

Gaussian Processes for Systematic Mitigation

A Bayesian Method to Mitigate the Effects of Unmodelled Time-Varying
Systematics for Global 21-cm Cosmology Experiments

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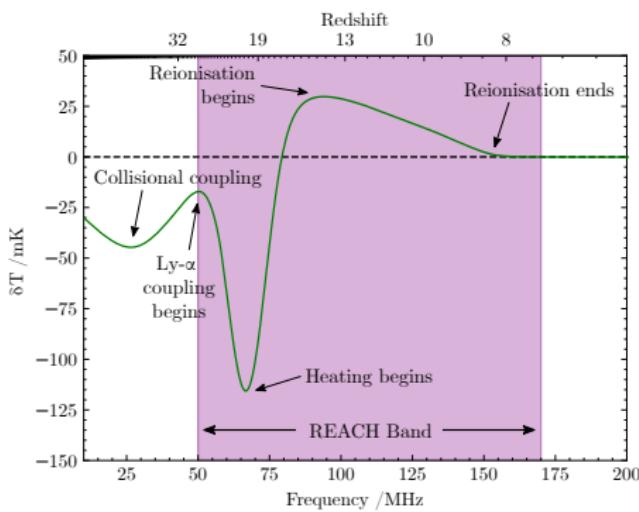


Table of Contents

- 1 Introduction
- 2 Systematics in the REACH System
- 3 Gaussian Processes
- 4 Results
- 5 Further Work



21-cm Cosmology



- Aim of 21-cm cosmology experiments is to measure emission from neutral hydrogen
- Neutral hydrogen has a hyperfine transition at $\nu = 1420$ MHz
- Intensity of 21-cm signal relative to the CMB at high redshift informs us about physics of the Cosmic Dawn



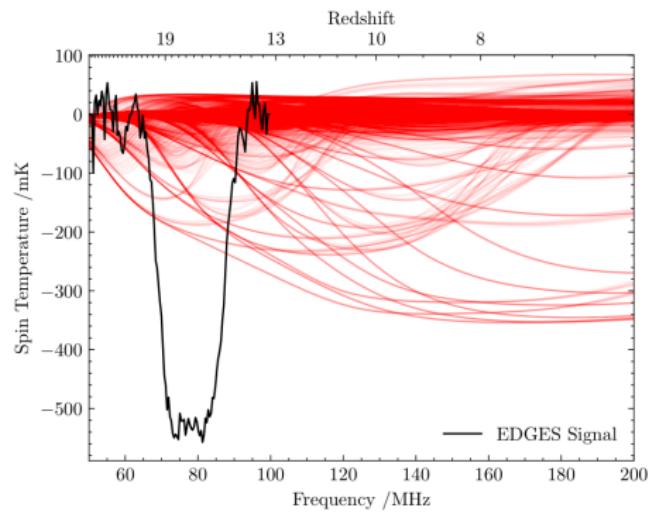
EDGES (Experiment to Detect the Global EoR Signature)



- EDGES is a low frequency radio experiment to detect the global sky-averaged 21-cm signal
- Collaboration between ASU and MIT
- Located in the Murchison Radio-astronomy Observatory in Western Australia



EDGES (Experiment to Detect the Global EoR Signature)



- Claimed detection of 21-cm signal at 78 MHz (Bowman+18)
- Unusually deep signal, requiring exotic physics
- Concerns that there is a residual systematic in the data (Hills+18, Sims and Pober 2020)



REACH (Radio Experiment for the Analysis of Cosmic Hydrogen)

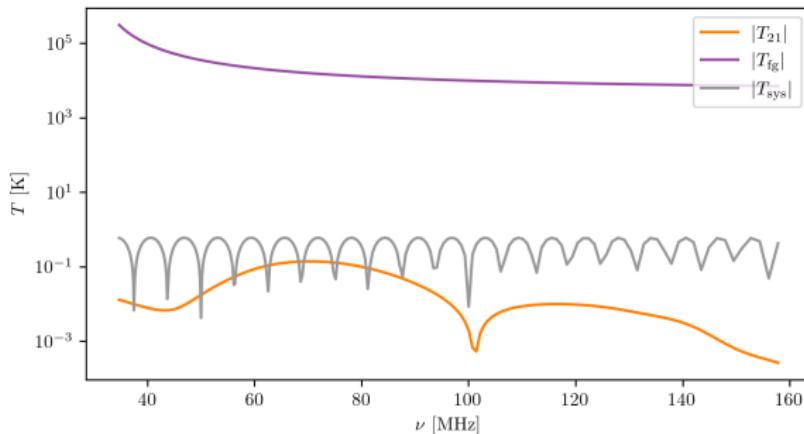


Credit: Saurabh Pegwal

- REACH is a global signal experiment designed to verify the EDGES detection
- Cambridge University, Stellenbosch University and others
- Located in the Karoo Desert on the SARAO facility in South Africa



