$v \in S_1 + S_2$ o gen $\Pi \Rightarrow \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + 0 . \begin{bmatrix} 1 \\ 6 \\ 3 \\ 0 \end{bmatrix}$

completar con otro vector u

tal que u sea linealmente independiente a v

y $\notin S_1 + S_2 \rightarrow$ un canónico $\Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\Pi =$ gen $\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

no es único ya que

$v = a \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} - b \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + c \begin{bmatrix} 1 \\ 6 \\ 3 \\ 0 \end{bmatrix}$

$\Rightarrow$ ni canónico es único

Guía 2

2.2) Comprobar que $T\left(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\right) = [T(e_1) \ \ T(e_n)] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = e_1 x_1 + \dots + e_n x_n \rightarrow T(e_1 x_1 + \dots + e_n x_n)$

$= T(e_1 x_1) + \dots + T(e_n \cdot x_n)$

como $x_1 \dots x_n$ es un escalar $k \in \mathbb{R}$,

$x_1 . T(e_1) + \dots + x_n T(e_n) =$

$= \underbrace{[T(e_1) \ \dots \ T(e_n)]}_{A_T} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

$T(x) = A_T . x$

2.3) $T: \mathbb{R}^5 \to \mathbb{R}^5$ def. x:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + x_2 + x_3 - 2x_4 + x_5 \\ -x_1 + 3x_3 - 4x_4 + 2x_5 \\ -x_1 + 3x_3 - 5x_4 + 3x_5 \\ -x_1 + 3x_3 - 6x_4 + 4x_5 \\ -x_1 + 3x_3 - 6x_4 + 4x_5 \end{bmatrix}$$

a) $Nu(T)$

b) $Img(T)$

c) $T(x) = b$ con $b = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$

$$A_T = \begin{bmatrix} -1 & 1 & 1 & -2 & 1 \\ -1 & 0 & 3 & -4 & 2 \\ -1 & 0 & 3 & -5 & 3 \\ -1 & 0 & 3 & -6 & 4 \\ -1 & 0 & 3 & -6 & 4 \end{bmatrix}$$

a)

$$nul(A) = gen\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} = Nu(T)$$

b)

$$Col(A) = gen\left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -5 \\ -6 \\ -6 \end{bmatrix} \right\} = Img(T)$$

c) $b \in Img(T)$?

$$\underset{x_1}{-2}\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \underset{x_2}{-}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \underset{x_4}{0} \cdot \begin{bmatrix} -2 \\ -4 \\ -5 \\ -6 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \checkmark \to \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$$A_T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \to T^{-1}(w) = v_p + Nu(T)$$

$$T(v_p) = w$$

$$v_p = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow T(x) = b \Rightarrow \boxed{x = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + gen\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}}$$

Escaneado con CamScanner

2.4) Sea $T: \mathbb{P}_3[x] \to \mathbb{R}^3$

$T(p) = [\,p(0) \quad p(1) \quad p(2)\,]^T$

a) por qué T es una TL?

b) hallar $nu(T)$

c) para cada $j \in \mathbb{I}_3$, $T(p) = e_j$ admite solución → $T(p) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (...)

d) $T(p) = \begin{bmatrix} 3 \\ 6 \\ 36 \end{bmatrix}$?

$p = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad q = b_0 + b_1 x + b_2 x^2 + b_3 x^3$

a) Prop. 1) $T(p+q) = T(p) + T(q)$

2) $T(k.p) = k.T(p)$

$p + q = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3$

1) $T(p+q) = \begin{bmatrix} a_0 + b_0 \\ a_0 + b_0 + a_1 + b_1 + a_2 + b_2 + a_3 + b_3 \\ a_0 + b_0 + 2a_1 + 2b_1 + 4a_2 + 4b_2 + 8a_3 + 8b_3 \end{bmatrix} = \underbrace{\begin{bmatrix} a_0 \\ a_0 + a_1 + a_2 + a_3 \\ a_0 + 2a_1 + 4a_2 + 8a_3 \end{bmatrix}}_{T(p)} + \underbrace{\begin{bmatrix} b_0 \\ b_0 + b_1 + b_2 + b_3 \\ b_0 + 2b_1 + 4b_2 + 8b_3 \end{bmatrix}}_{T(q)}$ ✓

2) $k.p = k(a_0 + a_1 x + a_2 x^2 + a_3 x^3)$

$T(kp) = k\,T(p)$ ✓

b) $nu(T) \to T(v) = 0^{\mathbb{R}^3}$

$\begin{cases} a_0 = 0 \\ a_0 + a_1 + a_2 + a_3 = 0 \\ a_0 + 2a_1 + 4a_2 + 8a_3 = 0 \end{cases}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 4 & 8 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{array}\right] \to \begin{array}{l} a_0 = 0 \\ a_1 = 2a_3 \\ a_2 = -3a_3 \end{array}$

$\mathrm{Nu}(T) = \text{gen}\{2x - 3x^2 + x^3\}$

c) $T(P_1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $T(P_2) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $T(P_3) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

∘ $T^{-1}(w) = v + \mathrm{Nu}(T) \,/\, T(v) = w$

$P_1(0) = 1$
$P_1(1) = 0 \rightarrow (x-1)$ $\Rightarrow$ $a(x-1)(x-2) = P(x)$
$P_1(2) = 0 \rightarrow (x-2)$

