Guia 2

✓		1	A B C ✓
		2	✓ ?
✓	■	3	A B C ✓
		4	A B C D
	◇	5	A B C D
?	■	6	?
		7	
✓?	■	8	A B C D E ✓ ?
	◇	9	
✓		10	A B ✓ C ?
		11	
✓	■	12	A ✓ B ×
		13	A B C
		14	A B C D
		15	
		16	
✓ ?	■	17	A B C D ✓? E ?
✓ ?	■	18	A B ✓?
		19	
?	■	20	A ✓ B ?
		21	
		22	
		23	A B C D E

→ Proyecciones y simetrías

24 A B
25 A B C D E F G H I
26 A B C
27 A B C D E

NOTA

HOJA N°

FECHA

1) $T_1 : \mathbb{R}^3 \to \mathbb{R}$ $\quad T_1\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right) = -3x_2 + 2x_3$

A) $T(x) + T(y) = T(x+y)$

$T(x) \cdot a = T(x \cdot a)$

$$(-3x_2 + 2x_3) + (-3y_2 + 2y_3) = -3(x_2 + y_2) + 2(x_3 + y_3)$$

$$-3x_2 + 2x_3 - 3y_2 + 2y_3 = -3x_2 - 3y_2 + 2x_3 + 2y_3 \quad \checkmark$$

$$a(-3x_2 + 2x_3) = -3(a \cdot x_2) + 2(a \cdot x_3)$$

$$-3ax_2 + 2ax_2 = -3ax_2 + 2ax_3 \quad \checkmark$$

$\mathbb{R}^3 \to \mathbb{R}^2$

B) $T_2\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right) = \begin{pmatrix} -3x_2 + 2x_3 \\ 3x_1 - x_3 \end{pmatrix}$

$$\begin{pmatrix} -3x_2 + 2x_3 + (-3y_2 + 2y_3) \\ 3x_1 - x_3 + (3y_1 - y_3) \end{pmatrix} = \begin{pmatrix} -3(x_2 + y_2) + 2(x_3 + y_3) \\ 3(x_1 + y_1) - (x_3 + y_3) \end{pmatrix}$$

$$\begin{pmatrix} -3x_2 - 3y_2 + 2x_3 + 2y_3 \\ 3x_1 + 3y_1 - x_3 - y_3 \end{pmatrix} = \begin{pmatrix} -3x_2 - 3y_2 + 2x_3 + 2y_3 \\ 3x_1 + 3y_1 - x_3 - y_3 \end{pmatrix} \quad \checkmark$$

$$a\begin{pmatrix} -3x_2 + 2x_3 \\ 3x_1 - x_3 \end{pmatrix} = \begin{pmatrix} -3(a \cdot x_2) + 2(a \cdot x_3) \\ 3(x_1 \cdot a) - (x_3 \cdot a) \end{pmatrix}$$

$$\begin{pmatrix} -3 \cdot a \cdot x_2 + 2 \cdot a \cdot x_3 \\ 3 \cdot a \cdot x_1 - a \cdot x_3 \end{pmatrix} = \begin{pmatrix} -3 \cdot a \cdot x_2 + 2 \cdot a \cdot x_3 \\ 3 \cdot a \cdot x_1 - a \cdot x_3 \end{pmatrix} \quad \checkmark$$

NOTA

c) $\mathbb{R}^3 \to \mathbb{R}^3$

$$T_3\left(\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\right) = \begin{pmatrix}-3x_2+2x_3\\3x_1-x_3\\-2x_1+x_2\end{pmatrix}$$

$$\begin{pmatrix}-3x_2+2x_3\\3x_1-x_3\\-2x_1+x_2\end{pmatrix} + \begin{pmatrix}-3y_2+2y_3\\3y_1-y_3\\-2y_1+y_2\end{pmatrix} = \begin{pmatrix}-3(x_2+y_2)+2(x_3+y_3)\\3(x_1+y_1)-(x_3+y_3)\\-2(x_1+y_1)+(x_2+y_2)\end{pmatrix}$$

$$\begin{pmatrix}-3x_2-3y_2+2x_3+2y_3\\3x_1+3y_1-x_3-y_3\\-2x_1-2y_1+x_2+y_2\end{pmatrix} = \begin{pmatrix}-3x_2-3y_2+2x_3+2y_3\\3x_1+3y_1-x_3-y_3\\-2x_1-2y_1+x_2+y_2\end{pmatrix} \checkmark$$

$$a\begin{pmatrix}-3x_2+2x_3\\3x_1-x_3\\-2x_1+x_2\end{pmatrix} = \begin{pmatrix}-3(a.x_2)+2(a.x_3)\\3(a.x_1)-(a.x_3)\\-2(a.x_1)+(a.x_2)\end{pmatrix}$$

$$\begin{pmatrix}-3.a.x_2+2.a.x_3\\3.a.x_1-a.x_3\\-2.a.x_1+a.x_2\end{pmatrix} = \begin{pmatrix}-3.a.x_2+2.a.x_3\\3.a.x_1-a.x_3\\-2.a.x_1+a.x_2\end{pmatrix} \checkmark$$

