison of Wall Treatment for Non-Dimensional Y-Reynolds Stess at x/H = 1

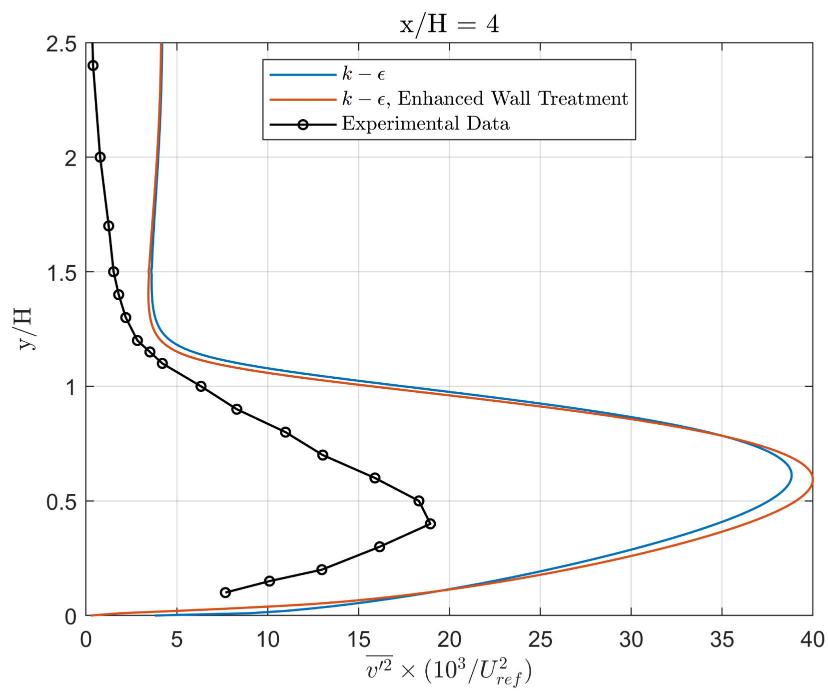

Fig. 74 Comparison of Wall Treatment for Non-Dimensional Y-Reynolds Stess at x/H = 4

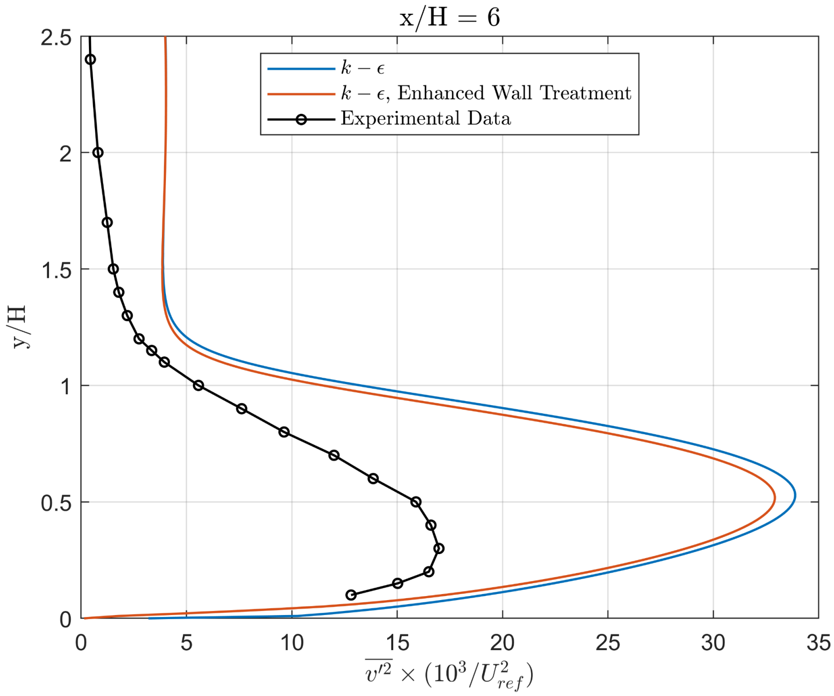

Fig. 75 Comparison of Wall Treatment for Non-Dimensional Y-Reynolds Stess at x/H = 6

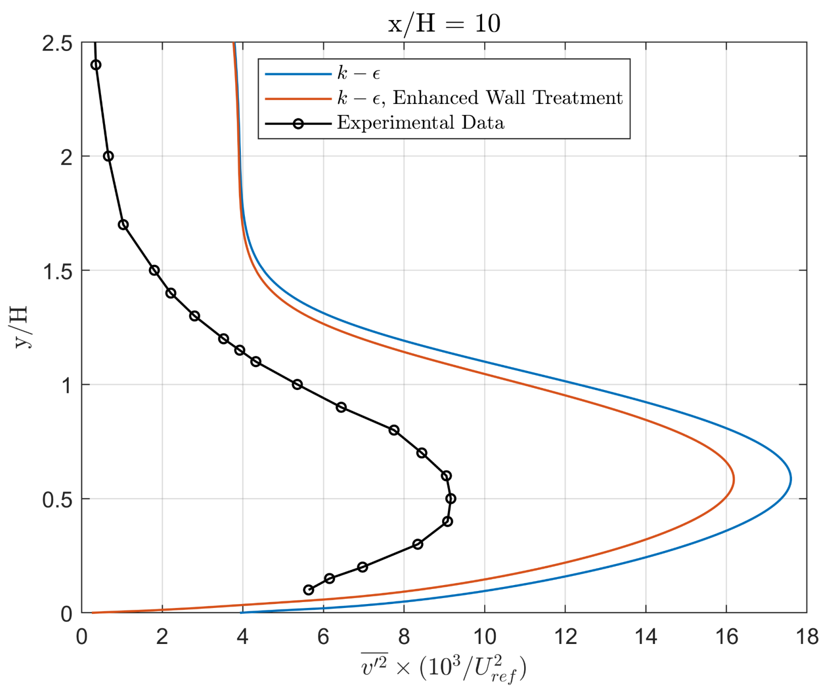

Fig. 76 Comparison of Wall Treatment for Non-Dimensional Y-Reynolds Stess at x/H = 10

6.5 Non-Dimensional XY-Reynolds Stress Results

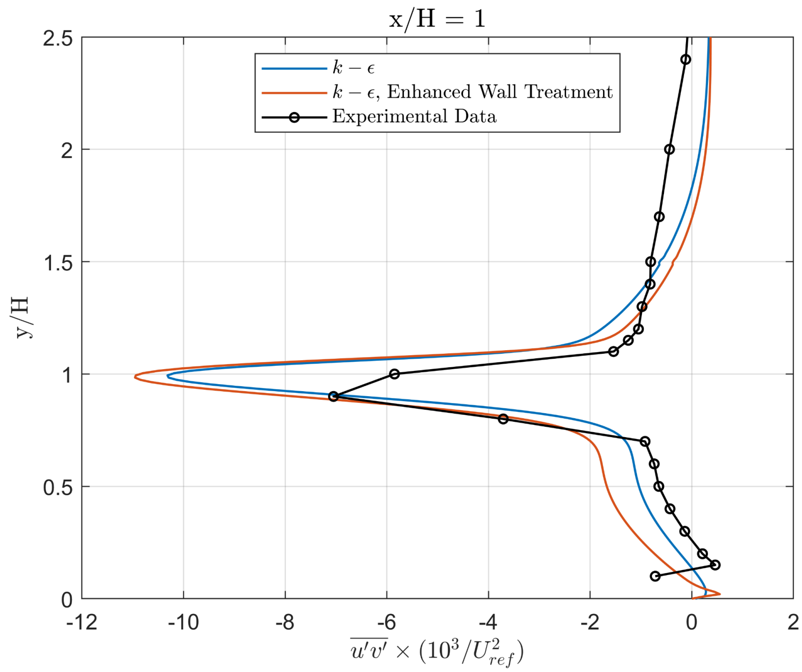

Fig. 77 Comparison of Wall Treatment for Non-Dimensional XY-Reynolds Stess at x/H = 1