$a(0-1)(0-2) = 1$

$a(2) = 1 \rightarrow a = \frac{1}{2}$

$P_1 = \frac{1}{2}(x-1)(x-2) \rightarrow \boxed{\mathrm{Sol}(e_1) = \underbrace{\frac{1}{2}(x-1)(x-2)}_{P\ \text{particular}} + \lambda(2x - 3x^2 + x^3)}$

$P_2(0) = 0$
$P_2(1) = 1 \rightarrow a(x-2)x = P_2(x)$
$P_2(2) = 0$

$\Rightarrow \boxed{\mathrm{Sol}(e_2) = -x(x-2) + \lambda(2x - 3x^2 + x^3)}$

$a(1-2) \cdot 1 = 1$

$a(-1) = 1 \rightarrow a = -1$

$P_3(0) = 0$
$P_3(1) = 0 \rightarrow a(x-1)x$
$P_3(2) = 1$

$a(2-1)2 = 1 \Rightarrow \boxed{\mathrm{Sol}(e_3) = \frac{1}{2}x(x-1) + \lambda(2x - 3x^2 + x^3)}$

$2a = 1 \rightarrow a = \frac{1}{2}$

d) $T(P) = \begin{bmatrix} 3 \\ 6 \\ 36 \end{bmatrix} \rightarrow$ $P(0) = 3$, $P(1) = 6$, $P(2) = 36$ $\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 1 & 1 & 1 & 1 & 6 \\ 1 & 2 & 4 & 8 & 36 \end{array}\right]$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 2 & 6 & 27 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 & -21/2 \\ 0 & 0 & 1 & 3 & 27/2 \end{array}\right] \rightarrow$ $a_0 = 3$, $a_1 = 2a_3 - \frac{21}{2}$, $a_2 = -3a_3 + \frac{27}{2}$

$\Rightarrow \boxed{\mathrm{Sol}\left(\begin{bmatrix} 3 \\ 6 \\ 36 \end{bmatrix}\right) = 3 - \frac{21}{2}x + \frac{27}{2}x^2 + \lambda(2x - 3x^2 + x^3)}$

P: $\boxed{a_3 = 0}$

2.5) $T: \mathbb{R}_3[x] \to \mathbb{R}_3[x]$

$$T(p) = p + (1-x)p'$$

a) -

b) Base de $Nu(T)$

c) Base de $Img(T)$

d) $q = 1 + x + x^2 - x^3 \in Img(T) \quad T(p) = q$

$p = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$p' = a_1 + 2a_2 x + 3a_3 x^2 \longrightarrow T(p) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$+ [(1-x)(a_1 + 2a_2 x + 3a_3 x^2)]$

$a_1 + 2a_2 x + 3a_3 x^2$

$-a_1 x - 2a_2 x^2 - 3a_3 x^3$

$T(p) = a_1 + a_0 +$

$(a_1 + 2a_2 - \cancel{a_1})x +$

$(a_2 + 3a_3 - \cancel{2}a_2)x^2$

$(a_3 - 3a_3)x^3$

$T(p) = (a_1 + a_0) + (2a_2)x + (3a_3 - a_2)x^2 + (-2a_3)x^3$

$$\begin{bmatrix} a_1 + a_0 & 2a_2 & 3a_3 - a_2 & -2a_3 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \end{bmatrix}$$

$a_1 + a_0 = 0 \quad 2a_2 = 0 \quad 3a_3 - a_2 = 0 \quad -2a_3 = 0$

$$\boxed{a_1 = -a_0 \quad a_2 = 0 \quad a_3 = 0 \quad a_3 = 0}$$

$nul(T) = a_0(1 - x) \Rightarrow gen\{1 - x\}$

c) Sea $T\ V \to W$ y $B = \{v_1, v_2, \dots v_n\}$ base de V, $Img(T) = gen\{T(B)\}$

Si extiendo B con $nu(T)$ obtengo $gen\{T(B)\}$ LI (no es necesario una base

$B = \{1 - x, x^3, x^2, 1\}$

$T(1-x) \rightarrow \begin{matrix} a_0 = 1 \\ a_1 = -1 \end{matrix} \Rightarrow T(1-x) = 0$

$T(x^3) \rightarrow a_3 = 1 \Rightarrow 3x^2 - 2x^3$

$T(x^2) \rightarrow a_2 = 1 \rightarrow 2x - x^2$

$T(1) \rightarrow a_0 = 1 \Rightarrow 1$

$\text{Img}(T) = \text{gen}\{3x^2 - 2x^3,\ 2x - x^2,\ 1\}$

d)

$q \in \text{Img}\,T?$ $\quad a(3x^2 - 2x^3) + b(2x - x^2) + c = 1 + x + x^2 - x^3$

$$\begin{cases} c = 1 \\ 2b = 1 \\ 3a - b = 1 \\ -2a = -1 \end{cases} \Rightarrow \begin{matrix} c = 1 \\ b = \frac{1}{2} \\ a = \frac{1}{2} \end{matrix} \quad \frac{3}{2} - \frac{1}{2} = 1 \checkmark$$

