Resolución

① $p(x) = 6x^2 + 3x + 1 \qquad B_1 = \{ x^2+x+1,\ x^2+x,\ x^2 \}$

a ojo: $6x^2+3x+1 = 1\,(x^2+x+1) + 2\,(x^2+x) + 3\,x^2$

$\Rightarrow [p]^{B_1} = [1\ 2\ 3]^T$

$M_{B_1}^{B_2}\,[p]^{B_1} = [p]^{B_2}$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \\ 13 \end{bmatrix} \Rightarrow [6x^2+3x+1]^{B_2} = [10\ 13\ 13]^T$$

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② $W = \{ A \in \mathbb{R}^{2\times 2} : A^T = A \} \longrightarrow \dim = 3$

$B = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\} \qquad T: W \longrightarrow \mathbb{R}^3$

$S = \text{gen}\left\{ \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \right\}$. Hallar $T^{-1}(S)$

•) S subesp. de $\mathbb{R}^3 \Rightarrow T^{-1}(S)$ subesp. de W

$S = \left\{ \alpha \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}, \alpha \in \mathbb{R} \right\} \Rightarrow T^{-1}(S) = \left\{ \alpha\, T^{-1}\left(\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}\right),\ \alpha \in \mathbb{R} \right\}$

•) $T^{-1}\left(\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}\right) = \left\{ A \in W \,/\, T(A) = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \right\} \rightarrow$ busco usando coordenadas

$[T]_B^E\,[A]^B = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \rightarrow$ llamo: $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = [A]^B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -2 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \rightarrow \text{S.N.H}: \begin{bmatrix} a \\ b \\ c \end{bmatrix} = x_p + \text{nul}\left([T]_B^E\right)$$

•) a ojo: $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow x_p = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

•) $\text{nul}\left([T]_B^E\right)$:
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -2 \\ 2 & 1 & 0 \end{bmatrix} \xrightarrow[\frac{1}{2}F_2]{} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \xrightarrow{F_1+F_2} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ \cancel{2} & \cancel{1} & \cancel{0} \end{bmatrix} \Rightarrow$$

$$\rightarrow \begin{cases} 2x_1 + x_2 = 0 \\ x_1 + x_3 = 0 \end{cases} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ -2x_1 \\ x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

coordenadas

$$\therefore \text{nul}\left([T]_B^E\right) = \text{gen}\left\{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right\} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\therefore A = 1\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \lambda\left[1\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + 1\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\therefore T^{-1}\left(\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}\right) = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \right\}.$$

$$\text{•)} \quad T^{-1}(S) = \alpha T^{-1}\left(\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}\right) = \alpha\left[\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\right] =$$

$$= \alpha \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \underbrace{\alpha\lambda}_{=\beta} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \qquad \alpha, \beta \in \mathbb{R}.$$

$$\therefore T^{-1}(S) = \text{gen}\left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \right\}$$

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(3) $\Pi = \Pi_{S_1 S_2}$ $\quad S_1 = \{x_1 - x_2 = 0\} = \text{gen}\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$

$S_2 = \text{gen}\left\{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\right\}$

$$\mathbb{R}^3 \xrightarrow{\Pi} \mathbb{R}^3$$

$$B \begin{cases} (1\,1\,0) \rightarrow (1\,1\,0) \\ (0\,0\,1) \rightarrow (0\,0\,1) \\ (1\,-1\,1) \rightarrow (0\,0\,0) \end{cases} \qquad [\Pi]_B^E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{cases} \Pi(e_1) = [\Pi]_E^E \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = ? \\ \Pi(e_2) = [\Pi]_E^E \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = ? \\ \Pi(e_3) = e_3 \end{cases} \qquad [\Pi]_E^E = [\Pi]_B^E \underbrace{M_E^B}_{(M_B^E)^{-1}}$$

·) $M_E^B = \left(M_B^E\right)^{-1} = \begin{bmatrix}1&0&1\\1&0&-1\\0&1&1\end{bmatrix}^{-1} =$ cuentas... $= \begin{bmatrix}1/2&1/2&0\\-1/2&1/2&1\\1/2&-1/2&0\end{bmatrix}$

·) $[\Pi]_E^E \left[\begin{array}{c|c}1&0\\0&1\\0&0\end{array}\right] = \begin{bmatrix}1&0&0\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}1/2&1/2&0\\-1/2&1/2&1\\1/2&-1/2&0\end{bmatrix}\left[\begin{array}{c|c}1&0\\0&1\\0&0\end{array}\right]$

$= \begin{bmatrix}1&0&0\\1&0&0\\0&1&0\end{bmatrix}\left[\begin{array}{c|c}1/2&1/2\\-1/2&1/2\\1/2&-1/2\end{array}\right] = \begin{array}{cc}\left[\begin{array}{c|c}1/2&1/2\\1/2&1/2\\-1/2&1/2\end{array}\right]\\ \quad v_1 \quad v_2\end{array}$

