MAE 565: Rocket Propulsion

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Final Project Report

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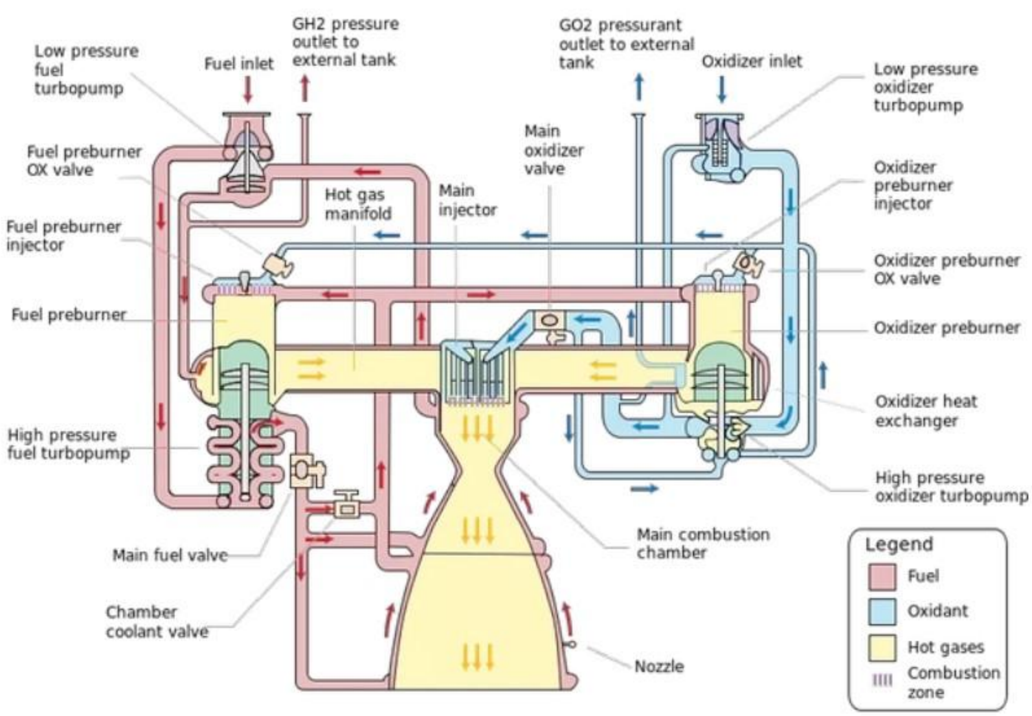
Table of Contents

1. Introduction

1.1 Project Description

This project outlines the propellant feed system for NASA's Space Shuttle's main engines, RS-25. Additionally, the propellant system was designed to be used in the Space Launch System (SLS), which launched on November 16th, 2022. The RS-25 has a multitude of sections which are outlined in Figure 1-1. The system begins with both low-pressure oxygen and fuel turbopump (LPOTP and LPFTP respectively), and from here the system has direct lines to a high-pressure oxygen and fuel turbopump (HPOTP and HPFTP respectively). The low and high turbopumps consist of a pump and a turbine that drive liquid and gas through the feed system. Continuing forward, the feed system gets more complex as the flows are directed in a multitude of directions. For the HPOTP side, shown in blue in Figure 1, an additional boost pump is added and some of the flow is split between going to the preburners and going to the LPOTP turbine. For the HPFTP side, shown in red in Figure 1, the flow from the pump splits to go to the thrust chamber cooling, the expansion nozzle cooling, and then the preburners. From here, the main injectors take in both the GH2 and LO2 and equate the pressures going into the thrust chamber. The thrust chamber is where the combustion takes place and results in the flow going into the expansion nozzle. Finally, the expansion nozzle results in both a frozen flow and a shifting equilibrium flow. Using these flows, we can calculate a variety of variables for both the sea level and vacuum cases.

Figure 1-1 RS-25 Propellant Feed System



1.2 Report Outline

This final project report will be organized by each section of the RS-22 propellant feed system, as stated above. Every input into the corresponding excel file will be included in this report and referred to by the row and column name. For example, referencing a value of LPOTP temperature will be stated by the value in the spreadsheet being D37. Additionally, since there are both green (transferred values) and blue (calculated values) cells, the referenced values will be indicated by an abbreviation. For example, referencing a value for LPOTP temperature, which is a green cell, will be stated as GN-D37.

2. Low-Pressure Oxygen Turbopump (LPOTP)

2.1 Pump Section: Inlet

Beginning with the LPOTP, it is stated in *MAE 565 Project F22* in the LPOTP section that the flow was LO2, which results in BL-D35 being a liquid.

For the LPOTP inlet pressure in cell GN-D36, the LO2 tankage will provide a pressure of 689 kPa. Similarly, for the inlet temperature for cell GN-D37, the LO2 tankage will be providing the liquid at 90 K.

2.2 Pump Section: Outlet

Moving through the LPOTP, the pump will be pressurizing the LO2 and will cause BL-D42 to result in 2789 kPa. This was calculated using Eq. 1, and further rearranged in Eq. 2.

$$\Delta P = P_2 - P_1 \tag{1}$$

$$P_2 = \Delta P + P_1 \tag{2}$$

$$P_2 = 2.1 \cdot 10^3 kPa + 689\, kPa$$

$$P_2 = 2789\, kPa$$

For the temperature exiting the LPOTP in cell BL-D43, it will only be increasing to 91 K, shown in Eq. 3. This equation was given in *MAE 565 Project F22* in the background on liquid turbopumps. This equation uses the given values for LPOTP pump efficiency, LO2 density, and LO2-specific heat.

$$(T_2 - T_1) = \left(\frac{1-\eta_p}{\eta_p}\right)\left(\frac{P_{t2}-P_{t1}}{\rho C_v}\right) \tag{3}$$

$$T_2 = \left(\frac{1-.632}{.632}\right)\left(\frac{2789\, kPa - 689\, kPa}{(1141\, \frac{kg}{m^3})(1669\, \frac{J}{kg-K})}\right) + 90\, k$$

$$T_2 = 91\, K$$

Finally, the LPOTP pump power in cell BL-D44 can be calculated using Eq. 4. The actual pump power results in 1169 kW generated. Similar to the temperature, this equation was given in *MAE 565 Project F22* in the background on liquid turbopumps.

$$|\overline{W}|_{actual} = \overline{m}(\frac{1}{\eta_p})(\frac{P_{t2} - P_{t1}}{\rho}) \tag{4}$$

$$|\overline{W}|_{actual} = (401\frac{kg}{s})(\frac{1}{.632})(\frac{2789\ kPa - 689\ kPa}{(1141\ \frac{kg}{m^3})})$$

$$|\overline{W}|_{actual} = 1168\ kW$$

2.3 Turbine Section: Inlet

Referring to Figure 1-1, the LPOTP section has the pump on top of the turbine and they both lead to the HPOTP later in the feed system. From this HPOTP feed system, an additional plumbing line is added to the system which comes back to the LPOTP turbine. This high-pressure LO2 from the HPOTP will be driving the turbine. Due to these circumstances, the flow for the turbine will remain liquid from the LPOTP pump, and the LO2 temperature and the LO2 pressure will be coming from the HPOTP.

The LO2 temperature in cell GN-D47, coming from cell BL-D149, results in the inlet temperature being 97 K. This will be demonstrated in section 5.1 HPOTP Pump section inlet.

For the LO2 pressure at the LPOTP turbine inlet, cell GN-D48, this value will be 29.6 MPa. This is a given value in the HPOTP which is the pressure exiting the HPOTP pump.

Finally, the LO2 mass flow rate through the turbine for cell BL-D49 will equate to 77.2 kg/s and is calculated using Eq. 5 and rearranged in Eq. 6. This equation was given in *MAE 565 Project F22* in the background on liquid turbines.

$$|\overline{W}|_{actual} = \overline{m}(\frac{1}{\eta_T})(\frac{P_{t1} - P_{t2}}{\rho}) \tag{5}$$

$$\overline{m} = \frac{|\overline{W}|_{actual}}{\overline{m}(\frac{1}{\eta_T})(\frac{P_{t1} - P_{t2}}{\rho})} \tag{6}$$

$$\overline{m} = \frac{1168\ kW}{(\frac{1}{.644})(\frac{29.6 \cdot 10^3\ kPa - 2789\ kPa}{(1141\ \frac{kg}{m^3})})}$$

$$\overline{m} = 77.2\ \frac{kg}{s}$$

2.4 Turbine Section: Outlet

Moving through the LPOTP turbine, the pressure exiting the turbine will be equivalent to the LPOTP pump outlet. Therefore GN-D52 will be 2789 kPa, just as BL-D42 showed in Eq. 2.