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2) Ejemplo $k = n = \cancel{4}\,3$ (2.1B)

$$T\left(\begin{pmatrix}x_1\\x_2\\x_3\\ \cancel{x_4}\end{pmatrix}\right) = \left(\underbrace{\begin{pmatrix}T(1)\\T(0)\\T(0)\end{pmatrix}\begin{pmatrix}T(0)\\T(1)\\T(0)\end{pmatrix}\begin{pmatrix}T(0)\\T(0)\\T(1)\end{pmatrix}}_{3\times3}\right)\underbrace{\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}}_{3\times1} \quad 3\times1$$

$$A_T = \begin{pmatrix}T(1) & T(0) & T(0)\\T(0) & T(1) & T(0)\\T(0) & T(0) & T(1)\end{pmatrix} \quad ?$$

$\tilde{B}$) 2.1.A)

$$\begin{pmatrix}0 & -3 & 2\end{pmatrix}\cdot\underbrace{\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}}_{3\times1} = -3x_2+2x_3$$

2.1.B)

$$\underbrace{\begin{pmatrix}0 & -3 & 2\\3 & 0 & -1\end{pmatrix}}_{2\times3}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}-3x_2+2x_3\\3x_1-x_3\end{pmatrix}$$

NOTA 2.1.C)

$$\begin{pmatrix}0 & -3 & 2\\3 & 0 & -1\\-2 & 1 & 0\end{pmatrix}\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}-3x_2+2x_3\\3x_1-x_3\\-2x_1+x_2\end{pmatrix}$$

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2.3) $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -x_1 + x_2 + x_3 - 2x_4 + x_5 \\ -x_1 + 3x_3 - 4x_4 + 2x_5 \\ -x_1 + 3x_3 - 5x_4 + 3x_5 \\ -x_1 + 3x_3 - 6x_4 + 4x_5 \\ -x_1 + 3x_3 - 6x_4 + 4x_5 \end{pmatrix}$

$$\begin{pmatrix} -1 & 1 & 1 & -2 & 1 \\ -1 & 0 & 3 & -4 & 2 \\ -1 & 0 & 3 & -5 & 3 \\ -1 & 0 & 3 & -6 & 4 \\ -1 & 0 & 3 & -6 & 4 \end{pmatrix} \begin{matrix} \\ -F_1 \\ -F_1 \\ -F_1 \\ -F_1 \end{matrix} \quad \begin{pmatrix} -1 & 1 & 1 & -2 & 1 \\ 0 & -1 & 2 & -2 & 1 \\ 0 & -1 & 2 & -3 & 2 \\ 0 & -1 & 2 & -4 & 3 \\ 0 & -1 & 2 & -4 & 3 \end{pmatrix} \begin{matrix} \\ \\ -F_2 \\ -F_2 \\ -F_2 \end{matrix}$$

$$\begin{pmatrix} -1 & 1 & 1 & -2 & 1 \\ 0 & -1 & 2 & -2 & 1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & -2 & 2 \end{pmatrix} \quad \begin{matrix} -x_1 + x_2 + x_3 - 2x_4 + x_5 \\ -x_2 + 2x_3 - 2x_4 + x_5 \\ -x_4 + x_5 \end{matrix} \quad A$$

~~5~~ $\mathbb{R}^5 \to \mathbb{R}^5$ Base $\mathbb{R}^5$ = $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$\mathrm{Im}(T) = \mathrm{gen}\{T(v_1), T(v_2) \dots\}$

$T\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 + 0 \dots \\ -1 + 0 \dots \\ -1 + 0 \dots \\ -1 + 0 \dots \\ -1 + 0 \dots \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$ $\quad T\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$T\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \\ 3 \\ 3 \end{pmatrix}$ $\quad T\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ -5 \\ -6 \\ -6 \end{pmatrix}$ $\quad T\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 4 \end{pmatrix}$

~~Im~~ Base $(\mathrm{Im}_{(T)}) = \left\{ \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ -4 \\ -5 \\ -6 \\ -6 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 4 \end{pmatrix} \right\}$

Esto no es L.I.

(*)

NOTA

Nucleo: $\{x \in \mathbb{R}^5 \,/\, T(x) = 0_{\mathbb{R}^5}\}$ $\qquad T(x) = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}$

(A)
$$-x_1 + x_2 + x_3 - 2x_4 + x_5 = 0$$
$$-x_2 + 2x_3 - 2x_4 + x_5 = 0$$
$$-x_4 + x_5 = 0 \Rightarrow [x_4 = x_5]$$

$$-x_2 + 2x_3 - 2x_4 + x_4 = 0 \Rightarrow [2x_3 - x_4 = x_2]$$
$$-x_1 + (2x_3 - x_4) + x_3 - 2x_4 + x_4 = 0$$
$$-x_1 + 3x_3 - 2x_4 = 0 \Rightarrow [3x_3 - 2x_4 = x_1]$$

$$\begin{pmatrix}3x_3 - 2x_4\\ 2x_3 - x_4\\ x_3\\ x_4\\ x_4\end{pmatrix} \longrightarrow \text{Base Nucleo}(T) = \left\{\begin{pmatrix}3\\2\\1\\0\\0\end{pmatrix}\begin{pmatrix}-2\\-1\\0\\1\\1\end{pmatrix}\right\} \checkmark$$