$q \in \text{Img}(T)$

$T(p) = q \Rightarrow T^{-1}(q) = p + \text{Nu}(T)$

$$\begin{cases} a_0 + a_1 = 1 \\ 2a_2 = 1 \\ -a_2 + 3a_3 = 1 \\ -2a_3 = -1 \end{cases} \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & -1 & 3 & 1 \\ 0 & 0 & 0 & -2 & -1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 6 & 3 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 1/2 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 \end{array}\right] \rightarrow \begin{matrix} a_3 = \frac{1}{2} \\ a_2 = \frac{1}{2} \\ a_0 = 1 - a_1 \end{matrix}$$

$p = 1 + \frac{1}{2}x^2 + \frac{1}{2}x^3$ con $a_1 = 0$ (particular)

$$\boxed{\text{sol}(T(p) = q) = 1 + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \underbrace{\lambda(-x+1)}_{\text{Nu}(T)}}$$

2.6) $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} b x_3 - x_2 \\ x_1 - a x_3 \\ a x_2 - b x_1 \end{bmatrix}$$

hallar $a, b \in \mathbb{R}$ tales que $Im(T) = gen\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

$Im(T) = gen\{T(E)\}$ siendo $E = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$

$$gen\left\{\begin{bmatrix} 0 \\ 1 \\ -b \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ a \end{bmatrix}, \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix}\right\} = gen\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$$

Para [illegible] $\to$ $b = 1$, $a = 1$ ¿ $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \in gen\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$?

$$-\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad = \quad \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \in gen\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$$

$$\boxed{b = 1 \qquad a = 1}$$

$$\begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix} \in Im(T) \;? \quad \left[\begin{array}{cc|c} 0 & -1 & 2 \\ 1 & 0 & 2 \\ -1 & 1 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right]$$

$$\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix} \quad \checkmark$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 - x_2 \\ x_1 - x_3 \\ x_2 - x_1 \end{bmatrix} \rightsquigarrow \begin{cases} x_3 - x_2 = 2 \\ x_1 - x_3 = 2 \\ x_2 - x_1 = -4 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 0 & -1 & 1 & 2 \\ 1 & 0 & -1 & 2 \\ -1 & 1 & 0 & -4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & -1 & 1 & 2 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & -1 & -2 \\ 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \to nul(T) = \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases} = gen\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$$

$$\to x_3 = 0 \Rightarrow \begin{matrix} x_1 = 2 \\ x_2 = -2 \end{matrix} \Rightarrow \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} = \text{Particular}$$

$W \Rightarrow T(w) = \begin{bmatrix} 2 \\ 2 \\ -4 \end{bmatrix} = \left| \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} + \text{gen}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \right|$

2.7) $T : \mathbb{V} \to \mathbb{W}$ $\quad V = \{ v_i ; i \in \mathbb{I}_n \} \subset \mathbb{V}$ $\quad$ n puntos de $\mathbb{V}$

comprobar que la imagen de la cápsula convexa de V por T es la cápsula convexa de la imagen de V por T. En símbolos, $T(C(V)) = C(T(V))$

$$C(V) = \sum_{i=1}^{n} p_i v_i \quad \text{con de} \quad \sum_{i=1}^{n} p_i = 1 \quad \text{con} \quad p_1, p_2 \ldots p_n \in \mathbb{R}^+$$

$$T(C(V)) = T\left(\sum_{i=1}^{n} p_i v_i \right) \quad \text{x prop} \quad T(kv) = k\,T(v) \text{ con } k \in \mathbb{R}$$

$$= \sum_{i=1}^{n} p_i\, T(v_i) = C(T(V))$$

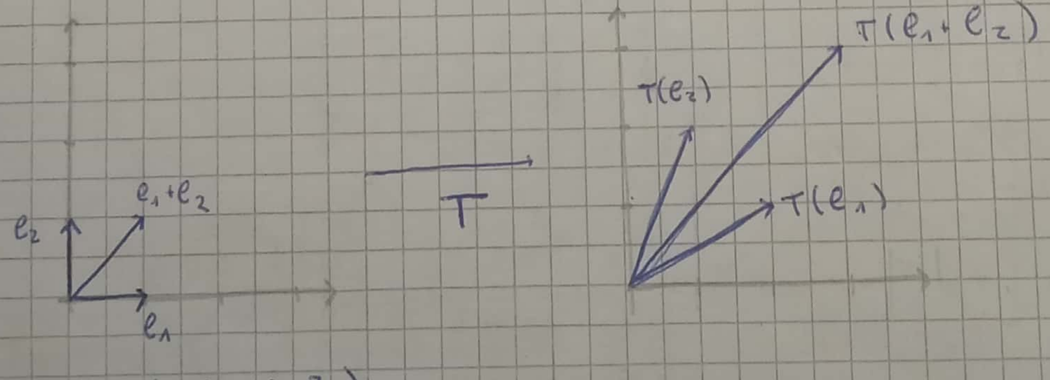
2.8) $T : \mathbb{R}^2 \to \mathbb{R}^2$ $\quad$ definida como $T(x) := A.x$ $\quad A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad$ hallar y graficar la Img por T de $R \subset \mathbb{R}^2$ definida por:

a) $R = \{ e_1, e_2, e_1 + e_2 \}$

$T(e_1) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

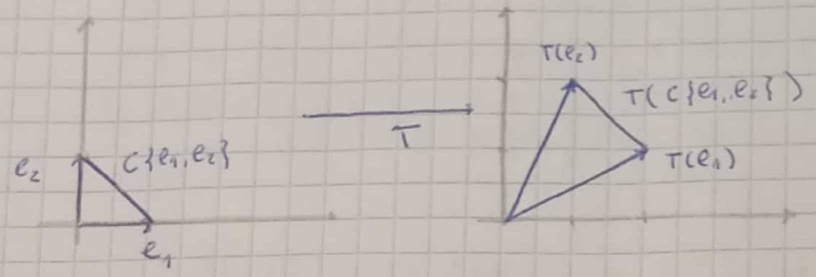
$T(e_1 + e_2) = T(e_1) + T(e_2) = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $\quad T(e_2) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$



b) $R = C(\{ e_1, e_2 \})$

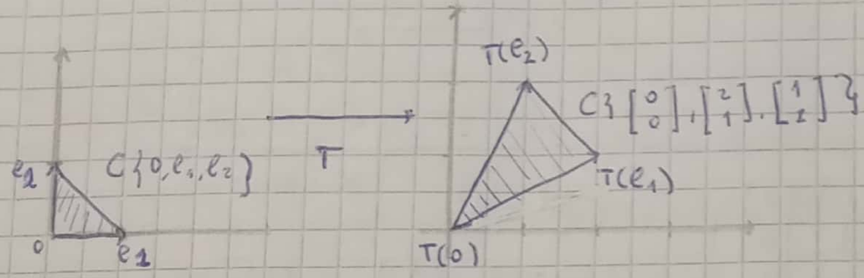
$C(\{e_1, e_2\}) = p_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + p_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad p_1, p_2 \in \mathbb{R}^+$ $\quad p_1 + p_2 = 1$

$T(R) = T(C\{e_1,e_2\}) = P_1\begin{bmatrix}2\\1\end{bmatrix} + P_2\begin{bmatrix}1\\2\end{bmatrix} = C\left\{\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}\right\}$



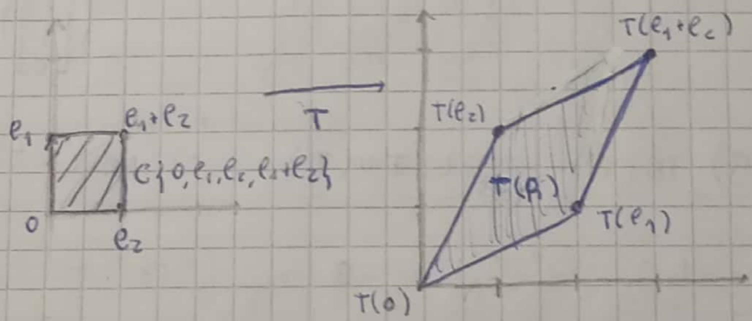
c) $R = C(\{0,e_1,e_2\})$

$T(R) = C\left(\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}\right\}\right) = P_1\begin{bmatrix}0\\0\end{bmatrix} + P_2\begin{bmatrix}2\\1\end{bmatrix} + P_3\begin{bmatrix}1\\2\end{bmatrix}$



d) $R = C(\{0,e_1,e_2,e_1+e_2\})$

$T(R) = C\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}3\\3\end{bmatrix}\right\}$

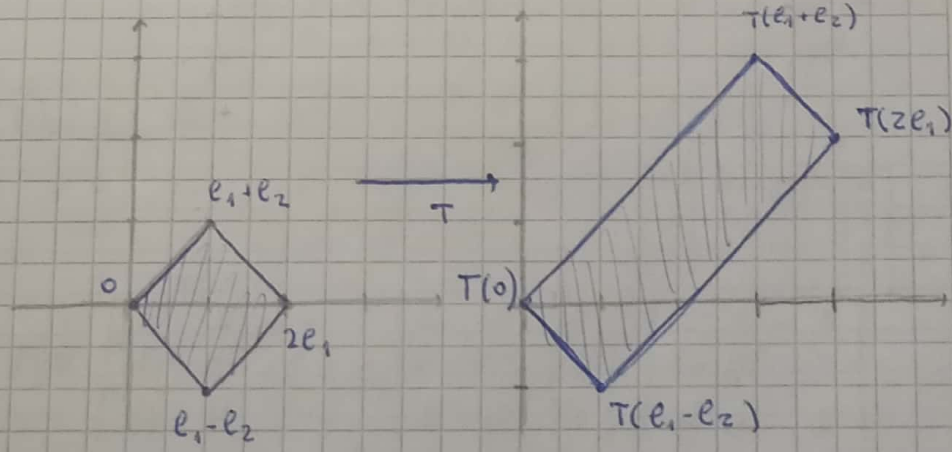


e) $R = C(\{0, e_1+e_2, e_1-e_2, 2e_1\})$

$e_1 - e_2 = \begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$

$T(e_1-e_2) = \begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$

$T(2e_1) = 2T(e_1) = \begin{bmatrix}4\\2\end{bmatrix}$



2.9) Hallar $a \in \mathbb{R}$ para los que existe $T: \mathbb{R}^3 \to \mathbb{R}^3$ tal que:

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\a\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}1\\0\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\end{bmatrix} \qquad T\left(\begin{bmatrix}-1\\-1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix} \qquad T\left(\begin{bmatrix}1\\-1\\-1\end{bmatrix}\right) = \begin{bmatrix}5\\1\\a^2\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1 & 1 & -1 & 1\\1 & 0 & -1 & -1\\1 & -1 & 0 & -1\end{array}\right] \to \left[\begin{array}{ccc|c}1 & 1 & -1 & 1\\0 & -1 & 0 & -2\\0 & -2 & 1 & -2\end{array}\right] \to \left[\begin{array}{ccc|c}1 & 0 & -1 & -1\\0 & 1 & 0 & 2\\0 & 0 & 1 & 2\end{array}\right] \to \left[\begin{array}{ccc|c}1 & 0 & 0 & 1\\0 & 1 & 0 & 2\\0 & 0 & 1 & 2\end{array}\right]$$

$$\begin{bmatrix}1\\1\\1\end{bmatrix} + \begin{bmatrix}2\\0\\-2\end{bmatrix} + \begin{bmatrix}-2\\-2\\0\end{bmatrix} = \begin{bmatrix}1\\-1\\-1\end{bmatrix} \checkmark \quad \to \quad \begin{bmatrix}1\\a\\1\end{bmatrix} + \begin{bmatrix}2\\0\\2\end{bmatrix} + \begin{bmatrix}2\\4\\6\end{bmatrix} = \begin{bmatrix}5\\1\\a^2\end{bmatrix}$$

$$\begin{cases}5 = 0\\ a + 4 = 1\\ 9 = a^2\end{cases} \Rightarrow \quad a = -3 \qquad (-3)^2 = 9 \checkmark \quad \to \quad \boxed{a = -3}$$

2.10) $B = \left\{ \underbrace{\begin{bmatrix}1\\1\\0\end{bmatrix}}_{B_1}, \underbrace{\begin{bmatrix}1\\-1\\0\end{bmatrix}}_{B_2}, \underbrace{\begin{bmatrix}0\\0\\1\end{bmatrix}}_{B_3} \right\}$ $\qquad T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\-3/2\\2\end{bmatrix} \qquad T\left(\begin{bmatrix}1\\-1\\0\end{bmatrix}\right) = \begin{bmatrix}-3\\9/2\\-6\end{bmatrix} \qquad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}2\\-3\\4\end{bmatrix}$$

a) $Nu(T)$ $\qquad$ b) $Im(T)$ $\qquad$ c) Hallar $T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right)$ y calcular $T\left(\begin{bmatrix}2\\3\\5\end{bmatrix}\right)$

$$[T(B_1)\ T(B_2)\ T(B_3)]\,[v]^B = [T(v)]^{\varepsilon} \qquad T_{\varepsilon}^{\varepsilon} \qquad T_{\varepsilon}^{B} \cdot M_{B}^{\varepsilon} = T_{\varepsilon}^{\varepsilon}$$

$$\begin{bmatrix}1 & -3 & 2\\-3/2 & 9/2 & -3\\2 & -6 & 4\end{bmatrix} \qquad \underbrace{\left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\1 & -1 & 0 & 0 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]}_{M_{\varepsilon}^{B}} \to \left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\0 & -2 & 0 & -1 & 1 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right] \to \left[\begin{array}{ccc|ccc}1 & 1 & 0 & 1 & 0 & 0\\0 & 1 & 0 & 1/2 & -1/2 & 0\\0 & 0 & 1 & 0 & 0 & 1\end{array}\right]$$

$$(M_{\varepsilon}^{B})^{-1} = M_{B}^{\varepsilon}$$

$$\begin{bmatrix}1 & -3 & 2\\-3/2 & 9/2 & -3\\2 & -6 & 4\end{bmatrix} \cdot \begin{bmatrix}1/2 & 1/2 & 0\\1/2 & -1/2 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-1 & 2 & 2\\3/2 & -3 & -3\\-2 & 4 & 4\end{bmatrix} = [T]_{\varepsilon}^{\varepsilon}$$

$$[T]_{\varepsilon}^{\varepsilon} \cdot \begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}1\\-3/2\\2\end{bmatrix} \checkmark \qquad [T]_{\varepsilon}^{\varepsilon} \cdot \begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}-3\\9/2\\-6\end{bmatrix} \checkmark \qquad [T]_{\varepsilon}^{\varepsilon} \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}2\\-3\\4\end{bmatrix} \checkmark$$

$$Nu(T) = Nul\left([T]^{E}_{E}\right) \rightarrow \begin{bmatrix} -1 & 2 & 2 \\ 3/2 & -3 & -3 \\ -2 & 4 & 4 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 2 \\ 1 & -2 & -2 \\ -2 & 4 & 4 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a_1 = 2a_2 + 2a_3 \rightarrow \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 2a_2 + 2a_3 \\ a_2 \\ a_3 \end{bmatrix} \Rightarrow \boxed{gen\left\{\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}}$$

$Nu(T)$

$$Im(T) = col(A) \rightarrow C_2 = C_3$$

$$C_2 = -2C_1 \Rightarrow \boxed{gen\left\{\begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}\right\}} \quad Im(T)$$

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -1 & 2 & 2 \\ 3/2 & -3 & -3 \\ -2 & 4 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_2 + 2x_3 - x_1 \\ 3/2\,x_1 - 3x_2 - 3x_3 \\ -2x_1 + 4x_2 + 4x_3 \end{bmatrix}$$

$$T\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} ? \quad \begin{matrix} 2.3 + 2.5 - 2 = 6 + 10 - 2 = 14 \\ 3 - 3.3 - 3.5 = 3 - 9 - 15 = -21 \\ -2.2 + 4.3 + 4.5 = -4 + 12 + 20 = 28 \end{matrix} \rightarrow \boxed{T\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} = \begin{bmatrix} 14 \\ -21 \\ 28 \end{bmatrix}}$$

