MAE 598: Analysis and Modeling of Fluid Flows

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Final Project Report

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1. Introduction

1.1 Project Description

This project outlines a comparison between different vortex identification methods for a NASA experiment characterizing flow behind a NACA 0012 airfoil [3], seen in Fig. 1. The flow conditions were established from the experiments of Chow, J.S., Zilliac, G.G., and Bradshaw, P (1997) [1]. The experimental results from the study will be converted to 2D and five different vortex identification methods will be compared against each other at three positions along the z-axis (spanwise) of the airfoil. The methods used were vorticity, λ_2-criterion, Q-criterion, Δ-criterion, and the Rortex method. Additionally, the data will be Gaussian filtered and the results will be compared against the original data.

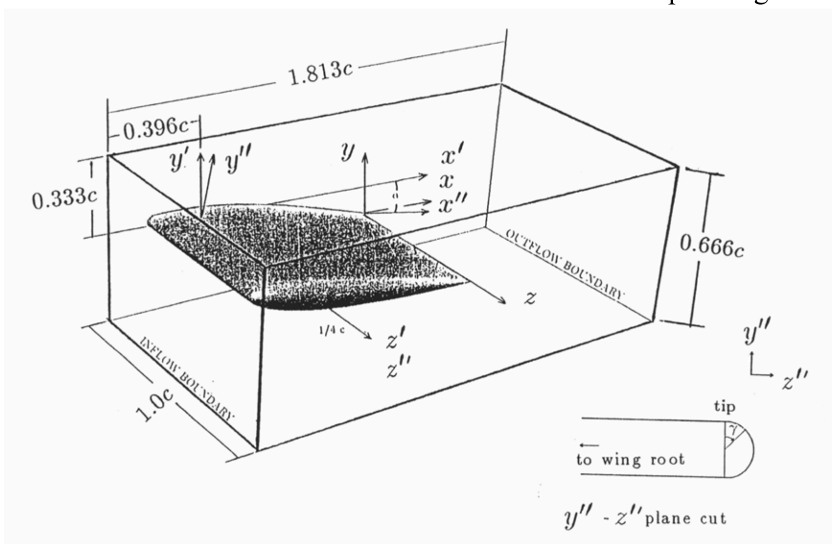


Fig. 1 NACA 0012 Airfoil Model [1]

1.2 Report Outline

This final project report will be organized starting with an overview of the experiment conducted by Chow, J.S., Zilliac, G.G., and Bradshaw, P (1997) [1] and vortex identification theory. Then, the results of the methods will be compared against each other, before and after benign filtered. Finally, the report will have a final discussion/conclusion to the overall project, references used, and the appendix with the MATLAB code.

2. Background

2.1 Description

This project details calculations and comparison between data that was collected by Chow, J.S., Zilliac, G.G., and Bradshaw, P (1997) [1].

2.2 Experiment Set-up

The experiment was conducted in the low-speed wind tunnel at the Fluid Mechanics Laboratory (FML) at NASA Ames Research Center, where a model is shown in Fig. 2. The freestream velocity was set at 170 ft/s and the maximum turbulence level was measured at .15%. The area of the wind tunnel section is 32 by 48 inches, where a half-wing model of a 4-foot chord wingtip of a NACA 0012 wing section was used. The airfoil was constructed out of aluminum and pitched at an angle of attack of 10 degrees [3].

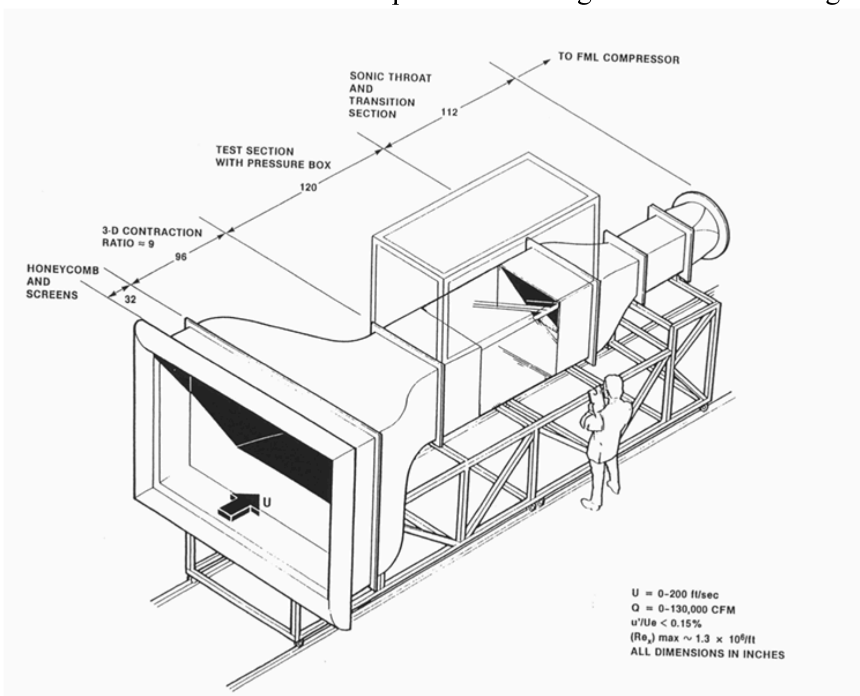


Fig. 2 FML Low-Speed Wind Tunnel [A]

In terms of measurement instruments and techniques, the experiment used hot-wire anemometry, particle image velocimetry (PIV), pressure probes, laser doppler velocimetry (LDV), and flow visualization. Using these instruments and techniques is vital for accurately capturing the complex behaviors of the turbulent flows in the wingtip vortices. Additionally, they offer a high resolution to allow the derivation of quantitative data for the identification of vortices [1]. Combining them all provides a comprehensive view of the vortex dynamics and offers an advancement in the understanding of aircraft designs.

2.3 Results

The experiment yielded a variety of results, however, for this project the data retrieved from the wind tunnel are all that is necessary to calculate the various vortex identification methods. The data was given as a zip file that included 2 readme's and 15 data files. The file used for this project was the TAKALL.DAT which includes the mean velocity information at every x, y, and z position.

3. Vertex Identification Methods

3.1 Description

Vortex identification is crucial in fluid dynamics, specifically for understanding turbulence or aerodynamic performance. A vortex is a region in a fluid where the flow rotates around an axis. There are a multitude of identification methods, each having advantages and disadvantages. Selecting the correct method allows for better prediction of fluid behavior and optimization of designs.

3.2 Vorticity

Vorticity is defined as the curl of the velocity field in a fluid flow. Mathematically, it is expressed in Eq. 1.

$$\omega = \nabla \times \bar{v} \tag{1}$$

In Eq. 1, the vorticity (ω) is equal to the gradient of the magnitude of the velocity vector. Large vorticity indicates regions where the fluid is undergoing significant rotational movement. Vorticity is particularly useful for analyzing turbulent flows, where it helps identify and visualize complex flow structures such as eddies and vortices [4].

However, using vorticity alone to identify vortices has limitations. While iso-surfaces of vorticity can highlight areas with high rotational speeds, they do not necessarily differentiate between actual vortex rotation and parallel shear movements, such as those caused by no-slip conditions at solid boundaries. For example, near the no-slip walls in a laminar boundary layer, shear-induced vorticity can be high, yet it may not correspond to the swirling motion typically associated with vortices. This issue complicates the use of vorticity for vortex identification, as the selection of threshold values for defining vorticity iso-surfaces lacks a clear physical basis and can be arbitrary. Moreover, in three-dimensional flows, vortex lines derived from vorticity fields can be cluttered and irregular, making it difficult to pinpoint the exact locations and extent of vortical structures [6]. Thus, while vorticity is a powerful tool for visualizing flow patterns, additional criteria are often needed to accurately identify and characterize vortices in fluid dynamics studies.

3.3 λ_2-Criterion

The λ_2-criterion is a widely used vortex identification method valued for its ability to pinpoint vortex cores in complex flow fields. This criterion is based on the eigenvalues of the symmetric part of the square of the velocity gradient tensor, shown in Eqs. 2 through 4.

$$(S + \Omega)^2 \tag{2}$$

Where,

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

For the λ_2-criterion, a region is identified as a vortex if the second eigenvalue of the tensor is negative, satisfying $\lambda_1 > \lambda_2 > \lambda_3$ and $\lambda_2 < 0$ [6].

Despite being widely used, the λ_2-criterion does not provide a direct interpretation of vortex regions. It often requires subjective judgment in the selection of threshold values for λ_2. This subjectivity can affect the accuracy and consistency of vortex identification across different studies or flow conditions. Additionally, the vortex core lines identified using the λ_2-criterion can sometimes appear discontinuous. This discontinuity arises because these lines are composed of segments that may not seamlessly connect, especially in turbulent or highly unsteady flows [6]. Therefore, while the λ_2-criterion is effective for highlighting areas likely to be influenced by vortical movements, it may require supplementary analysis or additional criteria to confirm the presence and characteristics of vortices.

3.4 Q-Criterion

The Q-criterion is an essential vortex identification technique utilized primarily for detecting coherent vortices in turbulent flows. It defines vortices based on the second invariant of the velocity gradient tensor, where Q is shown in Eq. 5.

$$Q = \frac{1}{2}\left(|\Omega|^2 - |S|^2\right) > 0 \tag{5}$$

The fundamental principle of the Q-criterion is for $Q > 0$, the local rotation dynamics are represented by the rotation rate tensor. The rotation-rate tensor dominates over the deformation characteristics of the flow, indicated by the strain-rate tensor [5]. This condition is pivotal for confirming the presence of a vortex, as it suggests that the rotating motion is strong enough to maintain the vortex structure against the stretching and folding actions of the strain field. The Q-criterion has been extensively applied in numerical simulations to study large-scale vortices, providing critical insights into the dynamics and structure of turbulent flows [6].

3.5 Δ-Criterion

The Δ-criterion is an analytical approach used to identify vortices by focusing on the complex eigenvalues of the velocity gradient tensor, in Eq. 6. The Δ-criterion is then shown in Eq. 7.

$$\nabla \bar{v} \tag{6}$$

$$\Delta = \left(\frac{Q}{3}\right)^3 + \left(\frac{R}{2}\right)^2 > 0 \tag{7}$$

This criterion specifically assesses the discriminant of the characteristic equation derived from the tensor, which aids in distinguishing between swirling motions and shear flows. The fundamental idea is that Δ >

0 indicates the presence of complex roots and potential vortical structures. The advantage of the Δ-criterion is its sensitivity to any rotational behavior, as a positive Δ suggests a stronger spiraling motion within the fluid. This makes it particularly useful in scenarios where vortices may not be strongly defined or are emerging within the flow, providing an early indication of rotational dynamics.
However, similar to other vortex identification methods, the Δ-criterion has its limitations. Primarily its dependency on the selection of thresholds for Δ values, which can be somewhat arbitrary and may vary with flow conditions.

3.6 The Rortex Method

The Rortex method is a relatively recent development in the field of vortex identification, designed to enhance the precision of detecting vortical structures within fluid flows. Rortex offers a nuanced approach by focusing on the relative magnitude of the rotational part of the velocity gradient tensor compared to its entire magnitude. This method mathematically defines the Rortex in Eq. 8 [2].

$$R = \frac{|Q|}{|\nabla \bar{v}|} \tag{8}$$

The core idea behind Rortex is that it quantifies the intensity of local rotation by comparing it to the overall dynamics of the fluid at each point. This comparison helps in distinguishing true rotational movements from mere shear or strain effects, which may appear similar but have different implications in fluid dynamics. By evaluating Eq. 8, Rortex effectively highlights regions where rotational motions are dominant relative to the general flow movements, thus identifying potential vortices with greater accuracy.

Rortex is particularly valuable in complex flow situations where traditional methods like the Q-criterion or λ_2-criterion might either miss subtle vortical patterns or misclassify non-vortical regions as vortices due to local shear or strain. his method's ability to provide a more direct measure of vortex strength makes it an important tool in the advanced visualization and analysis of fluid flows [2].

4. Vortex Identification Results

4.1 Description

As discussed in section 2, the half NACA 0012 wing was used inside the wind tunnel. The given data points along the span of the airfoil were from 0 to 28 inches. By separating this wing into three main sections, near the wall, middle, and tip, a set of ranges for the z-axis can be created to transform the data into 2D. These distributions can be seen in Table 1.