$\left.\begin{array}{l}\Pi(e_1) = v_1\\ \Pi(e_2) = v_2\\ \Pi(e_3) = e_3\end{array}\right\}$ triángulo formado por v_1, v_2 y $e_3 =$

$= \left\{x \in \mathbb{R}^3 \,/\, x = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 e_3 \text{ con } \alpha_1+\alpha_2+\alpha_3 = 1,\ \alpha_i \geq 0\ i=1,2,3\right\}$

combinaciones convexas

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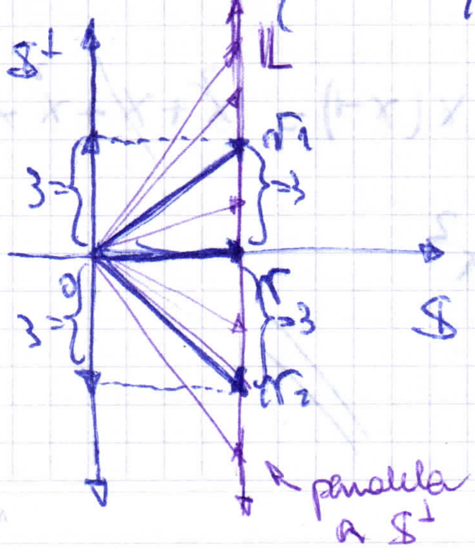
④ $(\mathbb{R}^3, \langle\cdot,\cdot\rangle)$ p.i. canónico

$\mathbb{S} = \text{gen}\{[1\ -1\ 0]\ [1\ 1\ 4]\}$

·) $r = [2\ -1\ 2] = \frac{3}{2}[1\ -1\ 0] + \frac{1}{2}[1\ 1\ 4]$ (a ojo)

·) Todos los $\left\{x \in \mathbb{R}^3 \,/\, P_{\mathbb{S}}(x) = \begin{bmatrix}2\\-1\\2\end{bmatrix}\right\} \longrightarrow$ recta $\mathbb{L}$



$\mathbb{L} = \{x = r + \mathbb{S}^\perp\}$

$\mathbb{S}^\perp = \text{gen}\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix} \times \begin{bmatrix}1\\1\\4\end{bmatrix}\right\} = \text{gen}\left\{\begin{bmatrix}2\\2\\-1\end{bmatrix}\right\}$

$x \in \mathbb{L} \Rightarrow x = \begin{bmatrix}2\\-1\\2\end{bmatrix} + \underbrace{\lambda\begin{bmatrix}2\\2\\-1\end{bmatrix}}_{\in \mathbb{S}^\perp}$

de estos x, buscamos los que

están a distancia $= 3$ de S.

Son los que tienen su proyección a $S^{\perp}$ de norma $= 3$

$$P_{S^{\perp}}(x) = \lambda \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \longrightarrow \| P_{S^{\perp}}(x) \| = \left\| \lambda \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \right\| = |\lambda| \underbrace{\left\| \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \right\|}_{=3} = 3$$

$$\Rightarrow |\lambda| = 1 \Rightarrow \lambda = \pm 1.$$

$$\therefore \; r_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}, \quad r_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$$

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(5) $(\mathbb{R}_2[x], \langle \cdot , \cdot \rangle)$ $\quad \langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$

$$S = \text{gen}\{ \underbrace{x(x+1)}_{p_1} ; \underbrace{x(x-1)}_{p_2} \}$$

p_1: raíces 0 y -1

p_2: raíces 0 y 1

·) $p_1 \perp p_2$ pues $\langle p_1, p_2 \rangle = 0\, p_2(-1) + 0 \cdot 0 + p_1(1) \cdot 0 = 0$

·) $p(x) = x^2 + 1 \longrightarrow P_S(p) = \dfrac{\langle p, p_1 \rangle}{\|p_1\|^2} p_1 + \dfrac{\langle p, p_2 \rangle}{\|p_2\|^2} p_2$

$\langle p_1, p_1 \rangle = \|p_1\|^2 = p_1(1)\, p_1(1) = 2 \cdot 2 = 4$

$\langle p_2, p_2 \rangle = \|p_2\|^2 = p_2(-1)\, p_2(-1) = 2 \cdot 2 = 4$

$\langle x^2 + 1, p_1 \rangle = 2 \cdot 0 + 0 + 2 \cdot 2 = 4$

$\langle x^2 + 1, p_2 \rangle = 2 \cdot 2 = 4$

$$\therefore P_S(x^2+1) = \frac{4}{4} x(x+1) + \frac{4}{4} x(x-1) = x^2 + \cancel{x} + x^2 - \cancel{x}$$

$$P_S(x^2+1) = 2x^2$$

NOTA