The exiting LO2 temperature for cell BL-D52 results in 102 K, and is calculated in Eq. 7. This equation was given in *MAE 565 Project F22* in the background on liquid turbines.

$$(T_2 - T_1) = (1 - \eta_T)(\frac{P_{t1} - P_{t2}}{\rho C_v}) \tag{7}$$

$$T_2 = (1 - .644)(\frac{29.6 \cdot 10^3\ kPa - 2789\ kPa}{(1141\ \frac{kg}{m^3})(1669\ \frac{J}{kg-K})}) + 97\ k$$

$$T_2 = 101\ K$$

Finally, the resulting LPOTP turbine power will be equivalent to the LPOTP pump power. This then means that cell GN-D54 will be 1168 kW, just as BL-D44 showed in Eq. 4.

3. Low-Pressure Fuel Turbopump (LPFTP)

3.1 Pump Section: Inlet

Switching sides of the feed system from the oxidizer to the fuel, the LPFTP section can be seen in Figure 1-1 as the red section. The flow is known to be liquid as stated in the *MAE 565 Project F22* in the LPFTP section that the flow is LH2.

Similar to the LPOTP pump section, the LPFTP inlet pressure in cell GN-D59, the LH2 tankage will be providing a pressure of 207 kPa. Similarly, for the inlet temperature for cell GN-D60, the LO2 tankage will be providing the liquid at 20 K.

3.2 Pump Section: Outlet

Moving through the LPFTP, the pump will be pressurizing the LH2 and will cause BL-D64 to result in 1807 kPa. This was calculated using Eq. 1, and further rearranged in Eq. 2 but shown in Eq. 8.

$$P_2 = 1.6 \cdot 10^3 kPa + 207\ kPa \tag{8}$$

$$P_2 = 1807\ kPa$$

For the temperature exiting the LPFTP in cell BL-D65, it will only be increasing to 21 K, using Eq. 3, but shown in Eq. 9. This equation was given in *MAE 565 Project F22* in the background on liquid turbopumps. This equation uses the given values for LPOTP pump efficiency, LO2 density, and LO2-specific heat.

$$T_2 = \left(\frac{1-.674}{.674}\right)\left(\frac{1807\ kPa - 207\ kPa}{(70.8\ \frac{kg}{m^3})(9668\ \frac{J}{kg-K})}\right) + 20\ k \tag{9}$$

$$T_2 = 21\ K$$

Finally, the LPFTP pump power in cell BL-D67 can be calculated using Eq. 5, but shown in Eq. 10. The actual pump power results in 2.25 MW generated. Similar to the temperature, this equation was given in *MAE 565 Project F22* in the background on liquid turbopumps.

$$|\overline{W}|_{actual} = (67.1\ \frac{kg}{s})\left(\frac{1}{.674}\right)\left(\frac{1807\ kPa - 207\ kPa}{(70.8\ \frac{kg}{m^3})}\right) \tag{10}$$

$$|\overline{W}|_{actual} = 2.25\ MW$$

3.3 Turbine Section: Inlet

Referring to Figure 1-1, the LPFTP section has the pump on top of the turbine, but unlike the LPOTP, the LPFTP's pump and turbine lead to different areas. From this, the LPFTP pump goes to the HPFTP pump and the LPFTP turbine goes to the HPFTP turbine. Additionally, a plumbing line goes back into the turbine from the regenerative cooling plumbing that comes from the HPFTP section. Due to these circumstances, the flow for the turbine will be gaseous, and the GH2 temperature and the GH2 pressure will be coming from the HPFTP regenerative cooling.

The GH2 pressure in cell GN-D70, coming from *MAE 565 Project F22* in the regenerative cooling section, results in the inlet pressure being 32500 kPa. Additionally, this is a given value in the HPOTP regenerative colling section which is the temperature exiting the thrust chamber cooling.

For the GH2 temperature at the LPFTP turbine inlet, cell GN-D71, this value will be 269 K. This is a given value in the HPOTP regenerative colling which is the temperature exiting the thrust chamber cooling.

Finally, the GH2 mass flow rate through the turbine for cell BL-D72 will equate to 13.3 kg/s and is calculated using Eq. 11 and rearranged in Eq. 12. This equation was given in *MAE 565 Project F22* in the background on gas turbines.

$$|\overline{W}|_T = \overline{m} \cdot C_P(T_{t1} - T_{t2}) \tag{11}$$

$$\overline{m} = \frac{|\overline{W}|_T}{C_P(T_{t1} - T_{t2})} \tag{12}$$

$$\overline{m} = \frac{2.25\ MW}{(14340\ \frac{J}{kg-K})(269\ K - 257\ K)}$$

$$\overline{m} = 13.3\ \frac{kg}{s}$$

3.4 Turbine Section: Outlet

Moving through the LPFTP turbine, a pressure ratio is given to be 1.3. Knowing the GH2 pressure at the turbine inlet, the GH2 pressure exiting the turbine can be calculated to be 25000 kPa, shown in Eq. 13 and rearranged in Eq. 14.

$$P_{ratio} = \frac{P_{Inlet}}{P_{Outlet}} = \frac{P_1}{P_2} \tag{13}$$

$$P_2 = \frac{P_1}{P_{ratio}} \tag{14}$$

$$P_2 = \frac{325000\ kPa}{1.3}$$

$$P_2 = 25000\ kPa$$

The exiting GH2 temperature for cell BL-D77 results in 257 K, and is calculated in Eq. 15. This equation was given in *MAE 565 Project F22* in the background on gas turbines.

$$\frac{T_{t2}}{T_{t1}} = 1 - \eta_T\left[1 - \left(\frac{P_{t2}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{15}$$

$$T_2 = \left\{1 - (.536)\left[1 - \left(\frac{25000}{32500}\right)^{\frac{1.483-1}{1.483}}\right]\right\} \cdot 269$$

$$T_2 = 257\ K$$

Finally, the resulting LPFTP turbine power will be equivalent to the LPFTP pump power. This then means that cell GN-D78 will be 2.25 MW, just as BL-D67 showed in Eq. 10.

4. High-Pressure Fuel Turbopump (HPFTP)

4.1 Pump Section: Inlet

Referencing Figure 1-1 again, the LPFTP leads directly to the HPFTP through both the pump and the turbine. We can conclude since the LPFTP pump outlet is left with LH2, that the HPFTP will have liquid flowing into it (LH2).

For the HPFTP pump inlet, we are given a pressure drop between the LPFTP and HPFTP to be 398 kPa. This then allows us to calculate the inlet pressure in cell GN-D84 for the HPFTP pump to be 1409 kPa, as seen in Eq. 16.

$$P_2 = P_1 - \Delta P \tag{16}$$

$$P_2 = 1807\,kPa - 398\,kPa$$

$$P_2 = 1409\,kPa$$

Using similar logic for the liquid flow, it can be determined that the LH2 mass flow rate going through the pump will be 67.1 kg/s for cell GN-D85, as it was given for the LPFTP section.

The temperature entering the HPFTP pump for cell GN-D89 can be assumed to be 21 K, as it is traveling from the LPFTP pump and no temperature loss/gain was given. This was previously calculated in Eq. 9.

4.2 Pump Section: Outlet

Moving through the pump, the pressure increase was given to be 41.7 MPa. Using this we can calculate the LH2 HPFTP pump outlet pressure for cell BL-D87 to be 43109 kPa, as shown in Eq. 17.

$$P_2 = P_1 + \Delta P \tag{17}$$

$$P_2 = 1409kPa + 41.7 \cdot 10^3\,kPa$$

$$P_2 = 43109\,kPa$$

For the HPFTP pump outlet temperature we can use Eq. 3 to calculate cell BL-D90, which results in 41 K as shown in Eq. 18.

$$T_2 = \left(\frac{1-.758}{.758}\right)\left(\frac{43109\,kPa - 1409\,kPa}{(70.8\,\frac{kg}{m^3})(9668\,\frac{J}{kg-K})}\right) + 21\,k \tag{18}$$

$$T_2 = 41\,K$$

Calculating the HPFTP pump power than for cell BL-D91, this can be seen in the HPFTP turbine section, as the HPFTP turbine power is given as 47.2 MW. As discussed previously, turbine power = pump power, and therefore, the HPFTP pump power will equate and equal 47.2 MW.