Global Experiment Challenges



Credit: Harry Bevins

- **Galactic foregrounds**
Smooth synchrotron emission
 $\sim 10^5$ K
- **Instrumental systematics**
Generally sinusoidal
 $\sim 10^0$ K
- **21-cm Signal**
Not spectrally smooth
 $\sim 10^{-2}$ K



Time Varying Systematics in the REACH System



- Some systematics are expected to be static, others vary with time
 - Reflections from the soil vary with rainfall (Bevins+21, 22)
 - Impedances in the system are temperature dependent
 - Improperly modelled beams can cause systematics which vary with the galactic foreground



Modelling Time Varying Systematics

- Model systematic as a sinusoid whose amplitude is modulated by the power of the galactic foreground

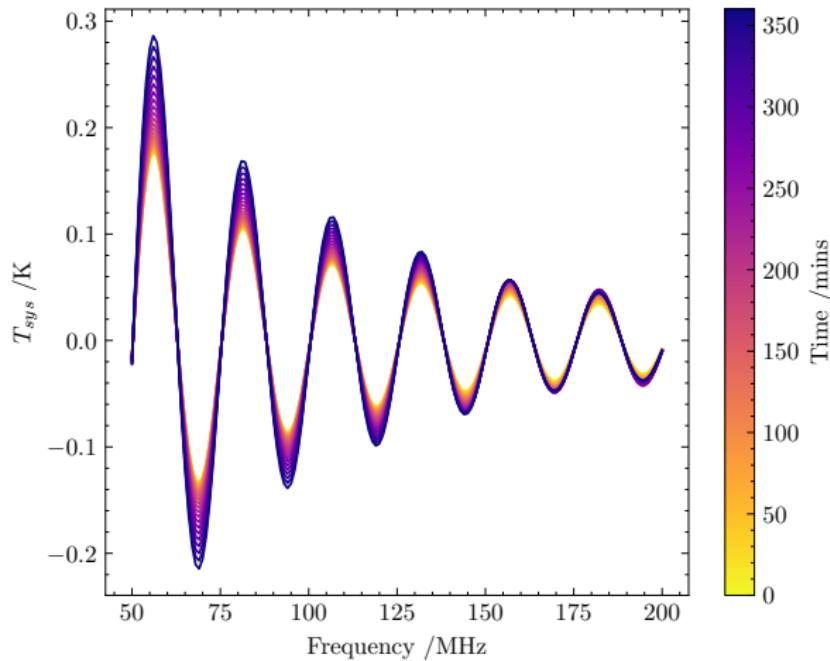
$$T_{\text{sys}}(\nu) = A_{\text{sys}} \left(\frac{\nu}{\nu_{0,\text{sys}}} \right)^{-\alpha_{\text{sys}}} \sin \left(\frac{2\pi\nu}{P_{\text{sys}}} + \phi_{\text{sys}} \right) \quad (1)$$

- Systematic amplitude is varied as

$$A_{\text{sys}}(t_j) = A_{\text{sys}}(t_0) \cdot \frac{T_{\text{fg}}(\nu = \nu_0, t = t_j)}{T_{\text{fg}}(\nu = \nu_0, t = t_0)} \quad (2)$$



Modelling Time Varying Systematics



Standard REACH Pipeline

- Fully Bayesian forward model of antenna beam, galactic foregrounds and 21-cm signal (Anstey+21, 22)
- Uses a Gaussian likelihood with uncorrelated noise, σ_0

$$\log \mathcal{L} = \sum_i -\frac{1}{2} \log(2\pi\sigma_0^2) - \frac{1}{2} \left(\frac{T_{\text{data}}(\nu_i) - (T_{\text{fg}}(\nu_i) + T_{21}(\nu_i) + T_{\text{CMB}})}{\sigma_0} \right)^2 \quad (3)$$

- Will be referred to as the “Standard Pipeline”



Gaussian Processes

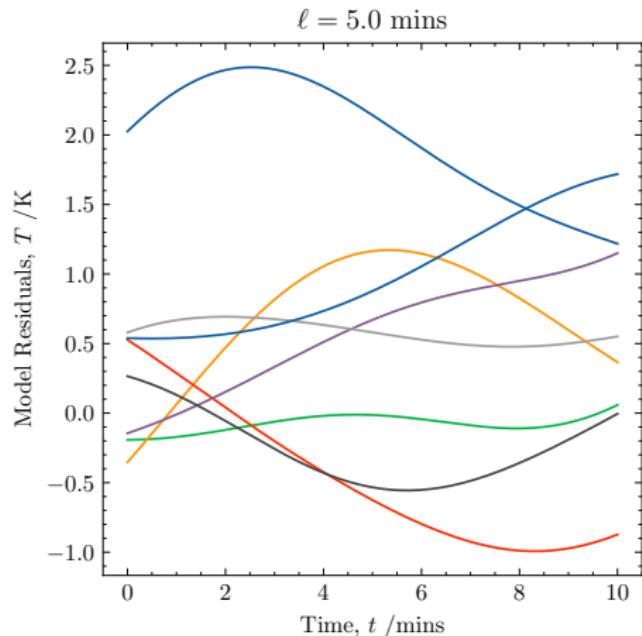
- Define a GP as a collection of random variables which have consistent joint Gaussian distributions (Rasmussen 2004)
- Distribution is defined by the covariance matrix, or covariance function

