3.2) verificar que $\langle x, y \rangle = y^T G x$ define un producto interno para $\mathbb{R}^2$

d) $G \in G_4 = \left\{ \underbrace{\begin{bmatrix} a & b \\ b & c \end{bmatrix}}_{\text{simétrica}} : a > 0 \,,\ \det \begin{bmatrix} a & b \\ b & c \end{bmatrix} > 0 \right\}$

$a \cdot c - b^2 > 0$

• $\langle x, y \rangle = y^T \begin{bmatrix} a & b \\ b & c \end{bmatrix} x$

Rep

$\boxed{\cdot\ \langle x, y \rangle = \langle y, x \rangle}$ $\quad \cdot\ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\begin{bmatrix} a y_1 + y_2 b & b y_1 + c y_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a x_1 + b x_2 & b x_1 + c x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$x_1 (a y_1 + b y_2) + x_2 (b y_1 + c y_2) = y_1 (a x_1 + b x_2) + y_2 (b x_1 + c x_2)$

✓ $\underline{a x_1 y_1} + \underline{b x_1 y_2} + \underline{b y_1 x_2} + \underline{c y_2 x_2} = \underline{a x_1 y_1} + \underline{b x_2 y_1} + \underline{b x_1 y_2} + \underline{c x_2 y_2}$

$\boxed{\cdot\ \langle \lambda x, y \rangle = \lambda \langle x, y \rangle}$

$\begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix} = \lambda \left(x_1 (a y_1 + b y_2) + x_2 (b y_1 + c y_2) \right)$

$\begin{bmatrix} a y_1 + b y_2 & b y_1 + c y_2 \end{bmatrix} \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix} = \lambda x_1 (a y_1 + b y_2) + \lambda x_2 (b y_1 + c y_2)$

✓ $\lambda \left(x_1 (a y_1 + b y_2) + x_2 (b y_1 + c y_2) \right) = \lambda \left(x_1 (a y_1 + b y_2) + x_2 (b y_1 + c y_2) \right)$

$\boxed{\cdot\ \langle x, x \rangle = 0 \iff x = 0}$

$a x_1^2 + 2 b x_1 x_2 + c x_2^2$

si $x_2 = 0 \rightarrow a x_1^2 > 0$

si $x_1 = 0 \rightarrow b x_2^2 > 0$

$\forall\ x = 0 \rightarrow 0$ ✓

$a x_1^2 + c x_2^2 + 2b x_1 x_2 > 0$ $\quad \cdot \; a.c - b^2 > 0$

$a\left(x_1^2 + \frac{2b}{a} x_1 x_2\right) + c x_2^2$

1 $\; c - \frac{b^2}{a} > 0$

$a\left(\left(x_1 + \frac{b}{a} x_2\right)^2 - \frac{b^2}{a^2} x_2^2\right) + c x^2$

$\underbrace{a\left(x_1 + \frac{b}{a} x_2\right)^2}_{>0} + \underbrace{\left(-\frac{b^2}{a} + c\right)}_{>0} \underbrace{x^2}_{>0} > 0$ ✓

x 1

$\cdot \; \langle x + z, y \rangle = \langle x, y \rangle + \langle z, y \rangle \qquad z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$[y_1 \; y_2] \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \end{bmatrix} = \dots$

$[a y_1 + b y_2 \quad b y_1 + c y_2] \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \end{bmatrix}$

$a x_1 y_1 + b x_1 y_2 + a z_1 y_1 + b z_1 y_2 + a x_2 y_1 + b x_2 y_2 + b z_2 y_1 + c z_2 y_2$

$\underbrace{\left[a x_1 y_1 + b x_1 y_2 + a x_2 y_1 + b x_2 y_2\right]}_{\langle x, y \rangle} + \underbrace{\left[a z_1 y_1 + b z_1 y_2 + b z_2 y_1 + c z_2 y_2\right]}_{\langle z, y \rangle}$ ✓

3.4) $(V, \langle \cdot, \cdot \rangle)$ $\mathbb{R}$-espacio euclídeo de dim = 3

$B = \{u_i : i \in I_3\} \subset \{u \in V : \|u\| = 1\}$ una base de V

Tal que $\|u_i + u_j\|^2 = 2 + \sqrt{3}$ $\quad \|u_i - u_j\|^2 = 2 - \sqrt{3}$ $\quad i \neq j$

a) Hallar G_B $\qquad$ b) Hallar $\Theta := [\arccos(\langle u_i, u_j \rangle)]$ $i \in I_3$ $j \in I_3$

c) Construir Δ con vértices $0, u_1, u_2 - \lambda u_1$ con $\lambda \in \mathbb{R}$ rectángulo

d) área del Δ $0, u_1, u_2$

e) área del Δ u_1, u_2, u_3

$$\|u_i + u_j\|^2 = \langle u_i + u_j, u_i + u_j \rangle$$

$$= \langle u_i, u_i \rangle + \langle u_i, u_j \rangle + \langle u_j, u_i \rangle + \langle u_j, u_j \rangle$$

$$2 + \sqrt{3} = \underbrace{\langle u_i, u_i \rangle}_{} + \underbrace{\langle u_j, u_j \rangle}_{} + 2\langle u_i, u_j \rangle$$

$$1 + 1 + 2\langle u_i, u_j \rangle$$

$$\cancel{2} + \sqrt{3} = \cancel{2} + 2\langle u_i, u_j \rangle$$

$$\boxed{\langle u_i, u_j \rangle = \frac{\sqrt{3}}{2}}$$

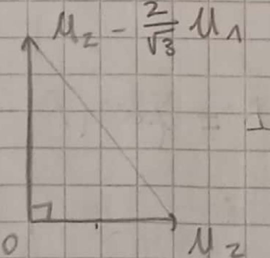
$$\|u_i - u_j\|^2 = \langle u_i - u_j, u_i - u_j \rangle$$

$$\underbrace{\langle u_i, u_i \rangle}_{1} - \langle u_i, u_j \rangle - \langle u_j, u_i \rangle + \underbrace{\langle u_j, u_j \rangle}_{1}$$

$$\cancel{2} - \sqrt{3} = \cancel{2} - 2\langle u_i, u_j \rangle \Rightarrow \langle u_i, u_j \rangle = \frac{\sqrt{3}}{2}$$

