Guia 6 . Episodio 20, 21 , 22

1 A ✓? B C

2

3

4 A ✓? B C

5

6 A B C

7

8

9 ✓

6.1) a) Hallar DVS
Determinar bases ortonormales de subespacios fundamentales
Determinar matrices de proyección.

A) $A = \begin{pmatrix} 3 & 5 \\ -1 & 1 \\ 5 & 3 \end{pmatrix} \rightarrow rg(A) = 2$

Autovalores de $A^T . A = \begin{pmatrix} 35 & 29 \\ 29 & 35 \end{pmatrix}$

$\begin{vmatrix} 35-x & 29 \\ 29 & 35-x \end{vmatrix} \Rightarrow (35-x)(35-x) - 29^2 = \cancel{1225 - 35x}$

$35^2 - 2.35.x + x^2 - 29^2$

$x^2 - 70x + 384 = 0$

$\hookrightarrow x_1 = 6, \ x_2 = 64$

Valores singulares: $\sigma_1 = 8 \ , \ \sigma_2 = \sqrt{6}$

Autoespacios de $A^T . A$

$S_{\lambda=64}) \ \begin{pmatrix} -29 & 29 \\ 29 & -29 \end{pmatrix} \rightarrow -x_1 + x_2 = 0 \rightarrow x_2 = x_1$

$gen\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$

$S_{\lambda=6}) \ \begin{pmatrix} 29 & 29 \\ 29 & 29 \end{pmatrix} \rightarrow x_1 + x_2 = 0 \rightarrow -x_1 = x_2 \quad gen\left\{\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right\}$

Formamos un BON de $\mathbb{R}^2$ $\quad \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\rangle = -1 + 1 = 0 \ \checkmark$

$\left\{ \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \right\} \leftarrow$ BON $\mathbb{R}^2$.

$\rightarrow V = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

Construimos U: $u_1 = \dfrac{A . v_1}{\sigma_1} \quad \dfrac{\begin{pmatrix} 3 & 5 \\ -1 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}}{8} = \cancel{\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = u_1}$

$\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = u_1$

$$u_2 = \frac{A \cdot v_2}{\sigma_2} = \dots \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{pmatrix}$$

cuidado ¿traspuesta?

Busco a mano u_3: $u_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$A = U \cdot \Sigma \cdot V^T \qquad A = \underbrace{\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{3} & 1 \\ 1/\sqrt{2} & -1/\sqrt{3} & 0 \end{pmatrix}}_{3\times 3} \underbrace{\begin{pmatrix} 8 & 0 \\ 0 & \sqrt{6} \\ 0 & 0 \end{pmatrix}}_{3\times 2} \underbrace{\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}}_{2\times 2}$$

3x2

Col(A)   Nul(A^T)

Fil(A)

$\emptyset$ → Nul(A)

DVS reducido: $rg(A) = 2$

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 8 & 0 \\ 0 & \sqrt{6} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Único Nul es trivial, $\{0_{\mathbb{R}^2}\}$

Matrices proyección (P_S)

- $P_{Fil(A)} = V_r \cdot V_r^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- $P_{Col(A)} = U_r \cdot U_r^T = \underbrace{\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix}}_{3\times 2} \underbrace{\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix}}_{2\times 3} = \begin{pmatrix} 5/6 & 1/3 & 1/6 \\ 1/3 & 1/3 & -1/3 \\ 1/6 & -1/3 & 5/6 \end{pmatrix}$

$$\frac{1}{6} \begin{pmatrix} 5 & 2 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix}$$

- $P_{Nul(A)} = I - P_{Fil(A)} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

- $P_{Nul(A^T)} = I - P_{Col(A)} = \begin{pmatrix} -1 & 2 & 1 \\ 1 & -4 & -2 \\ 1 & -2 & -1 \end{pmatrix} \frac{1}{6}$

6.4) A) Teniendo en cuenta solución de 6.1) A).