$$\mathbf{C}_{ij} = K(t_i, t_j) \quad (4)$$

- K is called the “kernel” – form can be chosen freely



Squared Exponential Kernel



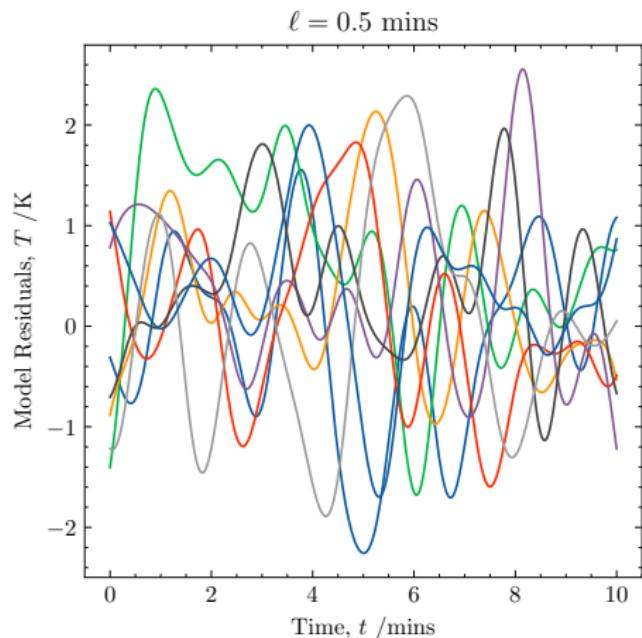
- In this work I use a squared exponential kernel with a white noise term

$$\mathbf{C}_{ij} = \sigma_{0,\text{GP}}^2 + \sigma_{\text{SE}}^2 \exp\left(-\frac{|t_i - t_j|^2}{2\ell^2}\right) \quad (5)$$

- Uncorrelated noise term, scale factor of the kernel and the covariance length
- Define smooth functions



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Gaussian Processes in the REACH Pipeline

- Hereafter called the “Gaussian Process pipeline” (Kirkham+24)

$$\mathcal{L}(\theta) = \frac{1}{\sqrt{(2\pi)^n |\mathbf{C}|}} \exp \left(-\frac{1}{2} (\mathbf{D} - \mathbf{M}(\theta))^T \mathbf{C}^{-1} (\mathbf{D} - \mathbf{M}(\theta)) \right) \quad (6)$$

- **D** is the data vector, **M** is the model (foreground, beam model, signal model)
- Gaussian process covariance matrix, **C**, fits covariance of model residuals between time bins



Simulating Data

- We model the global signal as a Gaussian

$$T_{\text{sg}}(\nu) = -A_{21} \exp\left(-\frac{(\nu - \nu_{0,21})^2}{2\sigma_{21}^2}\right) \quad (7)$$

- Simulated data produced with signal parameters $A_{21} = 0.155$ K, $\sigma_{21} = 15$ MHz and $\nu_{0,21} = 85$ MHz
- 21-cm signal is fitted for using Gaussian model

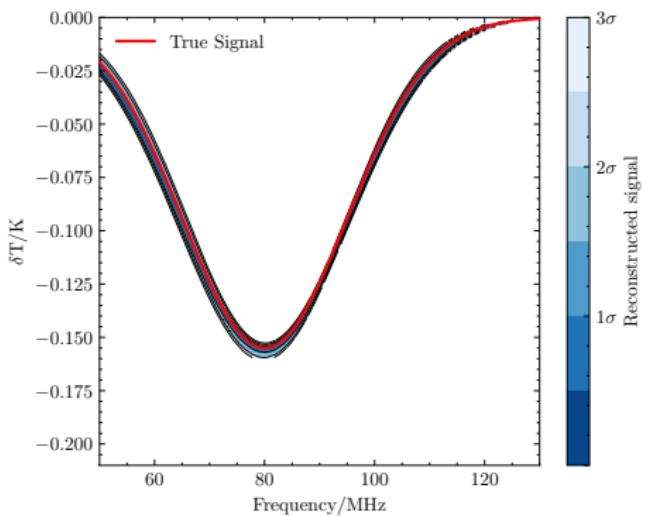


Varying Systematic parameters

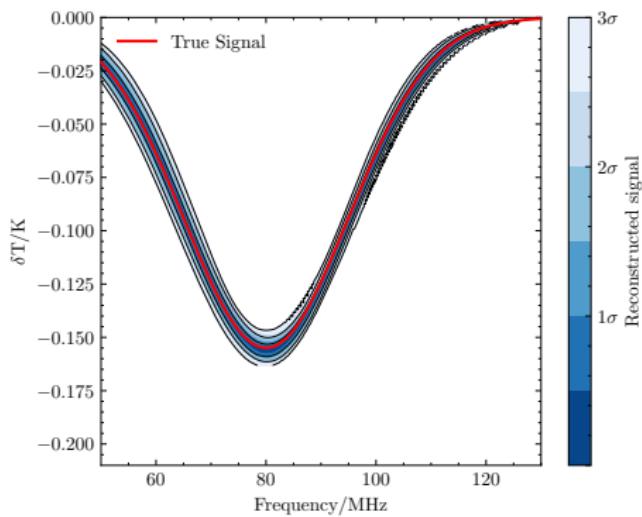
- Will test both pipelines with a range of systematics
- Data generated for 24 time bins of length 15 minutes – 6 hours of observation in total
- Posteriors and evidences were then calculated using POLYCHORD