a) $G_B = \begin{bmatrix} 1 & \frac{1}{2}\sqrt{3} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & 1 & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2}\sqrt{3} & 1 \end{bmatrix} \Rightarrow \frac{1}{2}\begin{bmatrix} 2 & \sqrt{3} & \sqrt{3} \\ \sqrt{3} & 2 & \sqrt{3} \\ \sqrt{3} & \sqrt{3} & 2 \end{bmatrix}$

b) $\Theta = \frac{1}{6}\begin{bmatrix} 0 & \pi & \pi \\ \pi & 0 & \pi \\ \pi & \pi & 0 \end{bmatrix}$



c) $\langle u_2 - \lambda u_1, u_2 \rangle = 0$

$\langle u_2, u_2 \rangle - \langle \lambda u_1, u_2 \rangle = 0$

$1 - \lambda \langle u_1, u_2 \rangle = 0$

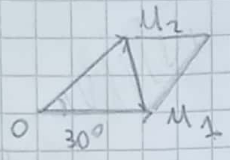
$1 - \lambda \frac{\sqrt{3}}{2} = 0$

$1 = \lambda \frac{\sqrt{3}}{2}$

$\boxed{\lambda = \frac{2}{\sqrt{3}}}$

Escaneado con CamScanner

d) área de Δ $0, u_1, u_2$



$$\text{área } \Delta = \frac{\sqrt{\det G_{\{u_1, u_2\}}}}{2}$$

$$\det G_{\{u_1, u_2\}} = \det \begin{bmatrix} \langle u_1, u_1 \rangle & \langle u_1, u_2 \rangle \\ \langle u_2, u_1 \rangle & \langle u_2, u_2 \rangle \end{bmatrix}$$

$$= \det \left[\frac{1}{2} \begin{bmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 2 \end{bmatrix} \right] = \frac{1}{2}(4 - 3) = \frac{1}{2}$$

$$\text{área } \Delta = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

e) área de Δ u_1, u_2, u_3

$$\text{área del } \Delta = \frac{1}{2}\sqrt{\det G_B} = \frac{1}{2}\sqrt{\left| \frac{1}{2} \begin{bmatrix} 2 & \sqrt{3} & \sqrt{3} \\ \sqrt{3} & 2 & \sqrt{3} \\ \sqrt{3} & \sqrt{3} & 2 \end{bmatrix} \right|} \; *1$$

$$*1 \quad (-1)^2 \cdot 2 \cdot \det \begin{bmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 2 \end{bmatrix} + (-1)^3 \cdot \sqrt{3} \cdot \det \begin{bmatrix} \sqrt{3} & \sqrt{3} \\ \sqrt{3} & 2 \end{bmatrix} + (-1)^4 \cdot \sqrt{3} \det \begin{bmatrix} \sqrt{3} & 2 \\ \sqrt{3} & \sqrt{3} \end{bmatrix}$$

$$2 \cdot 1 - \sqrt{3} \cdot (2\sqrt{3} - 3) + \sqrt{3} \cdot (3 - 2\sqrt{3})$$

$$2 - 2\sqrt{3}(2\sqrt{3} - 3) = 2 - \underbrace{4 \cdot 3}_{12} + 6\sqrt{3} = -10 + 6\sqrt{3}$$

$$\boxed{\frac{1}{2}\sqrt{3\sqrt{3} - 5}}$$

3.7) $\langle \cdot , \cdot \rangle$ definido por $\langle x, y \rangle = y^T \begin{bmatrix} 2 & -2 & 0 \\ -2 & 5 & 4 \\ 0 & 4 & 6 \end{bmatrix} x$

$$S_1 = \{ x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0 \}$$

$$S_2 = \{ x \in \mathbb{R}^3 : x_1 - x_3 = 0 \}$$

a) Hallar $\left[P_{S_1^\perp} \right]_E^E$ y $\left[P_{S_2^\perp} \right]_E^E$

$$S_1 = \text{gen} \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\} \qquad S_2 = \text{gen} \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\left\langle x, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\rangle = 0 \Rightarrow \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 5 & 4 \\ 0 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -7 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\left\langle x, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\rangle = 0$$

$$\Downarrow$$

$$\begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 5 & 4 \\ 0 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -7 & -4 \\ -2 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -5 & -8 \\ -2 & 1 & -2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0 & 1 & 8/5 \\ 1 & 0 & 9/5 \end{bmatrix}$$

$$x_2 = -\frac{8}{5} x_3$$

$$x_1 = -\frac{9}{5} x_3$$

$$S_1^\perp = \text{gen} \left\{ \begin{bmatrix} 9 \\ 8 \\ -5 \end{bmatrix} \right\}$$

$$\left\langle x, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle = 0 \rightarrow \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 5 & 4 \\ 0 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\left\langle x, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\rangle = 0 \rightarrow \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 5 & 4 \\ 0 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 5 & 4 \\ 2 & 2 & 6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 7 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 10/7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 11/7 \\ 0 & 1 & 10/7 \end{bmatrix} \Rightarrow \begin{cases} x_1 = -11/7\, x_3 \\ x_2 = -10/7\, x_3 \end{cases}$$

$$S_2^\perp = \text{gen} \left\{ \begin{bmatrix} -11 \\ -10 \\ 7 \end{bmatrix} \right\}$$

$$[P_{S_1^\perp}]^{\mathcal{E}}_{\mathcal{E}} = \frac{\left\langle x, \begin{bmatrix}9\\8\\-5\end{bmatrix}\right\rangle}{\left\langle \begin{bmatrix}9\\8\\-5\end{bmatrix}, \begin{bmatrix}9\\8\\-5\end{bmatrix}\right\rangle}\begin{bmatrix}9\\8\\-5\end{bmatrix} \qquad [P_{S_2^\perp}]^{\mathcal{E}}_{\mathcal{E}} = \frac{\left\langle x, \begin{bmatrix}-11\\-10\\7\end{bmatrix}\right\rangle}{\left\langle \begin{bmatrix}-11\\-10\\7\end{bmatrix}, \begin{bmatrix}-11\\-10\\7\end{bmatrix}\right\rangle}\begin{bmatrix}-11\\-10\\7\end{bmatrix}$$

$$[9\ 8\ -5]\begin{bmatrix}2&-2&0\\-2&5&4\\0&4&6\end{bmatrix} = \begin{bmatrix}2\\2\\2\end{bmatrix}^T \qquad [-11\ -10\ 7]\begin{bmatrix}2&-2&0\\-2&5&4\\0&4&6\end{bmatrix} = \begin{bmatrix}-2\\0\\2\end{bmatrix}^T$$