$A^+ = V_r \cdot \Sigma_r^{-1} \cdot U_r^T$

$A_x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$A^+ = \underbrace{\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 8 & 0 \\ 0 & \sqrt{6} \end{pmatrix}}_{\begin{pmatrix} 8 & -\sqrt{6} \\ 8 & \sqrt{6} \end{pmatrix}} \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix}$$

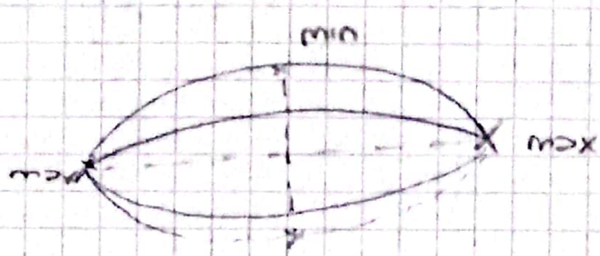
$$= \begin{pmatrix} 3 & -1 & 5 \\ 5 & 1 & 3 \end{pmatrix}$$

$$\hat{x} = A^+ \cdot b \rightarrow \begin{pmatrix} 3 & -1 & 5 \\ 5 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \hat{x} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$$

HOJA N

FECHA

6.6)



si esta relleno → el minimo es $D_{\mathbb{R}^n}$.

NOTA

6.8)

B) $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix} \longrightarrow A^T \cdot A = \begin{pmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{pmatrix}$

$$\begin{vmatrix} 13-x & 12 & 2 \\ 12 & 13-x & -2 \\ 2 & -2 & 8-x \end{vmatrix} = (13-x)^2(8-x) + 12(-2)2 + 2 \cdot 12 \cdot (-2) - (13-x) \cdot 4 - (-2)(-2)(13-x) - (8-x) \cdot 12 \cdot 12$$

$$(13-x)^2(8-x) - 96 - 52 + 4x - 52 + 4x - 1152 + 144x = 0$$

$$(13^2 - 26x + x^2)(8-x) + 152x - 1352 = 0$$

$$\cancel{1352} - 208x + 8x^2 - 169x + 26x^2 - x^3 + 152x \cancel{-1352} = 0$$

$$-x^3 + 34x^2 - 225x = 0 \longrightarrow \cancel{17 + \sqrt{514}},\ \cancel{17 - \sqrt{514}},\ \cancel{0}$$

$\cancel{1) \quad A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}}$ $\quad \cancel{\begin{vmatrix} 3-x & 2 \\ 2 & 3-x \end{vmatrix} = (3-x)^2 - 4 = 0}$

$\cancel{9 - 6x + x^2 - 4 = 0}$

$\cancel{x^2 - 6x + 5 = 0 \to x_1 = 5, \ x_2 = 1}$

$$-x^3 + 34x^2 - 225x = 0 \longrightarrow x = 25, \quad x = 9, \quad x = 0.$$

$\cancel{x(-x^2 + 34x - 225)}$

$\cancel{x(x(-x+34) - 225)}$

$\lambda_1 = 25)$ $\begin{pmatrix} -12 & 12 & 2 \\ 12 & -12 & -2 \\ 2 & -2 & -17 \end{pmatrix} \begin{matrix} /2 \\ +F_1 \\ \cdot 3 \end{matrix} \Bigg] \begin{pmatrix} -6 & 6 & 1 \\ 6 & -6 & -51 \\ 0 & 0 & 0 \end{pmatrix} + F_1 \Bigg] \begin{pmatrix} -6 & 6 & 1 \\ 0 & 0 & -50 \\ 0 & 0 & 0 \end{pmatrix}$

$$\left.\begin{matrix} -x_1 + x_2 = 0 \to x_1 = x_2 \\ -x_3 = 0 \end{matrix}\right\} \text{gen}\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right\}$$

$\lambda_2 = 9)$ $\cancel{\begin{pmatrix} 4 & 3 & -7 \\ 3 & 4 & -11 \\ -7 & -11 & -1 \end{pmatrix} \begin{matrix} \cdot 4 - 3F_1 \\ \cdot 4 + 7F_1 \end{matrix} \Bigg] \begin{pmatrix} 4 & 3 & -7 \\ 0 & 0 & -23 \\ 0 & -12 & -53 \end{pmatrix} \to x_1 = 0, \ x_2 = 0, \ x_3 = 0}$