~~C) $b = \begin{pmatrix}1\\2\\2\\2\\2\end{pmatrix}$ ¿ $b \in Im(T)$?~~

Base $Im(T)$ =

(x) Algoritmo columnas ...
$$\begin{pmatrix}-1 & 1 & 1 & -2 & 1\\ 0 & -1 & 2 & -2 & 1\\ 0 & 0 & 0 & -1 & 1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix} \longrightarrow \left\{\begin{pmatrix}-1\\-1\\-1\\-1\\-1\end{pmatrix}, \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} \begin{pmatrix}-2\\-4\\-5\\-6\\-6\end{pmatrix}\right\}$$

C) $b = \begin{pmatrix}1\\2\\2\\2\\2\end{pmatrix}$ ¿ $b \in Im(T)$?
$$\begin{array}{c} \left(\begin{array}{ccc|c}-1 & 1 & -2 & 1\\ -1 & 0 & -4 & 2\\ -1 & 0 & -5 & 2\\ -1 & 0 & -6 & 2\\ -1 & 0 & -6 & 2\end{array}\right) \end{array}\begin{array}{l} \\ -F_1\\ -F_1\\ -F_1\\ -F_1\end{array}$$

$$\begin{pmatrix}-1 & 1 & -2 & 1\\ 0 & -1 & -4 & 1\\ 0 & -1 & -5 & 1\\ 0 & -1 & -6 & 1\\ 0 & -1 & -6 & 1\end{pmatrix}\begin{array}{l} \\ \\ -F_2\\ -F_2\\ -F_2\end{array} \quad \begin{pmatrix}-1 & 1 & 0 & 1\\ 0 & -1 & 0 & 1\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \rightarrow \begin{array}{l} -a_1 + a_2 = 1\\ -a_2 = 1\\ -a_3 = 0\end{array}$$

$$-a_1 - 1 = 1 \Rightarrow a_1 = -2$$

Calcul vector & coord

$$-2\begin{pmatrix}-1\\-1\\-1\\-1\\-1\end{pmatrix} - 1\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} + 0 \ldots = \begin{pmatrix}1\\2\\2\\2\\2\end{pmatrix} \checkmark$$
$a_1 \qquad a_2 \qquad a_3$

NOTA

$\mathbb{R}^3 \to \mathbb{R}^3$

2.6) $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b\,x_3 - x_2 \\ x_1 - a\,x_3 \\ a\,x_2 - b\,x_1 \end{pmatrix}$

$\mathrm{Im}(T) = \mathrm{gen}\left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right] \qquad y = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}$

$\left(\begin{array}{cc|c} 0 & -1 & 2 \\ 1 & 0 & 2 \\ -1 & 1 & 4 \end{array}\right)_{+F_2} \Big\} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & -1 & 2 \end{pmatrix}_{+F_2} \Big\} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 8 \end{array}\right)$ INCOMPATIBLE

$\downarrow$

$-4!$

$\left(\begin{array}{cc|c} 0 & -1 & 2 \\ 1 & 0 & 2 \\ -1 & 1 & -4 \end{array}\right) \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ -1 & 1 & -4 \end{pmatrix}_{+F_1} \Big\} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ \cancel{0} & \cancel{1} & \cancel{-2} \end{pmatrix}$

$a_1 = 2$
$a_2 = -2 \quad \checkmark \to y \in \mathrm{Im}(T).$

$T(x) = y.$

Matriz: $\begin{pmatrix} 0 & -1 & b \\ 1 & 0 & -a \\ -1 & a & 0 \end{pmatrix} \to \begin{pmatrix} 0 & -1 & b \\ 1 & 0 & -a \\ -1 & a & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}$

$\downarrow$

¿ $\mathrm{gen}\left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ a \end{pmatrix} \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}\right]$?

$\to$ Buscamos a, b pra que este conjunto sea igual a $\mathrm{gen}\left\{\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$

$a = 1 \to \left\{\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} b \\ -1 \\ 0 \end{pmatrix}\right\} \quad -1\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + 1\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \underset{\downarrow}{=} \begin{pmatrix} b \\ -1 \\ 0 \end{pmatrix}$

si $b = -1$

Entonces la matriz es:
$$\begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ +1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 0 & -1 & -1 & 2 \\ 1 & 0 & -1 & 2 \\ +1 & 1 & 0 & -4 \end{array}\right) - F_2 \Big\} \left(\begin{array}{cccc} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 6 \\ 0 & 1 & -1 & 2 \end{array}\right) - F_2 \Big\}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & -2 & 8 \end{array}\right) \rightarrow -2x_3 = 8 \rightarrow x_3 = -4$$

$$x_2 + 4 = 6 \rightarrow x_2 = 2 \qquad x = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$$

$$x_1 + 4 = 2 \rightarrow x_1 = -2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 1 & 1 & 0 & -4 \\ 0 & -1 & -1 & 2 \end{array}\right) - F_2 \Big\} \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -6 \\ 0 & -1 & -1 & 2 \end{pmatrix} + F_2 \Big\} \begin{matrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & 0 & \end{matrix}$$

NOTA

2.8) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \qquad T(x) = A \cdot x$

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \longrightarrow T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ x_1 + 2 \cdot x_2 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A) Conjunto $R = \{ e_1, e_2, e_1 + e_2 \}$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

