Binary Neutron Star Gravitational Wave Bursts: A Post-Merger Model Using Principal Component Analysis

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Outline

1. Binary neutron star mergers are likely to result in the formation of a stable / quasi-stable, differentially rotating neutron star remnant [1, 2, 3, 4].
2. Transient non-axisymmetric deformations and f-mode oscillations $\rightarrow$ short (10–100 ms) burst of high-frequency ($\sim$kHz) gravitational wave (GW) emission.
3. Spectral properties of this burst carry finger prints of neutron star equation of state, particularly the dominant peak frequency f_{peak} [5, 6].
4. May be observable to $\sim$10–100 Mpc, with a matched filter.
5. Merger & post-merger phase are *not well-modelled* & unmodelled burst searches currently struggle to reconstruct the full time-frequency structure, which can be disjoint in the TF-plane.
6. We **propose a method to construct a phenomelogical waveform model, based on principal component analysis of numerical merger simulations**, to allow more **robust identification and characterisation** of this high frequency component of the GW signal from binary neutron star coalescence.

Principal Component Analysis & Approximate Waveform Modelling

Objective: Given a collection of numerical waveforms, with no analytic model, construct a *reduced* set of basis functions from which we can construct any one waveform to reasonable accuracy

1. Organise N waveforms, each containing M samples, from numerical simulations of binary neutron star mergers into an $M \times N$ data matrix, $\mathbf{X}$
2. Align dominant features (see below), and center the data (subtract the mean waveform, $\mathbf{h}$) to get centered data matrix
$$\mathbf{Y} = \mathbf{X} - \mathbf{h} \tag{1}$$
3. Find the eigenvectors $\mathbf{W}$ of the covariance matrix
$$\mathbf{C} = \mathbf{Y}^{\top}\mathbf{Y}, \tag{2}$$
4. Using $\mathbf{W}$, we can find the principal component decomposition of $\mathbf{X}$,
$$\mathbf{Z} = \mathbf{X}\mathbf{W}, \tag{3}$$
where $\mathbf{W}$ has been sorted in order of descending eigenvalues, and $\mathbf{Z}$ is the *score matrix*, first $p < N$ columns of which represent our reduced basis[1].
5. Implemented via singular value decomposition, $\mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{W}^{\top}$ so that,
$$\mathbf{Z} = \mathbf{X}\mathbf{W} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{W}\mathbf{W}^{\top} = \mathbf{U}\boldsymbol{\Sigma} \tag{4}$$

[1] One takes care, of course, to add the mean waveform $\mathbf{h}$ back on to the waveform reconstructed from $\mathbf{Z}$

Feature Alignment

Goal is to use PCA to model *variance* in data matrix $\mathbf{X} \rightarrow$ important to align common features so that PCA picks out true *differences* in waveforms.

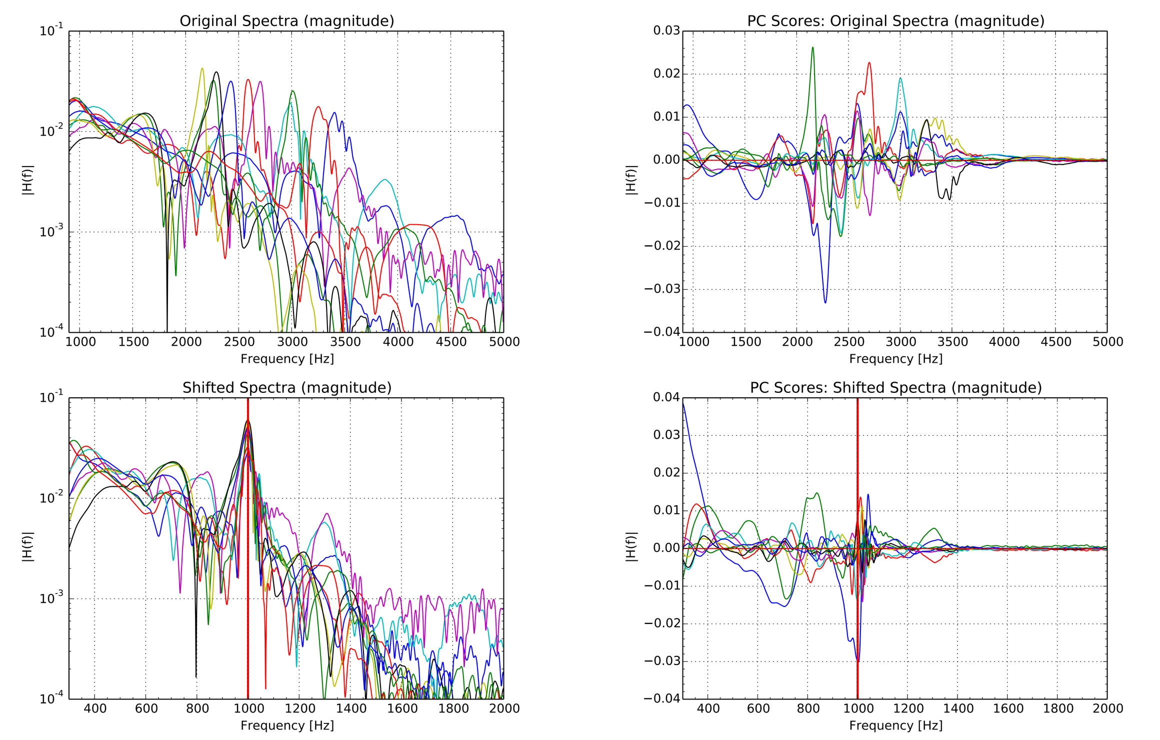
BNS merger waveforms exhibit varied broadband frequency content & large ($\sim$kHz) differences between post-merger oscillations frequencies. Natural to work in the frequency domain with amplitude and phase spectra.