Comparison of Signal Posteriors - No Systematic



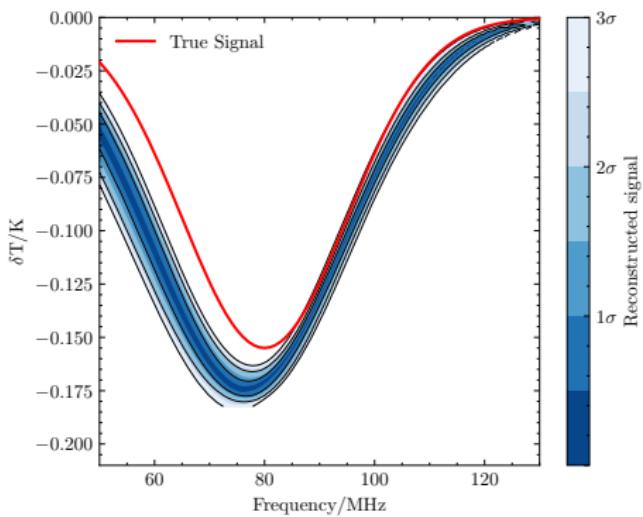
Standard pipeline



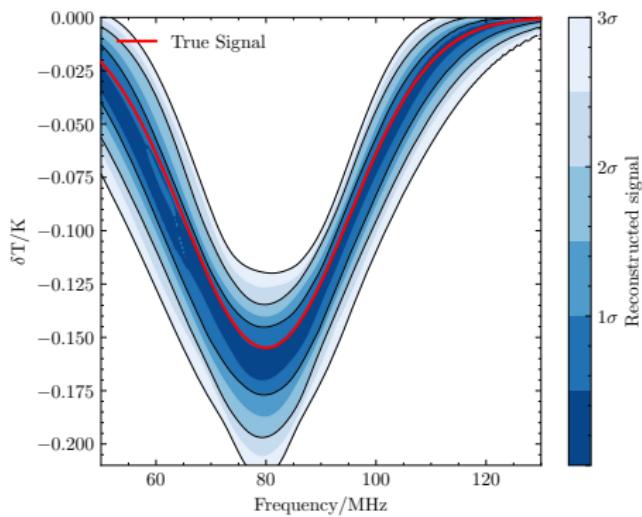
Gaussian Process pipeline



Comparison of Signal Posteriors - With Systematic



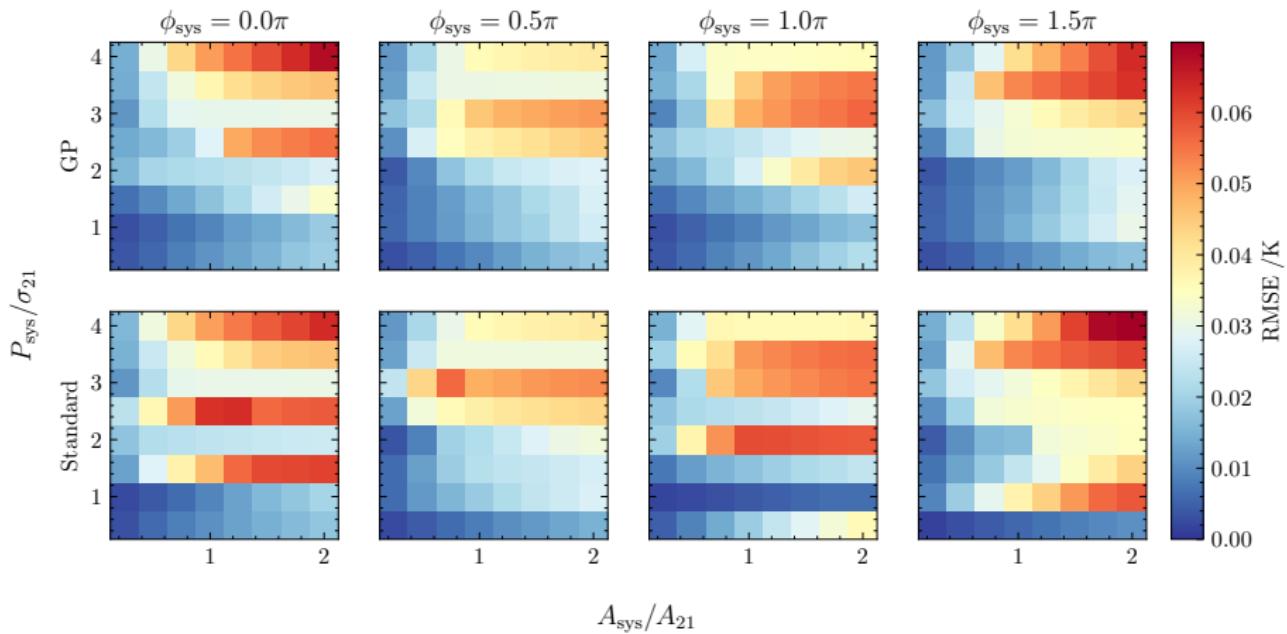
Standard pipeline



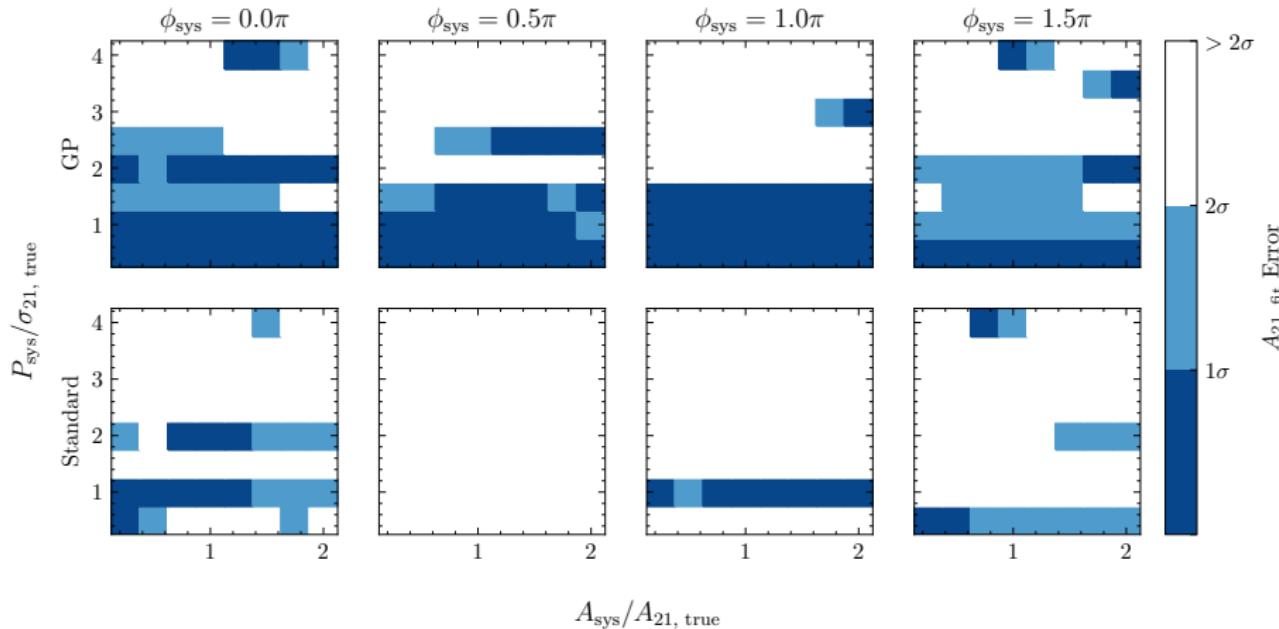
Gaussian Process pipeline



RMSE



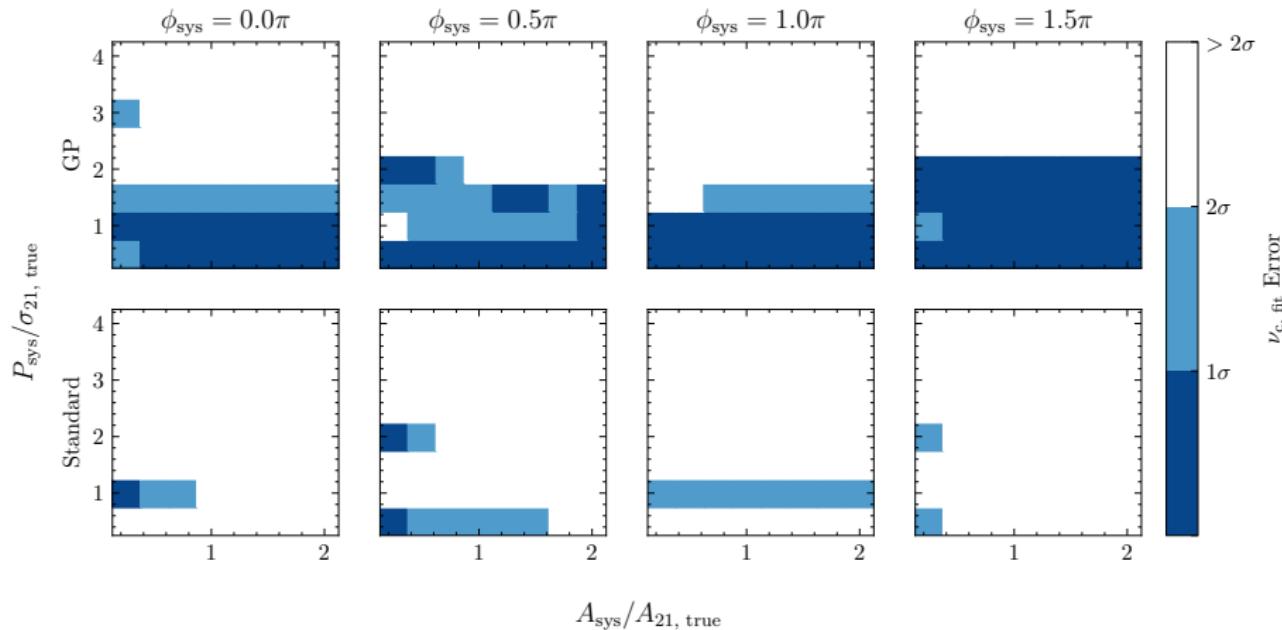
Fitted Signal Amplitude error



$A_{\text{sys}}/A_{21, \text{true}}$



Fitted Centre Frequency error