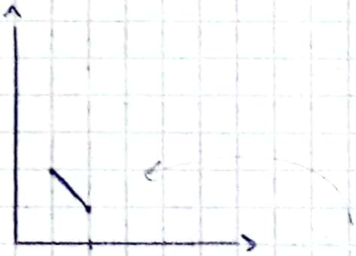
B) $R = C(\{e_1, e_2\})$

$T(x) \longrightarrow x = \lambda_1 \cdot e_1 + \lambda_2 \cdot e_2$

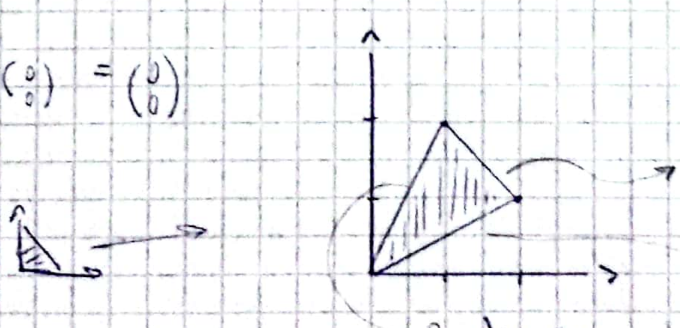
$\lambda_1 + \lambda_2 = 1$

$\lambda_1, \lambda_2 \geq 0$

Con estas condiciones sobre $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ construyo esta recta en la imagen



C) $T\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$



$T(x) = \lambda_1 \cdot e_1 + \lambda_2 \cdot e_2$

$\lambda_1 + \lambda_2 = 1$

$\lambda_1, \lambda_2 \geq 0$

$\lambda_2 \cdot e_2$

$0 \leq \lambda_2 \leq 1$

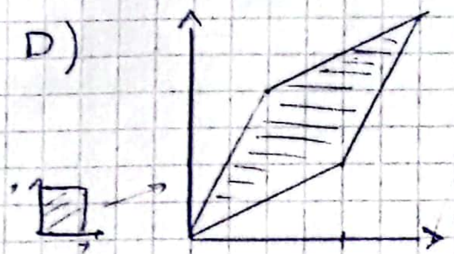
$\lambda_1 \cdot e_1$

$0 \leq \lambda_1 \leq 1$

$$\begin{cases} T(x) = \lambda_1 \cdot e_1 + \lambda_2 \cdot e_2 \\ 0 \leq \lambda_1 + \lambda_2 \leq 1 \end{cases}$$

✓?

D)



$R = C(\{0, e_1, e_2, e_1+e_2\})$

$\{T(x) = \lambda_1 . e_1 + \lambda_2 . e_2 \;/\; 0 \leq \lambda_1 , \lambda_2 \leq 1\}$

✓?

NOTA

2.10) B base $\mathbb{R}_3$: $B = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

$T: \mathbb{R}^3 \to \mathbb{R}^3 \; / \; T \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -3/2 \\ 2 \end{pmatrix}$

$T \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 9/2 \\ -6 \end{pmatrix} \quad T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$

Si la base es $B = \{v_1, v_2 \dots v_n\}$ entonces ~~Base~~ span de la imagen de T es $\{T(v_1), T(v_2) \dots T(v_n)\}$

$\text{~~Base~~}(Im(T))_{span} = \left\{ \begin{pmatrix} 1 \\ -3/2 \\ 2 \end{pmatrix} \begin{pmatrix} -3 \\ 9/2 \\ -6 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right\} \longrightarrow$ ¡Es L.D.!

1

No es base

Sea $T: V \to W$
Base B, Base C $\Big\}$ $[T(v)]^C = [T]^C_B \, [v]^B$

Las columnas de la matriz $[T]^C_B$ son las coordenadas de la base C de los transformados de los vectores de la base B.

Nucleo ~~Im~~ $T(x) = 0$ $\begin{pmatrix} 1 & -3 & 2 \\ -3/2 & 9/2 & -3 \\ 2 & -6 & 4 \end{pmatrix} \begin{matrix} \\ \times 2 \\ -2F_1 \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$\begin{pmatrix} 1 & -3 & 2 \\ -3 & 9 & -6 \\ 0 & 0 & 0 \end{pmatrix} + 3F_2 \Big\} \begin{pmatrix} 1 & -3 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to a - 3b + 2c = 0$

$a = 3b - 2c$

$(3b - 2c) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Big\}$ $\begin{matrix} 3b - 2c + b = 0 \\ 3b - 2c - b = 0 \end{matrix}$

$b \cdot \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} + c \cdot \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} \to Nu(T) = \left\{ \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} \right\}$

$T: V \to W$

$Dim(N_u(T)) + Dim(Im(T)) = Dim(W)$ $\quad 2 + X = 3$

$Dim(Im(T)) = 1.$

① Es L.D. $\longrightarrow$ 1 elemento es base de $Im(T)$.