4.3 HP LH2 Flow Splits: Thrust Chamber Cooling

From the HPFTP pump, three plumbing lines resulted from the exit. The first discussed is the line going to the thrust chamber cooling. It is given that 20.3% of the LH2 will be going to this route. Using this, we can calculate the LH2 mass flow rate for cell BL-D94 to be 13.6 kg/s using Eq. 19.

$$\overline{m}_{thrust\ Chamber} = \overline{m}_{HPFTP\ pump} \cdot Flow\,\% \tag{19}$$

$$\overline{m}_{thrust\ Chamber} = 67.1\frac{kg}{s} \cdot (20.3\% \cdot 100)$$

$$\overline{m}_{thrust\ Chamber} = 13.6\ \frac{kg}{s}$$

Since now temperature changes are happening across this route, the temperature entering the thrust chamber cooling in cell GN-D95 will be 41 K, as previously shown in Eq. 18.

Similarly, the pressure has no changes and for cell GN-D96, the pressure will be 43109 kPa, as shown in Eq. 17.

Due to the cooling of the thrust chamber, the temperature will increase and the pressure will decrease as the LH2 becomes GH2. These are given values in the spreadsheet.

4.4 HP LH2 Flow Splits: Expansion Nozzle Cooling

Similar to the thrust chamber cooling, the expansion nozzle cooling will be another route from the HPFTP pump which given that 42.4% of the LH2 will be going. Using Eq. 19, the mass flow rate through this route can be calculated for cell BL-D101 to be 28.5 kg/s, as shown in Eq. 20.

$$\overline{m}_{Expansion\ Nozzle} = 67.1\frac{kg}{s} \cdot (42.4\% \cdot 100) \tag{20}$$

$$\overline{m}_{Expansion\ Nozzle} = 28.5\ \frac{kg}{s}$$

As stated in the thrust chamber cooling, the temperature will have no changes across the route to entering the expansion nozzle cooling, and therefore in cell GN-D102, the temperature will be 41 K, as previously shown in Eq. 18.

Similarly, the pressure will have no changes and for cell GN-D102, the pressure will be 43109 kPa, as shown in Eq. 17.

As mentioned above for the cooling of the thrust chamber, the temperature will increase and the pressure will decrease as the LH2 becomes GH2. These are given values in the spreadsheet.

4.5 HP LH2 Flow Splits: Bypass Cooling

For the remaining 37.3% of the LH2 flow from the HPFTP pump, an LH2 mass flow rate bypassing both the thrust chamber and expansion nozzle cooling will be using Eq. 19. Therefore, cell BL-D108 will equate to 25 kg/s, as shown in Eq. 21.

$$\overline{m}_{Bypass} = 67.1\,\frac{kg}{s} \cdot (37.3\% \cdot 100) \tag{21}$$

$$\overline{m}_{Bypass} = 25\,\frac{kg}{s}$$

The resulting pressure and temperature after transferring from LH2 to GH2 were given. From here, a combined GH2 mass flow rate can be calculated. This combined mass flow rate can be seen in Figure 1-1, where the expansion nozzle cooling and the bypass cooling routes both come together to flow to the preburners. This then results in cell BL-D111 equating to 53.5 kg/s, and is shown in Eq. 22.

$$\overline{m}_{Combined\ GH2} = \overline{m}_{Expansion\ Nozzle} + \overline{m}_{Bypass} \tag{22}$$

$$\overline{m}_{Combined\ GH2} = 25\,\frac{kg}{s} + 28.5\,\frac{kg}{s}$$

$$\overline{m}_{Combined\ GH2} = 53.5\,\frac{kg}{s}$$

Additionally, the combined GH2 temperatures can be calculated using a given equation in *MAE 565 Project F22.* Therefore, in cell BL-D112, the GH2 temperature going to the preburners will equate to 154 K, as shown in Eq. 23. Inside Eq. 23, Eq. 24-26 are also represented and are stated as such. It should be noted that the CPs cancel in this case as both sections are GH2.

$$T = \left[Y_1 \cdot \left(\frac{C_{p1}}{C_p}\right) \cdot T_1\right] + \left[Y_2 \cdot \left(\frac{C_{p2}}{C_p}\right) \cdot T_2\right] \tag{23}$$

Where,

$$Y_1 = \frac{\overline{m}_1}{\overline{m}} \tag{24}$$

$$Y_2 = \frac{\overline{m}_2}{\overline{m}} \tag{25}$$

$$C_P = Y_1 \cdot C_{p1} + Y_2 \cdot C_{p2} \tag{26}$$

Therefore,

$$T = \left[\left(\frac{25\ \frac{kg}{s}}{53.5\ \frac{kg}{s}} \right) \cdot (28\ K) \right] + \left[\left(\frac{28.5\ \frac{kg}{s}}{53.5\ \frac{kg}{s}} \right) \cdot (265\ K) \right]$$

$$T = 154\ K$$

Finally, the pressure going into the preburners will be equivalent to the expansion nozzle cooling exit due to the flows being at the same pressure. Therefore, since the expansion nozzle cooling exit pressure is given, cell GN-D113 will be 35200 kPa.

4.6 Preburner

From section 4.5 HP LH2 Flow Splits: Bypass Cooling, a few values which have been combined are known. Due to a given value of 68% of the GH2 flow going to the preburner, we can use Eq. 19. Therefore, cell BL-D116 will equate to 36.4 kg/s, as shown in Eq. 27. Additionally, Eq. 27 will be using the mass flow rate calculated in Eq, 22, found to be 53.5 kg/s.

$$\overline{m}_{Bypass} = 53.5 \frac{kg}{s} \cdot (68\% \cdot 100) \tag{27}$$

$$\overline{m}_{Bypass} = 36.4 \frac{kg}{s}$$

Continuing, the temperature enting the HPFTP preburner is equated to the combined GH2 temperature found in Eq. 23, and cell GN-D117 will be 154 K.

Similarly, the pressure will also be the same as the pressure heading to the preburner, and as such cell, GN-D118 will equate to 35200 kPa.

Due to the O/F ratio given of .97, the LO2 mass flow rate entering the preburner from the LO2 boost pump, in cell BL-D120, will be 35.3 kg/s as shown in Eq. 28. This will be explained further in the HPOTP section, however referencing figure 1-1, it can be seen that not only does the HPFTP preburner take in the fuel (GH2) from the HPFTP pump, but also oxidizer (LO2) from the HPOTP boost pump.

$$\overline{m}_{LO2} = \overline{m}_{GH2} \cdot \frac{O}{F} ratio \tag{28}$$

$$\overline{m}_{LO2} = 36.4 \frac{kg}{s} \cdot .97$$

$$\overline{m}_{LO2} = 35.3 \frac{kg}{s}$$

Similarly, the temperature from the HPOTP boost pump will be shown further in the HPOTP section, but cell GN-D121 will be 100 K.

The same is said for the LO2 pressure from the HPOTP boost pump. This is a given value in the spreadsheet but cell GN-D122 is 50200 kPa.

Finally, the preburner product gas mass flux is the result of adding the GH2 Mass flow rate and the LO2 mass flow rate going into the preburner as per conservation of mass. Therefore, cell GN-D122 will be 71.6 kg/s, as shown in Eq. 29.

$$\overline{m}_{preburrner} = \overline{m}_{LO2} + \overline{m}_{GH2} \tag{29}$$

$$\overline{m}_{preburrner} = 36.4\,\frac{kg}{s} + 35.3\,\frac{kg}{s}$$

$$\overline{m}_{LO2} = 71.6\,\frac{kg}{s}$$

4.7 Turbine Section: Inlet

Transitioning from the preburner to the HPFTP turbine, the flow from the LPFTP turbine was stated to be a gas, and therefore this will continue being GH2.

The turbine inlet GH2 temperature will be equivalent to the preburner product gas temperature, and therefore cell GN-D134 will be 1117 K. This is due to the turbine being below the preburner and the resulting products will be entering the HPFTP turbine.

Similarly, the turbine inlet pressure will be equivalent to the preburner product gas pressure, and therefore cell GN-D135 will be 35500 kPa.

4.8 Turbine Section: Outlet

Due to the turbine pressure ratio being given as 1.52, the turbine outlet pressure can be calculated using Eq. 14, and shown in Eq. 30. Therefore, cell BL-D137 will equate to 23355 kPa.

$$P_2 = \frac{35500\,kPa}{1.52} \tag{30}$$

$$P_2 = 23355\,kPa$$

The turbine outlet mass flux will be equivalent to the preburner product mass flux, as this will be the mass flux through the turbine. Therefore, cell GN-D138 will be 71.6 kg/s, as shown in Eq. 29.