$$[2\ 2\ 2]\begin{bmatrix}9\\8\\-5\end{bmatrix} = 24 \qquad [-2\ 0\ 2]\begin{bmatrix}-11\\-10\\7\end{bmatrix} = 36$$

$$\frac{1}{24}\ 2x_1 + 2x_2 + 2x_3 \begin{bmatrix}9\\8\\-5\end{bmatrix} \qquad \frac{1}{36}(-2x_1 + 2x_3)\begin{bmatrix}-11\\-10\\7\end{bmatrix}$$

$$\frac{2}{24}_{12}(x_1 + x_2 + x_3)\begin{bmatrix}9\\8\\-5\end{bmatrix} \qquad \frac{2}{36}_{18}(-x_1 + x_3)\begin{bmatrix}-11\\-10\\7\end{bmatrix}$$

$$\boxed{[P_{S_1^\perp}]^{\mathcal{E}}_{\mathcal{E}} = \frac{1}{12}\begin{bmatrix}9&9&9\\8&8&8\\-5&-5&-5\end{bmatrix}} \qquad \boxed{[P_{S_2^\perp}]^{\mathcal{E}}_{\mathcal{E}} = \frac{1}{18}\begin{bmatrix}11&0&-11\\10&0&-10\\-7&0&7\end{bmatrix}}$$

b) Sea $b = \begin{bmatrix}1\\1\\2\end{bmatrix}$ inma $d(b, S_1^\perp)$ y $d(b, S_2)$

$$d(b, S_1^\perp) \to \|P_{S_1}(b)\| \to \|b - P_{S_1^\perp}(b)\| \qquad d(b, S_2) \to \|P_{S_2^\perp}(b)\|$$

$$P_{S_1^\perp}(b) = \frac{1}{12}\begin{bmatrix}9&9&9\\8&8&8\\-5&-5&-5\end{bmatrix}\cdot\begin{bmatrix}1\\1\\2\end{bmatrix} = \frac{1}{12}\begin{bmatrix}36\\32\\-20\end{bmatrix} = \begin{bmatrix}3\\8/3\\-5/3\end{bmatrix} \Rightarrow b - P_{S_1^\perp}(b) = \begin{bmatrix}1\\1\\2\end{bmatrix} - \begin{bmatrix}3\\8/3\\-5/3\end{bmatrix} = \begin{bmatrix}-2\\-5/3\\11/3\end{bmatrix}$$

$$\|b - P_{S_1^\perp}(b)\|^2 = \|P_{S_1}(b)\|^2 = [-2\ -5/3\ 11/3]\begin{bmatrix}2&-2&0\\-2&5&4\\0&4&6\end{bmatrix}\begin{bmatrix}-2\\-5/3\\11/3\end{bmatrix}$$

$$\left[-\frac{2}{3}\ \frac{31}{3}\ \frac{46}{3}\right]\begin{bmatrix}-2\\-5/3\\11/3\end{bmatrix} = \frac{4}{3} - \frac{155}{9} + \frac{506}{9} = \frac{121}{3}$$

$$\sqrt{\frac{121}{3}} = \frac{\sqrt{121}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{11}{3}\sqrt{3}}$$

Práctica 26/10 - Repaso Pre Parcial

1. Hallar la sol. por cuadrados mínimos de norma mínima de la ecuación. (?)

$$\underbrace{\begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}}_{b}$$

$$\hat{x} = (A^T A)^{-1} A^T \, b$$

$$A^T A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{pmatrix}$$

$$(A^T A)^{-1} = \left(\begin{array}{ccc|ccc} 2 & -2 & 2 & 1 & 0 & 0 \\ -2 & 5 & 1 & 0 & 1 & 0 \\ 2 & 1 & 5 & 0 & 0 & 1 \end{array}\right) \to \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1/2 & 0 & 0 \\ 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 3 & 3 & -1 & 0 & 1 \end{array}\right]$$

A no es de rango máximo

$b = P_{col(A)}(b)$?

$$\underbrace{\begin{pmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{pmatrix}}_{A^T A} \hat{x} = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{pmatrix}}_{A^T} \underbrace{\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}}_{b} = \begin{pmatrix} -2 \\ 9 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & -2 & 2 & -2 \\ -2 & 5 & 1 & 9 \\ 2 & 1 & 5 & 5 \end{array}\right) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & -1 \\ 0 & 3 & 3 & 7 \\ 0 & 3 & 3 & 7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 4/3 \\ 0 & 1 & 1 & 7/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{l} x_1 = \frac{4}{3} - 2x_3 \\ x_2 = \frac{7}{3} - x_3 \end{array}\right\} \quad \boxed{\hat{x} = \frac{1}{3}\begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix} + \text{gen}\left\{\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}\right\}}$$

NO es de norma min

norma min? $\to$ $\hat{x} \in Fil(A)$

$\cap \; Fil(A)$

2) $(V, \langle \cdot , \cdot \rangle)$ de dim = 2 con $B = \{v_1, v_2\}$

con $\|v_1\| = 1$ $\quad \|v_1 + v_2\| = 2 \cdot \sqrt{2}$ $\quad \|v_1 - v_2\| = \sqrt{2}$

calcular el área del triangulo v_1, v_2 y $v_1 + v_2$

$$\langle v_1 + v_2 , v_1 + v_2 \rangle = (2 \cdot \sqrt{2})^2 = 4 \cdot 2 = 8$$

$$\underbrace{\langle v_1 , v_1 \rangle}_{1} + 2\langle v_1 , v_2 \rangle + \underbrace{\langle v_2 , v_2 \rangle}_{\|v_2\|^2} = 8$$

$$\langle v_1 - v_2 , v_1 - v_2 \rangle = 2$$

$$\underbrace{\langle v_1 , v_1 \rangle}_{1} - 2\langle v_1 , v_2 \rangle + \underbrace{\langle v_2 + v_2 \rangle}_{\|v_2\|^2} = 2$$