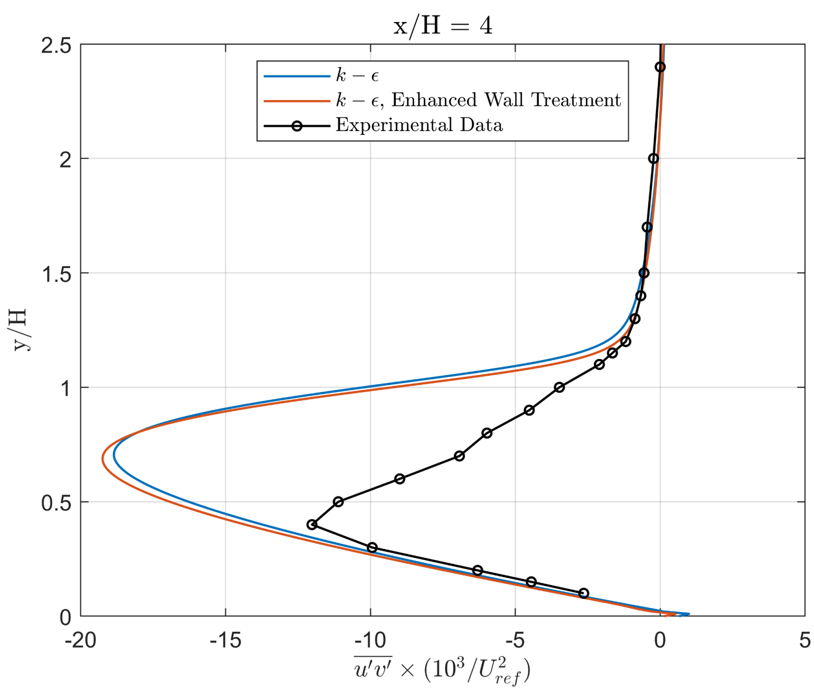

Fig. 78 Comparison of Wall Treatment for Non-Dimensional XY-Reynolds Stess at x/H = 4

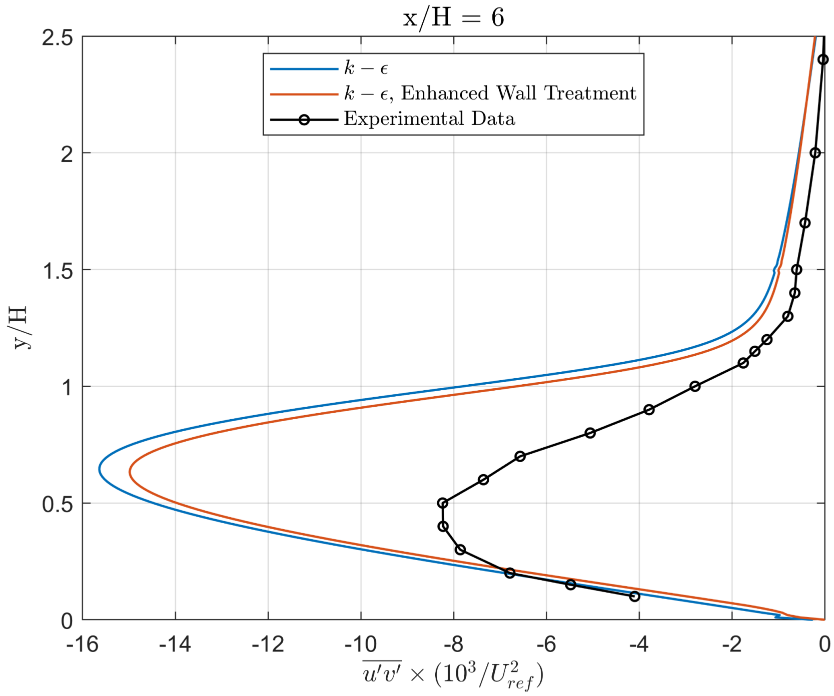

Fig. 79 Comparison of Wall Treatment for Non-Dimensional XY-Reynolds Stess at x/H = 6

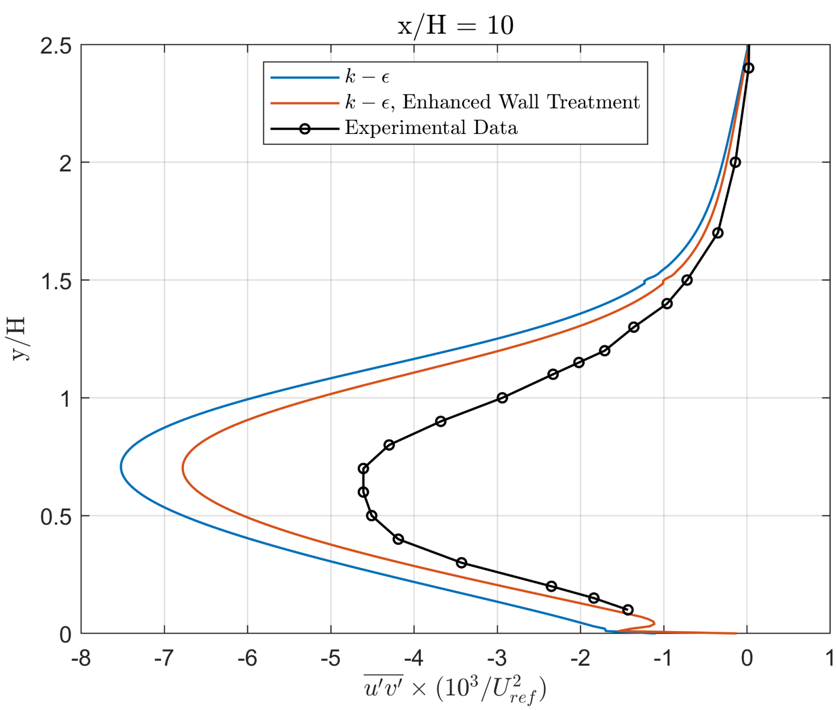

Fig. 80 Comparison of Wall Treatment for Non-Dimensional XY-Reynolds Stess at x/H = 10