Table 1 Wing Section Distribution

Section	Z Range
Wall	$z \leq 4$
Middle	$12 \leq z \leq 16$
Tip	$25 \leq z$

4.2 Vortex Identification Comparison at NACA 0012 Sections

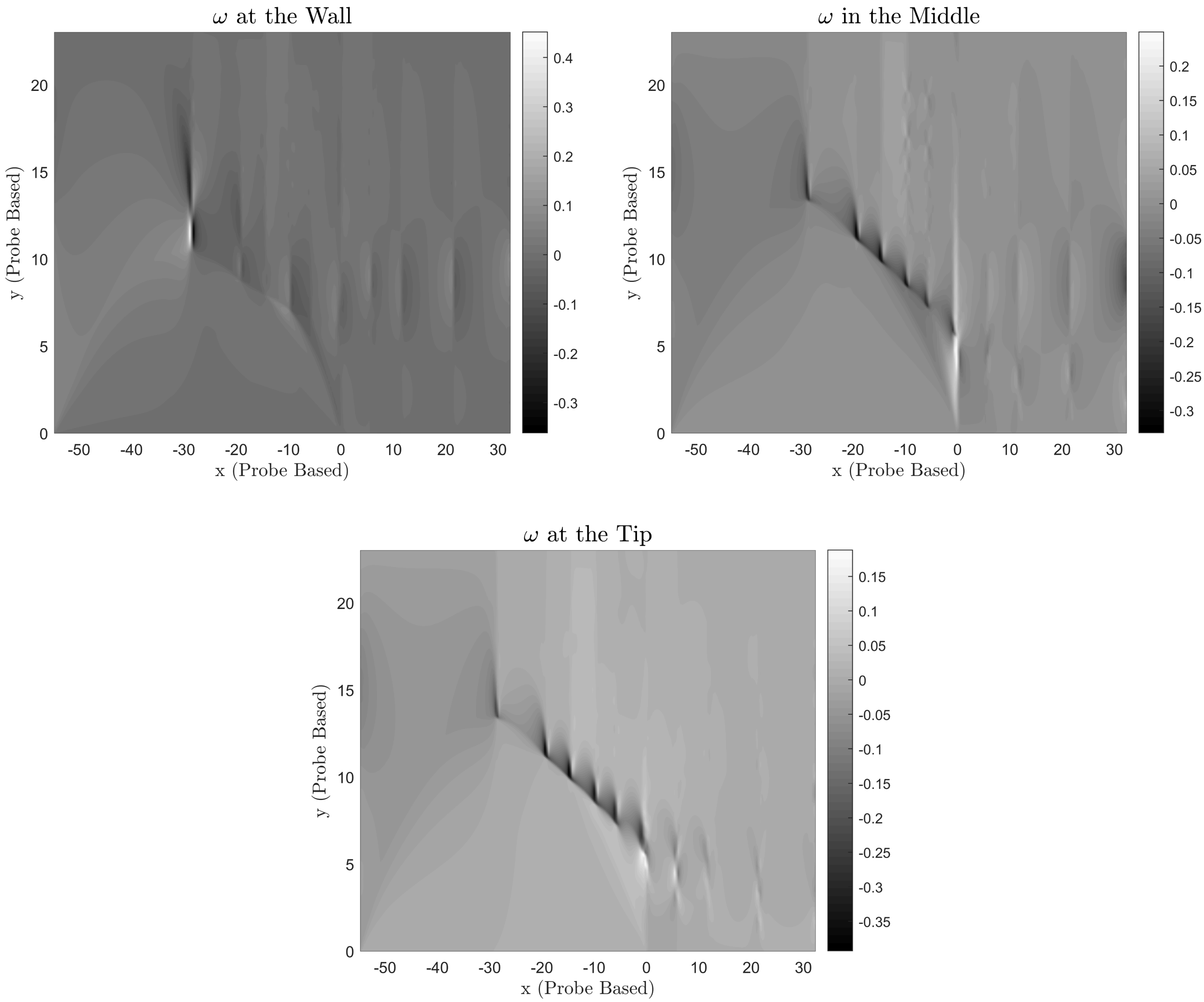


Fig. 3 Comparison of Vorticity at NACA 0012 Sections

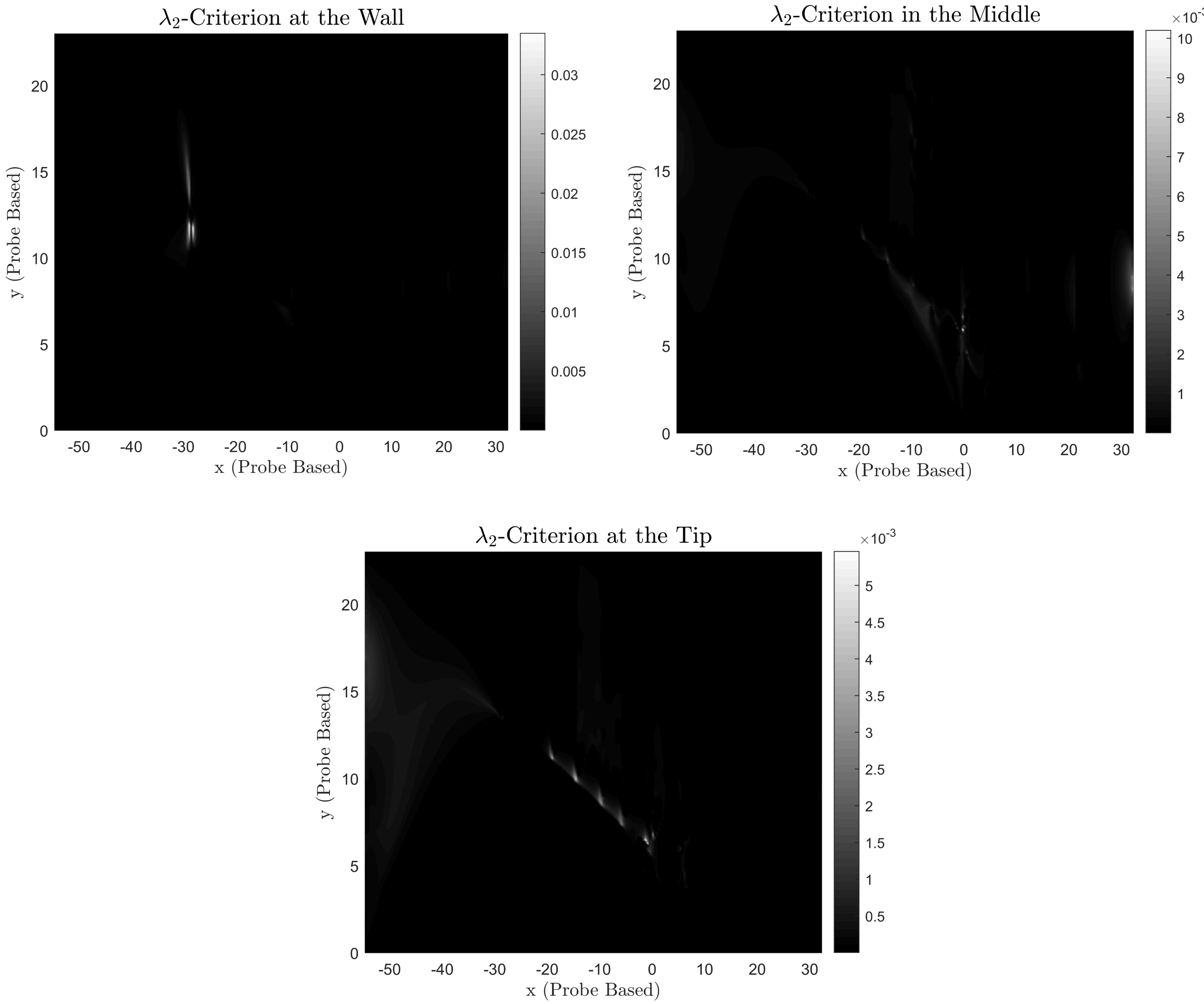


Fig. 4 Comparison of λ_2-Criterion at NACA 0012 Sections

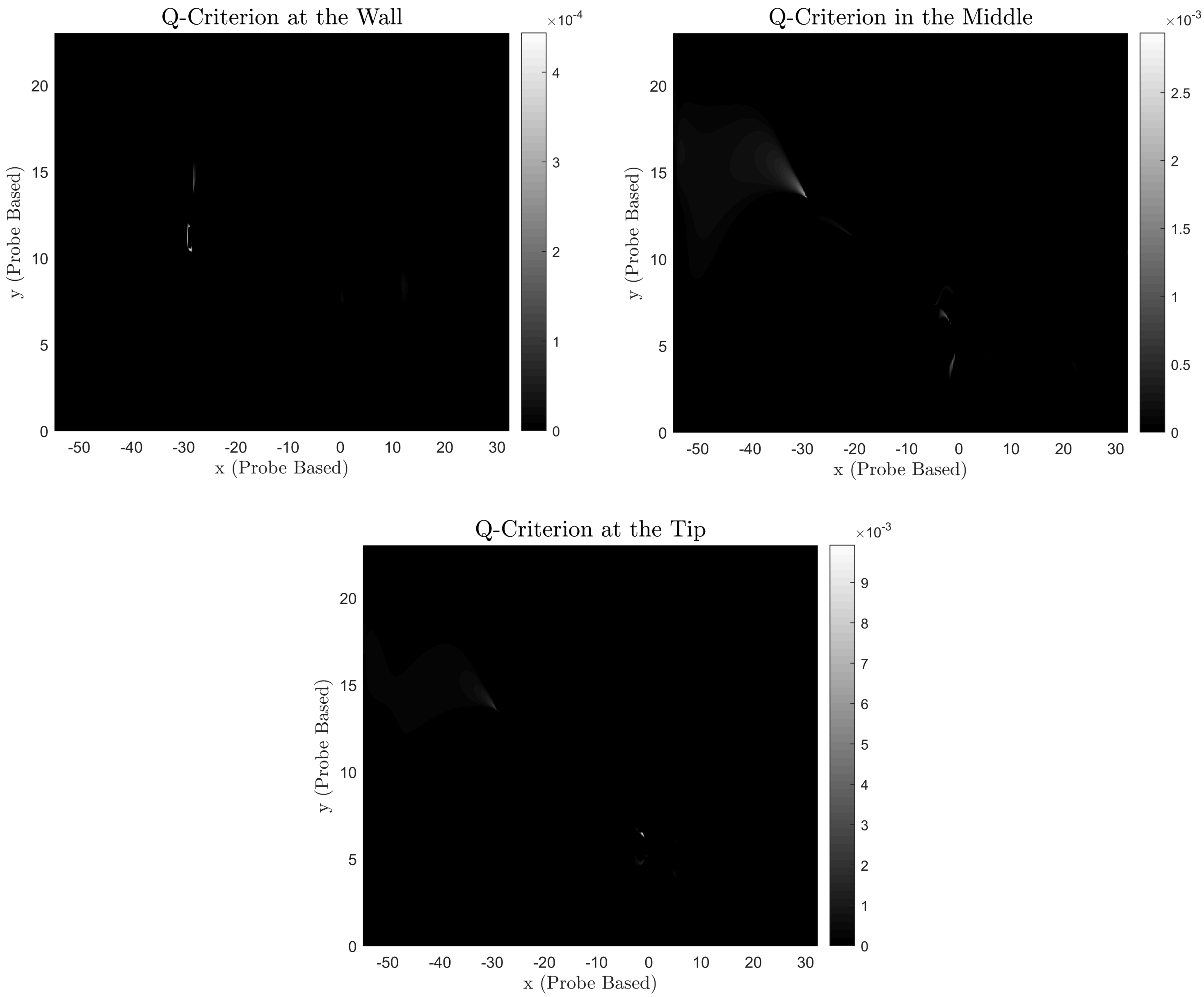


Fig. 5 Comparison of Q-Criterion at NACA 0012 Sections

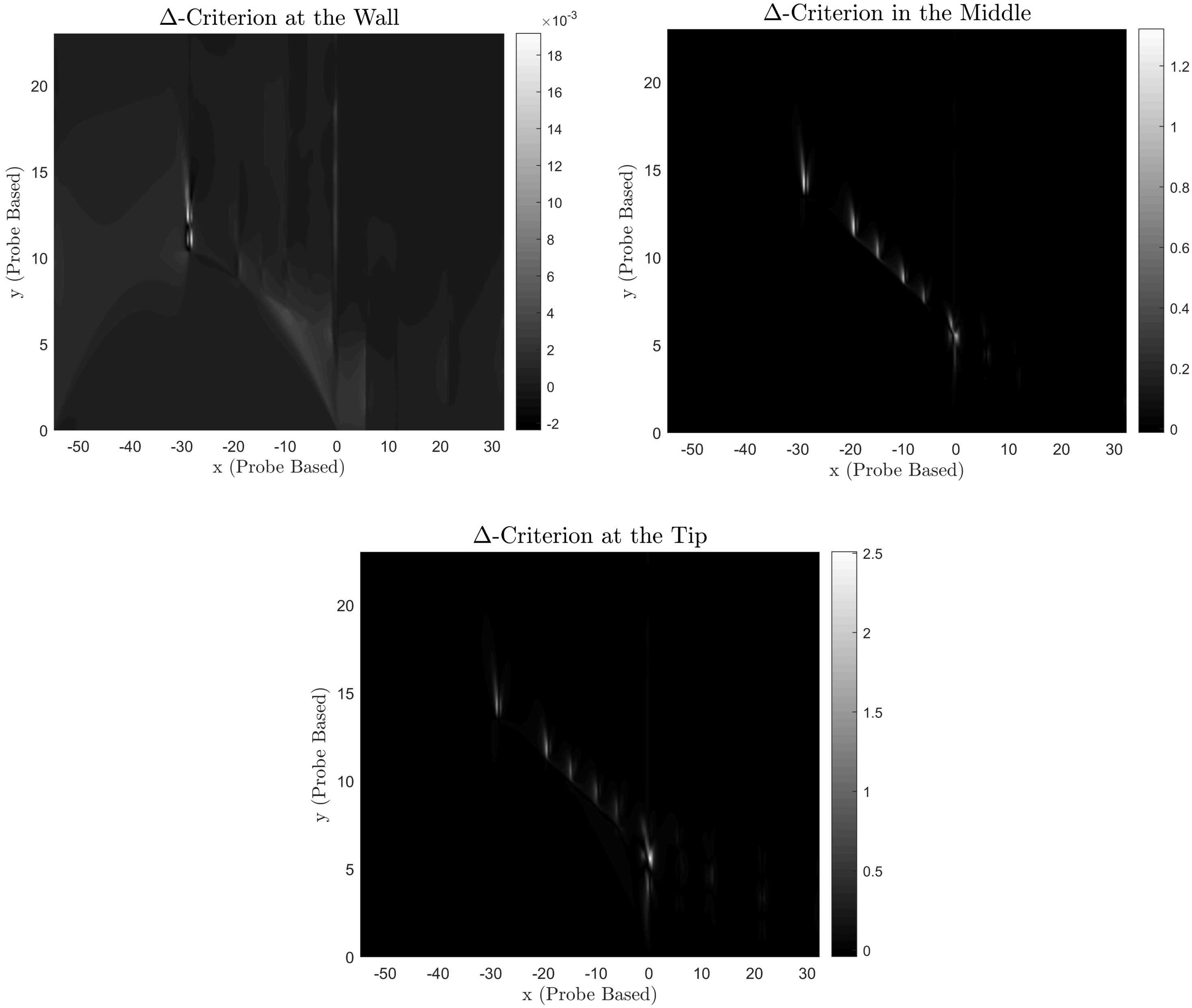


Fig. 6 Comparison of Δ-Criterion at NACA 0012 Sections

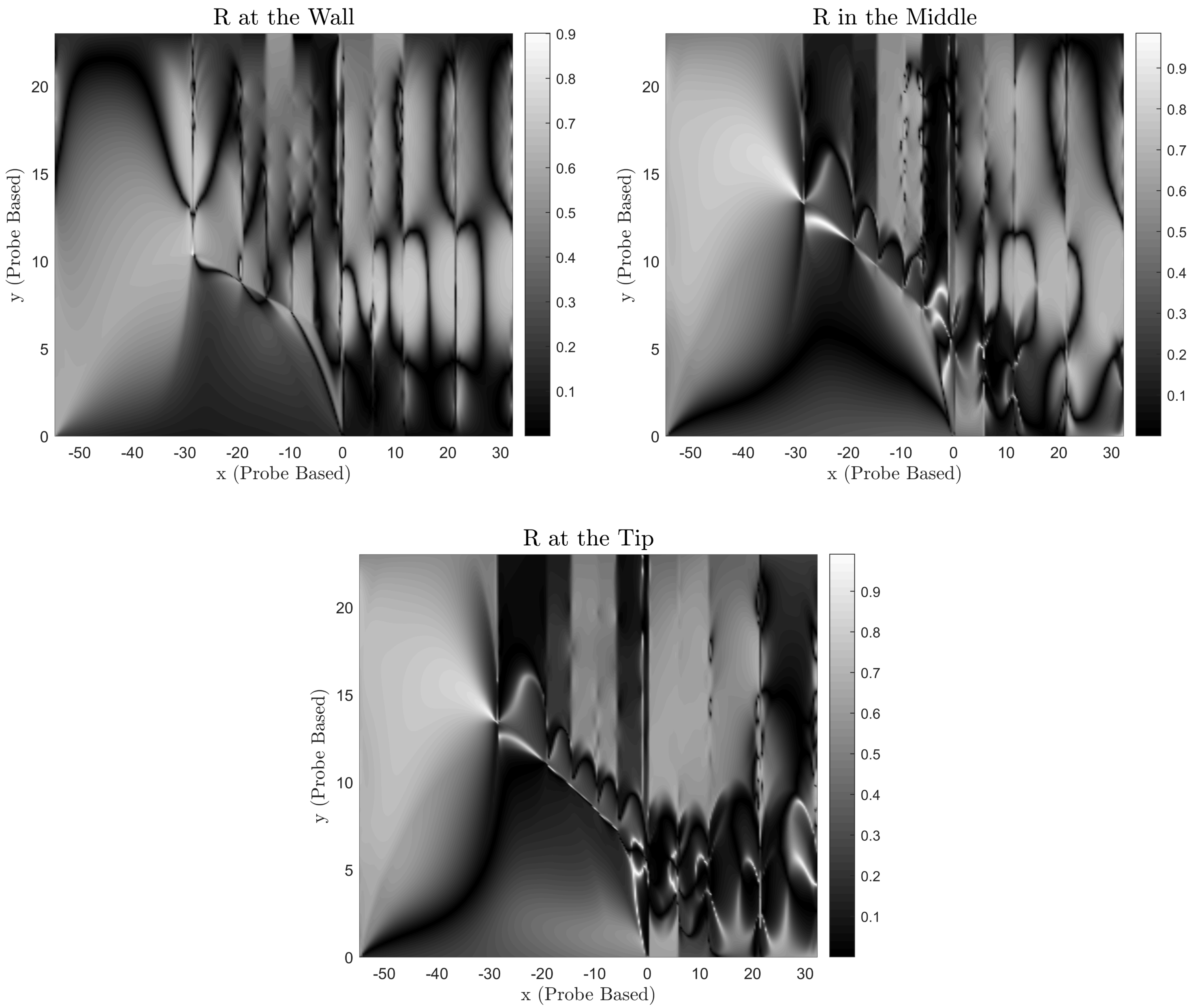


Fig. 7 Comparison of Rortex Method at NACA 0012 Sections

5. Filtered Vortex Identification Results

5.1 Description

The data discussed in sections 2 and 4 were filtered to determine if the results of the vortex identification methods would change. Especially, the data was Gaussian filtered which is a technique in image processing and signal analysis for smoothing and reducing noise of data. It does this while maintaining the edges, and other important aspects of the original data. This method utilizes a bell curve to create an average around the data point. There is a higher value of weight associated with values of data near the center and progressively less to those further away. This blurs the image, mitigating the effects of random variations in the data.

The Gaussian filter's strength is determined by the kernel or standard deviation. For this project, the kernel was set at 2 to ensure the blurring wasn't affecting the entire figure but only near the airfoil in the center.