Key component to our analysis: align the magnitude (and phase) spectra to a common reference frequency, prior to PCA. Alignment procedure:

1. Construct spectrum of waveform, $H(f) = A(f)\exp[i\phi(f)]$
2. Compute a new set of frequencies to scale $H(f)$ such that the dominant post-merger peak lies at f_{ref}: $f_{\mathrm{shift}} = \frac{f_{\mathrm{ref}}}{f_{\mathrm{peak}}} f$
3. Interpolate the spectrum $H(f) \rightarrow H(f_{\mathrm{shift}})$ & yield a new, shifted spectrum whose dominant post-merger oscillation frequency lies at f_{ref}
4. Perform PCA for magnitude & phase spectra (separately)
5. The original waveform can then be reconstructed by summation of the chosen principal component basis waveforms, followed by the inverse of the frequency shift operation to the desired f_{peak}.

Waveform Catalogue & Decomposition

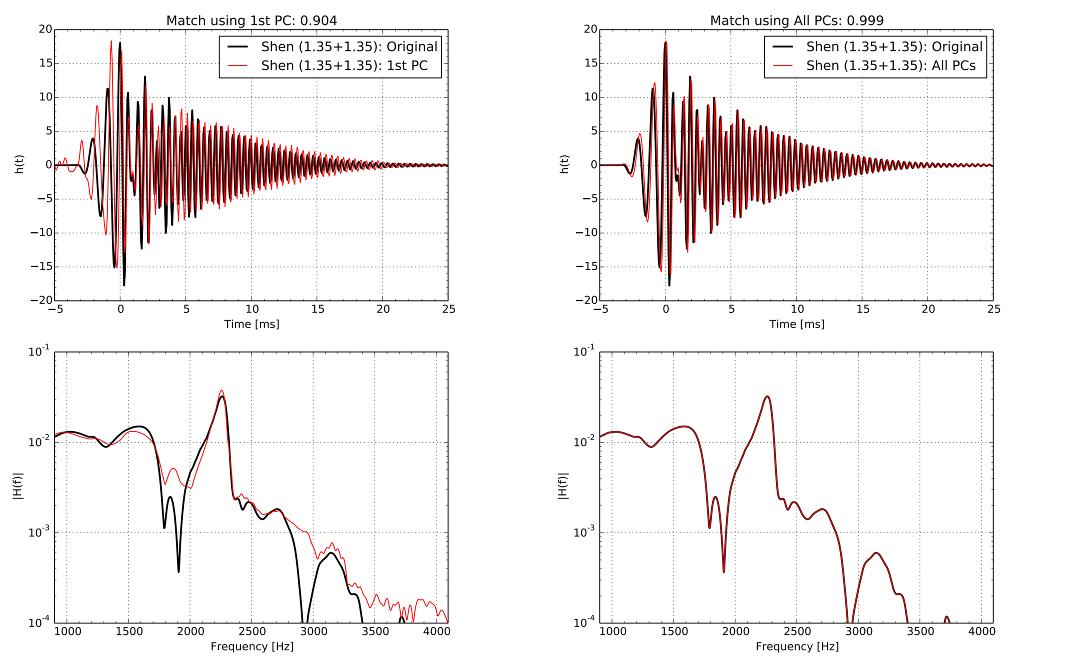
Here, we use the sample of numerical waveforms discussed in [7], which consists of **ten waveforms with 8 equations of state**. For the purposes of this study, the salient detail is that these **waveforms span the full space of frequencies, while sharing a generally similar morphology**. Figure 1a shows the spectra of full catalogue overlaid with each other. An example of a typical waveform in the time domain can be found in figure 2.