6.6 Non-Dimensional Turbulent Kinetic Energy Results

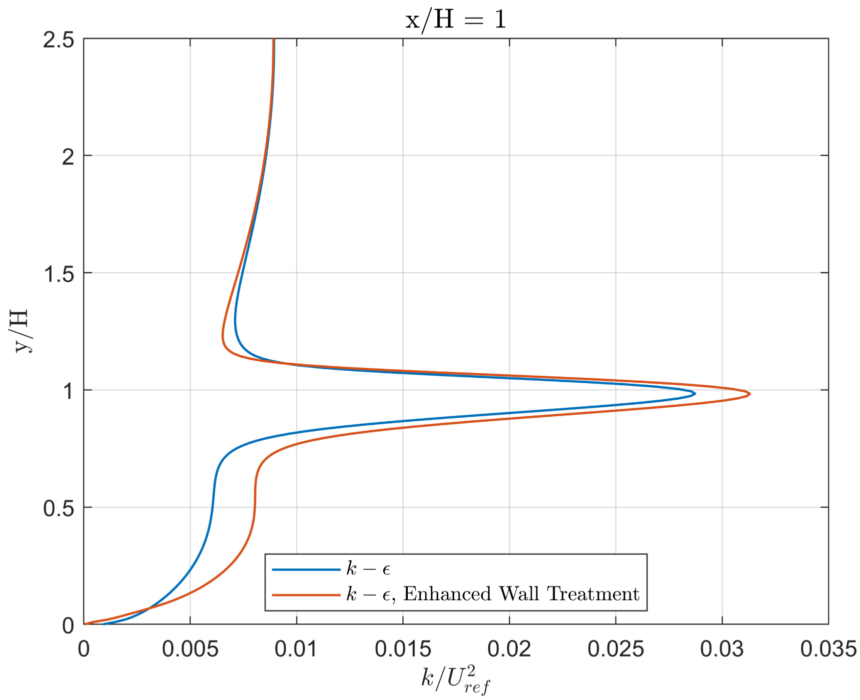

Fig. 81 Comparison of Wall Treatment for Non-Dimensional Turbulent Kinetic Energy at x/H = 1

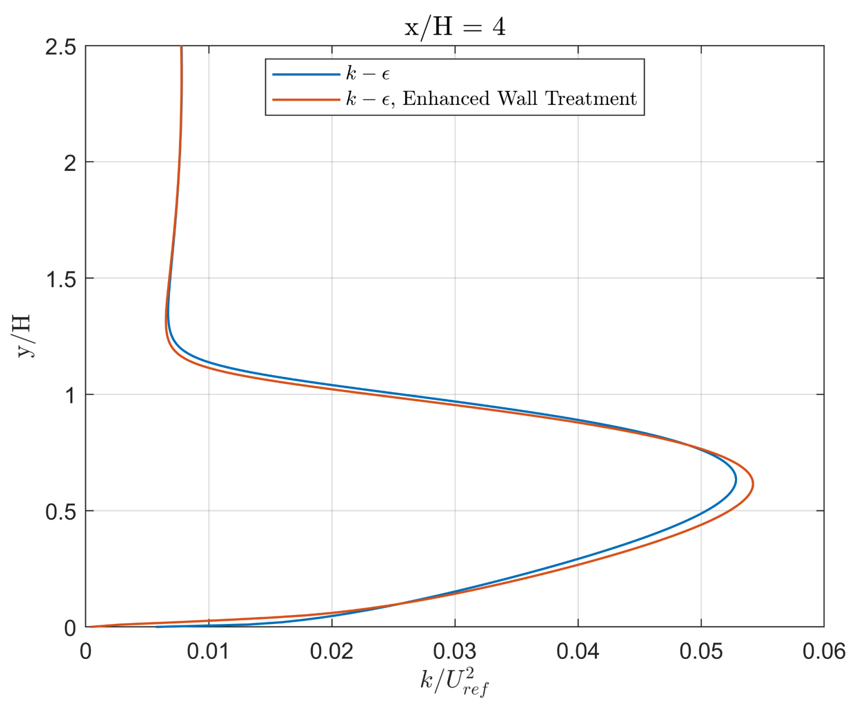

Fig. 82 Comparison of Wall Treatment for Non-Dimensional Turbulent Kinetic Energy at x/H = 4

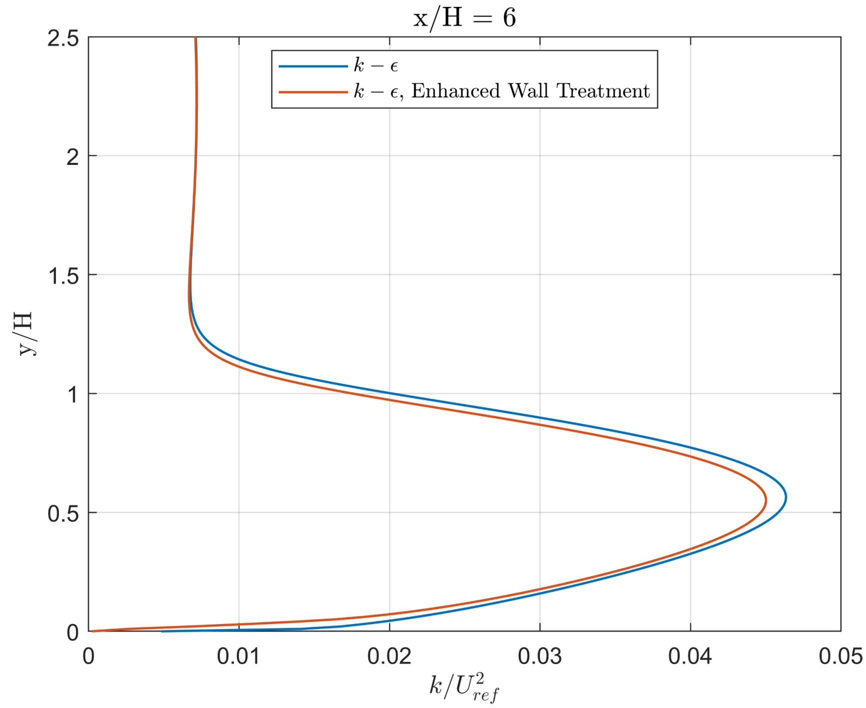

Fig. 83 Comparison of Wall Treatment for Non-Dimensional Turbulent Kinetic Energy at x/H = 6