5.2 Filtered Vortex Identification Comparison for Vorticity

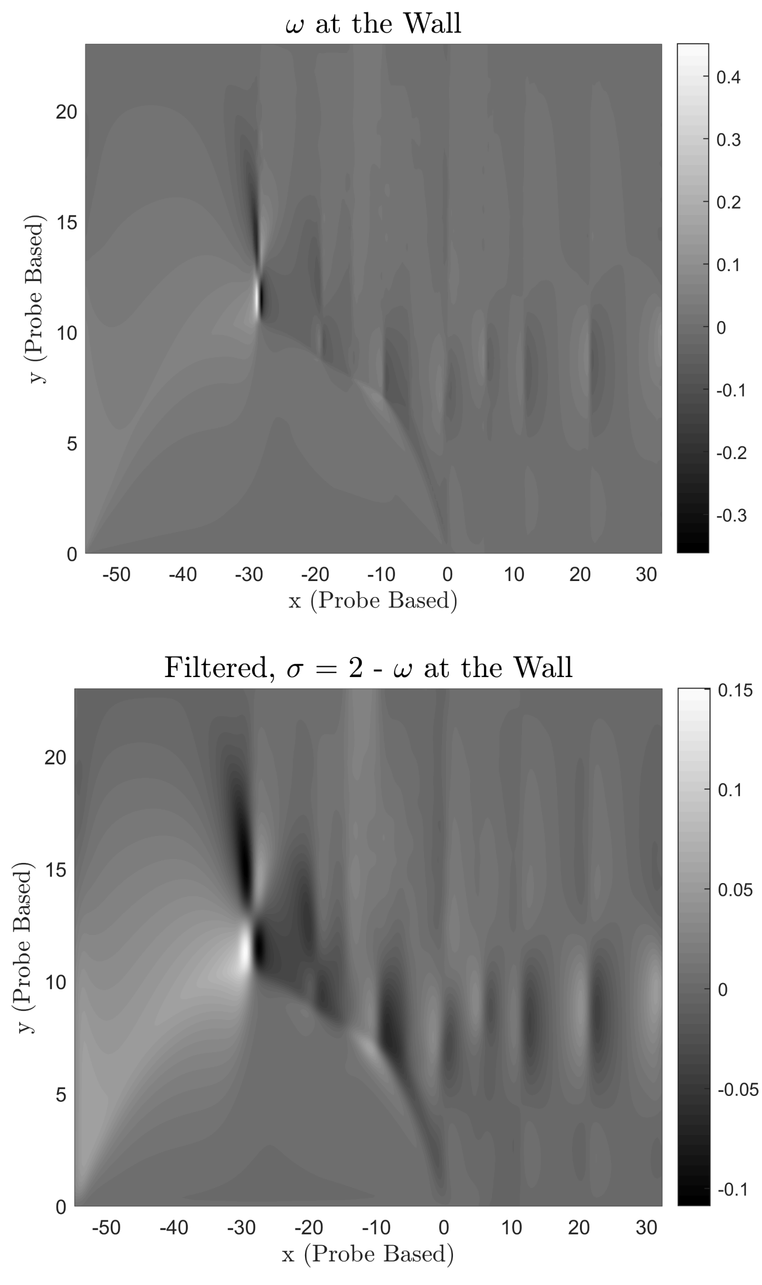


Fig. 8 Comparison of Filtered and Non-Filtered - Wall

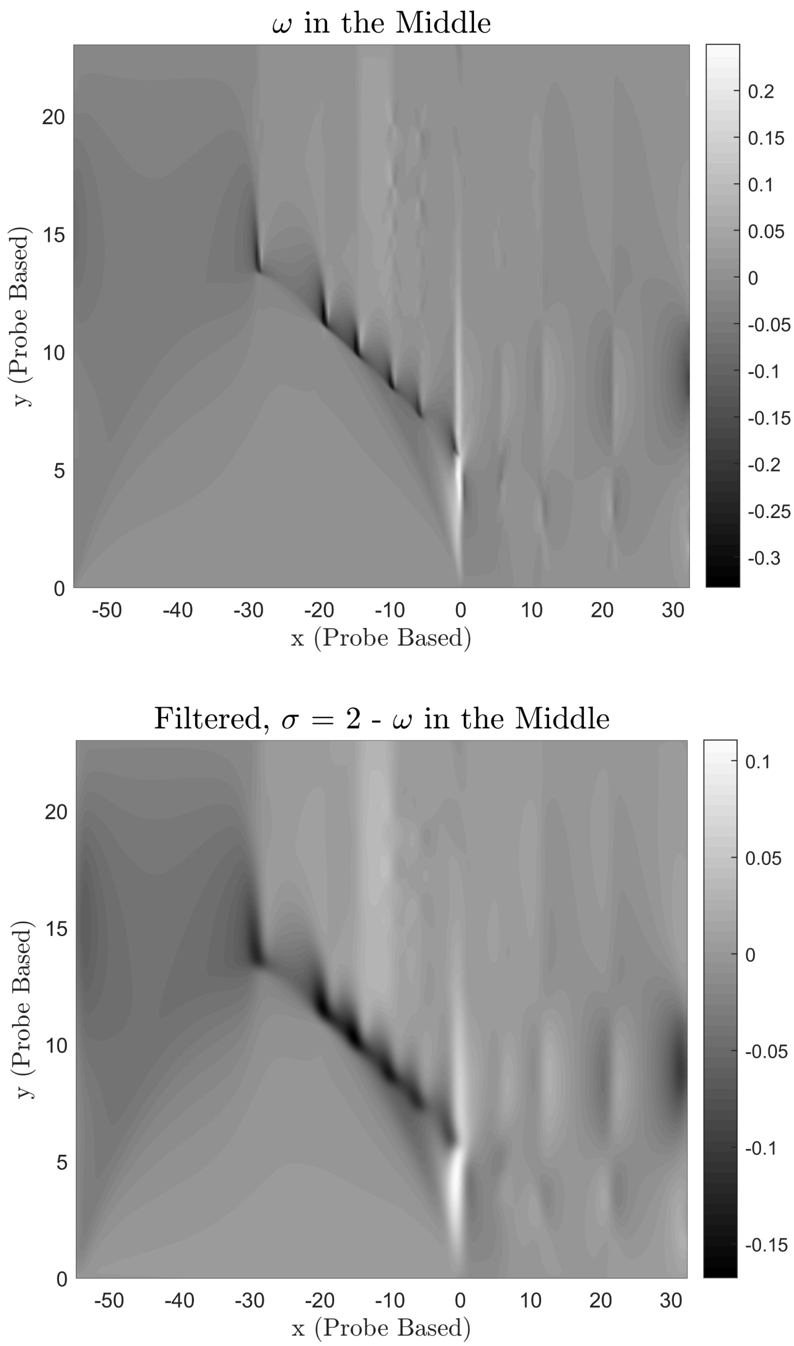


Fig. 9 Comparison of Filtered and Non-Filtered - Middle

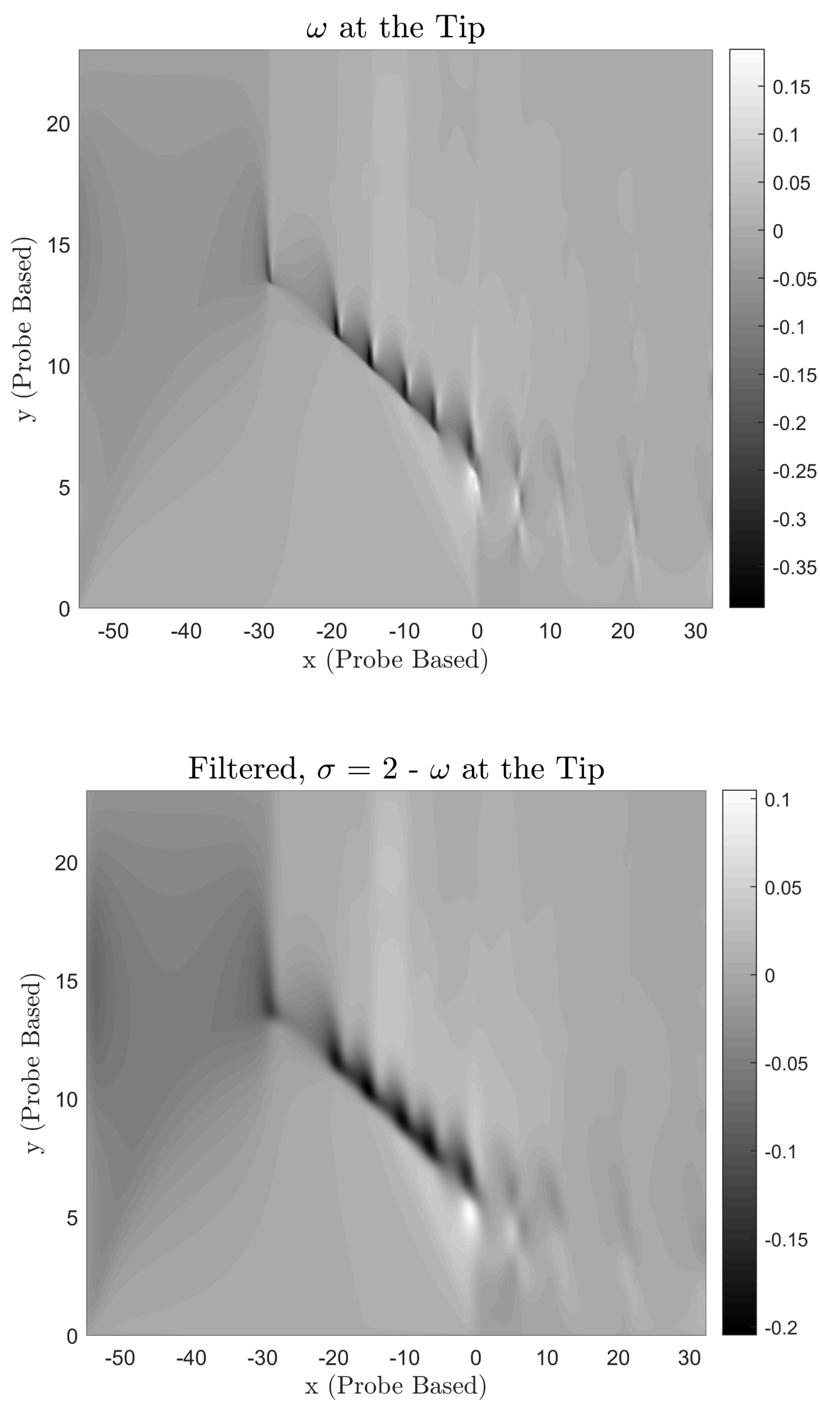
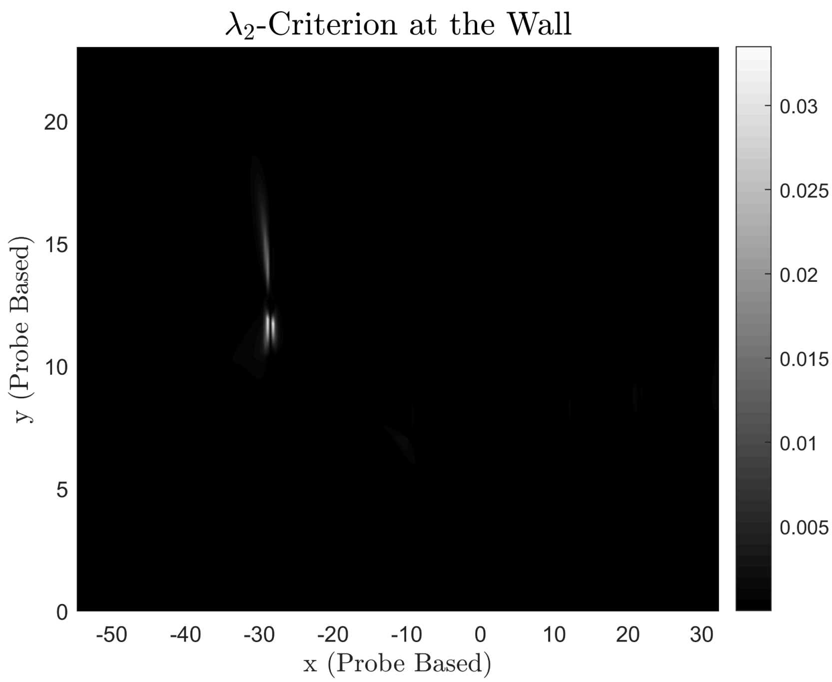


