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Chirplets In LALSimulation

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1 Introduction

This document describes a proposed chirplet waveform for use in `LALSimulation` and integration into `LALInference`. The proposed waveform is described in [1]. We'll just copy and build on that description.

2 Definition of Chirplets

2.1 Time Domain

Define the time-domain chirplet as:

$$\psi(t) \equiv A \exp\left\{-\frac{(2\pi f_0)^2}{Q^2}(t-t_0)^2\right\} \exp\left\{2\pi i[f_0 + \mathcal{D}/2(t-t_0)^2] + \phi_0\right\}, \tag{1}$$

where t_0 and f_0 are the center time and frequency, respectively and ϕ_0 is an arbitrary initial phase. Following [1], quality factor Q is,

$$Q = 2\sqrt{\pi} f_0 \tau, \tag{2}$$

where τ is the chirplet duration,

$$\tau \equiv 2\sqrt{\pi} \int_T (t-t_0)^2 |\psi(t)| \, \mathrm{d}t. \tag{3}$$

The amplitude A is a normalisation term,

$$A = \left(\frac{8\pi f_0^2}{Q^2}\right)^{1/4}, \tag{4}$$

which ensures $\int_T |\psi(t)|^2 \, \mathrm{d}t = 1$. The quantity $\mathcal{D}$ is the *chirp rate*, which controls the frequency evolution so that the instantaneous frequency evolves linearly as $f(t) = f_0 + \mathcal{D}(t-t_0)$. The waveform reduces to the familiar sine-Gaussian when $\mathcal{D} = 0$. It's worth emphasising here that these are linear chirps. It is easy to imagine reparameterising to allow for (e.g.,) power-law or polynomial frequency evolution[1].

With the basis waveform $\psi(t)$ thus defined, the polarisations for a time-domain linear chirplet in `LALSimulation` are,

$$h_+(t|h_{\mathrm{rss}}, \alpha, f_0, t_0, \phi_0, Q, \mathcal{D}) = \Re\left[h_{\mathrm{rss}} \cos(\alpha) \psi(t|f_0, t_0, \phi_0, Q, \mathcal{D})\right] \tag{5}$$

$$h_\times(t|h_{\mathrm{rss}}, \alpha, f_0, t_0, \phi_0, Q, \mathcal{D}) = \Im\left[h_{\mathrm{rss}} \sin(\alpha) \psi(t|f_0, t_0, \phi_0, Q, \mathcal{D})\right] \tag{6}$$

where α is a polarisation angle to control the ellipticity of the waveform and h_{rss} is the root-sum-squared amplitude,

$$h_{\mathrm{rss}} = \sqrt{\int_T |h_+(t)|^2 + |h_\times(t)|^2 \, \mathrm{d}t}. \tag{7}$$

2.2 Frequency Domain

Following [1], the frequency domain chirplet is,

$$\tilde{\psi}(f) = \mathcal{A} \exp\left[-\frac{\tilde{Q}^2}{4}\left(\frac{f-f_0}{f}\right)^2 + i\phi_0\right] \tag{8}$$

[1] see e.g., `http://docs.scipy.org/doc/scipy-0.14.0/reference/generated/scipy.signal.chirp.html`

where the amplitude normalisation $\mathcal{A}$ is,

$$\mathcal{A} = \left[\frac{\tilde{Q}^4}{Q^2}\frac{1}{2\pi f_0^2}\right]^{1/4}, \tag{9}$$

and $\tilde{Q}$ is the complex quality factor which carries the chirp rate $\tilde{D}$,

$$\tilde{Q} = Q\frac{\sqrt{z}}{|z|}, \tag{10}$$

and $z = 1 + i\mathcal{D}\tau^2$.

Then, as before, we have the frequency domain polarisations:

$$\tilde{h}_+(f|h_{\rm rss}, \alpha, f_0, \phi_0, Q, \mathcal{D}) = h_{\rm rss}\cos(\alpha)\tilde{\psi}(f|h_{\rm rss}, \alpha, f_0, \phi_0, Q, \mathcal{D}) \tag{11}$$

$$\tilde{h}_\times(f|h_{\rm rss}, \alpha, f_0, \phi_0, Q, \mathcal{D}) = h_{\rm rss}\sin(\alpha)\tilde{\psi}(f|h_{\rm rss}, \alpha, f_0, \phi_0 + \pi/2, Q, \mathcal{D}) \tag{12}$$

3 Examples

Figures 1 and 2 show linearly- and circularly-polarised waveforms, respectively, generated in the time-domain with the following call in python:

```
hp, hc = lalsim.SimBurstChirplet(Q, centre_frequency,
    chirp_rate, hrss, alpha, phi0, delta_t)
```

The right panels of the figures also show the PSDs. The epochs are computed such that the middle sample of the waveform is $t = 0$, as per the sine-Gaussian convention. We also confirm that the root-sum-squared strain (see equation 7) is unity as requested. This is computed using:

```
hrss=lalsim.MeasureHrss(hp,hc).
```

Finally, figure 3 shows the analytic Fourier domain waveform returned by,

```
hp, hc = lalsim.SimBurstChirpletF(Q, centre_frequency,
    chirp_rate, hrss, alpha, phi0, delta_f, delta_t),
```

as well as the FFT of the time-domain waveform from `SimBurstChirplet()`. We plot the real parts of the complex Fourier spectra in the left panel and the imaginary parts in the right panel. We confirm that the F-domain analytic waveform does indeed match the FFT of the T-domain waveform (ignoring the high-frequency phase-shift oscillations in the numerical waveform).