$Base(Im(T)) = \left\{ \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right\}$

c) $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$

NOTA

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FECHA

(?) $B = \left\{ \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \right\}$

$T \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ $\quad T \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ $\quad T \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

Imagen (T) para gen $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} \right\}$

- Consigamos vector $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}^B \longrightarrow \left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 2 & 1 & -2 & 0 \\ 1 & 2 & 2 & 1 \end{array} \right) \begin{matrix} \\ -2F_1 \\ 2(-F_1) \end{matrix}$

$\left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 0 & 5 & -4 & -2 \\ 0 & 6 & 3 & 1 \end{array} \right) \times 5 - (6.F_2) \quad \left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 0 & 5 & -4 & -2 \\ 0 & 0 & 39 & 17 \end{array} \right)$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/3 \end{array} \right) \quad \begin{matrix} \alpha = 1/3 \\ \beta = 0 \\ \gamma = 1/3 \end{matrix} \quad \begin{pmatrix} 1/3 \\ 0 \\ 1/3 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$T \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ -2/3 \\ 1/3 \end{pmatrix}$

- $\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}^B \longrightarrow \left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 2 & 1 & -2 & 1 \\ 1 & 2 & 2 & 5 \end{array} \right) \begin{matrix} \\ -F_1 \\ -\frac{1}{2}F_1 \end{matrix} \quad \left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 0 & 3 & -3 & 0 \\ 0 & 3 & 3/2 & 9/2 \end{array} \right) \begin{matrix} \\ \\ -F_2 \end{matrix}$

$\left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & -3/2 & 9/2 \end{array} \right) \longrightarrow -\frac{3}{8}\gamma = \frac{9}{8} \Rightarrow \boxed{\gamma = \cancel{-3}}$ 1

NOTA

$$3\beta - 3.(-3) = 0 \Rightarrow 3\beta = -9 \Rightarrow \boxed{\beta = \cancel{-3}\ 1}$$

$$2\alpha - 2(-3) + (-3) = 1 \Rightarrow 2\alpha + 6 - 3 = 1$$

$$2\alpha = -2 \Rightarrow \boxed{\alpha = \cancel{-1}\ 1}$$

$$\cancel{\begin{pmatrix}-1\\-3\\-3\end{pmatrix}} \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\1\\5\end{pmatrix} = \cancel{-1}\,1 \begin{pmatrix}2\\2\\1\end{pmatrix} \cancel{-3}\,1 \begin{pmatrix}-2\\1\\2\end{pmatrix} \cancel{-3}\,1 \begin{pmatrix}1\\-2\\2\end{pmatrix}$$

$$T\begin{pmatrix}1\\1\\5\end{pmatrix} = \cancel{-1}\,1 \begin{pmatrix}2\\-1\\-1\end{pmatrix} \cancel{-3}\,1 \begin{pmatrix}-1\\2\\-1\end{pmatrix} \cancel{-3}\,1 \begin{pmatrix}-1\\-1\\2\end{pmatrix} = \cancel{\begin{pmatrix}4\\-2\\-2\end{pmatrix}} \begin{pmatrix}0\\0\\0\end{pmatrix}$$

A) gen $\left\{\begin{pmatrix}1\\-2\\1\end{pmatrix}\right\}$

B) Preimagen de T del subespacio

$$\{y \in \mathbb{R}^3 : y_1 - y_3 = 0,\ y_1 + y_2 + y_3 = 0\}$$

$$y_1 = y_3 \qquad \left.\begin{matrix} y_1 + y_2 + y_1 = 0 \\ y_2 = -2y_1 \end{matrix}\right\} \quad y_1 \begin{pmatrix}1\\-2\\1\end{pmatrix}$$

Subespacio = gen $\begin{pmatrix}1\\-2\\1\end{pmatrix}$

$$T(x) = \alpha\begin{pmatrix}2\\-1\\-1\end{pmatrix} + \beta\begin{pmatrix}-1\\2\\-1\end{pmatrix} + \gamma\begin{pmatrix}-1\\-1\\2\end{pmatrix} = \begin{pmatrix}1\\-2\\1\end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & -1 & 1 \\ -1 & 2 & -1 & -2 \\ -1 & -1 & 2 & 1 \end{array}\right) \longrightarrow \begin{matrix} \alpha = 1/2 \\ \beta = -1/2 \\ \alpha\gamma = 1/2 \end{matrix}$$

$$x = \frac{1}{2}\begin{pmatrix}2\\2\\1\end{pmatrix} + \frac{-1}{2} + \begin{pmatrix}-2\\1\\2\end{pmatrix} + \frac{-1}{2}\begin{pmatrix}1\\-2\\2\end{pmatrix} = \cancel{\begin{pmatrix}5/2\\-1\\1/2\end{pmatrix}}$$

MAL

NOTA

HOJA N°

FECHA

Subespacio S =

2.12) B) $\{ y \in \mathbb{R}^3 : y_1 - y_3 = 0 \ , \ y_1 + y_2 + y_3 = 0 \}$

$y_1 = y_3 \rightarrow y_1 + y_2 + y_1 = 0 \Rightarrow y_2 = -2y_1$

$gen \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\} \rightarrow \underset{S}{Base} = \left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\}$

$T^{-1}(S) = \{ x \in \mathbb{R}^3 \ / \ T_{(x)} = S \}$

"Preimagen de S"

T Repasar matriz de transformación lineal (según bases canónicas).

a) $T^{-1}(s) \rightarrow [T]_{E}^{E} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

Base canónica

Base canónica

Saca de Tincho

$\frac{1}{9} \left(\begin{array}{ccc|c} 5 & 5 & -2 & 1 \\ -7 & 2 & 1 & -2 \\ 2 & 7 & 1 & 1 \end{array} \right) \cdots > \left(\begin{array}{ccc|c} 1 & 0 & -1/5 & 36/15 \\ 0 & 1 & -1/5 & -9/15 \\ 0 & 0 & 0 & 0 \end{array} \right)$

$x_1 - \frac{1}{5} x_3 = \frac{36}{15} \qquad x_2 - \frac{1}{5} x_3 = -9/15$

$x_1 = \frac{12}{5} + \frac{1}{5} x_3 \qquad x_2 = -\frac{3}{5} + \frac{1}{5} x_3 \longrightarrow$

$T^{-1}(s) = gen \left\{ \begin{pmatrix} 1/5 \\ 1/5 \\ 1 \end{pmatrix} \right\} + \begin{pmatrix} 12/5 \\ -3/5 \\ 0 \end{pmatrix}$

NOTA

2.14) A) $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ $\quad T(x) = A \cdot x$ $\quad A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$

Escribir matriz de T con respecto a ~~espacios~~ bases canónicos.