Fig. 10 Comparison of Filtered and Non-Filtered - Tip

5.3 Filtered Vortex Identification Comparison for λ_2-Criterion



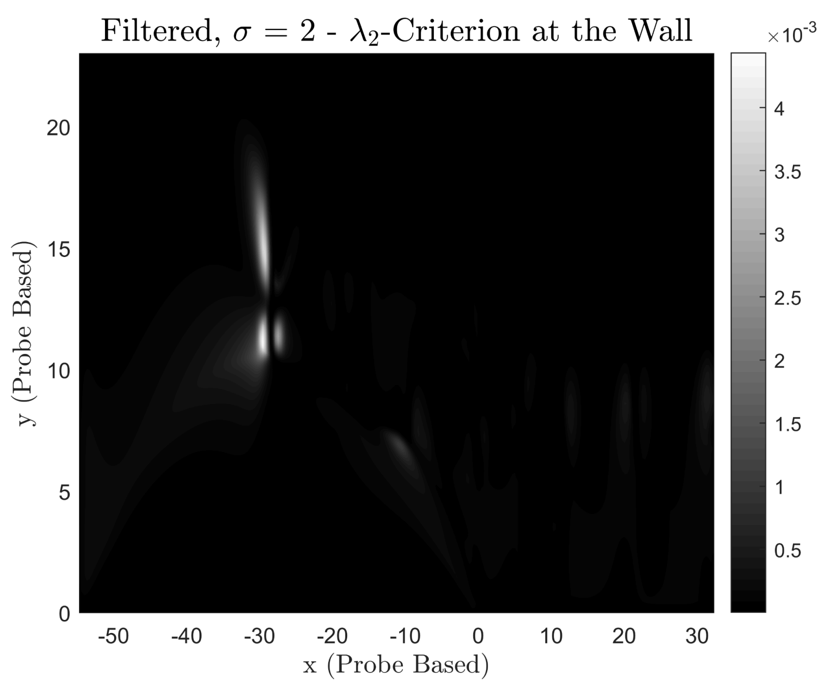


Fig. 11 Comparison of Filtered and Non-Filtered - Wall

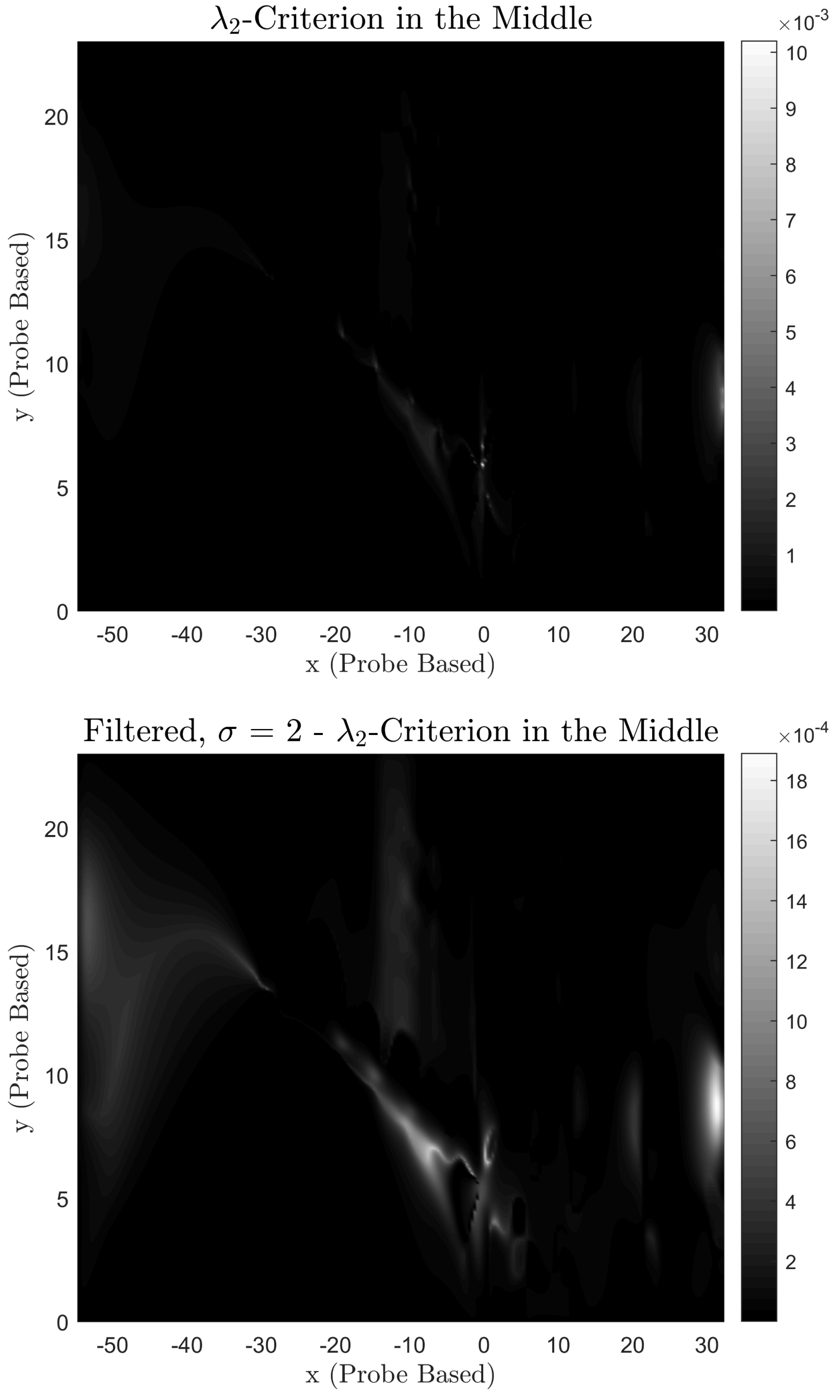


Fig. 12 Comparison of Filtered and Non-Filtered - Middle

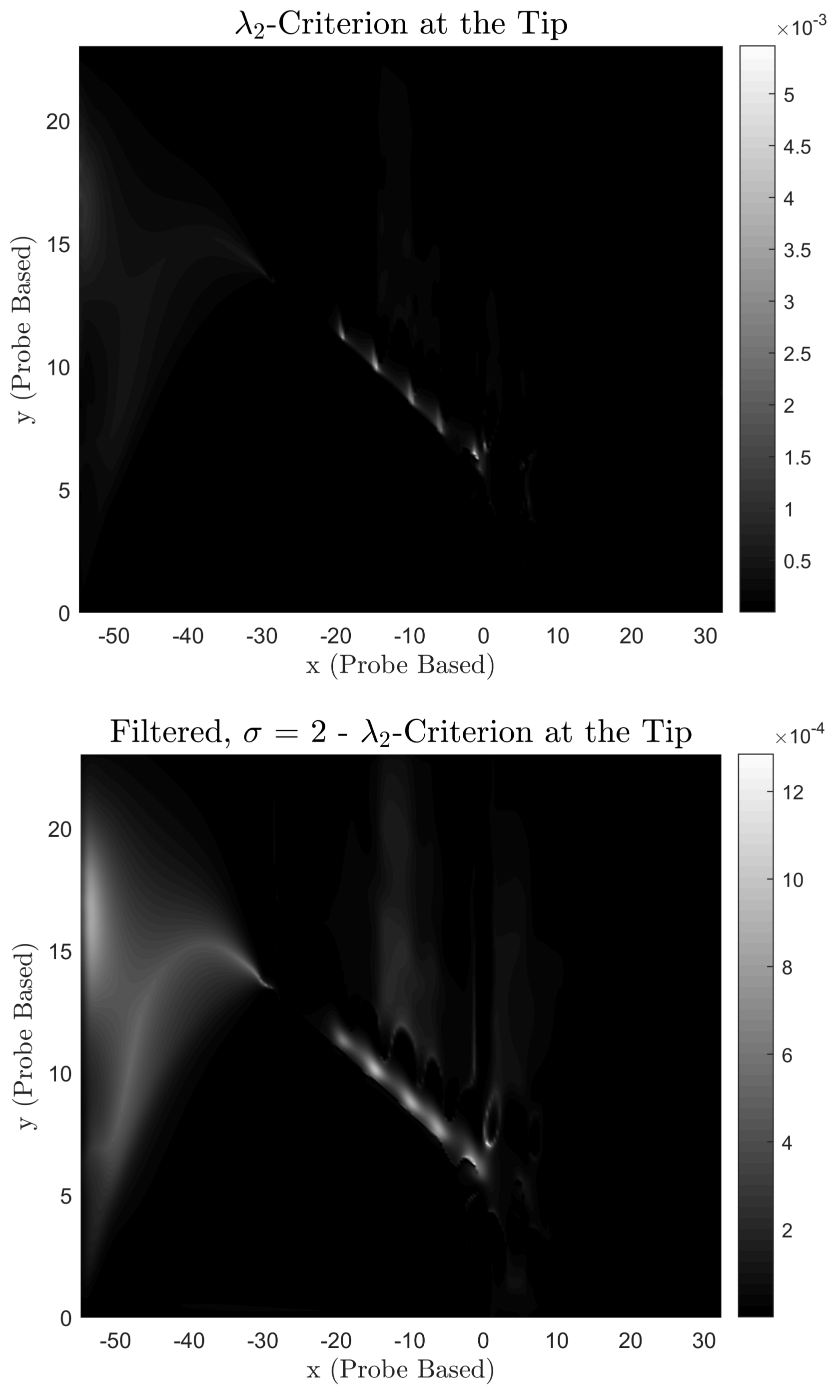
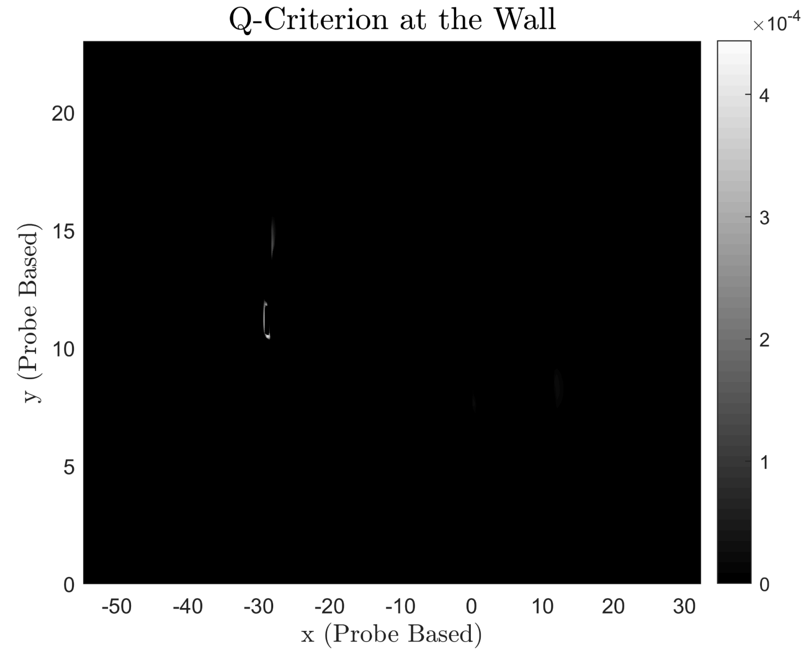