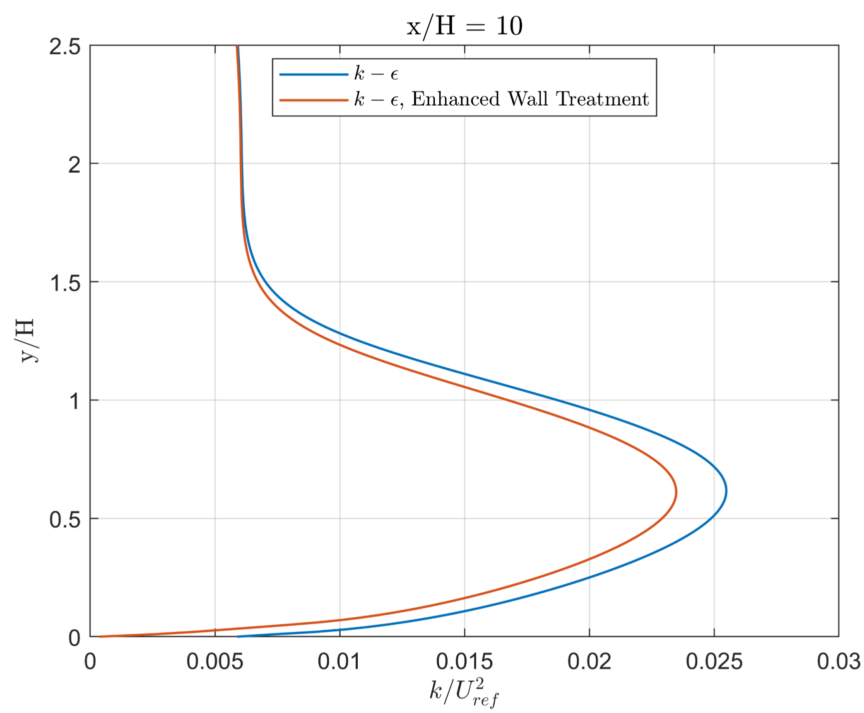

Fig. 84 Comparison of Wall Treatment for Non-Dimensional Turbulent Kinetic Energy at x/H = 10

6.7 Non-Dimensional Turbulent Dissipation Rate Results

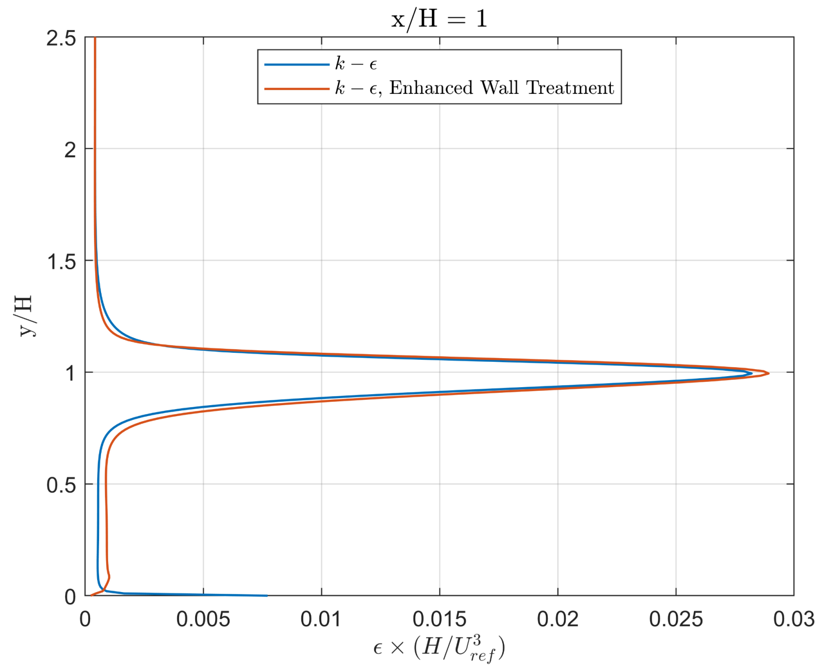

Fig. 85 Comparison of Wall Treatment for Non-Dimensional Turbulent Dissipation Rate at x/H = 1

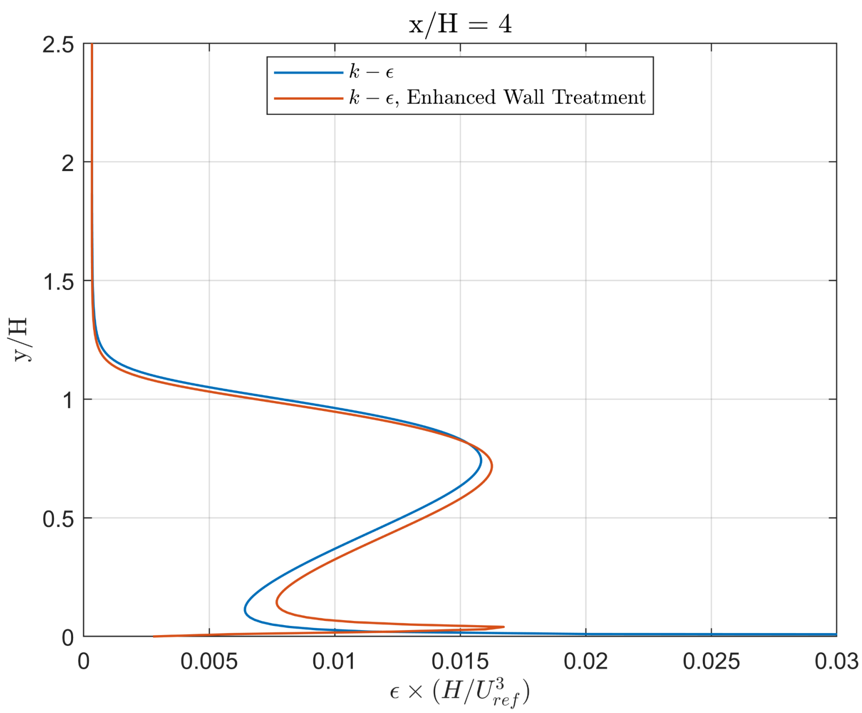

Fig. 86 Comparison of Wall Treatment for Non-Dimensional Turbulent Dissipation Rate at x/H = 4

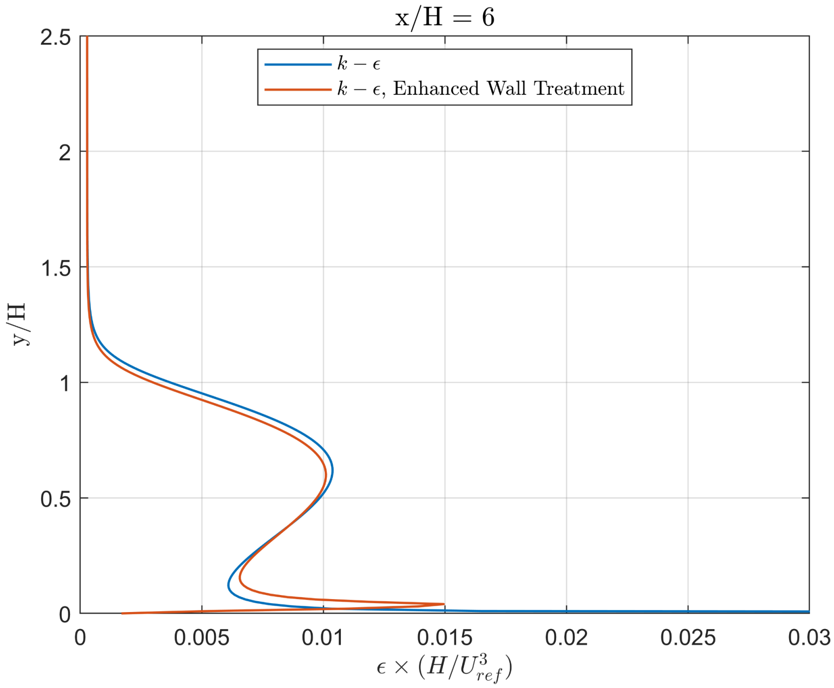

Fig. 87 Comparison of Wall Treatment for Non-Dimensional Turbulent Dissipation Rate at x/H = 4