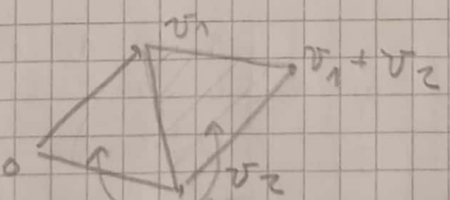
$$\|v_2\|^2 = 1 + 2\langle v_1 , v_2 \rangle = 4 = \langle v_2 , v_2 \rangle$$

$$2 + 4\langle v_1 , v_2 \rangle = 8$$

$$4\langle v_1 , v_2 \rangle = 6$$

$$\langle v_1 , v_2 \rangle = \frac{3}{2}$$

$$G_B = \begin{bmatrix} 1 & 3/2 \\ 3/2 & 4 \end{bmatrix}$$



área $\Delta = \frac{1}{2}\sqrt{\det G_B} = \frac{1}{2}\sqrt{4 - \frac{9}{4}}$

$$= \frac{1}{2}\sqrt{\frac{7}{4}} = \boxed{\frac{\sqrt{7}}{4}}$$

$$\hat{x} = A(A^TA)^{-1}A^Tb$$

$$A^TAx = A^Tb$$

$$A^TA\left(\alpha_1\begin{bmatrix}1\\0\\2\end{bmatrix} + \alpha_2\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = A^Tb$$

$$A^TA\begin{bmatrix}1 & 0\\0 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix} = A^Tb$$

* $\hat{x}$ es de norma mínima si $\hat{x} \in Fil(A)$

$$A^TA \cdot \left([v_1\ v_2 \dots v_n] \cdot [\hat{x}]^B\right) = A^Tb$$

con $\{v_1, v_2 \dots v_n\}$ base de $Fil(A)$

$$\begin{pmatrix}1 & 0 & 2\\-1 & 2 & 0\\0 & 1 & 1\end{pmatrix} \sim \begin{bmatrix}1 & 0 & 2\\0 & 2 & 2\\0 & 1 & 1\end{bmatrix} \to \begin{bmatrix}1 & 0 & 2\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix} \to Fil(A) = gen\left\{\begin{pmatrix}1\\0\\2\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$$

$$\begin{bmatrix}2 & -2 & 2\\-2 & 5 & 1\\2 & 1 & 5\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\0 & 2 & 1\\2 & 0 & 1\end{bmatrix}\begin{bmatrix}2\\4\\1\end{bmatrix}$$

$$\begin{bmatrix}6 & 0\\0 & 6\\12 & 6\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\end{bmatrix} = \begin{bmatrix}-2\\9\\5\end{bmatrix} \to \left[\begin{array}{cc|c}6 & 0 & -2\\0 & 6 & 9\\12 & 6 & 5\end{array}\right] \to \left[\begin{array}{cc|c}1 & 0 & -1/3\\0 & 1 & 3/2\\1 & 1/2 & 5/12\end{array}\right]$$

$$P_{col(A)}(b) \Rightarrow [P_{col(A)}(b)]^B = \begin{bmatrix}\langle b, \begin{bmatrix}1\\-1\\0\end{bmatrix}\rangle\\ \langle b, \begin{bmatrix}0\\2\\1\end{bmatrix}\rangle\end{bmatrix} \quad b = \begin{pmatrix}2\\4\\1\end{pmatrix}$$

$$[P_{col(A)}(b)]^B = \begin{bmatrix}-2\\9\end{bmatrix}$$

$$\alpha_1 = -\frac{1}{3} \quad \alpha_2 = \frac{3}{2}$$

$$\downarrow$$

$$-\frac{1}{3}\begin{bmatrix}1\\0\\2\end{bmatrix} + \frac{3}{2}\begin{bmatrix}0\\1\\1\end{bmatrix} = \boxed{\begin{bmatrix}-1/3\\3/2\\5/6\end{bmatrix} = \hat{x}\ min}$$

$$\operatorname{fil}(A) = \operatorname{gen}\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\} \longrightarrow \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$

$$\hat{x} \in \operatorname{gen}\left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\} + \frac{1}{3} \begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 4 \\ 7 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2\lambda + 4/3 \\ -\lambda + 7/3 \\ \lambda \end{bmatrix}$$

$$\operatorname{fil}(A) = \{ \bar{x} \in \mathbb{R}^3 : -2x_1 - x_2 + x_3 = 0 \}$$

$$-2\left(-2\lambda + \frac{4}{3}\right) - \left(-\lambda + \frac{7}{3}\right) + \lambda = 0$$

$$4\lambda - \frac{8}{3} + \lambda - \frac{7}{3} + \lambda = 0$$

$$6\lambda - \frac{15}{3} = 0 \longrightarrow \boxed{\lambda = \frac{5}{6}}$$

$$\frac{5}{6} \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 4/3 \\ 7/3 \\ 0 \end{bmatrix} = \begin{bmatrix} -10/6 + 4/3 \\ -5/6 + 7/3 \\ 5/6 \end{bmatrix} = \begin{bmatrix} -2/6 \\ 9/6 \\ 5/6 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 3/2 \\ 5/6 \end{bmatrix}$$

$$\boxed{\hat{x}_{min} = \begin{bmatrix} -1/3 \\ 3/2 \\ 5/6 \end{bmatrix}}$$

3) $(\mathbb{R}^{2\times 2}, \langle \circ, \circ \rangle)$ con $\langle A, B \rangle = Tr(B^t A)$

hallar la matriz simétrica más cercana a $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}_b$ y calcular su distancia.

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \in gen\left\{ \underset{A_1}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}, \underset{A_2}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}, \underset{A_3}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} \right\} = S$$

sea $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ entonces:

$$\langle A, B \rangle = Tr\left(\begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \right) = Tr\begin{pmatrix} b_{11}a_{11} + b_{21}a_{21} & x \\ x & b_{12}a_{12} + b_{22}a_{22} \end{pmatrix}$$

$$\langle A, B \rangle = b_{11}a_{11} + b_{21}a_{21} + b_{12}a_{12} + b_{22}a_{22}$$

$$\begin{cases} \langle A_1, B \rangle = 0 \longrightarrow b_{12} + b_{21} = 0 \rightarrow b_{12} = -b_{21} \\ \langle A_2, B \rangle = 0 \longrightarrow b_{11} = 0 \\ \langle A_3, B \rangle = 0 \longrightarrow b_{22} = 0 \end{cases}$$

$$S^\perp = gen\left\{ \underset{A^\perp}{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}} \right\}$$