 $A_{\text{sys}}/A_{21}, \text{true}$

Gaussian Process Regression

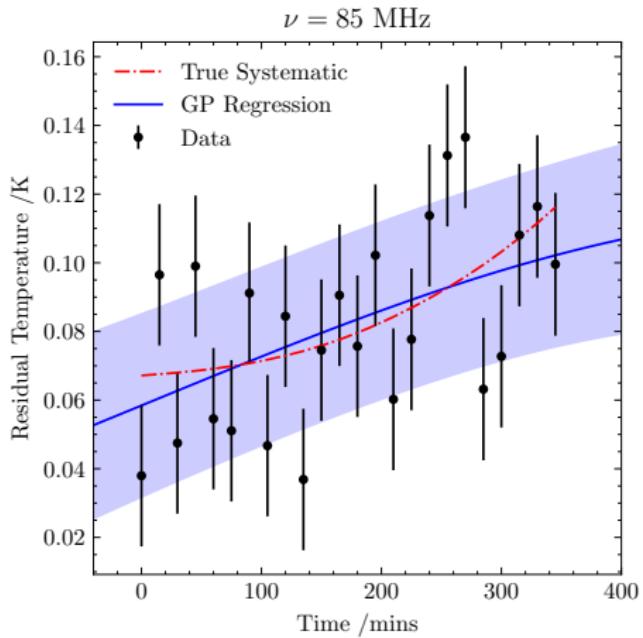
- Can also use Gaussian Processes for regression
- Mean function of the Gaussian Process posterior given by

$$\mu(\mathbf{t}_{\text{pred}}) = K(\mathbf{t}_{\text{pred}}, \mathbf{t}_{\text{data}})K(\mathbf{t}_{\text{data}}, \mathbf{t}_{\text{data}})^{-1}\mathbf{T}_{\text{data}}, \quad (8)$$

- Can be used to predict temperature, \mathbf{T}_{pred} , of model residuals at some time, \mathbf{t}_{pred}