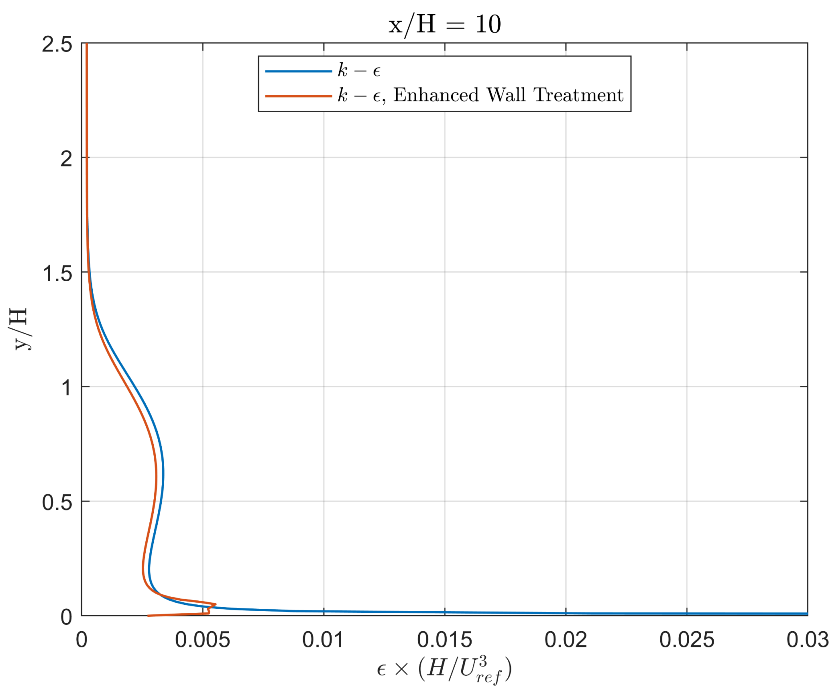

Fig. 88 Comparison of Wall Treatment for Non-Dimensional Turbulent Dissipation Rate at x/H = 10

6.8 DISCUSSION

Question 11: Does the choice of these the wall functions have any significant effect on the results from these three turbulence models for this problem?

Answer 11: For k-ε standard wall treatment and k-ε enhanced wall treatment, the results of sections 6.1 to 6.7 show there isn't a significant effect in comparison to each other. The enhanced wall treatment is barely recognizable for each solution variable, except for just a few particular x/H plots where it has slightly different results. Therefore, the enhanced wall treatment has no significant effect on improving the results.

Question 12: Do your results show that the "enhanced wall treatment" improves the accuracy of the model results when compared to the experimental data?

Answer 12: Based on the results, it doesn't appear that the k-ε enhanced wall treatment is improving the accuracy of the model in comparison to the experimental data. As noted in answer 11, there are sections of x/H where there is a slight difference, in which the enhanced wall treatment is closer to the experimental data. This was expected based on the understanding of near-wall boundary conditions for the k and ε equations. Even though the mesh is fine for this simulation, to obtain truly significant results a finer mesh would need to be added directly at the wall, below the log-law. Overall, the added solution time of the enhanced wall treatment model for the results to be non-significant, proves that it is not worth doing to obtain a more accurate result in this current mesh configuration for the backward-facing step.

Question 13: How does the magnitude of the differences in the model results for these two wall treatments compare with the magnitude of the overall differences between the model results and the experimental data?

Answer 13: The magnitude of the differences in the results of the k-ε standard wall treatment and k-ε enhanced wall treatment is not comparable to the differences between the k-ε models and the experimental data. As mentioned above, the k-ε models are very similar and are almost indistinguishable in a majority of the plots, and therefore have very small differences. In contrast, it was discussed above that the k-ε model, and the other two models for that matter, are not particularly accurate to the experimental data. Therefore, the differences between the models themselves and the experimental data are not comparable to the differences between the k-ε models alone.

7. Final Results/Discussion

Upon completing the ANSYS Fluent simulations, non-dimensionalizing the data, and comparing the different models together, it was very interesting to see the assumptions and constraints of turbulence modeling compared to experimental data. In the future, it would be interesting to continue this with a multitude of different scenarios, other than the backward-facing step. Additionally, adding different meshing parameters (finer in particular points of areas), different turbulence models (such as SST), and changing the coefficients for the equations could change the results shown in the above sections.

In summary, this report details the equations necessary to understand the k-ε, k-ω, and RSTM models. The calculations and graph results were all completed using MATLAB, from data that was achieved by simulations in ANSYS Fluent. The report provides a scenario of a backward-facing step for the turbulence models to be simulated and compares those models to the experimental results of Driver & Seegmiller (1995). The results showed the models performing rather poorly in comparison to the experimental results in a majority of the solution parameters.

Finally, thank you to Professor Werner Dahm for his in-depth and passionate lectures over this past semester. Without the guidance and support, completing this project would not have been possible.

8. References

[1] Driver, D. M., Seegmiller, H.L., "Features of a Reattaching Turbulent Shear Layer in Divergent Channel Flow," AIAA Journal, Vol. 23, No. 2, 1985