Finally, the turbine outlet gas temperature will be calculated using Eq. 15, and shown in Eq. 31. Cell BL-D140 will result in 1029 K.

$$T_2 = \left\{1 - (.770)\left[1 - \left(\frac{23355}{35500}\right)^{\frac{1.35-1}{1.35}}\right]\right\} \cdot 1117 \tag{31}$$

$$T_2 = 1029\ K$$

5. High-Pressure Oxygen Turbopump (HPOTP)

5.1 Pump Section: Inlet

Moving back to the right side of Figure 1-1 to the HPTOP section, the HPOTP pump will be receiving the LO2 from the LPOTP.

As such, the LO2 pressure for the HPOTP will be equivalent to the pressure traveling from the LPOTP, which means cell GN-D144 will be 2.8 MPa.

Additionally, the LO2 mass flow rate for the HPOTP will be the combination of the mass flow rate from the LPOTP pump and turbine. This is due to the additional plumbing line from the HPOTP, which goes to the LPOTP turbine, which adds an additional mass flow rate, as discussed in the LPOTP turbine section. Therefore, cell GN-D144 will be 478.2 kg/s, as shown in Eq. 32.

$$\overline{m}_{LO2\ HPOTP} = \overline{m}_{LO2\ Pump} + \overline{m}_{Turbine} \tag{32}$$

$$\overline{m}_{preburrner} = 401\ \frac{kg}{s} + 77.2\ \frac{kg}{s}$$

$$\overline{m}_{LO2} = 478.2\ \frac{kg}{s}$$

The temperature entering the pump will be the same as the temperature exiting the pump in the LPOTP section, as calculated in Eq. 3. Therefore, cell GN-D148 will be 91 K. The reason for this is due to the HPOTP adding to the turbine and making the temperature in the LPOTP turbine 102 K, but this is due to the HPOTP, and therefore we would be in a loop in itself if we used this temperature as the value for entering the HPOTP pump.

5.2 Pump Section: Outlet

Moving through the HPOTP pump, an outlet temperature can be calculated using Eq. 3, and therefore cell BL-D149 results in 97 K, as shown in Eq. 33.

$$T_2 = \left(\frac{1-.681}{.681}\right)\left(\frac{29.6\cdot10^3\ kPa - 2.8\cdot10^3\ kPa}{(1141\ \frac{kg}{m^3})(1669\ \frac{J}{kg-K})}\right) + 91\ k \tag{33}$$

$$T_2 = 97\ K$$

Finally, the pump power created in the HPOTP can be calculated using Eq. 4, and cell BL-D150 will be 16.5 MW as shown in Eq. 34.

$$|\overline{W}|_{actual} = (478.2\frac{kg}{s})(\frac{1}{.681})(\frac{29.6\cdot10^3\,kPa-2.8\cdot10^3\,kPa}{(1141\,\frac{kg}{m^3})}) \tag{34}$$

$$|\overline{W}|_{actual} = 16.5\,MW$$

5.3 Boost Pump

From the HPOTP about 10% of the LO2 flow, 48.4 kg/s, will be routed to a boost pump below the HPOTP pump. Therefore, the flow will continue being LO2.

The pressure entering the boost pump will then be the resulting exiting pressure from the HPOTP pump, and cell GN-D153 will be 29.6 MPa.

Similarly, the temperature going into the boost pump will be coming from the HPOTP pump exit. As such, the temperature in cell GN-D154 will be 97 K.

Exiting the boost pump, the temperature can be calculated using Eq. 3, and shown in Eq. 35. Therefore, cell BL-D157 will result to 100 K.

$$T_2 = (\frac{1-.803}{.803})(\frac{50.2\cdot10^3\,kPa-29.6\cdot10^3\,kPa}{(1141\,\frac{kg}{m^3})(1669\,\frac{J}{kg-K})}) + 97\,k \tag{35}$$

$$T_2 = 100\,K$$

Finally, the resulting boost pump power can be calculated using Eq. 4, and seen in Eq. 36. Cell BL-D159 will result to 1.1 MW.

$$|\overline{W}|_{actual} = (48.3\frac{kg}{s})(\frac{1}{.803})(\frac{50.2\cdot10^3\,kPa-29.6\cdot10^3\,kPa}{(1141\,\frac{kg}{m^3})}) \tag{36}$$

$$|\overline{W}|_{actual} = 1.1\,MW$$

5.4 Preburner

From the boost pump, both the HPOTP and HPFTP preburners receive LO2. Due to the 68% of GH2 going to the HPFTP preburner, then 32% of the GH2 will be headed to the HPOTP preburner section. Due to this, we can use Eq. 19 to calculate the GH2 mass flow rate going to the HPOTP preburner. Therefore, cell BL-D162 will equate to 17.1 kg/s, as shown in Eq. 37. Additionally, Eq. 23 will be using the mass flow rate calculated in Eq, 22, found to be 53.5 kg/s.

$$\overline{m}_{Bypass} = 53.5\frac{kg}{s} \cdot (32\% \cdot 100) \qquad (37)$$

$$\overline{m}_{Bypass} = 17.1\ \frac{kg}{s}$$

Continuing, the temperature enting the HPOTP preburner is equated to the combined GH2 temperature found in Eq. 23, and cell GN-D162 will be 154 K.

Similarly, the pressure will also be the same as the pressure heading to the preburner, and as such cell GN-D164will equate to 35200 kPa.

The temperature from the HPOTP boost pump, shown in Eq. 35, will make cell GN-D167 be 100 K.

The same is said for the LO2 pressure from the HPOTP boost pump. This is a given value in the spreadsheet but cell GN-D168 is 50200 kPa.

Finally, the preburner product gas mass flux is the result of adding the GH2 Mass flow rate and the LO2 mass flow rate going into the preburner as per conservation of mass. Therefore, cell GN-D169 will be 28.4 kg/s, using Eq. 29 and shown in Eq. 38.

$$\overline{m}_{preburrner} = 17.1\ \frac{kg}{s} + 11.3\ \frac{kg}{s} \qquad (38)$$

$$\overline{m}_{LO2} = 28.4\ \frac{kg}{s}$$

5.6 Turbine Section: Inlet

Due to the preburner resulting in a gas, the HPOTP turbine will have an incoming flow of gas.

With this the HPOTP turbine power, as seen in cell GN-D179, will be equal to the HPOTP pump plus the boost power. This equates to 17.6 MW, as shown in Eq. 39. The HPOTP pump is shown in Eq. 34, and the boost pump power is shown in Eq. 36, respectively.

$$|\overline{W}|_{Turbine} = |\overline{W}|_{Pump} + |\overline{W}|_{boost} \qquad (36)$$

$$|\overline{W}|_{actual} = 16.5\ MW\ +\ 1.1\ MW$$

$$|\overline{W}|_{actual} = 17.6\ MW$$

The turbine inlet LO2 temperature will be equivalent to the preburner product gas temperature, and therefore cell GN-D180 will be 836 K.

Similarly, the turbine inlet pressure will be equivalent to the preburner product gas pressure, and therefore cell GN-D181 will be 34400 kPa.

5.7 Turbine Section: Outlet

For the outlet of the HPOTP turbine, the pressure will need to equate that of the HPFTP turbine in order to make the main injectors have equivalent pressures to ensure no pressure difference in the system. Therefore, GN-D182 will be 23355 as seen calculated in Eq. 30.

Therefore, working backward of Eq. 30, the pressure ratio can be calculated to be 1.47 for BL-D182 as shown in Eq. 37.

$$P_{ratio} = \frac{34400\ kPa}{23355\ kPa} \tag{37}$$

$$P_{ratio} = 1.47$$

The turbine outlet mass flux then will be equivalent as the HPOTP preburner mass flux, which will make GN-D184 result to 28.4 kg/s.