$P_S = I - P_{S^\perp}$

$$P_{S^\perp}(b) = \frac{\langle A^\perp, b \rangle}{\langle A^\perp, A^\perp \rangle} A^\perp = \frac{\overset{-2}{2-4}}{2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$P_S(b) = b - P_{S^\perp}(b) = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\boxed{d(S, b) = \| P_{S^\perp}(b) \| = \sqrt{2}} \qquad \boxed{P_S(b) = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}}$$

4- $\mathbb{R}^3$ con $\langle x,y\rangle = y^T \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} x$

Hallar $[P_{S_1+S_2}]^{\varepsilon}_{\varepsilon}$ con $S_1 = gen\{\begin{pmatrix}1\\1\\1\end{pmatrix}\}$ $S_2 = \{x \in \mathbb{R}^3$ tal que $\begin{cases} x_1 + x_2 - 4x_3 = 0 \\ x_1 - x_2 = 0 \end{cases}\}$

$S_1 = \begin{bmatrix} a \\ a \\ a \end{bmatrix}$ con $a \in \mathbb{R}$

$\begin{bmatrix} 1 & 1 & -4 \\ 1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & -4 \\ 0 & -2 & 4 \end{bmatrix}$

$S_1 \cap S_2 = \begin{cases} a + a - 4a = 0 \\ a - a = 0 \checkmark \end{cases}$

$\begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & -2 \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \end{bmatrix}$

$S_2 = gen\{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\}$

$-2a = 0 \Leftrightarrow a = 0$

$S_1 \cap S_2 = 0$

$S_1 + S_2 = gen\{\underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}_{v_2}\}$

$\langle x, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\rangle = 0 \longrightarrow [1\ 1\ 1] \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} x = [4\ 3\ 2]\, x = 0$

$\langle x, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\rangle = 0 \longrightarrow [2\ 2\ 1] \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} x = [8\ 6\ 2]\, x = 0$

$\begin{bmatrix} 4 & 3 & 2 \\ 8 & 6 & 2 \end{bmatrix} x = 0 \mapsto \begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{cases} x_3 = 0 \\ x_1 = -\frac{3}{4} x_2 \end{cases} \Rightarrow (S_1 + S_2)^{\perp} = gen\{\begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}\}$

$[P_{S^\perp}]^{\varepsilon}_{\varepsilon} = \dfrac{\langle x, \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}\rangle}{\langle \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}\rangle} \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$

$\rightarrow [-3\ 4\ 0] \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = [-5\ 5\ 0]$

$$\langle x, \begin{bmatrix}-3\\4\\0\end{bmatrix} \rangle = [-5 \; 5 \; 0] \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = -5x_1 + 5x_2$$

$$\langle \begin{bmatrix}-3\\4\\0\end{bmatrix}, \begin{bmatrix}-3\\4\\0\end{bmatrix} \rangle = [-5 \; 5 \; 0] \begin{bmatrix}-3\\4\\0\end{bmatrix} = 15 + 20 = 35$$

$$[P_{S^\perp}]_E^E = \frac{1}{35} \begin{bmatrix}15x_1 - 15x_2\\-20x_1 + 20x_2\\0\end{bmatrix} = \frac{1}{35}\begin{bmatrix}15 & -15 & 0\\-20 & 20 & 0\\0 & 0 & 0\end{bmatrix}$$

$$[P_S]_E^E = I - [P_{S^\perp}] = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \begin{bmatrix}3/7 & -3/7 & 0\\-4/7 & 4/7 & 0\\0 & 0 & 0\end{bmatrix}$$

$$= \frac{1}{7}\left[\begin{bmatrix}7 & 0 & 0\\0 & 7 & 0\\0 & 0 & 7\end{bmatrix} - \begin{bmatrix}3 & -3 & 0\\-4 & 4 & 0\\0 & 0 & 0\end{bmatrix}\right]$$

$$[P_S]_E^E = \frac{1}{7}\begin{bmatrix}4 & 3 & 0\\4 & 3 & 0\\0 & 0 & 7\end{bmatrix}$$

3.7)

b) ii) $P_{S_2^\perp}(b) = \frac{1}{18}\begin{bmatrix}11 & 0 & -11\\10 & 0 & -10\\-7 & 0 & 7\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix} = \frac{1}{18}\begin{bmatrix}-11\\-10\\7\end{bmatrix}$

$$\|P_{S_2^\perp}(b)\|^2 = \langle P_{S_2^\perp}(b), P_{S_2^\perp}(b) \rangle = \frac{1}{324} \cdot [-11 \; -10 \; 7] \begin{bmatrix}2 & -2 & 0\\-2 & 5 & 4\\0 & 4 & 6\end{bmatrix}\begin{bmatrix}-11\\-10\\7\end{bmatrix}$$

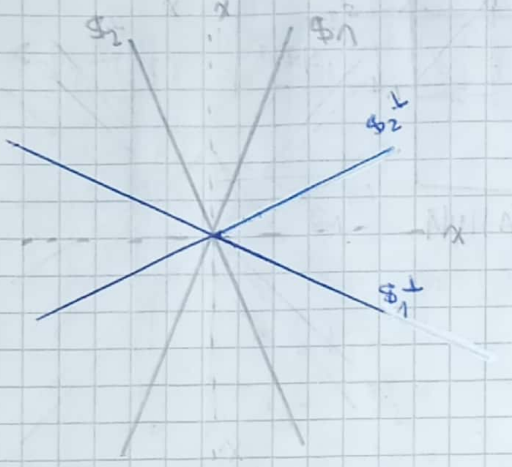
$$\frac{1}{324}[-2 \; 0 \; 2]\begin{bmatrix}-11\\-10\\7\end{bmatrix} = \frac{1}{324} = \frac{36}{324} = \frac{1}{9} \Rightarrow \boxed{\|P_{S_2^\perp}(b)\| = \frac{1}{3}}$$

c) Hallar todos los $x \in \mathbb{R}^3$ tal que $d(x, S_1) = d(x, S_2)$

$$\|P_{S_1^\perp}(x)\|^2 = \|P_{S_2^\perp}(x)\|^2$$

$$\langle P_{S_1^\perp}(x), P_{S_1^\perp}(x)\rangle = \langle P_{S_2^\perp}(x), P_{S_2^\perp}(x)\rangle$$

...