Fig. 13 Comparison of Filtered and Non-Filtered - Tip

5.3 Filtered Vortex Identification Comparison for Q-Criterion



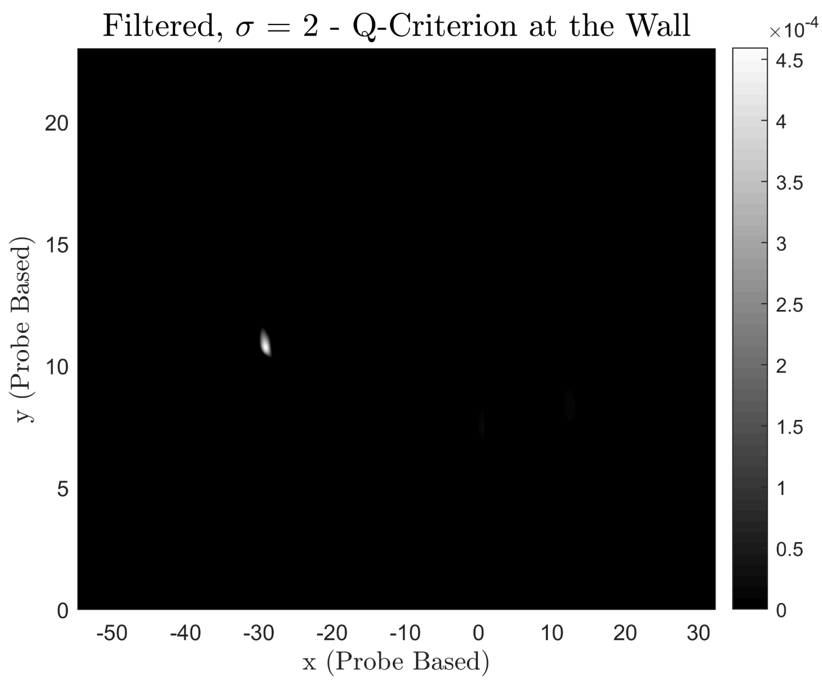


Fig. 14 Comparison of Filtered and Non-Filtered - Wall

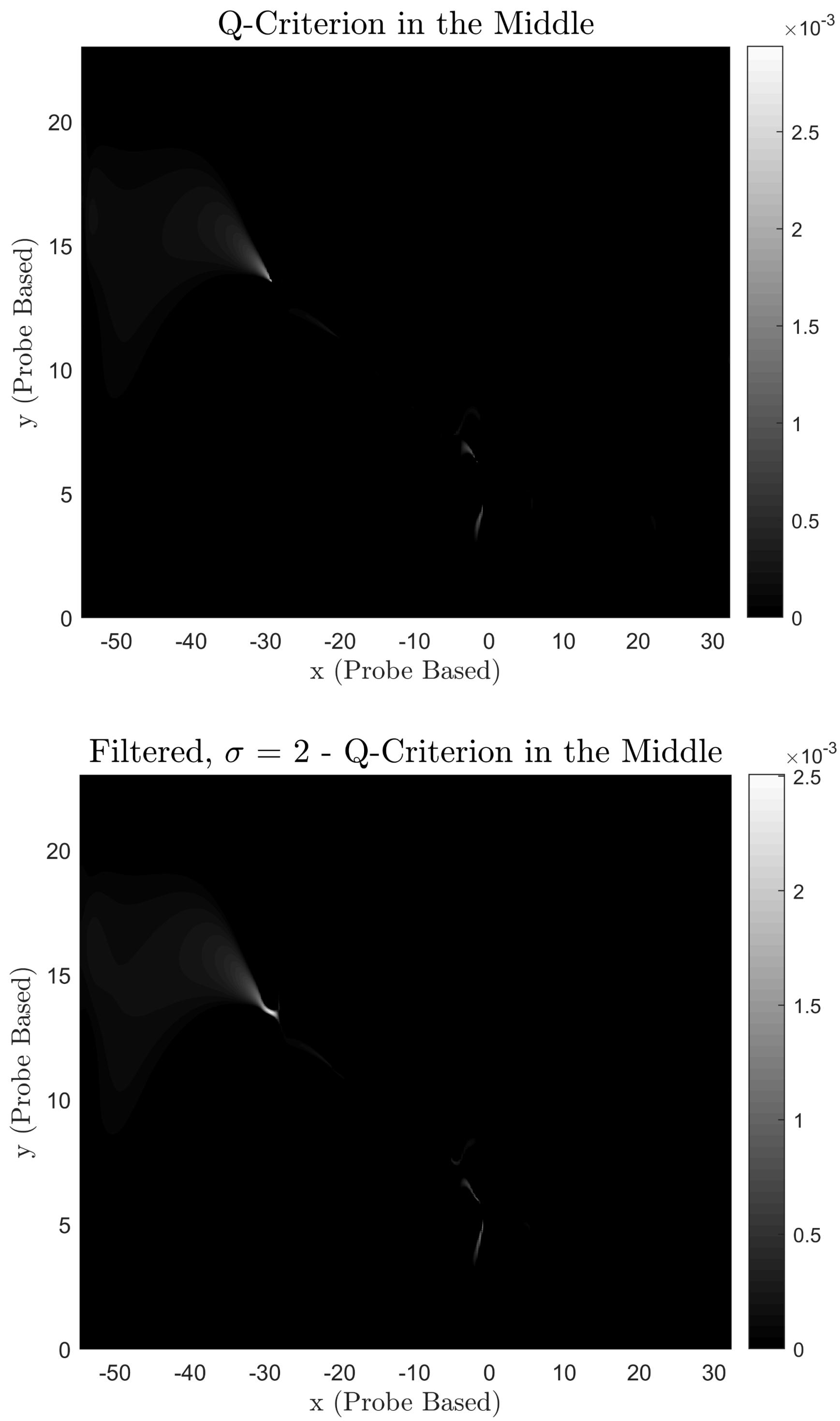


Fig. 15 Comparison of Filtered and Non-Filtered - Middle

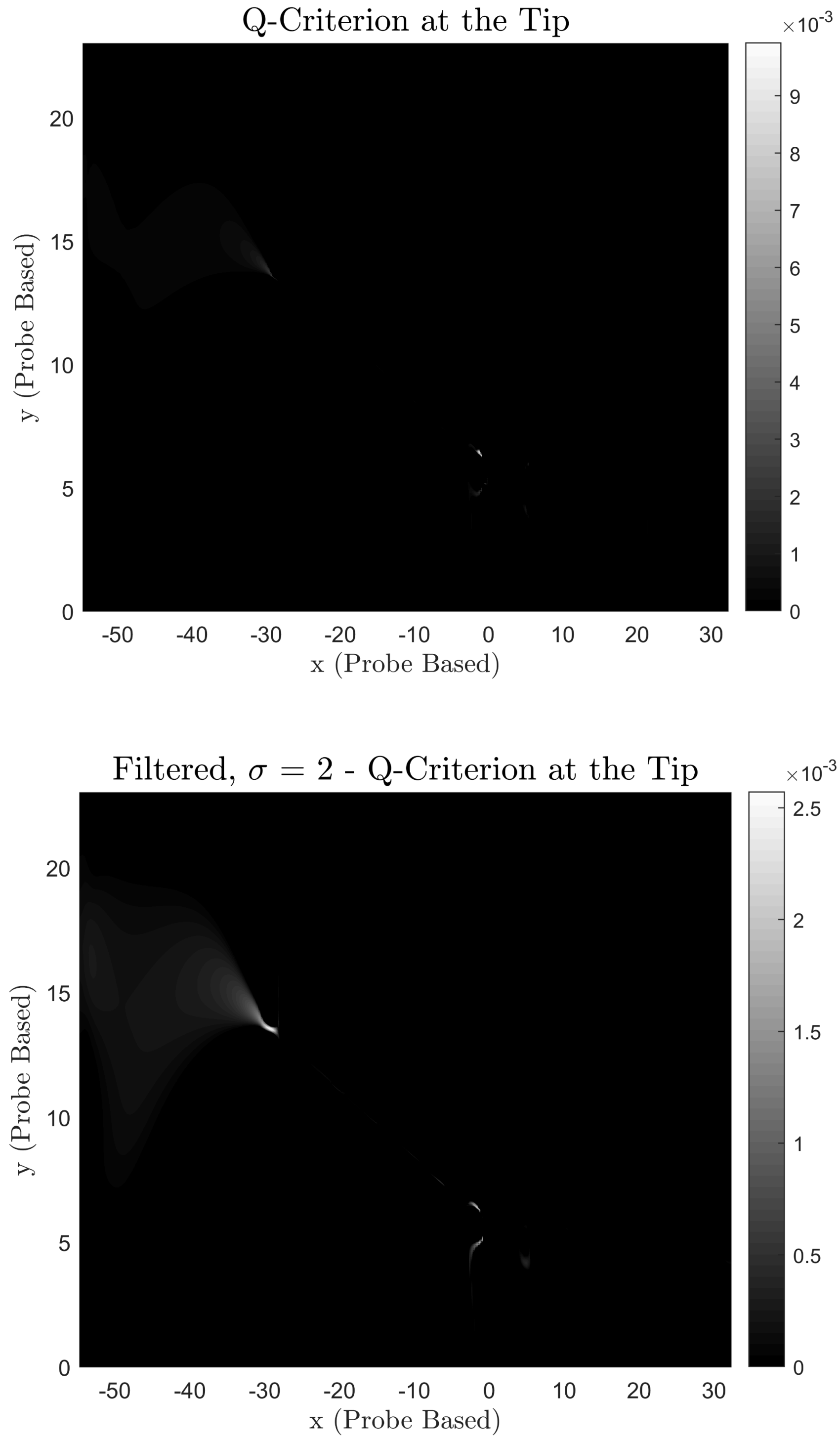


Fig. 16 Comparison of Filtered and Non-Filtered - Tip

5.4 Filtered Vortex Identification Comparison for Δ-Criterion

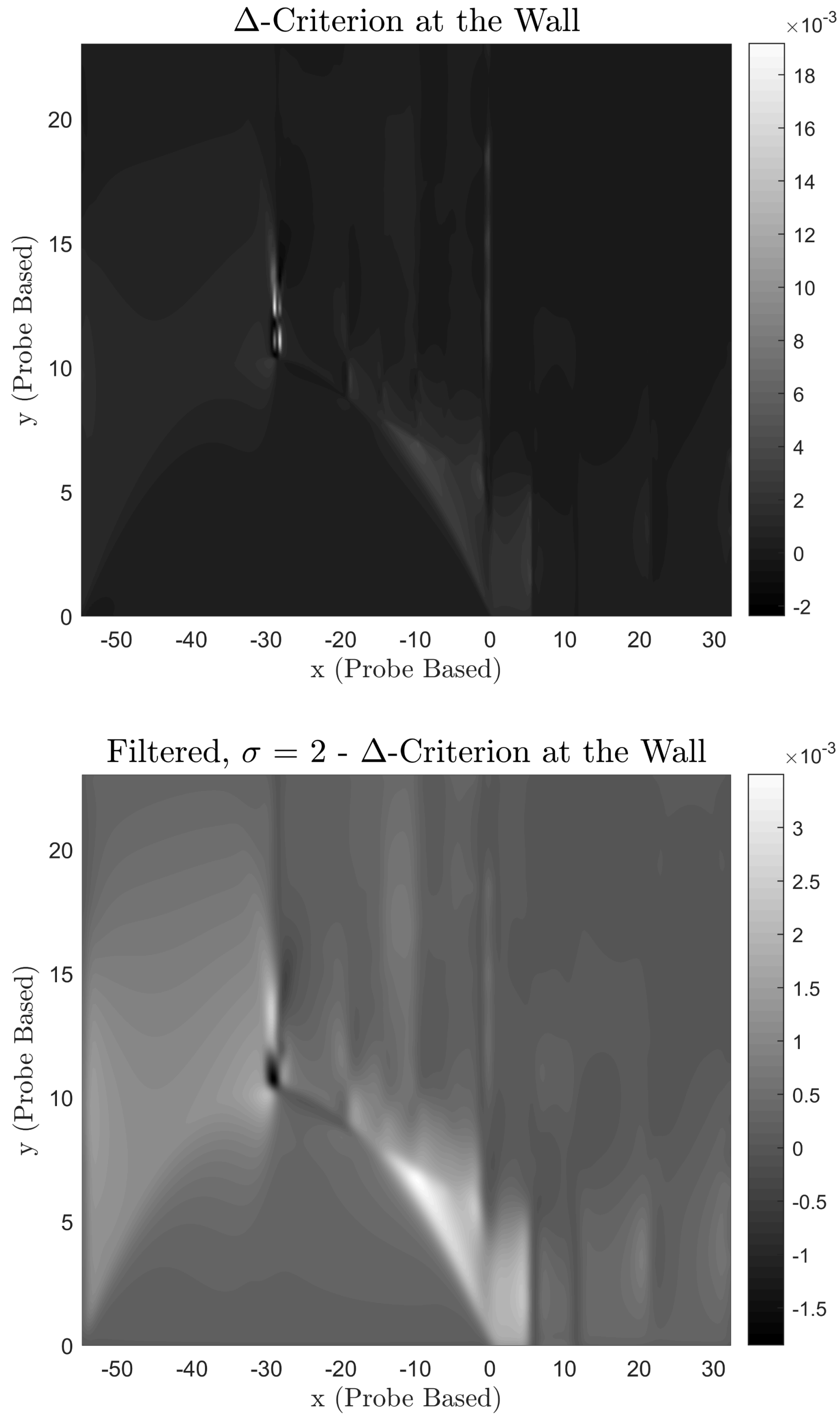
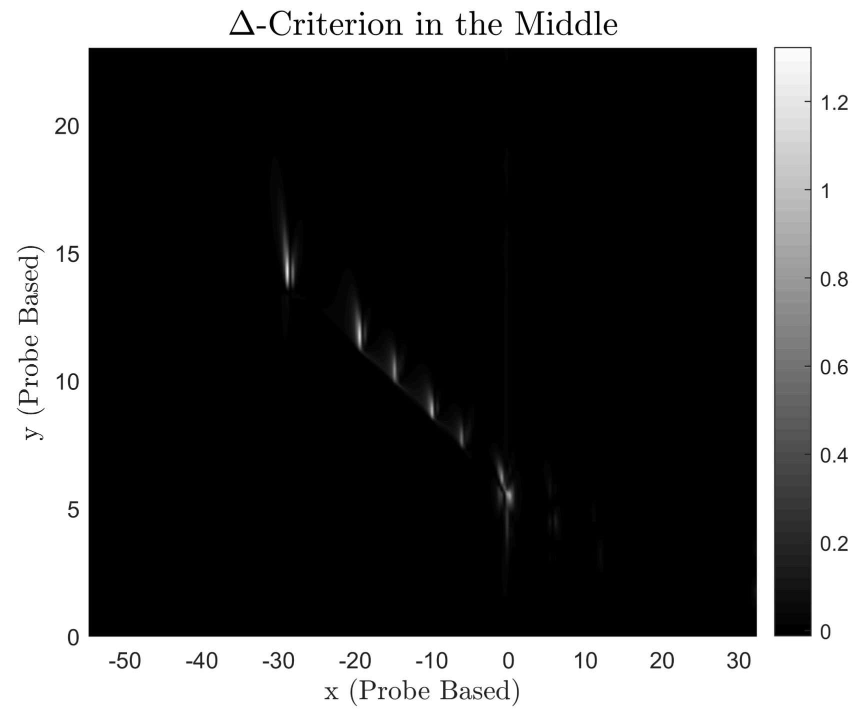