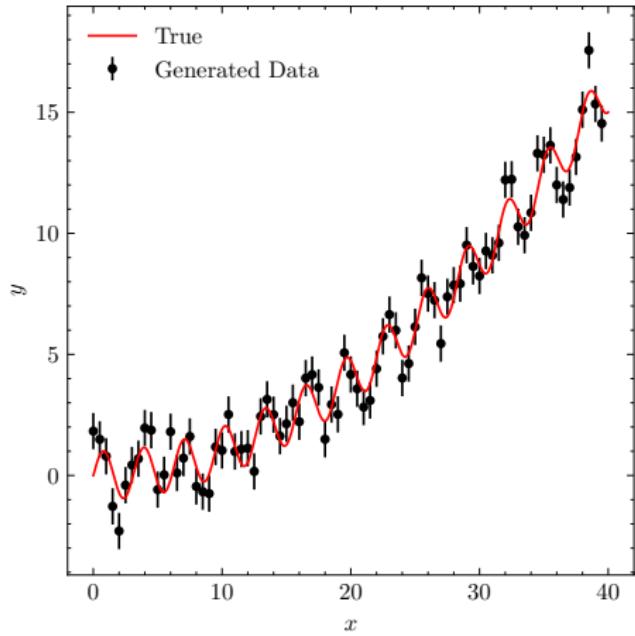
Gaussian Process Regression



- Use weighted mean Gaussian Process hyperparameters to get a smooth fit to model residuals
- Could possibly be used to inform future time-varying systematic models



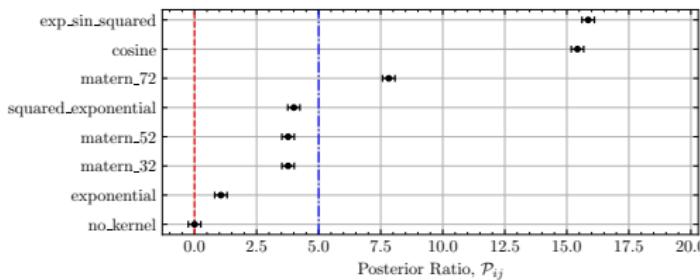
Automatic Kernel Selection



- Kernel choice is arbitrary so can use Bayes factor to inform us (Hee+15, Kroupa+24)
- Basic test with a quadratic curve with a sinusoidal residual
- Fit for quadratic but not sinusoid