3.8) $(V, \langle \cdot, \cdot \rangle)$ dim 3 $\quad B = \{v_1, v_2, v_3\}$

$$G_B = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$$

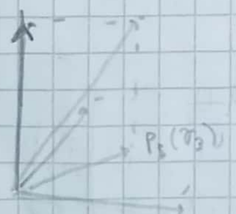
a) Hallar $\left[P_{gen\{v_1, v_2\}}\right]_B^B$

b) Hallar $P_{gen\{v_1, v_2\}}(v_3)$

$[P_S(v_1)]^B \quad [P_S(v_2)]^B \quad [P_S(v_3)]^B$

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad [P_S(v_3)]^B = G_B^{-1} \cdot \begin{bmatrix} \langle v_1, v_3 \rangle \\ \langle v_2, v_3 \rangle \\ 0 \end{bmatrix}$ (*1)

$P_{S^\perp}(v_3) = v_3 - P_S(v_3)$



$P_S(v_3) = a v_1 + b v_2$

$\langle v_1, v_3 - P_S(v_3) \rangle = 0 = \underbrace{\langle v_1, v_3 \rangle}_{1/3} - \langle v_1, P_S(v_3) \rangle = 0$

$\langle v_2, v_3 - P_S(v_3) \rangle = 0 = \langle v_2, v_3 \rangle - \langle v_2, P_S(v_3) \rangle = 0$

$\langle v_1, a v_1 + b v_2 \rangle = \frac{1}{3} \qquad \langle v_2, a v_1 + b v_2 \rangle = \frac{1}{4}$

$a \langle v_1, v_1 \rangle + b \langle v_1, v_2 \rangle = \frac{1}{3} \qquad a \langle v_2, v_1 \rangle + b \langle v_2, v_2 \rangle = \frac{1}{4}$

$a + \frac{1}{2} b = \frac{1}{3} \qquad \frac{1}{2} a + \frac{1}{3} b = \frac{1}{4}$

$$\left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \end{array}\right] \to \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1 & 2/3 & 2/2 \end{bmatrix} \to \begin{bmatrix} 1 & 1/2 & 1/3 \\ 0 & 1/6 & 1/6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/2 & 1/3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/6 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{matrix} a = -1/6 \\ b = 1 \end{matrix} \Rightarrow [P_S(v_3)]^B = \begin{bmatrix} -1/6 \\ 1 \\ 0 \end{bmatrix}$$

(*) Sea $B' = \{v_1, v_2\}$ base de S

$$G_{B'} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \Rightarrow G_{B'}^{-1} = 12 \begin{bmatrix} 1/3 & -1/2 \\ -1/2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}$$

$$\det = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$G_{B'}^{-1} \cdot \begin{bmatrix} \langle v_1, v_3 \rangle \\ \langle v_2, v_3 \rangle \end{bmatrix} = \begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix} \cdot \begin{bmatrix} 1/3 \\ 1/4 \end{bmatrix}$$

$$[P_S(v_3)]^{B'} = \begin{bmatrix} -1/6 \\ 1 \end{bmatrix} \to [P_S(v_3)]^B = \begin{bmatrix} -1/6 \\ 1 \\ 0 \end{bmatrix}$$

a) $$[P_S]_B^B = \begin{bmatrix} 1 & 0 & -1/6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

b) $P_S(v_3) = -\frac{1}{6} v_1 + 1 v_2$

c) Calcular de $d(v_3, S)$

$$P_{S^\perp}(v_3) = v_3 - P_S(v_3) = v_3 + \frac{1}{6} v_1 - v_2 \Rightarrow [P_{S^\perp}(v_3)]^B = \begin{bmatrix} 1/6 \\ -1 \\ 1 \end{bmatrix}$$

$$\| P_{S^\perp}(v_3) \| = d(v_3, S) = \sqrt{\langle P_{S^\perp}(v_3), P_{S^\perp}(v_3) \rangle}$$

$$\langle P_{S^\perp}(v_3), P_{S^\perp}(v_3) \rangle = [1/6 \ \ -1 \ \ 1] \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \begin{bmatrix} 1/6 \\ -1 \\ 1 \end{bmatrix}$$

$$= [0 \ \ 0 \ \ 1/180] \begin{bmatrix} 1/6 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{180}$$

$$\boxed{d(v_3, S) = \frac{1}{\sqrt{180}}}$$

d) Hallar todos los $v \in V$ tal que $d(v, v_1) = d(v, v_2)$

$$v - \frac{\langle v, v_i \rangle}{\langle v_i, v_i \rangle} v_i = P_{gen\{v_i\}^\perp}(v)$$

$\langle v_1, v_1 \rangle = 1 \qquad \langle v_2, v_2 \rangle = 1/3$

$$\langle v, v_i \rangle = \left([v_i]^B\right)^\perp G_B \, [v]^B$$

$$\langle v, v_1 \rangle = [1\ 0\ 0] \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = a_1 + \frac{a_2}{2} + \frac{a_3}{3}$$

$$\langle v, v_2 \rangle = [0\ 1\ 0] \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4}$$

$$P_{gen\{v_1\}^\perp}(v) = v - v_1 \cdot \left(a_1 + \frac{a_2}{2} + \frac{a_3}{3}\right)$$

$$P_{gen\{v_2\}^\perp}(v) = v - 3v_2\left(\frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4}\right) = v - v_2\left(\frac{3}{2}a_1 + a_2 + \frac{3}{4}a_3\right)$$

para que $d(v, v_1) = d(v, v_2) \rightarrow \|P_{gen\{v_1\}^\perp}(v)\| = \|P_{gen\{v_2\}^\perp}(v)\|$

$$\cancel{\|v\|} - \underbrace{\|v_1\|}_{\sqrt{1}}\left(a_1 + \frac{a_2}{2} + \frac{a_3}{3}\right) = \cancel{\|v\|} - \underbrace{\|v_2\|}_{\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}}\left(\frac{3}{2}a_1 + a_2 + \frac{3}{4}a_3\right)$$

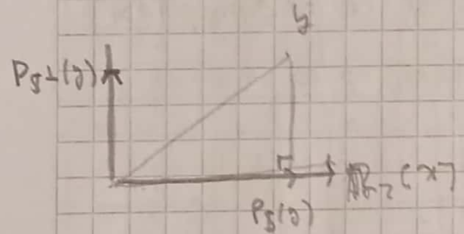
$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} = \frac{\sqrt{3}}{2}a_1 + \frac{\sqrt{3}}{3}a_2 + \frac{\sqrt{3}}{4}a_3$$