$\begin{pmatrix} 4 & 12 & 2 \\ 12 & 4 & -2 \\ 2 & -2 & -1 \end{pmatrix} \begin{matrix} /2 \\ -3F_1 \\ -\frac{1}{2}F_1 \end{matrix} \Bigg] \begin{pmatrix} 2 & 6 & 1 \\ 0 & -32 & -8 \\ 0 & -8 & -2 \end{pmatrix} \begin{matrix} /8 \\ -\frac{1}{4}F_2 \end{matrix} \Bigg] \begin{pmatrix} 2 & 6 & 1 \\ 0 & -4 & -1 \\ 0 & 0 & 0 \end{pmatrix} + F_2 \Bigg] \begin{pmatrix} 2 & 2 & 0 \\ 0 & -4 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$S_{\lambda_3} = 0)$ $\begin{pmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{pmatrix} \begin{matrix} \\ \times 13 - 12 \cdot F_1 \\ \times 13 - 2 \cdot F_1 \end{matrix} \sim \begin{pmatrix} 13 & 12 & 2 \\ 0 & 25 & -50 \\ 0 & -50 & 100 \end{pmatrix} \begin{matrix} \\ /25 \\ +2 \cdot F_1 \end{matrix} \sim \underbrace{\begin{pmatrix} 13 & 12 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}}$

$\begin{pmatrix} 13 & 13 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

$\cancel{13x_1 + 12x_2 + 2x_3 = 0}$

$\cancel{x_2 - 2x_3 = 0} \rightarrow$

$x_1 + x_2 = 0 \longrightarrow x_1 = -x_2$

$x_2 - 2x_3 = 0 \longrightarrow x_2 = 2x_3 \Rightarrow \frac{1}{2} x_2 = x_3$

$\text{gen}\left\{\begin{pmatrix} 1 \\ 1 \\ 1/2 \end{pmatrix}\right\} \longrightarrow \text{gen}\left\{\cancel{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}}\right\}$

v_3

$x_1 + 2x_3 = 0$
$x_2 - 2x_3 = 0$

$\left.\begin{matrix} x_1 = -2x_3 \\ x_2 = 2x_3 \end{matrix}\right\} \text{gen}\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$

$\circledast$ $\left.\begin{matrix} x_1 + x_2 = 0 \longrightarrow x_1 = -x_2 \\ 4x_2 + x_3 = 0 \longrightarrow x_3 = -4x_2 \end{matrix}\right\} \text{gen}\left\{\begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix}\right\}$

v_2

¿Son ortogonales? $\langle v_1, v_2 \rangle = 1(-1) + 1 \cdot 1 + 0 = 0$ ✓

$\langle v_1, v_3 \rangle = 1 \cdot -2 + 1 \cdot 2 = 0$ ✓

$\langle v_2, v_3 \rangle = \cancel{-1 \cdot 2 + 2 \cdot 1 - 4 = -4}$ ✗

$+1 \cdot -2 + 1 \cdot 2 - 4 = 0$ ✓

Base ortogonal de $\mathbb{R}^3$: $\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}\right\}$