$\mathcal{E}_{\mathbb{R}^2} = \left\{ \underset{v_1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \underset{v_2}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right\}$ "B" $\quad \mathcal{E}_{\mathbb{R}^3} = \left\{ \underset{w_1}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}, \underset{w_2}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}, \underset{w_3}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} \right\}$ "C"

. Y determinar si es mono, epi o iso.

¿$[T]_B^C$? $\rightarrow$ Cada columna: $[T(v)]^C = [T]_B^C \cdot [v]^B$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

$[T(v_1)]^C$

$$\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \cdots \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \cdots \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$$

$$[T]_B^C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{matrix} \cancel{\rightarrow \text{Dim}\,[T]_B^C = 2} \\ \rightarrow \text{Dim}\left(\text{Col}\,[T]_B^C\right) = 2 \end{matrix}$$

Para que sea epimorfismo: $\text{Col}\left([T]_B^C\right) = K^m$

T sea Epimorfismo $\iff \underset{(W)}{\dim(\mathbb{R}^3)} = \dim\left(\text{Col}\left([T]_B^C\right)\right)$

$3 \neq 2 \rightarrow$ No es epimorfismo

T es monomorfismo si: $\dim(V) = \dim(W)$

$\text{Nul}\left([T]_B^C\right) = \{0_W\}$

$\left(\dim(V) = 2 \neq \dim(W) = 3\right)$

NOTA

B) $T: \mathbb{R}^3 \to \mathbb{R}^2 \quad T(x) = x \cdot A \to A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$

$\underset{B}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \quad \underset{C}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$

$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ / $T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

...

$[T]_B^C = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix} \to \dim\left(Col\left([T]_B^C\right)\right) = 3$

Lo saco simplemente viendo las columnas.

T no es epimorfismo: $3 \neq \text{Dim}(W) \to 2$

No es mono: $\dim(V) \neq \dim(W)$.

C) $T: \mathbb{R}_3[x] \to \mathbb{R}^4$

$T(p) := \begin{pmatrix} p(0) \\ p(1) \\ p(10) \\ p(100) \end{pmatrix}$ ~~p(x) (p(20)) (p(30))~~

$\underset{B}{\text{Base}}\ \mathbb{R}_3[x] = \{1, x, x^2, x^3\}$ / $\underset{C}{\text{Base}}\ \mathbb{R}^4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$p(0) = a_0$
$p(1) = a_0 + a_1 + a_2 + a_3$
$p(10) = a_0 + 10 \cdot a_1 + 100 \cdot a_2 + 1000 \cdot a_3$
$p(100) = a_0 + 100 \cdot a_1 + 10000 \cdot a_2 + 1000000 \cdot a_3$

↓

Escrito de forma: $T(x) = A \cdot x \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 10 & 100 & 1000 \\ 1 & 100 & 10000 & 1000000 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 10 & 100 & 1000 \\ 1 & 100 & \cdots & \cdots \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ / $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 10 & 100 & 1000 \\ 1 & 100 & \cdots & \cdots \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 10 \\ 100 \end{pmatrix} \cdots$

$[T]_B^C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 10 & 100 & 1000 \\ 1 & 100 & 1000 & 1000000 \end{pmatrix}$

$\left.\begin{matrix} \dim(V) = \dim(W) \\ 4 = 4 \ \checkmark \end{matrix}\right\}$ Es mono.

NOTA

Hecho online
Revisar.

$\mathrm{Nul}\left([T]_B^C\right) = 0$, ~~ASS.~~

✓ $\mathrm{Dim} = 0$ → Si $\dim\left(\mathrm{Nul}\left([T]_B^C\right)\right) = 0 \iff$ es epimorfismo.

~~Al~~ Al ser mono. y epimorfismo, es isomorfismo.

2.17) D)

NOTA

2.17) Transformación lineal definida por:

$$[T]_B^C = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix}$$

Base B $\mathbb{R}_2[x]$ $= \left\{ \frac{1}{2}(x)(x-1),\ -x(x-2),\ \frac{1}{2}(x-1)(x-2) \right\}$

Base C $\mathbb{R}^3$ $= \left\{ \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \right\}$

CHECKEAR ¡ !