3.10) $\left(C([-1,1]), \langle \cdot , \cdot \rangle\right)$ $\quad \langle f, g \rangle := \int_a^1 f(x) g(x)\, dx$

a) HALLAR $P_{\mathbb{R}_2[x]}(y)$ con $y = \cos(\pi x)$

$S = \mathbb{R}_2[x] = \text{gen}\{1, x, x^2\}$ $\quad P_{\mathbb{R}_2[x]}(y) \in \mathbb{R}_2[x]$

$P_{\mathbb{R}_2[x]}(y) = a + bx + cx^2$



$\langle y - P_{\mathbb{R}_2[x]}(y), P_{\mathbb{R}_2[x]}(y) \rangle = 0$

$$\int_{-1}^{1} \left(\cos(\pi x) - a + bx + cx^2\right)\left(a + bx + cx^2\right) dx$$

3.14) $A \in \mathbb{R}^{3\times 3}$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

a) Hallar la solución por min. cuadrados de $A\hat{x} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ (a) y $A\hat{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ (b)

y determinar la norma mínima y calcular el error cuadrático

$$\min_{x \in \mathbb{R}^3} \| b - Ax \|^2$$

Para que sea sol. por mínimos cuadrados debe cumplir la ecuación

normal $A^T A \hat{x} = A^T b$ y como A no es de rango máx

la solución no es mínima

$$A^T = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 - b_3 \\ b_3 - b_2 \\ b_1 - b_2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & b_1 - b_3 \\ -1 & 2 & 1 & b_3 - b_2 \\ 1 & 1 & 2 & b_1 - b_2 \end{array}\right] \sim \left[\begin{array}{ccc|c} \cancel{0} & \cancel{-3} & \cancel{-3} & \cancel{-b_1 - b_3 + 2b_2} \\ 0 & 3 & 3 & b_3 + b_1 - 2b_2 \\ 1 & 1 & 2 & b_1 - b_2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 - b_2 \\ 0 & 1 & 1 & \frac{b_3}{3} + \frac{b_1}{3} - \frac{2}{3} b_2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{2}{3} b_1 - \frac{1}{3} b_2 - \frac{b_3}{3} \\ 0 & 1 & 1 & \frac{b_1}{3} - \frac{2}{3} b_2 + \frac{b_3}{3} \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = \frac{1}{3}(2b_1 - b_2 - b_3) - x_3 \\ x_2 = \frac{1}{3}(b_1 - 2b_2 + b_3) - x_3 \end{array}$$

a, b (i)
$$\boxed{\hat{x} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} b + \text{gen}\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}}$$

$$\hat{x}_a = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad \hat{x}_b = \begin{bmatrix} 5/3 \\ 7/3 \\ 0 \end{bmatrix}$$

$$\min \hat{x} \in (\text{N}(A)) \rightarrow \text{gen}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \right\}$$

$$\alpha_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \beta \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} b$$

$b = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

$b = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 5/3 \\ 7/3 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5 \\ 7 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 2 \\ 1 & -1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & -1 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -3 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

1° $F_3 - F_1$ $\longrightarrow \frac{1}{3}\begin{bmatrix} 5 \\ 7 \\ -5 \end{bmatrix}$

2° $F_2 = -F_2 - F_3 + F_2$ $\longrightarrow \frac{1}{3}\begin{bmatrix} 5 \\ -7 \\ -12 \end{bmatrix}$

3° $F_3 = \frac{F_3}{3}$ $\longrightarrow \frac{1}{3}\begin{bmatrix} 5 \\ -7 \\ -4 \end{bmatrix}$

4° $F_2 + F_3$

$F_1 - F_3$ $\longrightarrow \frac{1}{3}\begin{bmatrix} 9 \\ -11 \\ -4 \end{bmatrix}$

$x_1 = 0$

$x_2 = -1$ $\to$ $\boxed{\text{Sol min} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}$ a) ✓

$x_3 = 1$

$\text{Sol min} = \frac{9}{3}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{11}{3}\begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$

$\boxed{\text{Sol min} = \frac{1}{3}\begin{bmatrix} 9 \\ 11 \\ 20 \end{bmatrix}}$ b) ✓

$$\left\| \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\|^2 = \left\| \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\|^2 = \boxed{9}$$

ε_{b_1} a) ✓

$$\left\| \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 9 \\ 11 \\ 20 \end{bmatrix} \right\|^2 = \left\| \begin{bmatrix} 0 \\ -14/3 \\ -14/3 \end{bmatrix} \right\|^2$$

$$\boxed{\varepsilon_{b_2} = \frac{392}{9}}$$

(2) $\lambda \cdot \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \cap \text{gen}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \right\} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$

$x_1 + x_2 - x_3 = 0$

$(-\lambda + 1) - \lambda + 2 - \lambda = 0$

$3 - 3\lambda \Rightarrow \lambda = 1 \longrightarrow \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ ✓

$-3\lambda = -3$

$\lambda = 1$

3.16)

x	-1	0	1	2	3
y	-14	-5	-4	1	22

Ajuste por $y = a_0 + a_1 x$

$$\underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}_{A} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -14 \\ -5 \\ -4 \\ 1 \\ 22 \end{bmatrix} \Rightarrow A^T A x = A^T b$$

$$\downarrow$$

$$x = (A^T A)^{-1} A^T b$$

$$A^T \cdot A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 15 \end{bmatrix} \longrightarrow 5 \cdot 15 - 5 \cdot 5 = 50$$

$$(A^T A)^{-1} = \frac{1}{50} \cdot \begin{bmatrix} 15 & -5 \\ -5 & 5 \end{bmatrix}. \qquad (A^T A)^{-1} \cdot A^T = \frac{1}{50} \begin{bmatrix} 15 & -5 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} 20 & 15 & 10 & 5 & 0 \\ -10 & -5 & 0 & 5 & 10 \end{bmatrix}$$

$$(A^T A)^{-1} A^T \cdot b = \begin{bmatrix} -39/5 \\ 39/5 \end{bmatrix}$$

$$\boxed{y = -\frac{39}{5} x + \frac{39}{5}}$$

$$\varepsilon = \| \underbrace{b - A\hat{x}} \|^2 = \| \quad \|^2 = \frac{2840}{25} = \frac{568}{5} = 113{,}6$$

$$\begin{bmatrix} -14 \\ -5 \\ -4 \\ 1 \\ 22 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -39/5 \\ 39/5 \end{bmatrix} = \begin{bmatrix} 8/5 \\ 14/5 \\ -20/5 \\ -34/5 \\ 32/5 \end{bmatrix}$$