(a) Magnitude Spectra (b) Principle Component Scores

Figure 1: *Top row*: magnitude spectra (left) and principal component scores (right) of the original waveforms. Note the wide range in peak locations and variety of peak locations in the scores. *Bottom row*: spectra following the frequency shifting procedure and the corresponding PC scores. Waveform frequencies have been scaled such that the peaks align at 1 kHz (red vertical line). Only a few scores now dominate.

Demonstration



(a) First principal component (b) All principal components

Figure 2: Example: reconstructing the Shen 1.35+1.35 $\mathrm{M}_{\odot}$ waveform using only the first principal component (left) and all principal components (right).

Characterisation

Two common figures of merit are useful for characterising the performance of the decomposition and reconstruction performance of our method:

Explained Variance (a.k.a, 'eigen-energy'): The eigenvalues of the covariance matrix describe how much variance of the data matrix is represented by each principal component. The normalised, cumulative sum is then the fraction of variance in the catalogue explained by number of principal components.

Template match : the usual figure of merit for assessing the quality of a matched-filter template,

$$\mathrm{match} = \frac{(a|b)}{\sqrt{(a|a)(b|b)}}, \text{ where } (a|b) = \int \frac{a(f)b^{*}(f)}{S(f)}\,\mathrm{d}f \tag{5}$$

Match is maximised over the relative start time and phase-offsets of a and b.

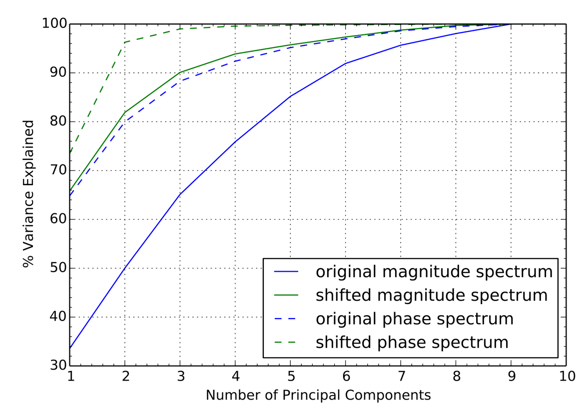
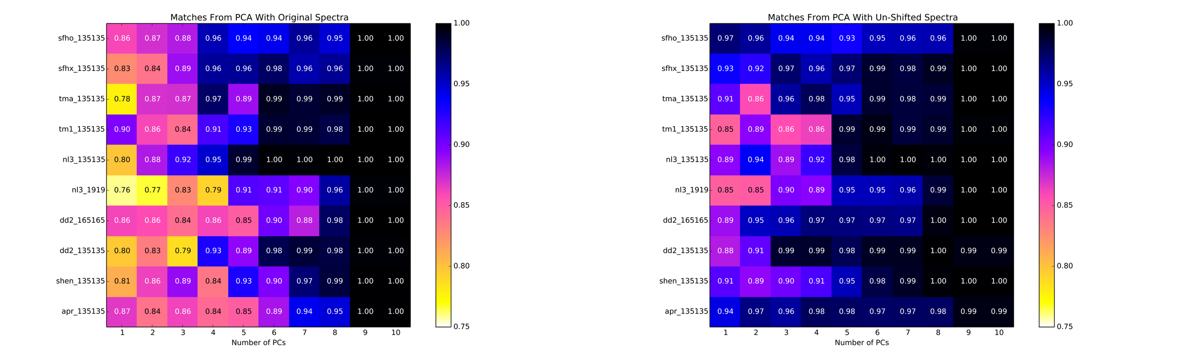


Figure 3: Explained Variance.

Figure 3 compares the explained variance as a function of number of principal components for the original and shifted spectra. We require about half as many principal components after applying our shifting procedure. Much of the variance in the original catalogue arises from a feature, the location of f_{peak}, which is easily aligned.



(a) Matches from PCA models using the original spectra (b) Matches from PCA models using the shifted spectra

Figure 4: Match as a function of number of basis functions used for all waveforms in the catalogue (vertical axes). Matches $\sim 90\%$ are achievable with even a small number of components.

Finally, figure 4 compares the matches for the waveforms in our catalogue using the original and shifted spectra. Provided we follow the frequency shifting procedure outlined, we find that **matches of $\sim 90\%$ and above are easily realised using a small number of principal components**, far fewer than would be required using the unshifted data.

Conclusions

- Have demonstrated an method to construct a low-dimensional and reasonably accurate model using principal component analysis and a novel feature alignment scheme.
- **Some caveats**: small catalogue & have only demonstrated ability to reconstruct *training* data.
- Applications include: burst parameter estimation follow-ups of BNS inspiral detections, Monte-Carlo simulations in un-modelled analysis and machine learning algorithms & astrophysical interpretation waveform reconstructions from other burst algorithms.

References

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