2.11) $B = \left\{\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}_{v_2}, \underbrace{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}}_{v_3}\right\}$ $\quad T: \mathbb{R}^3 \rightarrow \mathbb{R}_2[x]$

Tal que:

$$T(v_1) = 1 - x \quad T(v_2) = 1 + x^2 \quad T(v_3) = x + x^2$$

$p = 2 + x + 3x^2 \in Img(T)$? $\quad$ siendo $E = [1, x, x^2]$

Determinar $T^{-1}(p)$

$$[T(v_1)]^E = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad [T(v_2)]^E = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad [T(v_3)]^E = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$[T]^E_B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad [T]^E_B \cdot [v]^B = [p]^E = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$Im(T) = gen\{T(v_1), T(v_2), T(v_3)\}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ \cancel{0} & \cancel{1} & \cancel{1} & \cancel{3} \end{array}\right] \Rightarrow \begin{matrix} x_1 = 2 - x_2 \\ x_3 = 3 - x_2 \end{matrix} \Rightarrow \underbrace{\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}}_{x_p} + \alpha \underbrace{\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}}_{nu(T)}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \Rightarrow \begin{cases} x_1 + x_2 = 2 \\ -x_1 + x_3 = 1 \\ x_2 + x_3 = 3 \end{cases}$$

en Base B $\quad x_2 \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

① $B_{Im(T)} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

$$(-x_2 + 2) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + (-x_2 + 3) \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$T^{-1}(1)$

$$\begin{bmatrix} -x_2 + 2 \\ x_2 - x_2 + 3 \\ x_2 + x_2 - 3 \end{bmatrix} = \begin{bmatrix} -x_2 + 2 \\ 3 \\ 2x_2 - 3 \end{bmatrix} \rightarrow \boxed{x_2 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}} \quad x_2 \in \mathbb{R}$$

en base E

otro método

$[T]^E_E = [T]^E_B \, M^B_E$

$(M^E_B)^{-1}$

$\begin{matrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}$

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 1 & 1/2 & -1/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

2.12) $B = \left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \right\}$ base de $\mathbb{R}^3$ y sea $T: \mathbb{R}^3 \to \mathbb{R}^3$

definida por $\quad T\left(\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \quad T\left(\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \quad T\left(\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

a) Hallar la Im por T de gen $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} \right\}$

b) Hallar la preimagen por T del subespacio $\left\{ y \in \mathbb{R}^3 : \begin{cases} y_1 - y_3 = 0 \\ y_1 + y_2 + y_3 = 0 \end{cases} \right\}$

$[T]^{\mathcal{E}}_{B} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ $\qquad M^{\mathcal{E}}_{B} = \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$

$[T]^{\mathcal{E}}_{B} \cdot M^{B}_{\mathcal{E}} = [T]^{\mathcal{E}}_{\mathcal{E}} \qquad M^{B}_{\mathcal{E}} = (M^{\mathcal{E}}_{B})^{-1}$

$$M^{B}_{\mathcal{E}} = \frac{1}{9}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\frac{1}{9}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix} = \frac{1}{9} \cdot \begin{bmatrix} 5 & 5 & -2 \\ -7 & 2 & 1 \\ 2 & -7 & 1 \end{bmatrix} = [T]^{\mathcal{E}}_{\mathcal{E}}$$

a) Im T de S_1: $\text{gen}\left\{ \overbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}^{v_1}, \overbrace{\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}}^{v_2} \right\} = \text{gen}\left\{ T\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, T\begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} \right\}$

v_1: $M^{B}_{\mathcal{E}} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = [v_1]^{B}$

$* \quad [v]^{\mathcal{E}} = [T(v)]^{B}$

v_2: $M^{B}_{\mathcal{E}} \cdot \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 9 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [v_2]^{B}$

v_1: $T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} + \cancel{0\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}} + \frac{1}{3}\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \frac{1}{9}\begin{bmatrix} 5 & 5 & -2 \\ -7 & 2 & 1 \\ 2 & -7 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$

$$\boxed{\text{Im}(T(S_1)) = \text{gen}\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}}$$

a)

$T\left(\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}\right) = \frac{1}{9}\begin{bmatrix} 5 & 5 & -2 \\ -7 & 2 & 1 \\ 2 & -7 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

v_2: $T\left(\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{matrix} y_1 = y_3 \\ y_2 = -2y_3 \end{matrix} \Rightarrow gen\left\{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right\}$

$T^{-1}(S_2) = T^{-1}\left(gen\left\{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right\}\right) \rightarrow gen\left\{T^{-1}\left(\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\right)\right\}$

$[T]_{\mathcal{E}}^{\mathcal{E}} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$\frac{1}{9}\left[\begin{array}{ccc|c} 5 & 5 & -2 & 1 \\ -7 & 2 & 1 & -2 \\ 2 & -7 & 1 & 1 \end{array}\right] \sim \frac{1}{9}\left[\begin{array}{ccc|c} 1 & 1 & -2/5 & 1/5 \\ -7 & 2 & 1 & -2 \\ 2 & -7 & 1 & 1 \end{array}\right] \sim \frac{1}{9}\left[\begin{array}{ccc|c} 1 & 1 & -2/5 & 1/5 \\ 0 & 9 & -9/5 & -3/5 \\ 0 & -9 & 9/5 & 3/5 \end{array}\right] =$