9. Appendix

9.1 MATLAB Code

```
%% MAE 575 ; Chandler Hutchens

% Final Project

format compact ;
format long ;
close all ;
clear all ;
clc ;
dbstop if error ;

%% Loading Fluent Data

% Coarse Data
ANSYS.C(:,:,1) = importdata('k_e_coarse').data(:,:) ;
ANSYS.C(:,:,2) = importdata('k_w_coarse').data(:,:) ;
ANSYS.C(:,:,3) = importdata('RSTM_coarse').data(:,:) ;

% Medium Data
ANSYS.M(:,:,1) = importdata('k_e_medium').data(:,:) ;
ANSYS.M(:,:,2) = importdata('k_w_medium').data(:,:) ;
ANSYS.M(:,:,3) = importdata('RSTM_medium').data(:,:) ;

% Fine Data
ANSYS.F(:,:,1) = importdata('k_e_fine').data(:,:) ;
ANSYS.F(:,:,2) = importdata('k_w_fine').data(:,:) ;
ANSYS.F(:,:,3) = importdata('RSTM_fine').data(:,:) ;
ANSYS.F(:,:,4) = importdata('k_e_fine_ew').data(:,:) ;

%% Constants

H = 0.0127 ; % [m]
U_ref = 44.2 ; % [m/s]

%% Data Manipulation

% Rearranging columns of k-e data to match the columns of k-w data
ANSYS.C(:, [4 5], 1) = ANSYS.C(:, [5 4], 1) ;
ANSYS.M(:, [4 5], 1) = ANSYS.M(:, [5 4], 1) ;
ANSYS.F(:, [4 5], 1) = ANSYS.F(:, [5 4], 1) ;
ANSYS.F(:, [4 5], 4) = ANSYS.F(:, [5 4], 4) ;

% Rearranging columns of RSTM data to match the columns of k-w data
ANSYS.C(:, [4 5 6 7 8 9], 3) = ANSYS.C(:, [7 8 9 4 5 6], 3) ;
ANSYS.M(:, [4 5 6 7 8 9], 3) = ANSYS.M(:, [7 8 9 4 5 6], 3) ;
ANSYS.F(:, [4 5 6 7 8 9], 3) = ANSYS.F(:, [7 8 9 4 5 6], 3) ;

% Find lengths of each mesh
C_length = length(ANSYS.C) ;
M_length = length(ANSYS.M) ;
F_length = length(ANSYS.F) ;

% Find difference between meshes and fine mesh
C_diff = F_length - C_length ;
M_diff = F_length - M_length ;
```

```
% Assigning Data to 1 Structure Based
for i = 1:4

    if i < 4

        T(:,i,1) = [ANSYS.C(:,7,i); nan(C_diff,1)]/U_ref ;
        T(:,i,2) = [ANSYS.C(:,8,i); nan(C_diff,1)]/U_ref ;
        T(:,i,3) = [ANSYS.C(:,4,i); nan(C_diff,1)]*1000/U_ref^2 ;
        T(:,i,4) = [ANSYS.C(:,5,i); nan(C_diff,1)]*1000/U_ref^2 ;
        T(:,i,5) = [ANSYS.C(:,6,i); nan(C_diff,1)]*1000/U_ref^2 ;
        T(:,i,6) = [ANSYS.C(:,9,i); nan(C_diff,1)]/U_ref^2 ;
        T(:,i,7) = [ANSYS.C(:,10,i); nan(C_diff,1)]*H/U_ref^3 ;

        T(:,i+3,1) = [ANSYS.M(:,7,i); nan(M_diff,1)]/U_ref ;
        T(:,i+3,2) = [ANSYS.M(:,8,i); nan(M_diff,1)]/U_ref ;
        T(:,i+3,3) = [ANSYS.M(:,4,i); nan(M_diff,1)]*1000/U_ref^2 ;
        T(:,i+3,4) = [ANSYS.M(:,5,i); nan(M_diff,1)]*1000/U_ref^2 ;
        T(:,i+3,5) = [ANSYS.M(:,6,i); nan(M_diff,1)]*1000/U_ref^2 ;
        T(:,i+3,6) = [ANSYS.M(:,9,i); nan(M_diff,1)]/U_ref^2 ;
        T(:,i+3,7) = [ANSYS.M(:,10,i); nan(M_diff,1)]*H/U_ref^3 ;

    end

    T(:,i+6,1) = [ANSYS.F(:,7,i)]/U_ref ;
    T(:,i+6,2) = [ANSYS.F(:,8,i)]/U_ref ;
    T(:,i+6,3) = [ANSYS.F(:,4,i)]*1000/U_ref^2 ;
    T(:,i+6,4) = [ANSYS.F(:,5,i)]*1000/U_ref^2 ;
    T(:,i+6,5) = [ANSYS.F(:,6,i)]*1000/U_ref^2 ;
    T(:,i+6,6) = [ANSYS.F(:,9,i)]/U_ref^2 ;
    T(:,i+6,7) = [ANSYS.F(:,10,i)]*H/U_ref^3 ;

end


% Correcting factor of 10 from k-w uv-Reynolds stress
CF = [2 5 8] ;

for i = 1:length(CF)

    T(:,CF(i),5) = T(:,CF(i),5)*10 ;

end

%% Calculations

% Non-dimensionalizing and filling x values for Interpolant
x(:,1) = [ANSYS.C(:,2,1); nan(C_diff,1)]/H ;
x(:,2) = [ANSYS.M(:,2,1); nan(M_diff,1)]/H ;
x(:,3) = [ANSYS.F(:,2,1)]/H ;
x(:,4) = [ANSYS.F(:,2,4)]/H ;

% Non-dimensionalizing and filling x values for Interpolant
y(:,1) = [ANSYS.C(:,3,1) ; nan(C_diff,1)]/H ;
y(:,2) = [ANSYS.M(:,3,1) ; nan(M_diff,1)]/H ;
y(:,3) = [ANSYS.F(:,3,1)]/H ;
y(:,4) = [ANSYS.F(:,3,4)]/H ;

% Interpolant Data Correction
T(isnan(T))= -100 ;
```

```
x(isnan(x)) = -100 ;
y(isnan(y))= -100 ;

% Meshes + Models
Type = [1 1 1 2 2 2 3 3 3 4] ;

% Interpolant, Given in Project Description
for i = 1:10

    for j = 1:7

        F{i,j} = scatteredInterpolant(x(:,Type(i)),y(:,Type(i)),T(:,i,j),'linear','none') ;

    end

end

%% Part 1 and 2, Contour Plots

% heading = {'$k-\epsilon$ Model'; '$k-\omega$ Model'; 'RSTM'; '$k-\epsilon$ Model'; ...
%           '$k-\omega$ Model'; 'RSTM'; '$k-\epsilon$ Model'; '$k-\omega$ Model'; 'RSTM'} ;

% subheading = {'Coarse Mesh'; 'Coarse Mesh'; 'Coarse Mesh'; 'Medium Mesh'; ...
%              'Medium Mesh'; 'Medium Mesh'; 'Fine Mesh'; 'Fine Mesh'; 'Fine Mesh'} ;

% Plotting Each Contour Plot
for i = 1:9

    for j = 1:7

        % Plotting Inputs
        x_range = linspace(15, 27, 48*12*2) ;
        y_range = linspace(0, 2, 180*2*2) ;
        colorval = F{i,j}({x_range, y_range}) ;

        % Correction for x/H [0,12]
        x_range = x_range - 15 ;

        figure(10*i+j) ;
        h = pcolor(x_range, y_range, colorval') ;
        h.EdgeColor = 'none' ;
        axis equal ;
        xlim([0 12]) ;
        ylim([0 2]) ;
        colorbar ;