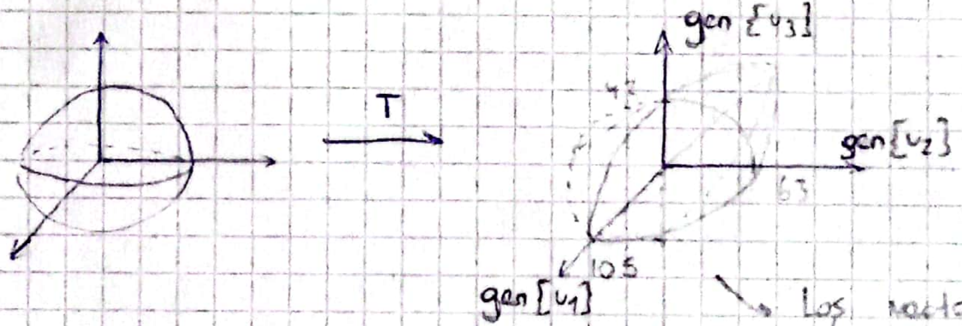
S_1 - Esfera

$$6.9)\quad A = 5\begin{pmatrix}-1\\2\\2\end{pmatrix}\overset{\sqrt{9}=3}{}\begin{pmatrix}2 & 6 & -3\end{pmatrix}\overset{\sqrt{49}=7}{} + 3\begin{pmatrix}2\\-1\\2\end{pmatrix}\overset{3}{}\begin{pmatrix}6 & 3 & 2\end{pmatrix}\overset{7}{}$$

$$+\ 2\begin{pmatrix}2\\2\\-1\end{pmatrix}_{3}\begin{pmatrix}3 & 2 & 6\end{pmatrix}_{7}$$

$$A = \underbrace{5\cdot 3\cdot 7}_{105}\underbrace{\begin{pmatrix}-1/3\\2/3\\2/3\end{pmatrix}}_{u_1}\begin{pmatrix}\frac{2}{7} & \frac{6}{7} & -\frac{3}{7}\end{pmatrix} + \underbrace{3\cdot 3\cdot 7}_{63}\underbrace{\begin{pmatrix}2/3\\-1/3\\2/3\end{pmatrix}}_{u_2}\begin{pmatrix}\frac{6}{7} & \frac{3}{7} & \frac{2}{7}\end{pmatrix} + \underbrace{2\cdot 3\cdot 7}_{42}\underbrace{\begin{pmatrix}2/3\\2/3\\-1/3\end{pmatrix}}_{u_3}\begin{pmatrix}\frac{3}{7} & \frac{2}{7} & \frac{6}{7}\end{pmatrix}$$

$$A = \begin{pmatrix}-1/3 & 2/3 & 2/3\\ 2/3 & -1/3 & 2/3\\ 2/3 & 2/3 & -1/3\end{pmatrix}\begin{pmatrix}105 & 0 & 0\\ 0 & 63 & 0\\ 0 & 0 & 42\end{pmatrix}\begin{pmatrix}2/7 & 6/7 & -3/7\\ 6/7 & 3/7 & 2/7\\ 3/7 & 2/7 & 6/7\end{pmatrix}$$



La imagen de la esfera unitaria es un elipsoide → ∈ Col(A)
LiDim 3 → Dim 3

→ Los vectores directores para dibujar la elipse son los de U.

$T(S_1) \neq \{$

$S_1\ \|x\| = 1$

$w_1 = \alpha\, \text{gen}\{u_1\}\ /\ w_2 = \beta\, \text{gen}\{u_2\}\ /\ w_3 = \gamma\, \text{gen}\{u_3\}$

$S_1 \ast \|x\| = 1 \qquad \alpha, \beta, \gamma \in \mathbb{R}$

Los extremos (vertices) del ~~la~~ elipsoide (o elipse, o segmento) son el valor de los valores singulares, es decir la raiz de los autovalores.

- Cuando no tengo DVS, debo formarla para saber la dirección de los ejes ~~de~~ ortogonales del elipsoide

$$T(S_1) = \left\{ z = w_1 + w_2 + w_3,\ AW,\ \cancel{w_1 \le 105},\ \cancel{w_2 \le 63},\ \cancel{w_3 \le 42} \right\}$$