$\frac{1}{9}\left[\begin{array}{ccc|c} 1 & 1 & -2/5 & 1/5 \\ 0 & 1 & -1/5 & -1/15 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \frac{1}{9}\left[\begin{array}{ccc|c} 1 & 0 & -1/5 & 4/15 \\ 0 & 1 & -1/5 & -1/15 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1/5 & 36/15 \\ 0 & 1 & -1/5 & -9/15 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x_1 = 12/5 + 1/5\, x_3 \\ x_2 = -3/5 + 1/5\, x_3 \end{cases} \Rightarrow T^{-1}(S_2) = gen\left\{\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}\right\} + \begin{bmatrix} 12/5 \\ -3/5 \\ 0 \end{bmatrix}$

2.14) $Nul(A^T)$

MARATÓN 22/10

0 1 2 3

$$B = \left\{ \frac{(x-1)(x-2)(x-3)}{-6}, \frac{x(x-2)(x-3)}{2}, \frac{x(x-1)(x-2)}{6}, -\frac{1}{2} x(x-1)(x-3) \right\}$$

M^E_B M^B_E canne $E = \{1, x, x^2, x^3\}$

$$M^B_E = \{ [1]^B \quad [x]^B \quad [x^2]^B \quad [x^3]^B \}$$

$$[p]^B = \begin{bmatrix} p(0) \\ p(1) \\ p(3) \\ p(2) \end{bmatrix} \rightarrow M^B_E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \\ 1 & 2 & 4 & 8 \end{bmatrix}$$

$$(M^B_E)^{-1} = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 3 & 9 & 27 & 0 & 0 & 1 & 0 \\ 1 & 2 & 4 & 8 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 3 & 9 & 27 & -1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 8 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 6 & 24 & -1 & -3 & 1 & 0 \\ 0 & 0 & 2 & 6 & -1 & -2 & 0 & 1 \end{array}\right] \sim \left[\dots \right] \sim M^E_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 11/6 & 3 & 11/3 & -3/2 \\ 1 & -5/2 & -1/2 & 2 \\ 1/6 & 1/2 & 1/6 & -1/2 \end{bmatrix}$$

$$(x^2 - 2x - x + 2)(x-3) = (x^3 - 3x^2 - 2x^2 + 6x - x^2 + 3x - 6)$$

$$-\frac{1}{6}(x^3 - 7x^2 + 8x - 6)$$

$$[p_1]^E = \left[1 \quad -\frac{3}{2} \quad \frac{7}{6} \quad -\frac{1}{6} \right]$$

$$(x^2 - 2x)(x-3) = x^3 - 3x^2 - 2x^2 + 6x$$

...

Sea $T : \mathbb{R}_3[x] \longrightarrow \mathbb{R}^2$

$$[T]_B^E = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 2 & 3 & -1 \end{bmatrix}$$

hallar las coordenadas [illegible] [illegible] de un [illegible] del $Nu(T)$)

$$Nu\left([T]_B^E\right) = \left[Nu(T)\right]^B$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{array}\right] \longrightarrow$$

$$x_1 = -2x_2 - 3x_3 \qquad x_4 = 0$$

$$\text{gen}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -11/6 & 3 & 1/3 & -7/2 \\ 1 & -3/2 & -1/2 & 0 \\ 1/6 & 1/3 & 1/6 & -1/2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2/3 \\ -9/2 \\ -7/6 \end{bmatrix}^E$$

$$\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -11/6 \\ -3/2 \\ -1/3 \end{bmatrix}^E$$

$$Nu(T) = \text{gen}\left\{ \begin{bmatrix} 3 \\ 7/3 \\ 7/2 \\ -11/6 \end{bmatrix}, \begin{bmatrix} 3 \\ 11/6 \\ 7/2 \\ 1/3 \end{bmatrix} \right\}$$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \qquad col(A) = \text{gen}\left\{ \underset{v_1}{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}}, \underset{v_3}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}} \right\}$$

base ortonormal (gramm - schimidt)

$$w_1 = v_1$$

$$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\|w_1\|^2} w_1$$

$$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\|w_1\|^2} w_1 - \frac{\langle v_3, w_2 \rangle}{\|w_2\|^2} w_2$$

$$w_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad w_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} - \frac{\left\langle \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle}{\underbrace{\left\langle \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle}_{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1 \\ 1/2 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\tilde{w}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$w_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{\overbrace{\left\langle \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle}^{0}}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{\overbrace{\left\langle \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\rangle}^{0}}{\underbrace{\left\langle \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\rangle}_{6}} \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$M = \left\{ \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$P_{col(A)}(x) = \langle x, u_1 \rangle u_1 + \langle x, u_2 \rangle u_2 + \langle x, u_3 \rangle u_3$$

$$= u_1^T x \, u_1 \ldots$$

$$= u_1 u_1^T x \ldots$$

$$= \left[u_1 u_1^T + u_2 u_2^T + u_3 u_3^T \right] x$$

$$\ldots = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 & 0 \\ -2 & 4 & 2 & 0 \\ 2 & 2 & 4 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$T = \begin{bmatrix} a & \vdots & \vdots \\ \vdots & a^2 & \vdots \\ \vdots & a+ & \ldots \end{bmatrix}$ u, v, w hallar $a \in \mathbb{R}$ tal que

area de la $C(u, v, w)$ por la Im de T sea X

Hallar una $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^4$

tal que $Nu(T) = \left\{ x \in \mathbb{R}^4 : \begin{array}{l} x_1 + x_2 - x_3 - x_4 = 0 \\ 2x_1 - x_2 + 2x_3 + x_4 = 0 \end{array} \right\}$

$$Im(T) = gen\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Hallar la imagen por T del triángulo de vértices