        colormap('jet') ;

        %[t,e] = title(heading(i),subheading(i)) ;
        xlabel('x/H','fontname','Times New Roman','interpreter','latex','fontsize',12)
        ylabel('y/H','fontname','Times New Roman','interpreter','latex','fontsize',12)

        %set(t, 'fontname', 'Times New Roman', 'interpreter', 'latex', 'fontsize', 15)
        %set(e, 'fontname', 'Times New Roman', 'interpreter', 'latex', 'fontsize', 12)
        set(figure(10*i+j),'Position',[100 100 1000 500])

        % Export Figures To MATLAB Folder
        exportgraphics(gca,sprintf('fig%d.png',(10*i+j)),'resolution',600)

        % Closes Figures So Doesn't Crash Computer
        set(gcf, 'visible', 'off')
```

```
    end

end

%% Given Driver & Seegmiller Data

% https://turbmodels.larc.nasa.gov/Backstep_validation/profiles.exp.dat

% y is y/H, u is u/Uref, v is v/Uref,
% uu is 1000*u'u'/Uref**2
% vv is 1000*v'v'/Uref**2
% uv is 1000*u'v'/Uref**2
% triple moments are 10000*u'u'u'/Uref**3 (for example)
% variables="ID","y","u","v","uu","vv","uv","uuu","uvv","vuu","vvv"

% zone x/H = 1
ds(:,:,1) = [
    2 0.10 -0.008 -0.005 3.18 2.94 -0.72 0.00 -0.35 0.14 0.19
    3 0.15 -0.017 -0.004 4.71 3.22 0.46 -0.46 -0.14 0.15 0.06
    4 0.20 -0.027 -0.001 5.04 3.29 0.21 -0.38 -0.11 0.22 -0.01
    5 0.30 -0.035 0.004 5.12 3.49 -0.14 -0.31 -0.21 0.31 0.05
    6 0.40 -0.035 0.014 4.50 3.67 -0.43 -0.33 -0.24 0.33 0.31
    7 0.50 -0.035 0.022 4.94 2.90 -0.65 -0.48 -0.25 0.32 0.20
    8 0.60 -0.027 0.028 4.56 3.00 -0.74 -0.61 -0.23 0.28 0.18
    9 0.70 -0.005 0.025 5.03 3.44 -0.92 -0.39 -0.15 -0.10 -0.04
    10 0.80 0.081 0.006 9.24 7.38 -3.71 2.63 2.22 -2.73 -2.24
    11 0.90 0.292 -0.027 13.80 11.77 -7.05 1.18 0.94 -0.65 -1.00
    12 1.00 0.519 -0.033 13.93 10.41 -5.85 -4.18 -2.73 2.91 2.32
    13 1.10 0.704 -0.023 6.18 3.79 -1.54 -1.19 -0.49 0.45 0.62
    14 1.15 0.740 -0.022 4.87 2.84 -1.25 -0.07 -0.16 0.12 0.25
    15 1.20 0.762 -0.023 4.27 2.51 -1.05 -0.21 -0.11 0.09 0.24
    16 1.30 0.793 -0.031 3.87 2.26 -0.98 -0.22 -0.11 0.14 0.16
    17 1.40 0.827 -0.027 3.38 2.05 -0.82 -0.26 -0.09 0.13 0.15
    18 1.50 0.851 -0.031 3.08 1.98 -0.81 -0.26 -0.11 0.14 0.13
    19 1.70 0.891 -0.034 2.49 1.57 -0.64 -0.25 -0.11 0.09 0.09
    20 2.00 0.942 -0.033 1.76 1.21 -0.44 -0.24 -0.09 0.11 0.09
    21 2.40 0.990 -0.031 0.91 0.67 -0.12 -0.11 -0.04 0.05 0.04
    22 2.80 1.009 -0.028 0.45 0.36 0.01 0.01 0.01 0.00 0.01] ;

% zone x/H = 4
ds(:,:,2) = [
    2 0.10 -0.171 -0.014 9.71 7.67 -2.64 4.26 2.49 -2.60 -2.47
    3 0.15 -0.154 -0.020 13.20 10.10 -4.45 10.50 4.70 -5.18 -4.94
    4 0.20 -0.134 -0.027 16.50 12.98 -6.30 13.67 5.61 -6.52 -5.34
    5 0.30 -0.057 -0.046 25.91 16.16 -9.94 16.29 4.72 -5.66 -4.41
    6 0.40 0.052 -0.068 32.92 18.95 -12.02 -4.83 -2.74 3.50 2.13
    7 0.50 0.169 -0.082 30.67 18.32 -11.11 -26.75 -8.98 11.86 6.02
    8 0.60 0.279 -0.093 23.24 15.90 -8.99 -19.61 -9.14 9.42 6.90
    9 0.70 0.381 -0.094 18.21 13.03 -6.93 -5.06 -4.23 3.71 4.03
    10 0.80 0.473 -0.097 17.03 10.98 -5.98 -3.65 -3.38 3.11 3.89
    11 0.90 0.568 -0.097 14.45 8.30 -4.52 -5.19 -2.98 2.45 3.04
    12 1.00 0.656 -0.092 11.55 6.33 -3.48 -6.85 -2.78 2.86 2.55
    13 1.10 0.730 -0.089 7.70 4.20 -2.10 -3.69 -1.47 1.52 1.57
    14 1.15 0.754 -0.087 6.27 3.52 -1.65 -2.27 -0.94 1.00 1.14
    15 1.20 0.778 -0.084 4.85 2.83 -1.19 -0.84 -0.42 0.48 0.71
    16 1.30 0.811 -0.083 3.80 2.19 -0.87 -0.44 -0.19 0.16 0.32
    17 1.40 0.835 -0.081 3.17 1.80 -0.67 -0.29 -0.11 0.13 0.20
    18 1.50 0.856 -0.079 2.86 1.52 -0.56 -0.31 -0.08 0.10 0.14
    19 1.70 0.895 -0.076 2.27 1.25 -0.45 -0.34 -0.10 0.14 0.13
```

```
    20 2.00 0.940 -0.072 1.47 0.78 -0.23 -0.22 -0.07 0.07 0.07
    21 2.40 0.978 -0.063 0.61 0.39 0.00 -0.06 -0.01 0.03 0.02
    22 2.80 0.986 -0.056 0.38 0.26 0.06 0.01 0.00 -0.01 0.00] ;

% zone x/H = 6
ds(:,:,3) = [
    2 0.10 0.147 -0.052 17.58 12.81 -4.10 0.03 0.65 -0.11 -2.05
    3 0.15 0.169 -0.061 19.50 15.02 -5.48 0.04 0.24 0.11 -1.31
    4 0.20 0.188 -0.067 19.53 16.50 -6.79 -2.21 -1.70 1.31 0.58
    5 0.30 0.248 -0.084 20.73 16.98 -7.86 -5.45 -4.28 3.36 3.25
    6 0.40 0.306 -0.091 19.70 16.60 -8.23 -6.80 -5.78 4.97 5.60
    7 0.50 0.369 -0.097 20.03 15.89 -8.24 -9.08 -7.74 7.14 6.87
    8 0.60 0.435 -0.099 18.36 13.86 -7.36 -8.93 -6.86 6.50 6.48
    9 0.70 0.501 -0.095 17.26 12.00 -6.57 -10.43 -7.51 7.32 7.26
    10 0.80 0.569 -0.094 14.96 9.63 -5.06 -10.37 -5.91 6.20 5.55
    11 0.90 0.632 -0.091 11.96 7.62 -3.79 -10.19 -4.74 5.11 4.09
    12 1.00 0.686 -0.089 8.88 5.57 -2.80 -7.74 -3.45 3.55 2.93
    13 1.10 0.730 -0.086 6.11 3.95 -1.76 -3.81 -1.72 1.93 1.87
    14 1.15 0.747 -0.086 5.23 3.35 -1.51 -2.68 -1.23 1.39 1.33
    15 1.20 0.763 -0.085 4.36 2.75 -1.25 -1.54 -0.74 0.84 0.79