Fig. 17 Comparison of Filtered and Non-Filtered - Wall



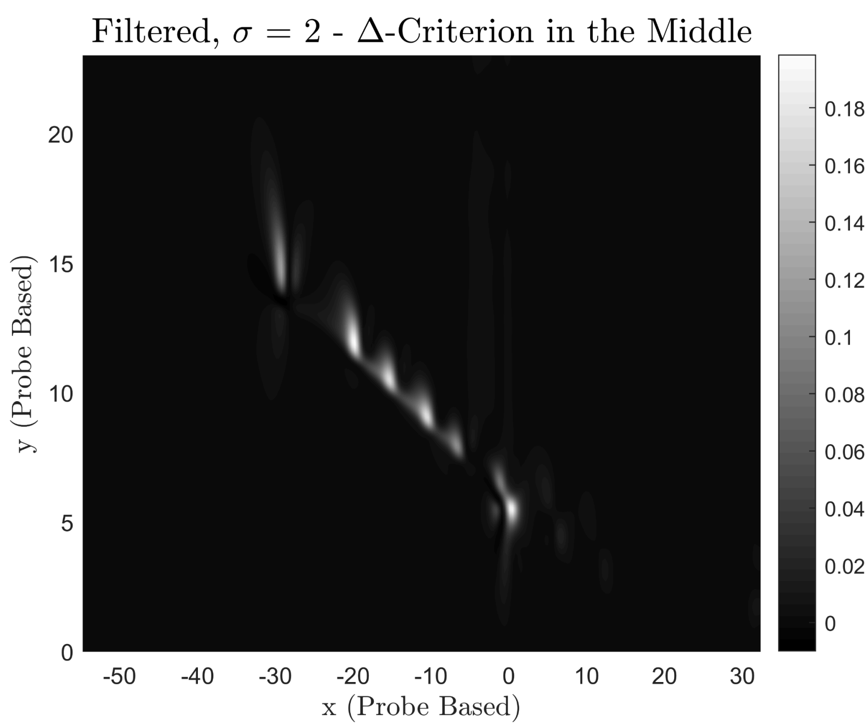


Fig. 18 Comparison of Filtered and Non-Filtered - Middle

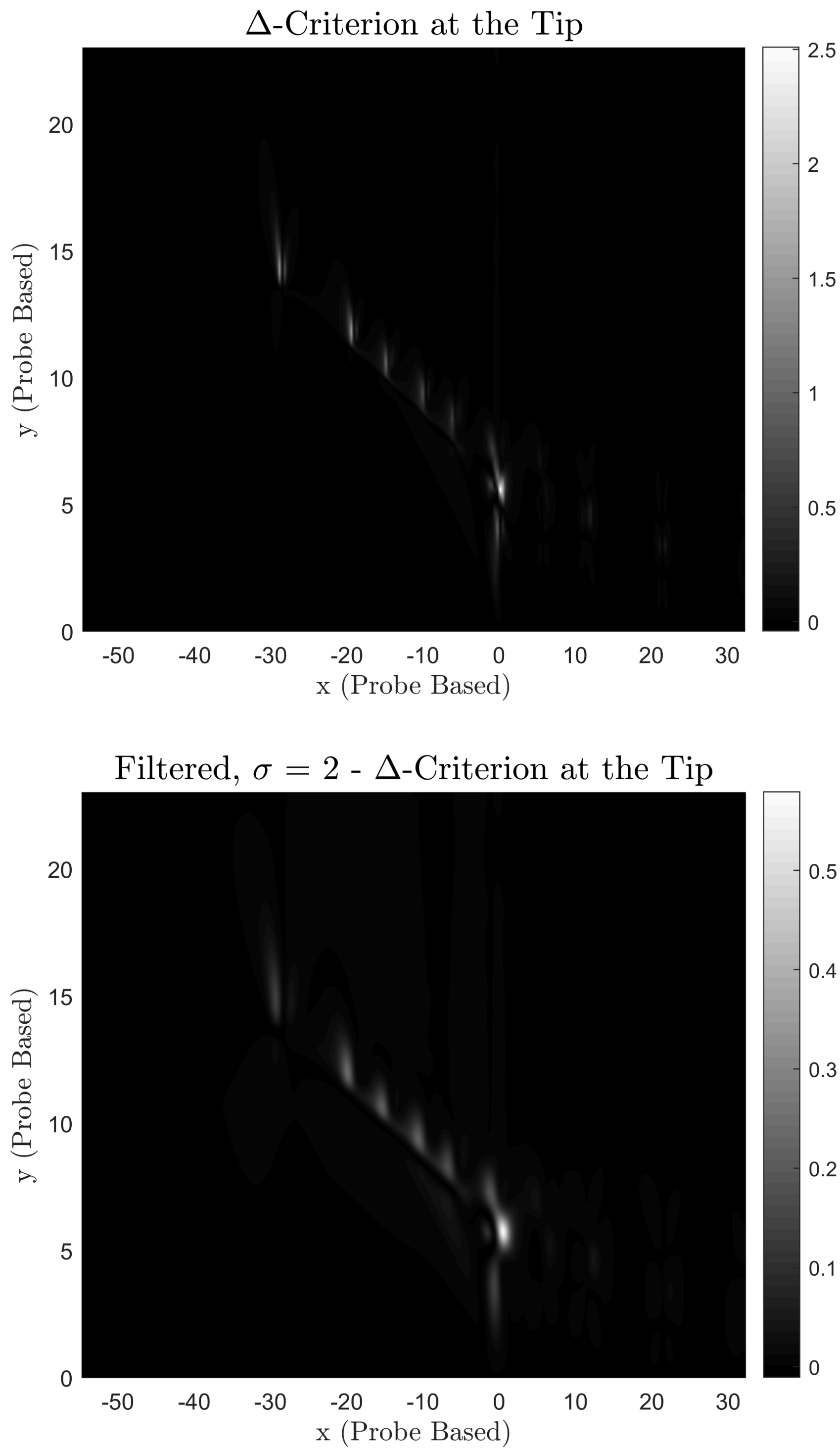


Fig. 19 Comparison of Filtered and Non-Filtered - Tip

5.5 Filtered Vortex Identification Comparison for Rortex Method

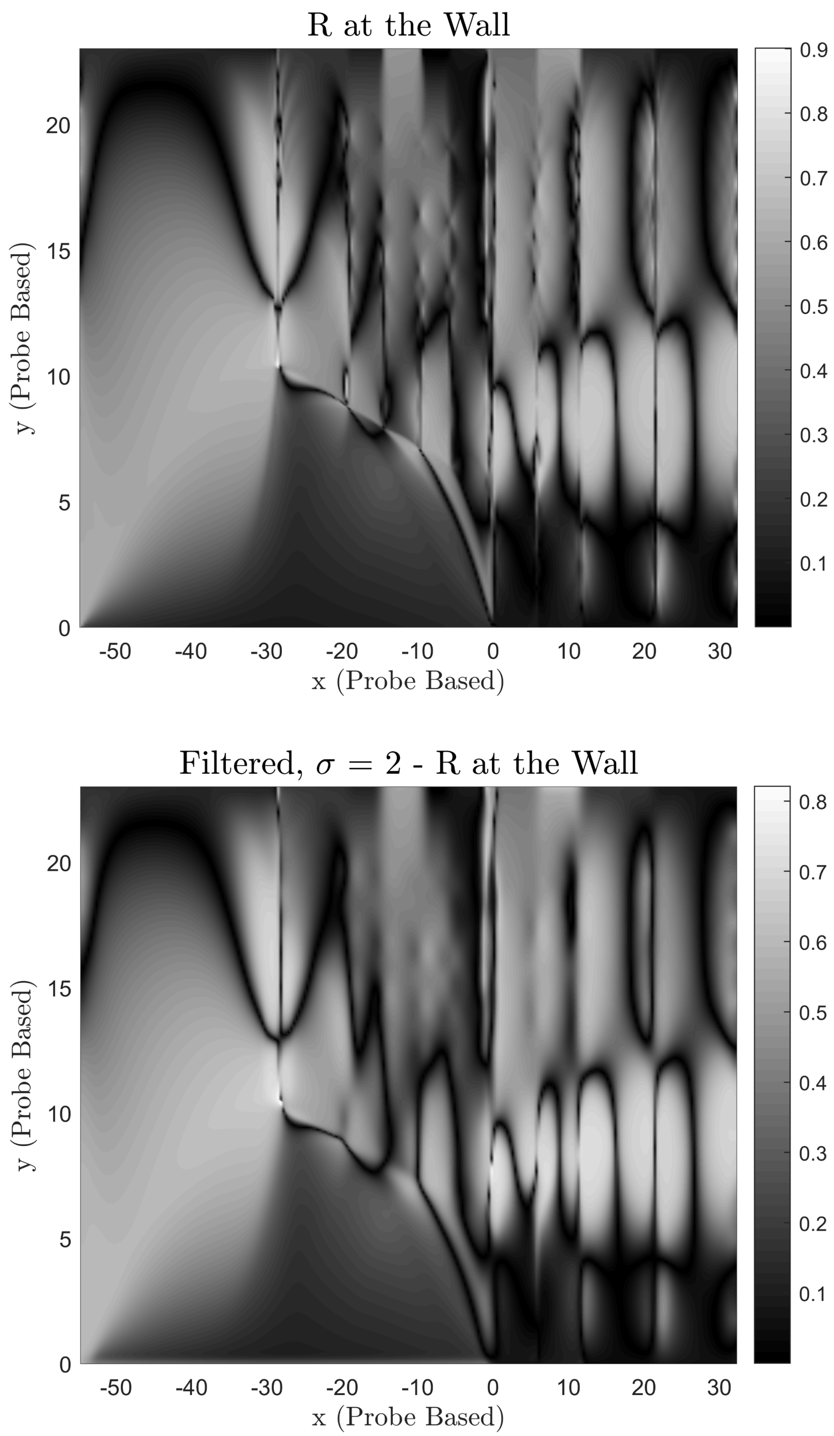


Fig. 20 Comparison of Filtered and Non-Filtered - Wall

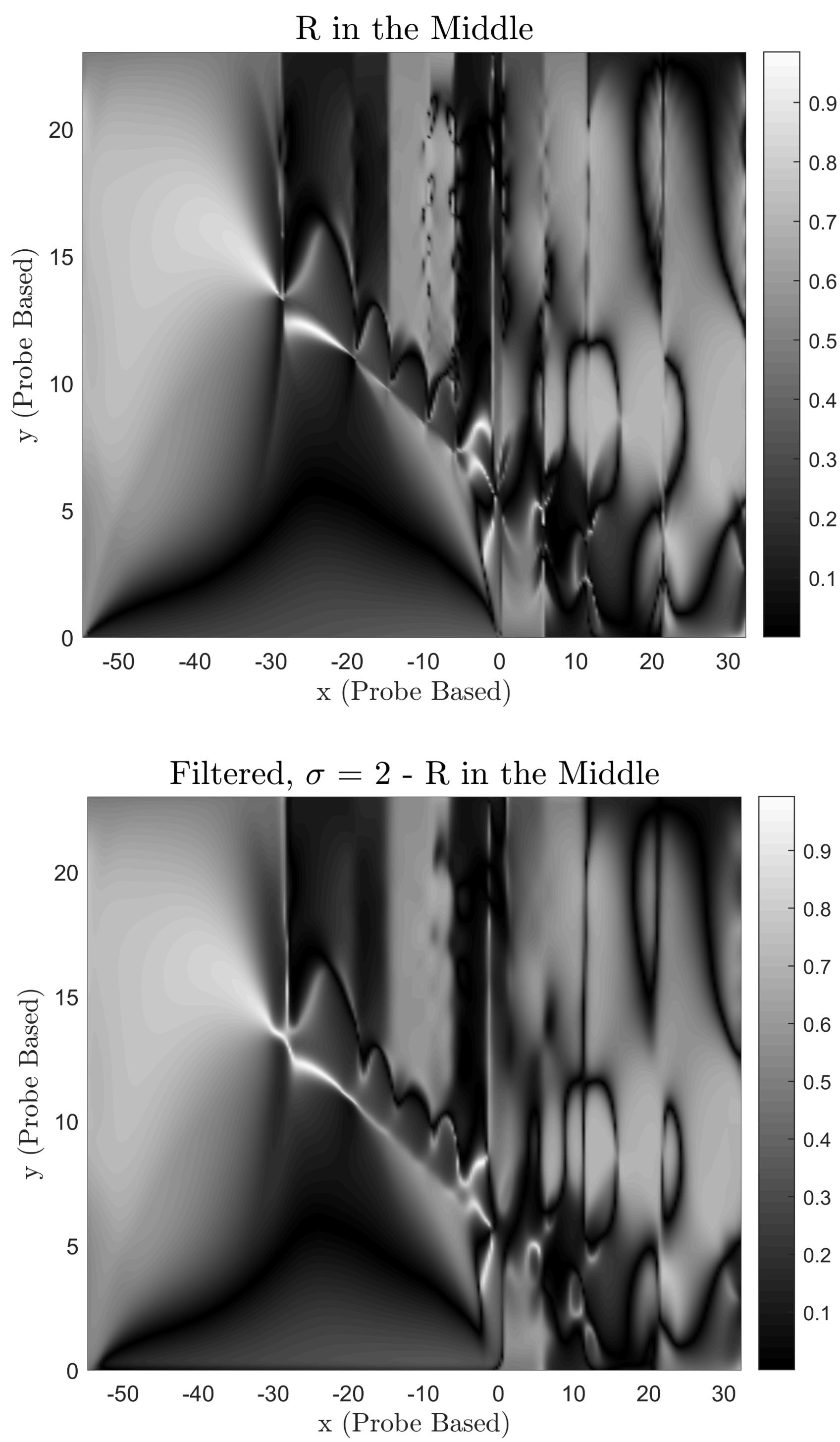


Fig. 21 Comparison of Filtered and Non-Filtered - Middle

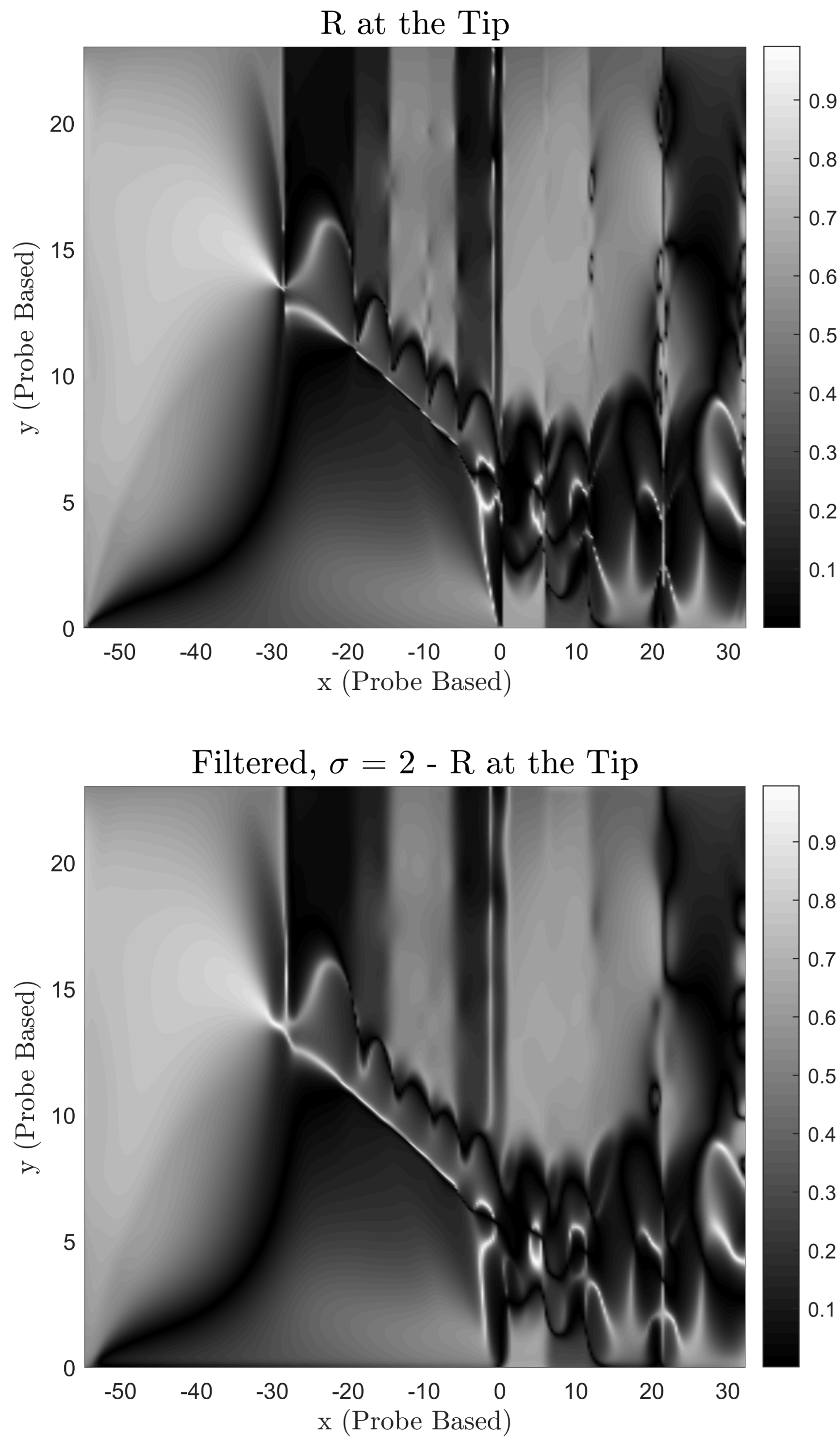


Fig. 22 Comparison of Filtered and Non-Filtered - Tip

6. Final Results/Discussion

Upon completing the calculations of the various vortex identification methods, Fig. 3 through 22 were created detailing comparisons between different sections and with filtering. Starting with the comparison between the sections of the NACA 0012 in section 4, it is clear to see that each of the positions on the airfoil generates a different flow behind it. For the vorticity, the wall had the largest values but was most concentrated at the airfoil's leading edge. For the middle section, the highest positive concentration value was half of the size of the wall and was at the end of the airfoil. However, the negative concentration value of the vorticity was at a multitude of sections down the back of the airfoil from $x = -30$ at the leading edge to $x = -10$. Then for the tip, the value of positive vorticity is rather low, but the negative vorticity is the highest of the three sections. Again, it can be seen that the areas on the back of the airfoil have this negative vorticity. Now, for λ_2 the values are extremely small making almost the entire plot black. At the wall, there is a singular concentration point directly at the leading edge, while the middle shows only one at the trailing edge. Contrasting both, the tip section details the value at a multitude of sections along the airfoil. For Q, the values are extremely small and are barely noticeable in the figures. Continuing, the Δ values are a little better than the previous two. For the wall, there are large streaks at the leading edge and below the trailing edge of the airfoil but these values are quite small. In the middle and at the tip, the values are larger and separated along the airfoil. Finally, for Rortex, the plots are significantly different than the previous methods. The plots detail a variety of contours and large streaks, however, they vary in the three sections. The airfoil shape is slightly visible at the wall but is much more distinct in the middle and tip sections.