The turbine outlet gas temperature will be calculated by rearranging Eq. 11, and shown in Eq. 38. Cell BL-D185 will result in 768 K.

$$T_{t2} = T_{t1} - \frac{|\overline{W}|_T}{\overline{m} \cdot C_p} \tag{38}$$

$$T_{t2} = 836\ K - \frac{17.6\ MW}{(28.4\ \frac{kg}{s}) \cdot (9073\ \frac{J}{kg \cdot K})}$$

$$T_{t2} = 768\ K$$

The turbine isentropic efficiency must be calculated, and as such can be seen in Eq. 39. This equation will rearrange Eq. 15 and results in BL-D186 to be .822 or 82.2% efficient.

$$\eta_T = \frac{1 - \frac{T_{t2}}{T_{t1}}}{\left[1 - \left(\frac{P_{t2}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{39}$$

$$\eta_T = \frac{1 - \frac{768\ K}{836\ K}}{\left[1 - \left(\frac{23355\ kPa}{34400\ kPa}\right)^{\frac{1.37-1}{1.37}}\right]}$$

$$\eta_T = .822$$

6. Main Injectors

6.1 GH2 Injectors: LPFTP and HPFTP Combination

The main injectors, as seen in Figure 1-1, combine several sections to create the combustion section for the thrust chamber. For this reason, the main injector section begins with referencing values calculated in previous areas such as the HPFTP and HPOTP.

The gas product mass flow rate from the HPFTP to the injectors begins with the preburner and subsequently the HPFTP turbine outlet. Therefore, GN-D189 will be 71.6 kg/s as calculated in Eq. 29.

The gas product temperature from the HPFTP to the injectors will similarly be coming from the HPFTP turbine exit. Therefore, GN-D190 will be 1029 K, as calculated in Eq. 31.

The gas product-specific heat from the HPFTP to the injectors comes from the given preburner product gas-specific heat value. This then makes GN-D191 equal to 8088 J/kg-K.

The gas product molecular weight from the HPFTP to the injectors comes from the given preburner product gas molecular weight value. This then makes GN-D192 equal to 3.97 g/mol.

The gas product pressure from the HPFTP to the injectors comes from the HPFTP turbine. This then makes GN-D193 equal to 23355 kPa, as calculated in Eq. 30.

The gas product H2 mass ratio from the HPFTP to the injectors comes from the given preburner product gas H2 mass ratio value. This then makes GN-D194 equal to .446.

The gas product H2O mass ratio from the HPFTP to the injectors come from the given preburner product gas H2O mass ratio value. This then makes GN-D195 equal to .554.

Another plumbing line is the LPFTP turbine to the HPFTP turbine and subsequently the injectors. The GH2 mass flow rate from the LPFTP will make GN-D197 equal to 13.3 kg/s, as calculated in Eq. 11.

The GH2 temperature from the LPFTP turbine to the injectors will make GN-D198 equal to 257 K, as calculated in Eq. 15.

The GH2 pressure from the LPFTP turbine to the injectors will make GN-D199 equal to 25000 kPa, as calculated in Eq. 13.

The GH2 specific heat from the LPFTP turbine to the injectors will make GN-D200 equal to 14340 J/kg-K, as given in fluid properties at the beginning of the spreadsheet.

The GH2 molecular weight from the LPFTP turbine to the injectors will make GN-D201 equal to 14340 J/kg-K, as given in *GH2 & LH2 Properties*.

Now that the given values from LPFTP and HPFTP are taken into account, the combination of these values will need to be calculated. For the combined GH2-rich mass flow rate from the fuel side, BL-D203 will equal 84.9 kg/s. This will be calculated using Eq. 40.

$$\overline{m}_{Combined} = \overline{m}_{HPFTP} + \overline{m}_{LPFTP} \tag{40}$$

$$\overline{m}_{Combined} = 71.6\ \frac{kg}{s} + 13.3\ \frac{kg}{s}$$

$$\overline{m}_{Combined} = 84.9\ \frac{kg}{s}$$

The combined GH2-rich temperature from the fuel side, BL-D204 will equal 837.7 K. This will be calculated using Eq. 23 but shown in Eq. 41.

$$T_{Combined} = \left[\left(\frac{71.6\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}\right) \cdot \left(\frac{8088\ \frac{J}{kg-K}}{C_{P\ combined}}\right) \cdot (1029\ K) \right] + \left[\left(\frac{13.3\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}\right) \cdot \left(\frac{14340\ \frac{J}{kg-K}}{C_{P\ combined}}\right) \cdot (257.2\ K) \right] \tag{41}$$

Where,

$$C_{P\ combined} = \left(\frac{71.6\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}\right) \cdot 8088\ \frac{J}{kg-K} + \left(\frac{13.3\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}\right) \cdot 14340\ \frac{J}{kg-K}$$

Therefore,

$$T_{Combined} = 837.7\ K$$

The combined GH2-rich pressure from the fuel side, BL-D205 will equal 23795 kPa. This will be calculated as shown in Eq. 42, and subsequently Eq. 43-47.

$$P_{Combined} = \left[X_1 \cdot P_1\right] + \left[X_2 \cdot P_2\right] \tag{42}$$

Where,

$$X_1 = \frac{\overline{n}_1}{\overline{n}} \tag{43}$$

$$X_2 = \frac{\overline{n}_2}{\overline{n}} \tag{44}$$

$$\overline{n}_1 = \frac{\overline{m}_1}{MW_1} \tag{45}$$

$$\overline{n}_2 = \frac{\overline{m}_2}{MW_2} \tag{46}$$

$$\bar{n} = \bar{n}_1 + \bar{n}_2 \tag{47}$$

Therefore,

$$P_{Combined} = \left[\left(\frac{\bar{n}_1}{\bar{n}}\right) \cdot 23355\ kPa \right] + \left[\left(\frac{\bar{n}_2}{\bar{n}}\right) \cdot 25000\ kPa \right]$$

$$P_{Combined} = 23795\ kPa$$

The combined GH2-rich H2 mass ratio from the fuel side, BL-D206 will equal .533. This will be calculated as shown in Eq. 48, a little deviated from Eq. 24.

$$Y_{H2\ combined} = \frac{(\bar{m}_{HPFTP} \cdot Y_{H2\ HPFTP}) + \bar{m}_{LPFTP}}{\bar{m}} \tag{48}$$

$$Y_{H2\ combined} = \frac{(71.6\ \frac{kg}{s} \cdot .446) + 13.3\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}$$

$$Y_{H2\ combined} = .533$$

The combined GH2-rich H2O mass ratio from the fuel side, BL-D207 will equal .467. This will be calculated as shown in Eq. 49, a little deviated from Eq. 25.

$$Y_{H2O\ combined} = \frac{(\bar{m}_{HPFTP} \cdot Y_{H2\ HPFTP})}{\bar{m}} \tag{49}$$

$$Y_{H2O\ combined} = \frac{(71.6\ \frac{kg}{s} \cdot .554)}{84.9\ \frac{kg}{s}}$$

$$Y_{H2O\ combined} = .467$$

The combined GH2-rich specific ratio from the fuel side, BL-D208 will equal 9066 J/kg-K. This was shown in Eq. 26 and will be calculated in Eq. 50.

$$C_{P\ combined} = \left(\frac{71.6\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}\right) \cdot 8088\ \frac{J}{kg-K} + \left(\frac{13.3\ \frac{kg}{s}}{84.9\ \frac{kg}{s}}\right) \cdot 14340\ \frac{J}{kg-K} \tag{50}$$

$$C_{P\ combined} = 9066\ \frac{J}{kg-K}$$

Finally, the combined GH2-rich H2O molecular weight from the fuel side, BL-D209 will equal 3.45 g/mol. This will be calculated in Eq. 51.

$$MW_{Combined} = \left[X_1 \cdot MW_1\right] + \left[X_2 \cdot MW_2\right] \tag{51}$$

$$MW_{Combined} = \left[\left(\frac{\bar{n}_1}{\bar{n}}\right) \cdot 3.97\ \frac{g}{mol}\right] + \left[\left(\frac{\bar{n}_2}{\bar{n}}\right) \cdot 2.02\ \frac{g}{mol}\right]$$

$$MW_{Combined} = 3.45\ g/mol$$

6.2 GH2 Injectors: Total Combination

Now that the HPFTP and LPFTP are combined, another combination is needed between that combination and the HPOTP. Before jumping into the total combination equations, the values previously calculated in the HPOTP need to be transferred to this section for easier reference.

The gas product mass flow rate from the HPOTP to the injectors begins with the preburner and subsequently the HPOTP turbine outlet. Therefore, GN-D211 will be 28.4 kg/s as calculated in Eq. 38.

The gas product temperature from the HPOTP to the injectors will similarly be coming from the HPOTP turbine exit. Therefore, GN-D212 will be 768 K, as calculated in Eq. 38.

The gas product pressure from the HPOTP to the injectors comes from the HPOTP turbine, which is equal to the HPFTP turbine. This then makes GN-D213 equal to 23355 kPa, as calculated in Eq. 30.

The gas product H2 mass ratio from the HPOTP to the injectors comes from the given preburner product gas H2 mass ratio value. This then makes GN-D214 equal to .549.