    16 1.30 0.794 -0.083 3.40 2.19 -0.80 -0.47 -0.23 0.25 0.44
    17 1.40 0.815 -0.082 2.87 1.79 -0.65 -0.33 -0.12 0.20 0.25
    18 1.50 0.834 -0.081 2.50 1.53 -0.61 -0.29 -0.10 0.14 0.14
    19 1.70 0.869 -0.077 2.01 1.24 -0.43 -0.27 -0.10 0.13 0.13
    20 2.00 0.915 -0.074 1.28 0.80 -0.21 -0.22 -0.06 0.08 0.04
    21 2.40 0.949 -0.066 0.53 0.44 -0.04 -0.01 0.00 0.01 0.02
    22 2.80 0.954 -0.060 0.42 0.33 0.03 0.02 0.02 0.00 0.00] ;

% zone x/h = 10
ds(:,:,4) = [
    2 0.10 0.418 -0.029 8.89 5.63 -1.43 2.05 0.73 -0.34 -0.46
    3 0.15 0.433 -0.027 9.97 6.15 -1.84 2.59 0.74 -0.35 -1.10
    4 0.20 0.447 -0.032 10.51 6.97 -2.35 2.19 0.83 -0.25 -0.99
    5 0.30 0.471 -0.036 11.57 8.34 -3.43 1.23 0.55 -0.24 -0.37
    6 0.40 0.496 -0.041 12.24 9.08 -4.19 0.67 -0.37 0.07 0.46
    7 0.50 0.526 -0.043 12.86 9.16 -4.51 -1.12 -1.29 0.99 1.47
    8 0.60 0.560 -0.045 13.37 9.05 -4.61 -3.54 -2.59 2.08 2.43
    9 0.70 0.596 -0.047 13.03 8.44 -4.61 -5.62 -3.18 3.10 3.00
    10 0.80 0.638 -0.047 12.00 7.75 -4.30 -7.23 -4.00 3.65 3.93
    11 0.90 0.676 -0.046 10.96 6.44 -3.68 -8.76 -3.81 3.96 3.37
    12 1.00 0.712 -0.047 9.04 5.35 -2.94 -8.54 -3.51 3.72 2.94
    13 1.10 0.745 -0.046 7.23 4.32 -2.33 -6.66 -2.57 2.72 2.36
    14 1.15 0.758 -0.045 6.44 3.92 -2.02 -5.85 -2.28 2.36 2.02
    15 1.20 0.772 -0.044 5.66 3.52 -1.71 -5.04 -2.00 2.01 1.68
    16 1.30 0.798 -0.042 4.22 2.80 -1.36 -2.94 -1.28 1.48 1.38
    17 1.40 0.819 -0.040 3.29 2.21 -0.96 -1.61 -0.81 0.80 0.96
    18 1.50 0.839 -0.039 2.54 1.80 -0.72 -0.83 -0.53 0.51 0.65
    19 1.70 0.871 -0.038 1.60 1.03 -0.35 -0.28 -0.09 0.11 0.15
    20 2.00 0.908 -0.034 0.96 0.66 -0.14 -0.13 -0.03 0.08 0.09
    21 2.40 0.929 -0.032 0.43 0.35 0.02 0.02 0.02 0.02 0.03
    22 2.80 0.933 -0.031 0.35 0.27 0.04 0.03 0.02 0.01 0.02] ;

%% Data Manipulation (pt 2)

e_y = ds(:,2) ;
e(:,:,1) = ds(:,3,:) ; % u
e(:,:,2) = ds(:,4,:) ; % v
e(:,:,3) = ds(:,5,:) ; % uu
e(:,:,4) = ds(:,6,:) ; % vv
e(:,:,5) = ds(:,7,:) ; % uv
```

```
%% Part 3: Line Plots, Models Comparison

% Vector for x/H and y/H
xH = [1 4 6 10] + 15 ;
yH = linspace(0, 2.5, 250)' ;

heading = {'x/H = 1', 'x/H = 4','x/H = 6','x/H = 10'} ;

label = {'$\overline{u} /U_{ref}$'; '$\overline{v}/U_{ref}$'; '$\overline{u"^2} \times (10^3/U_{ref}^2)$'; ...
    '$\overline{v"^2} \times (10^3/U_{ref}^2)$'; '$\overline{u"v"} \times (10^3/U_{ref}^2)$'; ...
    '$k/U_{ref}^2$'; '$\epsilon \times (H/U_{ref}^3)$'} ;

% Plotting
for i = 1:4

  for j = 1:7

    for k = 7:9

        % Evaluate interpolant along y/H at given x/H
        lineval(:,k-6, j,i) = F{k,j}({xH(i), yline})' ;

        figure(100+10*i+j)
        plot(lineval(:, k-6, j, i), yH, 'linewidth', 1)

        hold on

        t = title(heading(i)) ;
        xlabel(label(j),'interpreter','latex') ;
        ylabel('y/H','interpreter','latex') ;
        set(t,'interpreter','latex','fontsize',12) ;
        ylim([0 2.5]) ;
        grid on ;

        % Rescaling epsilon
        if j == 7

          xlim([0 0.05])

        end

    end

    % Plot Experimental Data
    if j <= 5

      plot(e(:,i,j), y_e, '-ok', 'linewidth', 1, 'markersize', 4)

    end

    legend('$k-\epsilon$','$k-\omega$','RSTM','Experimental Data','interpreter','latex','location','best')

    % Export Figures To MATLAB Folder
    exportgraphics(gca,sprintf('fig%d.png',(100+10*i+j)),'resolution',600)

    % Closes Figures So Doesn't Crash Computer
    set(gcf,'visible','off')

  end
```

```
end

%% Part 4: Line Plots, Wall Treatment

for i = 1:4

    for j = 1:7

        for k = 7:10

            % Evaluate interpolant along y/H at given x/H
            lineval(:, k-6, j,i) = F{k,j}({xH(i), yline})' ;

            figure(200+10*i+j)
            plot(lineval(:, k-6, j, i), yH, 'linewidth', 1)

            hold on

            t = title(heading(i)) ;
            xlabel(label(j), 'interpreter', 'latex') ;
            ylabel('y/H', 'interpreter', 'latex') ;
            set(t, 'interpreter', 'latex','fontsize', 12) ;
            ylim([0 2.5]) ;
            grid on ;

            % Rescaling Epsilon
            if j == 7

                xlim([0 0.03])

            end

        end

        % Plot Experimental Data
        if j <= 5

            plot(e(:,i,j), y_e, '-ok', 'linewidth', 1, 'markersize', 4)

        end

        legend('$k-\epsilon$', '$k-\epsilon$, Enhanced Wall Treatment', 'Experimental Data','interpreter', 'latex', 'location', 'best')

        % Export Figures To MATLAB Folder
        exportgraphics(gca,sprintf('fig%d.png',(200+10*i+j)),'resolution',600)

        % Closes Figures So Doesn't Crash Computer
        set(gcf,'visible','off')

    end

end
```