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}$$

$$Nu(T) : \begin{bmatrix} 1 & 1 & -1 & -1 \\ 2 & -1 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & -3 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1/3 & 0 \\ 0 & -1 & 4/3 & 1 \end{bmatrix}$$

$x_1 = -\frac{1}{3} x_3$

$x_2 = \frac{4}{3} x_3 + 1 x_4$

Extender una base de $\mathbb{R}^4$

$$B = \left\{ \begin{bmatrix} -1 \\ 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$w_1 \quad w_2 \quad w_3 \quad w_4$

$$gen\left\{ \begin{bmatrix} -1/3 \\ 4/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\} = gen\left\{ \begin{bmatrix} -1 \\ 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$T(w_1) = 0_{\mathbb{R}^4}$

$T(w_2) = 0_{\mathbb{R}^4}$

$T(w_3) = [1\ 2\ 1\ 2]^T$ $\rightarrow Im(T \circ T) = gen\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} \right\}$??

$T(w_4) = [1\ 1\ 1\ 1]^T$

$$[T]_B^E = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

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$T: \mathbb{R}_2[x] \to \mathbb{R}^3$ $\quad [T]_B^{\mathcal{C}} = \begin{bmatrix} 0 & 9 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{bmatrix}$ $\quad$ Hallar la sol. $T(p) = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$

$B = \{1+x,\ 1-x,\ 1-x+x^2\}$ $\quad \mathcal{C} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \right\}$

$M_B^{\mathbb{R}_2[x]} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad M_C^{\mathcal{E}} = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

$$[T]_{\mathbb{R}_2[x]}^{\mathcal{E}} = M_C^{\mathcal{E}} \cdot [T]_B^C \cdot M_{\mathbb{R}_2[x]}^B = M_C^{\mathcal{E}} [T]_B^C \cdot \left(M_B^{\mathbb{R}_2[x]}\right)^{-1}$$

$$[T]_{\mathbb{R}_2[x]}^{\mathcal{E}} = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 9 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -9 & 0 \\ 0 & 18 & -9 \\ 0 & 18 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -9/2 & 9/2 & 9 \\ 9 & -9 & -27 \\ 9 & -9 & -9 \end{bmatrix}$$

$$T(p) = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix} \to \begin{bmatrix} -9/2 & 9/2 & 9 \\ 9 & -9 & -27 \\ 9 & -9 & -9 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^{\mathbb{R}_2[x]} = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -9/2 & 9/2 & 9 & 0 \\ 9 & -9 & -27 & 6 \\ 9 & -9 & -9 & -6 \end{array}\right] \underset{G-J}{\rightsquigarrow} \left[\begin{array}{ccc|c} -9/2 & 9/2 & 9 & 0 \\ 0 & 0 & -9 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{cases} x_1 = -\frac{4}{3} + x_2 \\ x_3 = -\frac{2}{3} \end{cases}$$

$$\text{sol: } \begin{bmatrix} -4/3 \\ 0 \\ -2/3 \end{bmatrix}^{\mathbb{R}_2[x]} + \text{gen} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^{\mathbb{R}_2[x]}$$

$$\boxed{p \,/\, T(p) = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix} : -\frac{4}{3} - \frac{2}{3}x^2 + \underbrace{\text{gen}\{1+x\}}_{Nu(T)}}$$

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$$T(p) = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix} \qquad \text{Co}, \qquad [T]_B^C = \begin{bmatrix} 0 & 9 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{bmatrix}$$

$$C = \left\{ \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \right\} \qquad B = \{ 1+x,\ 1-x,\ 1-x+x^2 \}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & 2 & 0 \\ 2 & -1 & 2 & 6 \\ 2 & 2 & -1 & -6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 3 & 6 & 6 \\ 0 & 6 & 3 & -6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 1 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right] \qquad \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$$

$$\times 1 \qquad \begin{bmatrix} -4+4 \\ 2+4 \\ -4-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 0 & 9 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix} \quad \text{R}, \quad \begin{cases} 9x_2 = 0 \\ 3x_3 = -2 \\ -3x_3 = 2 \end{cases} \Rightarrow \begin{cases} x_2 = 0 \\ x_3 = -\frac{2}{3} \end{cases}$$

$$[x_p]^B = \begin{bmatrix} 0 \\ 0 \\ -\frac{2}{3} \end{bmatrix} + \text{gen} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \boxed{x_p = -\frac{2}{3} + \frac{2}{3}x - \frac{2}{3}x^2 + \text{gen}\{1+x\}}$$

$$-\frac{2}{3}(1+x)$$

$$\boxed{x_p = -\frac{4}{3} - \frac{2}{3}x^2 + \text{gen}\{1+x\}}$$