Automatic Kernel Selection



- Uses PolyChord to sample over all kernels using a choice parameter, c
- Uses the posterior ratio of c as a proxy for Bayes Factor

$$\mathcal{P}_{ij} = \log \frac{\Pr(c = j | \mathbf{D}, \mathbf{M})}{\Pr(c = i | \mathbf{D}, \mathbf{M})} \quad (9)$$



Summary

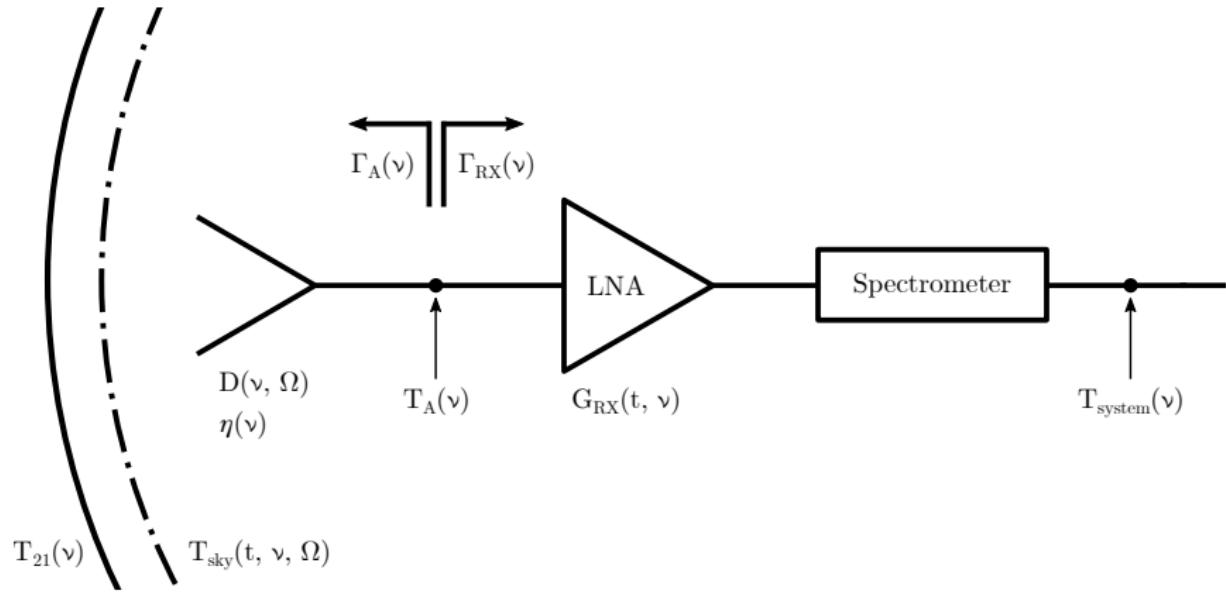
- Using Gaussian Processes to account for time correlated residuals improves fitting
- General method – no systematic model required
- Regression can inform future models of systematics
- Automatic Kernel Selection can use the data to select the most appropriate kernel