Now, looking at the filtered data compared to the non-filtered data it is very noticeable that the plots are slightly blurred and the values are always smaller than they originally were calculated to be. In a few cases, such as the vorticity at the wall, the areas of vortex concentration were intensified and extended for a more detailed view. Additionally, for Q where the plots were significantly small and detailed little to no concentrations, the filtered values remained small but were able to detail some of the concentration positions. The effects were rather drastic for the vortex identification methods, except for the Rortex. This method was mostly unchanged, with small clarity and roughness removed.

In summary, it is hard to classify a specific vortex method as the best for this project, as more investigation would be needed. Based on the values calculated, it appears that the λ_2 and Q criteria are not particularly useful for this flow as the resulting values didn’t meet the conditions to generate significant vortices. In contrast, the Rortex method detailed the flow very well. Now for the Gaussian filtered data, it appears that by filtering the data, the results became much clearer even with the plots being blurred. It eliminated any values over-weighted and smoothed out the entire data set. Given more time, additional vortex identification methods could be compared, and upgrading this project from 2D to 3D can further increase the accuracy.

7. References

[1] Chow, J. S., Zilliac, G. G., and Bradshaw, P., “Mean and turbulence measurements in the near field of a wingtip vortex,” *AIAA Journal*, vol. 35, Jan. 1997.

[2] Liu, C., Gao, Y., Tian, S., and Dong, X., “Rortex—a new vortex vector definition and vorticity tensor and vector decompositions,” *Physics of Fluids*, vol. 30, Mar. 2018.

[3] Rumsey, C., “EXP: Flow behind a NACA 0012 Wingtip,” *NASA* Available: https://turbmodels.larc.nasa.gov/Other_exp_Data/wingtip0012_exp.html.

[4] Wu, J. C., 1986.

[5] Zhan, J., Li, Y., Wai, W. O., and Hu, W., “Comparison between the Q Criterion and Rortex in the application of an in-stream structure,” *AIP Publishing* Available: https://pubs.aip.org/aip/pof/article/31/12/121701/957670/Comparison-between-the-Q-criterion-and-Rortex-in.

[6] Zhang, Y., Liu, K., Xian, H., and Du, X., “A review of methods for vortex identification in hydroturbines,” *Renewable and Sustainable Energy Reviews*, vol. 81, Jan. 2018.

8. Appendix

8.1 Wall Matlab Script

```
%% Given Data

filename = 'Z4.xlsx' ;  % Manipulated Data for Z = 0 to 4
data = readmatrix(filename) ;

% Extract columns
x = data(:, 1) ;  % x-coordinates
y = data(:, 2) ;  % y-coordinates
u = data(:, 4) ;  % u velocity, u/u_inf
v = data(:, 5) ;  % v velocity, v/u_inf

% Create a grid
mesh = 250 ;
xlim = linspace(min(x), max(x), mesh) ;
ylim = linspace(min(y), max(y), mesh) ;
[X, Y] = meshgrid(xlim, ylim) ;

% Interpolate
F_u = scatteredInterpolant(x, y, u, 'natural', 'none') ;
F_v = scatteredInterpolant(x, y, v, 'natural', 'none') ;
U = F_u(X, Y) ;
V = F_v(X, Y) ;

% Calculate partial derivatives using gradient function
[dU_dx, dU_dy] = gradient(U, ylim, xlim) ;
[dV_dx, dV_dy] = gradient(V, ylim, xlim) ;
```

8.2 Middle Matlab Script

```
%% Given Data

filename = 'Z14.xlsx' ;  % Manipulated Data for Z = 12 to 16
data = readmatrix(filename) ;

% Extract columns
x = data(:, 1) ;  % x-coordinates
y = data(:, 2) ;  % y-coordinates
u = data(:, 4) ;  % u velocity, u/u_inf
v = data(:, 5) ;  % v velocity, v/u_inf

% Create a grid
mesh = 250 ;
xlim = linspace(min(x), max(x), mesh) ;
ylim = linspace(min(y), max(y), mesh) ;
[X, Y] = meshgrid(xlim, ylim) ;

% Interpolate
F_u = scatteredInterpolant(x, y, u, 'natural', 'none') ;
F_v = scatteredInterpolant(x, y, v, 'natural', 'none') ;
U = F_u(X, Y) ;
V = F_v(X, Y) ;

% Calculate partial derivatives using gradient function
[dU_dx, dU_dy] = gradient(U, ylim, xlim) ;
[dV_dx, dV_dy] = gradient(V, ylim, xlim) ;
```

8.3 Tip Matlab Script

```
%% Given Data

filename = 'Z25.xlsx' ;  % Manipulated Data for Z = 12 to 16
data = readmatrix(filename) ;

% Extract columns
x = data(:, 1) ;  % x-coordinates
y = data(:, 2) ;  % y-coordinates
u = data(:, 4) ;  % u velocity, u/u_inf
v = data(:, 5) ;  % v velocity, v/u_inf

% Create a grid
mesh = 250 ;
xlim = linspace(min(x), max(x), mesh) ;
ylim = linspace(min(y), max(y), mesh) ;
[X, Y] = meshgrid(xlim, ylim) ;

% Interpolate
F_u = scatteredInterpolant(x, y, u, 'natural', 'none') ;
F_v = scatteredInterpolant(x, y, v, 'natural', 'none') ;
U = F_u(X, Y) ;
V = F_v(X, Y) ;

% Calculate partial derivatives using gradient function
[dU_dx, dU_dy] = gradient(U, ylim, xlim) ;
[dV_dx, dV_dy] = gradient(V, ylim, xlim) ;
```

8.4 Vorticity Matlab Script

```
%% Vortex Indentificaiton: Vorticity

% Calculate vorticity
w = dV_dx - dU_dy ;

figure;
h = pcolor(X,Y,w) ;
h.EdgeColor = 'none' ;
colorbar ;
shading interp ;
colormap('gray') ;

%Titles and axis labels
t = title('$\omega$ at the Wall') ;
set(t,'fontname','Times New Roman','interpreter','latex','fontsize',15) ;
xlabel('x (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;
ylabel('y (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;

% Export figure
exportgraphics(gca,sprintf('fig%d.png',1),'resolution',600)

% Can toggle visibility of figures
set(gcf,'visible','off') % When active, figures will go away
```

8.5 λ_2-Criterion Matlab Script

```
%% Vortex Indentificaiton: Lamda2

% Construct the velocity gradient tensor for each point
S = zeros(2, 2, numel(X));
S(1,1,:) = dU_dx(:);
S(1,2,:) = 0.5 * (dV_dy(:) + dV_dx(:));
S(2,1,:) = S(1,2,:);
S(2,2,:) = dV_dy(:);

% Calculate the symmetric part of the square of the velocity gradient tensor
A = zeros(size(S));
for k = 1:numel(X)
   S_ = reshape(S(:,:,k), 2, 2);
   A_ = S_' * S_;  % S transpose times S
   A(:,:,k) = 0.5 * (A_ + A_');  % Symmetric part
end

% Calculate eigenvalues at each point
lambda2 = zeros(size(X));
for k = 1:numel(X)
   A_ = reshape(A(:,:,k), 2, 2);
   eigenvalues = eig(A_);
   lambda2(k) = min(eigenvalues);
end

figure;
h = pcolor(X,Y,lambda2) ;
h.EdgeColor = 'none' ;
colorbar ;
shading interp ;
colormap('gray') ;

% negative Lambda-2 values indicate the presence of vortices
% Set color axis limits to focus on negative values only if they exist
if min(lambda2(:)) < 0
   caxis([min(lambda2(:)), 0]);  % Set the color limits to include negative values only
else
   caxis([min(lambda2(:)), max(lambda2(:))]);  % Set color limits to the range of lambda2 if no negative values exist
end

%Titles and axis labels
t = title('$\lambda_{2}$-Criterion at the Wall') ;
set(t,'fontname','Times New Roman','interpreter','latex','fontsize',15) ;
xlabel('x (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;
ylabel('y (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;

% Export figure
exportgraphics(gca,sprintf('fig%d.png',2),'resolution',600)

% Can toggle visibility of figures
set(gcf,'visible','off') % When active, figures will go away
```

8.6 Q-Criterion Matlab Script

```
%% Vortex Indentificaiton: Q-Criterion

% Symmetric and antisymmetric parts of the velocity gradient tensor
S = zeros(2, 2, numel(X));
Omega = zeros(2, 2, numel(X));
for k = 1:numel(X)
    gradU = [dU_dx(k), dU_dy(k); dV_dx(k), dV_dy(k)];
    S(:,:,k) = 0.5 * (gradU + gradU');
    Omega(:,:,k) = 0.5 * (gradU - gradU');
end

% Calculate Q-criterion
Q = zeros(size(X));
for k = 1:numel(X)
    S_mat = reshape(S(:,:,k), 2, 2);
    Omega_mat = reshape(Omega(:,:,k), 2, 2);
    Q(k) = 0.5 * (norm(Omega_mat, 'fro')^2 - norm(S_mat, 'fro')^2);
end

figure;
h = pcolor(X,Y,Q) ;
h.EdgeColor = 'none' ;
colorbar ;
shading interp ;
colormap('gray') ;
caxis([0, max(Q(:))]) ; % Only regions where Q>0 are of interest indicate the presence of vortical structures

% Titles and axis labels
t = title('Q-Criterion at the Wall') ;
set(t,'fontname','Times New Roman','interpreter','latex','fontsize',15) ;
xlabel('x (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;
ylabel('y (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;

% Export figure
exportgraphics(gca,sprintf('fig%d.png',3),'resolution',600)

% Can toggle visibility of figures
set(gcf,'visible','off') % When active, figures will go away
```

8.7 Δ-Criterion Matlab Script

```
%% Vortex Indentificaiton: Δ-Criterion

% Calculate Delta-criterion
Delta = zeros(size(X));
for k = 1:numel(X)
    gradU = [dU_dx(k), dU_dy(k); dV_dx(k), dV_dy(k)];
    traceA = trace(gradU);
    detA = det(gradU);
    Delta(k) = traceA^2 - 4 * detA;
end

figure;
h = pcolor(X,Y,Delta) ;
h.EdgeColor = 'none' ;
colorbar ;
shading interp ;
colormap('gray') ;
%caxis([-max(abs(Delta(:))), 0]);  % Focus on the negative range where Delta < 0
% which indicate the presence of complex eigenvalues and hence, vortex regions

% Titles and axis labels
t = title('$\Delta$-Criterion at the Wall') ;
set(t,'fontname','Times New Roman','interpreter','latex','fontsize',15) ;
xlabel('x (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;
ylabel('y (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;

% Export figure
exportgraphics(gca,sprintf('fig%d.png',4),'resolution',600)

% Can toggle visibility of figures
set(gcf,'visible','off') % When active, figures will go away
```

8.8 Rortex Method Matlab Script

```
%% Vortex Indentificaiton: Rortex

% Calculate Rortex
Rortex = zeros(size(X));
for k = 1:numel(X)
    gradU = [dU_dx(k), dU_dy(k); dV_dx(k), dV_dy(k)];
    Omega = 0.5 * (gradU - gradU');
    Rortex(k) = norm(Omega, 'fro') / norm(gradU, 'fro');
end

figure;
h = pcolor(X,Y,Rortex) ;
h.EdgeColor = 'none' ;
colorbar ;
shading interp ;
colormap('gray') ;

% Titles and axis labels
t = title('R at the Wall') ;
set(t,'fontname','Times New Roman','interpreter','latex','fontsize',15) ;
xlabel('x (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;
ylabel('y (Probe Based)','fontname','Times New Roman','interpreter','latex','fontsize',12) ;

% Export figure
exportgraphics(gca,sprintf('fig%d.png',5),'resolution',600)

% Can toggle visibility of figures
set(gcf,'visible','off') % When active, figures will go away
```

8.9 Gaussian Matlab Script

```
% Apply Gaussian filter to smooth the data
sigma = 2 ;  % Standard deviation of the Gaussian kernel
U_filtered = imgaussfilt(U, sigma) ;
V_filtered = imgaussfilt(V, sigma) ;
```