3.21) $A \in \mathbb{R}^{4\times 5}$

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 3 & 3 & 1 \\ 2 & 2 & 4 & 2 & 0 \\ 2 & 3 & 5 & 4 & 1 \end{bmatrix}$$

a) comprobar que $b = \begin{bmatrix} 3 \\ 4 \\ 6 \\ 7 \end{bmatrix} \in col(A)$

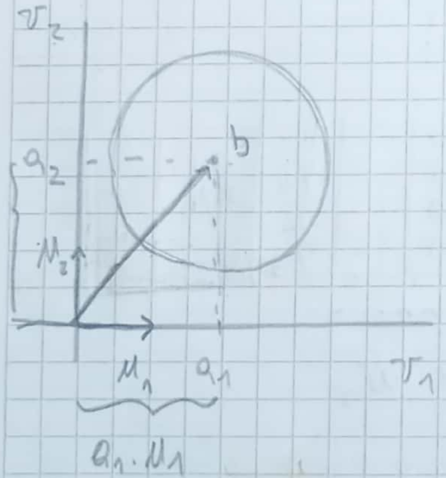
b) $\exists\ a_1, a_2 \in \mathbb{R}$ y $v_1, v_2 \in col(A)$ /

$\{y \in col(A) : d(b,y) = 1\} = \{(a_1 + \cos(\theta))\,\cancel{v_1}\,u_1 + (a_2 + sen(\theta))\,\cancel{v_2}\,u_2 : \theta \in [0, 2\pi]\}$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 2 & 1 & 0 & 3 \\ 1 & 2 & 3 & 3 & 1 & 4 \\ 2 & 2 & 4 & 2 & 0 & 6 \\ 2 & 3 & 5 & 4 & 1 & 7 \end{array}\right] \to \left[\begin{array}{ccccc|c} 1 & 1 & 2 & 1 & 0 & 3 \\ 0 & 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 0 & 1 & -1 & -1 & 2 \\ 0 & 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$Col(A) = gen\left\{ \underbrace{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}}_{v_1} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}}_{v_2} \right\}$ con $a_1' = 2$ $a_2' = 1$ tal que $a_1 u_1 + a_2 u_2 = b$

$$\begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \\ 7 \end{bmatrix} \checkmark$$



$u_1 = \frac{v_1}{\|v_1\|}$ $\quad \|v_1\| = \sqrt{1+1+4+4} = \sqrt{10}$

$\|v_2\| = \sqrt{1+4+4+9} = \sqrt{18}$

$u_2 = \frac{v_1}{\|v_2\|}$

$$\boxed{u_1 = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} \qquad u_2 = \frac{1}{\sqrt{18}} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} \qquad a_1 = 2\sqrt{10} \qquad a_2 = \sqrt{18}}$$

3.24) $A \in \mathbb{R}^{4\times 5}$ $A = \begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 1 & 3 & 5 & 2 & 1 \\ 2 & 3 & -5 & 10 & 0 \\ 2 & 4 & 0 & 8 & 1 \end{bmatrix}$

a) Hallar una Base ortogonal de $\mathbb{R}^4 \supseteq$ Base de col(A)

b) Hallar la Proyección ortogonal de $\mathbb{R}^4$ sobre col (A)

c) Calcular la distancia de $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ a col (A)

$$\begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 1 & 3 & 5 & 2 & 1 \\ 2 & 3 & -5 & 10 & 0 \\ 2 & 4 & 0 & 8 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 1 & 5 & -2 & 1 \\ 0 & -1 & -5 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 1 & 5 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -10 & 8 & 0 \\ 0 & 1 & 5 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$col(A) = gen\left\{ \underset{v_2}{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}} \underset{v_3}{\begin{bmatrix} 2 \\ 3 \\ 3 \\ 4 \end{bmatrix}} \cdot \underset{w_1}{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}} \right\}$$

Gramm-Schmidt

$$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\|w_1\|^2} w_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1/2 \\ 2 \\ 1/2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 \\ -1 \\ 4 \\ 1 \end{bmatrix}$$

$$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\|w_1\|^2} w_1 - \frac{\langle v_3, w_2 \rangle}{\|w_2\|^2} \cdot w_2$$

$$\begin{bmatrix} 2 \\ 3 \\ 3 \\ 4 \end{bmatrix} - \frac{7}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} - \frac{17}{11} \cdot \begin{bmatrix} 1 \\ -1/2 \\ 2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 5/11 \\ 3/11 \\ -1/11 \\ -3/11 \end{bmatrix} \longrightarrow \begin{bmatrix} 5 \\ 3 \\ -1 \\ 3 \end{bmatrix}$$

a) $B_{ON}(col(A)) = gen\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ -1 \\ -3 \end{bmatrix} \right\}$

b) $Nul(A^T) \perp Col(A)$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & 3 & 4 \\ 0 & 5 & -5 & 0 \\ 4 & 2 & 10 & 8 \\ 0 & 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -x_4$

$x_2 = -x_4$

$x_3 = -x_4$

$$\boxed{nul(A^T) = gen\left\{\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}\right\}}$$

$$P_{nul(A^T)}(\mathbb{R}_4) = \frac{\left\langle x, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}\right\rangle}{\underbrace{\left\langle \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}\right\rangle}_{4}} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = (x_1 - x_2 + x_3 + x_4)\begin{bmatrix} 1/4 \\ -1/4 \\ -1/4 \\ 1/4 \end{bmatrix}$$

$$[P_{Nul(A^T)}]^E_E = \frac{1}{4}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \sim [P_{Col(A)}]^E_E = I - [P_{Col(A)^\perp}]^E_E$$

$$[P_{col(A)}]^3_3 = \frac{1}{4}\left[\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\right] = \frac{1}{4}\begin{bmatrix} 3 & 1 & 1 & -1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ -1 & 1 & 1 & 3 \end{bmatrix}$$

c) $d(b, Col(A)) = \| P_{col(A)^\perp}(b) \| = \| b - P_{col(A)}(b) \|$

$$\frac{1}{4}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \to 0 \Rightarrow b \in Col(A) \longrightarrow \begin{bmatrix} -1+2 \\ -1+3-1 \\ -2+3 \\ -2+4-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$d(b, Col(A)) = 0$