The gas product H2O mass ratio from the HPOTP to the injectors comes from the given preburner product gas H2O mass ratio value. This then makes GN-D215 equal to .451.

The gas product-specific heat from the HPOTP to the injectors comes from the given preburner product gas-specific heat value. This then makes GN-D216 equal to 9073 J/kg-K.

The gas product molecular weight from the HPFTP to the injectors comes from the given preburner product gas molecular weight value. This then makes GN-D217 equal to 3.36 g/mol.

Now that the given values from HPOTP are taken into account, the combination of the previous combination and HPOTP values will need to be calculated. For the total combined GH2-rich mass flow, BL-D219 will equal 113.3 kg/s. This will be calculated using Eq. 52.

$$\overline{m}_{Total\ Combined} = \overline{m}_{combined\ 1} + \overline{m}_{HPOTP} \tag{51}$$

$$\overline{m}_{Total\ Combined} = 84.9\ \frac{kg}{s} + 28.4\ \frac{kg}{s}$$

$$\overline{m}_{Total\ Combined} = 113.3\ \frac{kg}{s}$$

The total combined GH2-rich H2 mass ratio, BL-D220 will equal .537. This will be calculated as shown in Eq. 52, a little deviated from Eq. 24.

$$Y_{Total\ H2\ combined} = \frac{(\overline{m}_{combined\ 1} \cdot Y_{H2\ combined\ 1}) + (\overline{m}_{HPOTP} \cdot Y_{H2\ HPOTP})}{\overline{m}} \tag{52}$$

$$Y_{Total\ H2\ combined} = \frac{(84.9\ \frac{kg}{s} \cdot .533) + (28.4\ \frac{kg}{s} \cdot .549)}{113.3\ \frac{kg}{s}}$$

$$Y_{Total\ H2\ combined} = .537$$

The total combined GH2-rich H2O mass ratio, BL-D221 will equal .463. This will be calculated as shown in Eq. 53, a little deviated from Eq. 25.

$$Y_{Total\ H2O\ combined} = \frac{(\overline{m}_{combined\ 1} \cdot Y_{H2O\ combined\ 1}) + (\overline{m}_{HPOTP} \cdot Y_{H2O\ HPOTP})}{\overline{m}} \tag{53}$$

$$Y_{Total\ H2O\ combined} = \frac{(84.9\ \frac{kg}{s} \cdot .467) + (28.4\ \frac{kg}{s} \cdot .451)}{113.3\ \frac{kg}{s}}$$

$$Y_{Total\ H2O\ combined} = .463$$

The total combined GH2-rich specific ratio, BL-D222 will equal 9068 J/kg-K. This was shown in Eq. 26 and will be calculated in Eq. 54.

$$C_{P\ Total\ combined} = \left(\frac{84.9\ \frac{kg}{s}}{113.3\ \frac{kg}{s}}\right) \cdot 9066\ \frac{J}{kg-K} + \left(\frac{28.4\ \frac{kg}{s}}{113.3\ \frac{kg}{s}}\right) \cdot 9073\ \frac{J}{kg-K} \tag{54}$$

$$C_{P\ combined} = 9068\ \frac{J}{kg-K}$$

The total combined GH2-rich temperature, BL-D223 will equal 820 K. This will be calculated using Eq. 23 but shown in Eq. 55.

$$T_{Total\ Combined} = \left[\left(\frac{84.9\ \frac{kg}{s}}{113.3\ \frac{kg}{s}} \right) \cdot \left(\frac{9066\ \frac{J}{kg-K}}{9068\ \frac{J}{kg-K}} \right) \cdot (837.7\ K) \right] + \left[\left(\frac{28.4\ \frac{kg}{s}}{113.3\ \frac{kg}{s}} \right) \cdot \left(\frac{9073\ \frac{J}{kg-K}}{9068} \right) \cdot (768\ K) \right] \quad (55)$$

$$T_{Total\ Combined} = 820\ K$$

The total combined GH2-rich H2O molecular weight, BL-D224 will equal 3.43 g/mol. This will be calculated in Eq. 56.

$$MW_{Total\ Combined} = \left[X_1 \cdot MW_1\right] + \left[X_2 \cdot MW_2\right] \quad (56)$$

$$MW_{Total\ Combined} = \left[\left(\frac{\bar{n}_1}{\bar{n}} \right) \cdot 3.45\ \frac{g}{mol} \right] + \left[\left(\frac{\bar{n}_2}{\bar{n}} \right) \cdot 3.36\ \frac{g}{mol} \right]$$

$$MW_{Total\ Combined} = 3.43\ g/mol$$

The total combined GH2-rich pressure, BL-D225 will equal 23682 kPa. This will be calculated using Eq. 42, but shown in Eq. 57.

$$P_{Total\ Combined} = \left[\left(\frac{\bar{n}_1}{\bar{n}} \right) \cdot 23795\ kPa \right] + \left[\left(\frac{\bar{n}_2}{\bar{n}} \right) \cdot 23355\ kPa \right] \quad (57)$$

$$P_{Combined} = 22682\ kPa$$

Finally, the GH2 pressure drop across the injectors for BL-D228 can be calculated to be 20600 kPa, as shown in Eq. 58.

$$P_{Exit} = P_{Total\ Combined} - P_{drop} \quad (58)$$

$$P_{Exit} = 23682\ kPa - 3082\ kPa$$

$$P_{Exit} = 20600\ kPa$$

6.3 LO2 INJECTORS

Not only is there the GH2 injectors from the preburners and turbines, but there are also LH2 injectors which are directly from the HPOTP pump. Again, some referenced values are all that is needed to begin this section.

The LO2 mass flow rate from the HPOTP to the injectors is the resultant flow from the pump which doesn't go to the LPOTP turbine or the boost pump. Therefore, GN-D230 will be 353 kg/s as calculated in Eq. 59.

$$\overline{m}_{LO2} = \overline{m}_{HPOTP\ Pump} - \overline{m}_{LPFTP\ Turbine} - \overline{m}_{Boost\ Pump} \tag{59}$$

$$\overline{m}_{LO2} = 478.2\ \frac{kg}{s} - 48.3\ \frac{kg}{s} - 77.2\ \frac{kg}{s}$$

$$\overline{m}_{Lo2} = 353\ \frac{kg}{s}$$

The LO2 temperature from the HPOTP to the injectors will be coming from the HPOTP pump exit. Therefore, GN-D231 will be 97 K, as calculated in Eq. 33.

The LO2 pressure from the HPOTP to the injectors will be coming from the HPOTP pump exit. Therefore, GN-D232 will be 29.6 MPa, as given in the HPOTP pump section.

Finally, the LO2 pressure drop across the injectors for BL-D235 can be calculated to be 20600 kPa, as shown in Eq. 60. This makes sense as the pressures going into the combustion process and the thrust chamber need to be equivalent.

$$P_{Exit} = P_{LO2} - P_{drop} \tag{60}$$

$$P_{Exit} = 29.6 \cdot 10^{3}\ kPa - 9000\ kPa$$

$$P_{Exit} = 20600\ kPa$$

7. Thrust Chamber

7.1 Entering Main Combustion Chamber

As mentioned above, the injectors are the process leading up to the combustion chamber which is located in this thrust chamber. To begin, transferring a few variables which will become important first will make for an easier reference.

From the GH2 injector's total combination section, the total combined GH2-rich mass flow, GN-D241 will equal 113.3 kg/s. This was previously calculated in Eq. 52.

From the GH2 injector's total combination section, the total combined GH2-rich H2 mass ratio, GN-D242 will equal .537. This was previously calculated in Eq. 52.

From the GH2 injector's total combination section, the total combined GH2-rich H2O mass ratio, GN-D243 will equal .463. This was previously calculated in Eq. 53.

From the GH2 injector's total combination section, the total combined GH2-rich temperature, GN-D244 will equal 820 K. This was previously calculated in Eq. 55.

From the LO2 injector's total combination section, the total combined LO2 mass flow, GN-D246 will equal 353 kg/s. This was previously calculated in Eq. 59.

Using the transferred values, the main combustion chamber's overall O/F mass ratio can be calculated. BL-D238 results in 3.11, as shown in Eq. 61.

$$\frac{O}{F}_{MCC\ Overall} = \frac{\bar{m}_{LO2}}{\bar{m}_{GH2}} \tag{61}$$

$$\frac{O}{F}_{MCC\ Overall} = \frac{353\ \frac{kg}{s}}{113.3\ \frac{kg}{s}}$$

$$\frac{O}{F}_{MCC\ Overall} = 3.11$$

The resulting O/F mass ratio entering the main combustion chamber will then, in turn, be equal to the main combustion chambers overall O/F mass ratio and make BL-D247 equal 3.11.