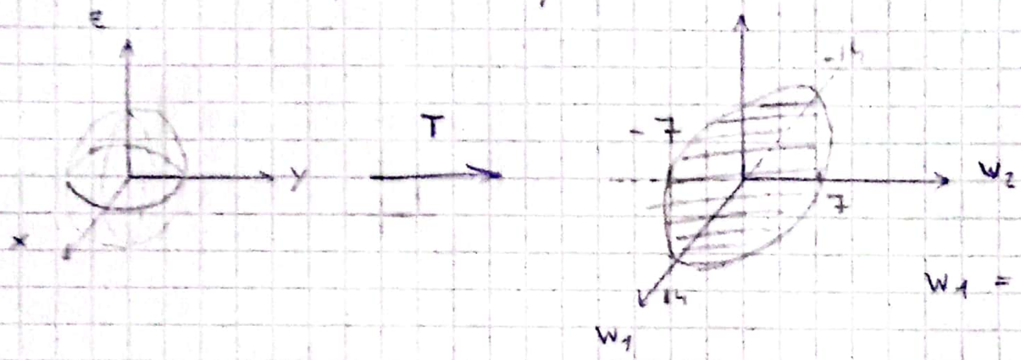
$$\frac{w_1}{105^2} + \frac{w_2}{63^2} + \frac{w_3}{42^2} = 1$$

B) $\frac{2}{9}\begin{pmatrix}-1\\8\\4\end{pmatrix}\underbrace{\begin{pmatrix}6 & 3 & 2\end{pmatrix}}_{\sqrt{49}=7} + \frac{1}{9}\begin{pmatrix}4\\4\\-7\end{pmatrix}\underbrace{\begin{pmatrix}2 & -6 & 3\end{pmatrix}}_{\sqrt{49}=7}$

$\sqrt{81} = 9$ $\quad$ $\sqrt{81} = 9$

$$\underbrace{\frac{2}{9}\cdot 9\cdot 7}_{14}\begin{pmatrix}-1/9\\8/9\\4/9\end{pmatrix}\begin{pmatrix}\frac{6}{7} & \frac{3}{7} & \frac{2}{7}\end{pmatrix} + \underbrace{\frac{1}{9}\cdot 9\cdot 7}_{7}\begin{pmatrix}4/9\\4/9\\-7/9\end{pmatrix}\begin{pmatrix}\frac{2}{7} & \frac{-6}{7} & \frac{3}{7}\end{pmatrix}$$

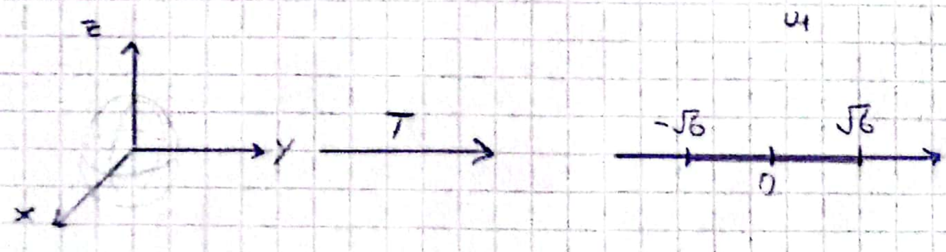
$$A = U\cdot\Sigma\cdot V^t = \begin{pmatrix}-1/9 & 4/9\\8/9 & 4/9\\4/9 & -7/9\end{pmatrix}\begin{pmatrix}14 & 0\\0 & 7\end{pmatrix}\begin{pmatrix}6/7 & 3/7 & 2/7\\2/7 & -6/7 & 3/7\end{pmatrix}$$



$$W_1 = \text{gen}\left\{\begin{pmatrix}-1/9\\8/9\\4/9\end{pmatrix}\right\}$$

$$W_2 = \text{gen}\left\{\begin{pmatrix}4/9\\4/9\\-7/9\end{pmatrix}\right\}$$

C) $A = \underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\sqrt{3}}\underbrace{\begin{pmatrix}1 & 0 & -1\end{pmatrix}}_{\sqrt{2}} = \underbrace{\sqrt{3}\cdot\sqrt{2}}_{\sqrt{6}}\underbrace{\begin{pmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{pmatrix}}_{u_1}\begin{pmatrix}1/\sqrt{2} & 0 & -1/\sqrt{2}\end{pmatrix}$



$$\text{gen}\left\{\begin{pmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{pmatrix}\right\}$$