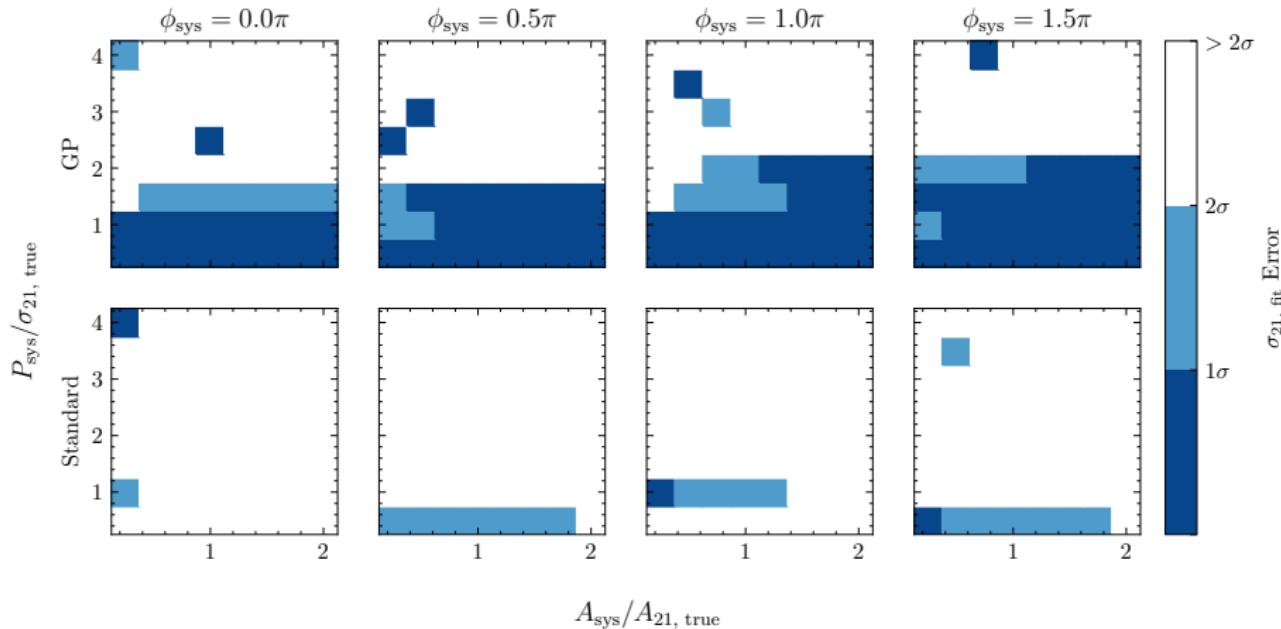
For more information



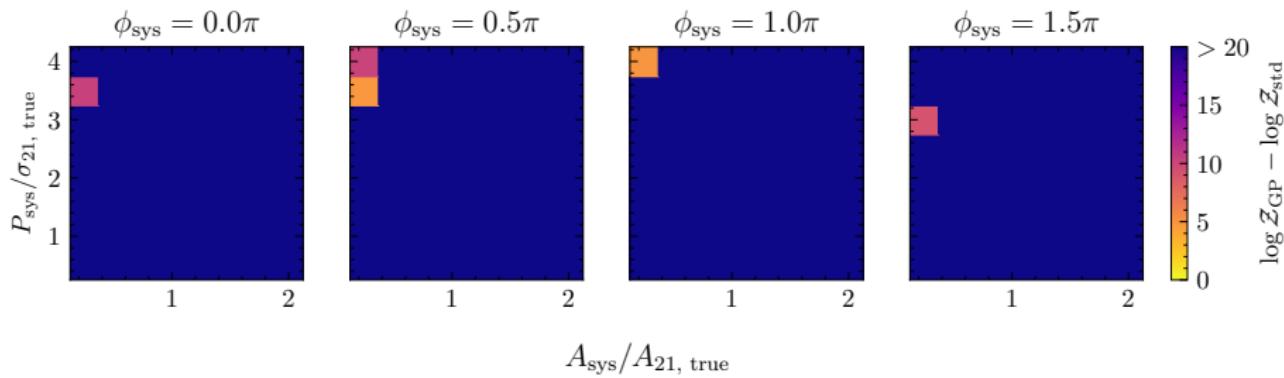
Antenna and Receiver System



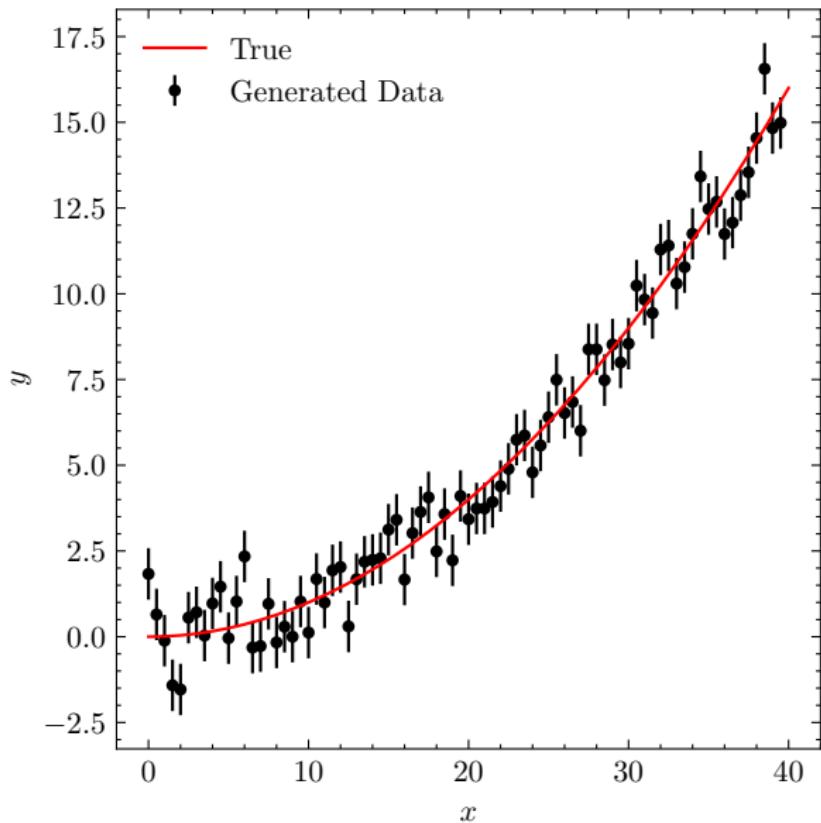
Signal Width



Bayes Factor



Automatic Kernel Selection - No Residual



Automatic Kernel Selection - No Residual