A) $\mathrm{Det}\left([T]_B^C\right) = -1 \neq 0 \longrightarrow$ Es isomorfismo. → Según Ana

B) ¿ $M^C_{\mathcal{E}\,\mathbb{R}^2[x]}$? $= [T]_B^C \cdot M_{\mathcal{E}}^B$

$\mathcal{E}\,\mathbb{R}_{2[x]} = \{1, x, x^2\}$ $\quad B = \left\{ \frac{1}{2}(x^2 - x),\ -x^2 + 2x,\ \frac{1}{2}(x^2 - 2x - x + 2) \right\}$

$$B = \Big\{ \underbrace{\tfrac{1}{2}x^2 - \tfrac{1}{2}}_{v_1},\ \underbrace{-x^2 + 2x}_{v_2},\ \underbrace{\tfrac{1}{2}x^2 - \tfrac{3}{2}x + 1}_{v_3} \Big\}$$

~~$[v_1]_{\mathcal{E}} = \{0, -\frac{1}{2}, \frac{1}{2}\}$ $[v_2]_{\mathcal{E}} = \{0,$~~

$[v_1]_{\mathcal{E}} = \left(0 \quad -\frac{1}{2} \quad \frac{1}{2}\right)$ / $[v_2]_{\mathcal{E}} = \left(0 \quad 2 \quad -1\right)$ / $[v_3]_{\mathcal{E}} = \left(1 \quad -\frac{3}{2} \quad \frac{1}{2}\right)$

$$M_B^{\mathcal{E}} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & 2 & -\frac{3}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix} \longrightarrow$$ Queremos $M_{\mathcal{E}}^B = \left(M_B^{\mathcal{E}}\right)^{-1} =$

calculadora

$$\begin{pmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

← Vectores Elementos de B escritos en coordenadas de $\mathcal{E}$

¿Cómo sacar inversa a mano?

$$\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 0 & -2 & -4 \\ 3 & -2 & -6 \end{pmatrix} = M^C_{\mathcal{E}\,\mathbb{R}_2[x]}$$

NOTA

$$M_b^{a} = M_c^{a} \cdot M_b^{c}$$

C) $M_B^{\varepsilon\, \mathbb{R}^3} = [T]_B^c \cdot M_C^{\varepsilon\, \mathbb{R}^3} \cdot [T]_B^c$

$$M_C^{\varepsilon} = \begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{pmatrix}$$

$$\cdot \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 3 \\ 3 & 2 & -3 \\ -6 & 5 & 9 \end{pmatrix} = M_B^{\varepsilon\, \mathbb{R}^3}$$

D) $M_{\varepsilon\, \mathbb{R}_2[x]}^{\varepsilon\, \mathbb{R}^3} = M_C^{\mathbb{R}^3} \; [T]_B^c \cdot M_{\mathbb{R}_2[x]}^{B}$

$$M_{\mathbb{R}[x]}^{\mathbb{R}^3} = \underset{A}{\begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{pmatrix}} \underset{B}{\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -2 & 2 & 3 \end{pmatrix}} \underset{C}{\begin{pmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}} = \begin{pmatrix} 7 & 4 & 4 \\ -2 & 4 & 10 \\ 8 & -7 & -19 \end{pmatrix}$$

E) ~~Im S = gen 2x~~ ~~S = gen {(2+3x+2x², 5+5x+4x²)}~~

$S = \text{gen}\{(2+3x+2x^2), (5+5x+4x^2)\}$. Son LI.

$Im = T[2+3x+2x^2], \; T(5+5x-4x^2)$.

¿Se puede hacer con la base canónica?

HALLAR LAS MATRICES

2.18) $\mathbb{R}^3 \xrightarrow{T_1} \mathbb{R}^3$ → TL del ejercicio 2.10

$\mathbb{R}^3 \xrightarrow{T_2} \mathbb{R}_2[x]$ $\begin{pmatrix} a \\ b \\ c \end{pmatrix} := (a+b) + (a+c)x + (b+c) \cdot x^2$

T_1 según Aca:
$$T_1 \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -a + 2b + 2c \\ \frac{3}{2}a - 3b - 3c \\ -2a + 4b + 4c \end{pmatrix}$$

A) $\left[T_1\right]^{E\mathbb{R}^3\, C}_{E\mathbb{R}^3\, B}$ → $T_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3/2 \\ -2 \end{pmatrix}$ / $T_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ / $T_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$

$$\left[T_1\right]^{C}_{B} = \begin{pmatrix} -1 & 2 & 2 \\ 3/2 & -3 & -3 \\ -2 & 4 & 4 \end{pmatrix}$$

$\left[T_2\right]^{E\mathbb{R}_2[x]}_{E\mathbb{R}^3}$: $T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1 + 1x$ / $T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 1 + x^2$ / $T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = x + x^2$

Suponiendo que base canónica es $\{1, x, x^2\}$ ⟶ $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \left[T_2\right]^{E\mathbb{R}_2[x]}_{E\mathbb{R}^3}$

$\left[T_2\right]^{-1}$ ⟶ Inversa $\left[T_2\right] = \begin{pmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \end{pmatrix} = \left[T_2^{-1}\right]^{E\mathbb{R}^3}_{E\mathbb{R}_2[x]}$

NOTA: ¿Falta justificar? ¿Qué es isomorfismo.

B) Repaso composición de transformaciones lineales:

$T_1: V \to W \qquad T_2: W \to U \qquad T_2 \circ T_1: V \to U$

$T_2(T_1(v))$

Si B base V, C base W y D base U.