Finally, the O2/H2 mass flux ratio entering the main combustion chamber will make BL-D248 equal to 5.8, as shown in Eq. 62.

$$\frac{O2}{H2}_{MCC} = \frac{\bar{m}_{LO2}}{\bar{m}_{GH2} \cdot Y_{H2}} \tag{62}$$

$$\frac{O2}{H2}_{MCC} = \frac{353\ \frac{kg}{s}}{(113.3\ \frac{kg}{s}) \cdot (.537)}$$

$$\frac{O2}{H2}_{MCC} = 5.8$$

7.2 Combustion Chamber

Moving now from entering the combustion chamber to the actual chamber itself, the combustion product gas pressure inside, GN-D252, will equal 20.6 MPa. This is equivalent to the main combustion chamber pressure, which is stated above.

The chamber itself has a given diameter of 45.1 cm and using this the cross-sectional area can be calculated. For BL-D260, the area is .16 m^2 and is shown in Eq. 63.

$$A_C = \pi \cdot (\frac{d}{2})^2 \tag{63}$$

$$A_C = \pi \cdot (\frac{45.1\ cm}{2})^2$$

$$A_C = .16\ m^2$$

Additionally, the throat diameter is given and using Eq. 63, BL-D262 can be calculated to be .054 m^2 as shown in Eq. 64.

$$A^* = \pi \cdot (\frac{26.2\ cm}{2})^2 \tag{64}$$

$$A^* = .054\ m^2$$

Using both BL-D260 and BL-D262 values, an area throat ratio can be calculated. BL-D263 then equals 2.96, as shown in Eq. 65. This is a good sanity check as this was also a given value in *MAE 565 Project F22* in the thrust chamber section.

$$Area\ Ratio = \frac{A_C}{A^*} \tag{65}$$

$$Area\ Ratio = \frac{.16\ m^2}{.054\ m^2}$$

$$Area\ Ratio = 2.96$$

Finally, the combustion product gas mass flux exiting the chamber can be calculated by taking the two flow rates going into the chamber, discussed previously, and adding them together per the conservation of mass. BL-D257 equals 466 kg/s and is shown in Eq. 66.

$$\overline{m}_{Exit} = \overline{m}_{LO2} + \overline{m}_{GH2} \tag{66}$$

$$\overline{m}_{Exit} = 353\ \frac{kg}{s} + 113.3\ \frac{kg}{s}$$

$$\overline{m}_{Exit} = 466\ \frac{kg}{s}$$

8. Expansion Nozzle

8.1 Nozzle Area

The expansion nozzle is a clever design at the end of the feed system which adds an extension to the nozzle for the thrust chamber. Additionally, it is the final material section of the feed system (excluding thrust calculations) and will be where a flow between frozen flow and shifting equilibrium flow is seen.

Before going into the different flows, the expansion nozzle exit area and area ratio should be calculated. Using Eq. 63, BL-D267 results in 4.17 m^2 as shown in Eq. 67.

$$A_e = \pi \cdot (\frac{d}{2})^2 \tag{67}$$

$$A_e = \pi \cdot (\frac{2.304\, m}{2})^2$$

$$A_e = 4.17\, m^2$$

Using this exit area and Eq. 65, BL-D268 can be calculated to give an area ratio of 77.3, shown in Eq. 68.

$$Area\ Ratio = \frac{A_e}{A^*} \tag{68}$$

$$Area\ Ratio = \frac{4.17\, m^2}{.054\, m^2}$$

$$Area\ Ratio = 77.3$$

8.2 Frozen Flow

Frozen flow refers to flow where the flow speeds are so fast that no changes in chemical composition can occur during a time where gas does from the throat to the nozzle exit. This then means that the inputs of gamma, MW, R, and Cp are all going to remain constant from the throat to the nozzle exit.

Since these values are taken as constant, the combustion product gas gamma entering the nozzle for GN-D271 can be taken from the given value of gamma entering the combustion chamber. Therefore, it results in 1.17.

Similarly, the molecular weight for GN-D272 will equal 10.13 g/mol, as it is given for the combustion chamber.

Moving to the calculations, the mach at the exit for BL-D273 can be calculated using the non-isentropic area-mach relation shown in Eq. 69. Using the area ratio calculated in Eq. 68, the given isentropic efficiency, and gamma value discussed above, the mach at the exit can be calculated using MATLAB. See section 11.1 in the appendix for matlab script using the function fzero to solve for the value. Mach exit will result in 4.27.

$$\frac{A_e}{A^*} = \frac{1}{M_e} \cdot \left\{ \left(\frac{2}{\gamma+1}\right) \cdot \left[1 + \frac{\gamma-1}{2} \cdot M_e^{\ 2}\right]\right\}^{\frac{1}{2}\cdot\frac{\gamma+1}{\gamma-1}} \cdot \left\{1 + \left(1 - \frac{1}{\eta_N}\right) \cdot \left(\frac{\gamma-1}{2}\right) \cdot M_e^{\ 2}\right\}^{\frac{-\gamma}{\gamma-1}} \tag{69}$$

$$M_e = 4.27$$

Continuing then, the non-isentropic pressure ratio can be calculated for BL-D274 to be .00114, as shown in Eq. 70.

$$\frac{P_e}{P_{t2}} = \left\{1 - \frac{1}{\eta_N} \cdot \left[\frac{(\gamma-1)\cdot M_e^{\ 2}}{2+(\gamma-1)\cdot M_e^{\ 2}}\right]\right\}^{\frac{\gamma}{\gamma-1}} \tag{70}$$

$$\frac{P_e}{P_{t2}} = \left\{1 - \frac{1}{.97} \cdot \left[\frac{(1.17-1)\cdot(4.27)^2}{2+(1.17-1)\cdot(4.27)^2}\right]\right\}^{\frac{1.17}{1.17-1}}$$

$$\frac{P_e}{P_{t2}} = .00114$$

Using the pressure ratio and the known value for Pressusre in the combustion chamber, the pressure at the exit for BL-D275 can be calculated to be 23.4 kPa, as shown in Eq. 71.

$$P_e = \frac{P_e}{P_{t2}} \cdot P_{t2} \tag{71}$$

$$P_e = (.00114) \cdot (20.6 \cdot 10^3\ kPa)$$

$$P_e = 23.4\ kPa$$

For the non-isentropic temperature at the exit of the expansion nozzle in cell BL-D276, using Eq. 72 the value results to 1923 K. Since Tte is equal to Tt2, the given temperature value in the combustion chamber (4904 K) can be used to calculate the temperature at the exit.

$$\frac{T_e}{T_{te}} = \left[1 + \frac{\gamma-1}{2} \cdot {M_e}^2\right]^{-1} \tag{72}$$

$$T_e = \left[1 + \frac{1.17-1}{2} \cdot (4.27)^2\right]^{-1} \cdot 4904\ K$$

$$T_e = 1923\ K$$

Finally, the velocity at the exit can be calculated using Eq. 73 (and subsequently Eq. 74 and 75). Therefore, BL-D277 equals 5803 m/s.

$$V_e = M_e \cdot a_e \tag{73}$$

$$a_e = \sqrt{\gamma \cdot T_e \cdot R} \tag{74}$$

$$R = \frac{\overline{R}}{MW} \tag{75}$$

$$R = \frac{8.3145}{10.13 \cdot 10^{-3}}$$

$$a_e = \sqrt{(1.17) \cdot (1923\ K) \cdot (820.77\ \frac{kg}{mol})}$$

$$V_e = (4.27) \cdot (1359\ \frac{m}{s})$$

$$V_e = 5803\ \frac{m}{s}$$

8.3 Shifting Equilibrium Flow

Shifting equilibrium flow refers to does that are low rough that gas is in chemical equilibrium with pressure and temperature along the nozzle. Along this path, a function for each of those values is used to compute local values such as gamma, MW, R, and Cp.

Due to the values being a function, 6 of the 7 values were given in the spreadsheet, and therefore only the velocity at the exit needed to be calculated. Using Eq. 73, BL-D284 will equal 6042 as shown in Eq. 76.

$$V_e = (4.213) \cdot (1434\ \frac{m}{s}) \tag{73}$$

$$V_e = 6042\ \frac{m}{s}$$

9. Thrust

9.1 Sea Level

Coming to the end of the calculations, the final values can be calculated to total the thrust and specific impulse for the RS-25 engine system. Due to the nature of the flows, the frozen and shifting equilibrium is only limited. Therefore, averaging them together would yield an incorrect result and so picking one of the flows to perform the calculations makes the most sense. Additionally, since shifting equilibrium was mostly given values from the functions, these are likely to have less error than the frozen flow which was all based on calculations from the entire spreadsheet.