- $[T_1]_B^C [v]^B = [T_1(v)]^C = [w]^C$
- $[T_2]_C^D [T_1]_B^C [v]^B = [T_2]_C^D [w]^C = [T_2(w)]^D = [u]^D$
- $[T_2 \circ T_1]_B^D [v]^B = [u]^D$
- $[T_2 \circ T_1]_B^D \cancel{[v]} = [T_2]_C^D [T_1]_B^C$

$T_1 \circ T_2^{-1} \longrightarrow T_1(T_2^{-1}) \qquad T_2^{-1}: \mathbb{R}_2[x] \to \mathbb{R}^3$

$T_1: \mathbb{R}^3 \to \mathbb{R}^3$

$$[T_1 \circ T_2^{-1}]_{E\mathbb{R}_2[x]}^{E\mathbb{R}^3} = [T_1]_{E\mathbb{R}^3}^{E\mathbb{R}^3} \cdot [T_2^{-1}]_{E\mathbb{R}_2[x]}^{E\mathbb{R}^3}$$

$$\begin{pmatrix} -1 & 2 & 2 \\ 3/2 & -3 & -3 \\ -2 & 4 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & -1/2 \end{pmatrix} = \begin{pmatrix} -1/2 & -1/2 & 5/2 \\ 3/4 & 3/4 & -15/4 \\ -1 & -1 & 5 \end{pmatrix}$$

hallar Base $(Nu(T_1 \circ T_2^{-1}))$

$$\left(\begin{array}{ccc|c} -\frac{1}{2} & -\frac{1}{2} & \frac{5}{2} & 0 \\ \frac{3}{4} & \frac{3}{4} & -\frac{15}{4} & 0 \\ -1 & -1 & 5 & 0 \end{array}\right) \begin{array}{l} \\ x\left(-\frac{1}{3}\right) - F_1 \\ -2 \cdot F_1 \end{array} = \left(\begin{array}{ccc|c} -1/2 & -1/2 & 5/2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$-\alpha - \beta + 5\gamma = 0 \longrightarrow \beta = -\alpha + 5\gamma \longrightarrow \text{gen}\left\{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}\right\}$$

En base $\mathbb{R}_2[x]$: Base $(Nu(T_1 \circ T_2^{-1})) = \{(1-x), (5x + x^2)\}$

¿Por qué? Como paso de un gen en $\mathbb{R}^3 \Rightarrow$ base $\mathbb{R}_2[x]$?

NOTA

HOJA N°

FECHA

2.20) $\mathbb{V} \to \mathbb{R}^3$ $\quad \underset{\text{de } \mathbb{V}}{\overset{\text{Base}}{B}} = \{v_1, v_2, v_3\}$

Subespacios S_1 y S_2: $\quad S_1 = gen\{\overset{b_1}{v_1 - 2v_2}, \overset{b_2}{v_1 + v_3}\} \to dim = 2$

$S_2 = gen\{\underset{b_3}{v_2 - v_3}\} \longrightarrow dim = 1$ } 3 ✓

A) Comprobar $S_1 \oplus S_2$: $\cancel{gen}$ $\{b_1, b_2, b_3\} \to$ Deben ser linealmente independientes.

$$\begin{pmatrix} v_1 & -2v_2 & \\ v_1 & & +3v_3 \\ & v_2 & -v_3 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -1 \end{pmatrix} -F_1 \Big\} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix} -F_2/2$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & -\frac{5}{4} \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \text{Son L.I.}$$

Ano lo hizo traspuesto. Está bien?

B) $B = \{v_1, v_2, v_3\}$, $C = \{v_1 - 2v_2, v_1 + v_3, v_2 - v_3\}$

$$[T]_B^B = [T]_C^B \cdot M_B^C$$

$$M_B^C = \left([v_1]^C, [v_2]^C [v_3]^C\right)$$

$$a_1(v_1 - 2v_2) + a_2(v_1 + v_3) + a_3(v_2 - v_3) = v_1$$

$$a_1 \cdot v_1 - 2a_1 \cdot v_2 + a_2 \cdot v_1 + a_2 \cdot v_3 + a_3 \cdot v_2 - a_3 \cdot v_3 = v_1$$

$$v_1(a_1 + a_2) + \cancel{v_2(-2a_1 + a_2 + a_3)} + \cancel{v_3(a_2 - a_3)} = v_1 + 0 v_2 + 0 v_3$$

$$\cancel{v_1(a_1 + a_2) = v_1} \implies \cancel{a_1 + a_2 = 1.}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} +2 \cdot F_1 \Big\} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 3 & 1 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix} \cdot 3 - F_2 \Big\} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & -4 & -2 \end{pmatrix}$$

$$-4a_3 = -2 \implies a_3 = \frac{1}{2} \;/\; 3 \cdot a_2 + \frac{1}{2} = 2 \implies a_2 = \frac{1}{2}$$

NOTA $\quad a_1 + \frac{1}{2} = 1 \implies a_1 = \frac{1}{2} \qquad [v_1]^B = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$

~~1 1 0 0~~ $\begin{pmatrix} 1 & 1 & 0 & 1 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \cdots\cdots \left[v_1\right]^{C} = \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \cdots \left[v_2\right]^{C} = \begin{pmatrix} -1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix} \cdots \left[v_3\right]^{C} = \begin{pmatrix} -1/3 \\ 1/3 \\ -2/3 \end{pmatrix}$

$\left[M_B^C\right] = \begin{pmatrix} \frac{1}{3} & \frac{-1}{3} & \frac{-1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \end{pmatrix}$

?