Beginning with the sea level calculations, the thrust is broken into two different components. The first is the jet thrust which is shown in Eq. 74 and calculated for BL-D287 to be 2816 kN.

$$T_{jet,\,SL} = \overline{m} \cdot V_e \tag{74}$$

$$T_{jet,\,SL} = (466\ \frac{kg}{s}) \cdot (6042\ \frac{m}{s})$$

$$T_{jet,\,SL} = 2816\ kN$$

The second component for the thrust is the pressure thrust, shown in Eq. 75 and calculated for BL-D288 to be -313kN.

$$T_{pressure,\,SL} = (P_e - P_\infty) \cdot A_e \tag{75}$$

$$T_{pressure,\,SL} = (26.1\ kPa - 101.325\ kPa) \cdot 4.17\ m^2$$

$$T_{pressure,\,SL} = -313\ kN$$

Adding these two values together then yield the nominal thrust at sea level for BL-D290 to be 2502 kN. With a given nozzle divergence thrust loss of .8%, the divergence corrected thrust for sea level is calculated to be 2482 kN for cell BL-D291, as shown in Eq. 76.

$$T_{Divergence\ Corrected,\,SL} = T_{Nominal,SL} - \left[T_{Nominal,SL} \cdot (Divergence\ \% \cdot 100)\right] \tag{76}$$

$$T_{Divergence\ Corrected,\,SL} = 2502\ kN \cdot [2502\ kN \cdot (.8\% \cdot 100)]$$

$$T_{Divergence\ Corrected,\,SL} = 2482\ kN$$

Using the calculated thrust values, the coefficient of thrust can now be calculated. Starting with the actual coefficient of thrust at sea level, see Eq. 77, for BL-D293 which results in 2.24.

$$C_{T\ Actual,SL} = \frac{T_{Divergence\ Corrected,\ SL}}{P_{t2} \cdot A^*} \tag{77}$$

$$C_{T\ Actual,SL} = \frac{2482\ kN}{(20.6 \cdot 10^3) \cdot (.054\ m^2)}$$

$$C_{T\ Actual,SL} = 2.24$$

From here, an ideal coefficient of thrust at sea level can be calculated for BL-D294 to be 2.54. Using Eq. 77, the calculation can be seen in Eq. 78 with only ideal values.

$$C_{T\ Ideal,SL} = \frac{T_{Jet\ Thrust,\ SL}}{P_{t2} \cdot A^*} \tag{78}$$

$$C_{T\ Ideal,SL} = \frac{2816\ kN}{(20.6 \cdot 10^3) \cdot (.054\ m^2)}$$

$$C_{T\ Ideal,SL} = 2.54$$

Using the resulting coefficient of thrust values, dividing them will yield nozzle coefficient of thrust efficiency at sea level. For BL-D295, this results in .882 as shown in Eq. 79.

$$C_{T\ Efficency,SL} = \frac{C_{T\ Actual,SL}}{C_{T\ Ideal,SL}} \tag{79}$$

$$C_{T\ Efficency,SL} = \frac{2.24}{2.54}$$

$$C_{T\ Efficency,SL} = .882$$

Finally, the specific impulse at sea level can be calculated using Eq. 80 and 81. This results in BL-D297 being 542.96 s.

$$I_{sp,SL} = \frac{V_{eq}}{g_{Earth}} \tag{80}$$

$$V_{eq} = \frac{T_{Divergence\ Corrected,SL}}{\overline{m}} \tag{81}$$

$$V_{eq} = \frac{2482\ kN}{466\ \frac{kg}{s}}$$

$$V_{eq} = 5.326\ \frac{km}{s}$$

$$I_{sp,SL} = \frac{5.262\ \frac{km}{s}}{9.81\ \frac{m}{s^2}}$$

$$I_{sp,SL} = 542.96\ s$$

9.2 VACUUM

Transitioning from Earths atmosphere to the vacuum of space, the equations change slightly for the calculations of the previous section. The first calculator for jet thrust will be equivalent, and therefore, BL-D299 will be 2816 kN.

The difference begins with the pressure thrust, using Eq. 75 but seen in Eq. 82 as BL-D300 is 109 kN.

$$T_{pressure,\ Vac} = (P_e - P_\infty) \cdot A_e \tag{82}$$

$$T_{pressure,\ Vac} = (26.1\ kPa - 0\ kPa) \cdot 4.17\ m^2$$

$$T_{pressure,\ Vac} = 109\ kN$$

Therefore, the resulting normal thrust in a vacuum will equate to 2925 kN for BL-D301. With a given nozzle divergence thrust loss of .8%, the divergence corrected thrust in a vacuum is calculated to be 2901 kN for cell BL-D303, using Eq. 76 but shown in Eq. 83.

$$T_{Divergence\ Corrected,\ Vac} = 2925\ kN \cdot [2925\ kN \cdot (.8\% \cdot 100)] \tag{83}$$

$$T_{Divergence\ Corrected,\ Vac} = 2901\ kN$$

Continuing to the coefficient of thrusts, the actual coefficient of thrust will be calculated using Eq. 77, but shown in Eq. 84. Therefore, BL-D305 will equal 2.61.

$$C_{T\,Actual,Vac} = \frac{T_{Divergence\,Corrected,\,Vac}}{P_{t2} \cdot A^{*}} \tag{84}$$

$$C_{T\,Actual,Vac} = \frac{2901\,kN}{(20.6 \cdot 10^{3}) \cdot (.054\,m^{2})}$$

$$C_{T\,Actual,Vac} = 2.61$$

A change comes again when discussing the ideal coefficient of thrust. This is a calculated value using Eq. 85, and BL-D306 becomes 2.84.

$$C_{T\,Ideal,Vac} = \gamma \cdot \left\{ \left(\frac{2}{\gamma-1}\right) \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \right\}^{\frac{1}{2}} \tag{85}$$

$$C_{T\,Ideal,Vac} = 1.109 \cdot \left\{ \left(\frac{2}{1.109-1}\right) \cdot \left(\frac{2}{1.109+1}\right)^{\frac{1.109+1}{1.109-1}} \right\}^{\frac{1}{2}}$$

$$C_{T\,Ideal,Vac} = 2.84$$

Therefore, calculating the coefficient of thrust efficiency will be using Eq. 79, and shown in Eq. 86

$$C_{T\,Efficency,\,Vac} = \frac{2.61}{2.84} \tag{86}$$

$$C_{T\,Efficency,\,Vac} = .919$$

Finally, the specific impulse in a vacuum can be calculated using Eq. 80 but shown in Eq 87. This results in BL-D309 being 639.7 s.

$$I_{sp,Vac} = \frac{6.225\,\frac{km}{s}}{9.81\,\frac{m}{s^{2}}} \tag{87}$$

$$I_{sp,Vac} = 639.7\,s$$

10. Final Results

Reviewing the results presented in this report, it is clear that there is potential for errors in a variety of areas. For instance, some equations being used are for non-isentropic but isentropic relations and assumptions were being in a variety of areas.

Additionally, using the shifting equilibrium values is a far limit and thus the thrust values seen are most likely a limit value. With these values, the specific impulse is rather high compared to a few values that were previously looked up.

When referencing the Thrust section, the values follow the correct trends. The ideal values are higher than the actual, resulting in the efficiency being less than 1, and the vacuum values being higher than the sea level ones due to the pressure losses.

In summary, this report details the RS-25 propellant feed system and all its subsystems such as the LPFTP/LPOTP, HPFTP/HPOTP, injectors, thrust chamber, and expansion nozzle. The calculations and excel cells can be found and referenced in *Hutchens_Chandler_565_Spreadsheet*.

11. Appendix

11.1 Mach Exit Matlab Code

```
%% Me
guess = 2 ;
gamma = 1.17 ;
AeAs = 77.3 ;
eta = .97 ;

Me2 = fzero(@(Me) (1/Me) * ((2/(gamma+1)*(1+(gamma-1)/2*Me^2))^((gamma+1)/(gamma-1)/2) *
(1+(1-1/eta)*(gamma-1)/2*Me^2)^(-gamma/(gamma-1)))- AeAs, guess)  ;
```