References

[1] É. Chassande Mottin, M. Miele, S. Mohapatra, and L. Cadonati. Detection of GW bursts with chirplet-like template families. *Classical and Quantum Gravity*, 27(19):194017, October 2010.

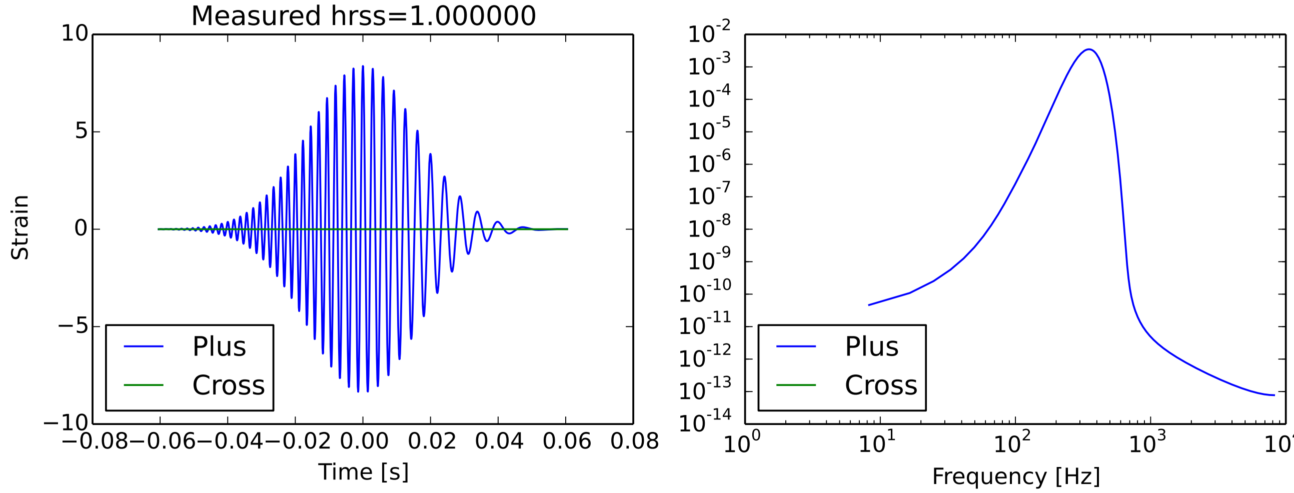

Figure 1: Linearly polarised chirplet with $h_{\mathrm{rss}} = 1$, $\alpha = 0$, $Q = 50$, $f_0 = 350\,\mathrm{Hz}$, $\mathcal{D} = -5000\,\mathrm{Hz\,s^{-1}}$.

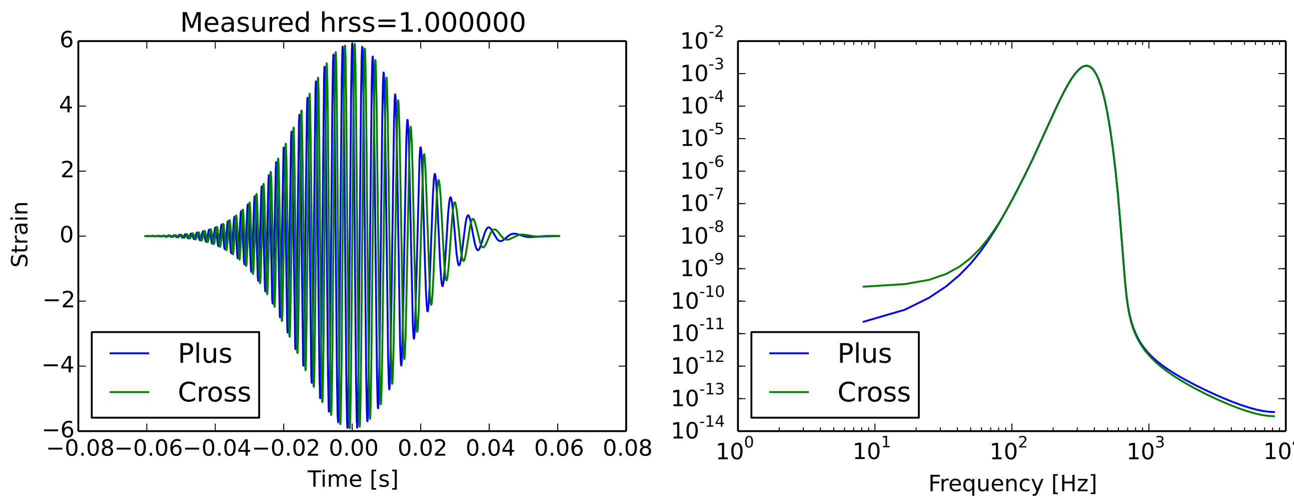

Figure 2: Circularly polarised chirplet with $h_{\mathrm{rss}} = 1$, $\alpha = \pi/4$, $Q = 50$, $f_0 = 350\,\mathrm{Hz}$, $\mathcal{D} = -5000\,\mathrm{Hz\,s^{-1}}$.

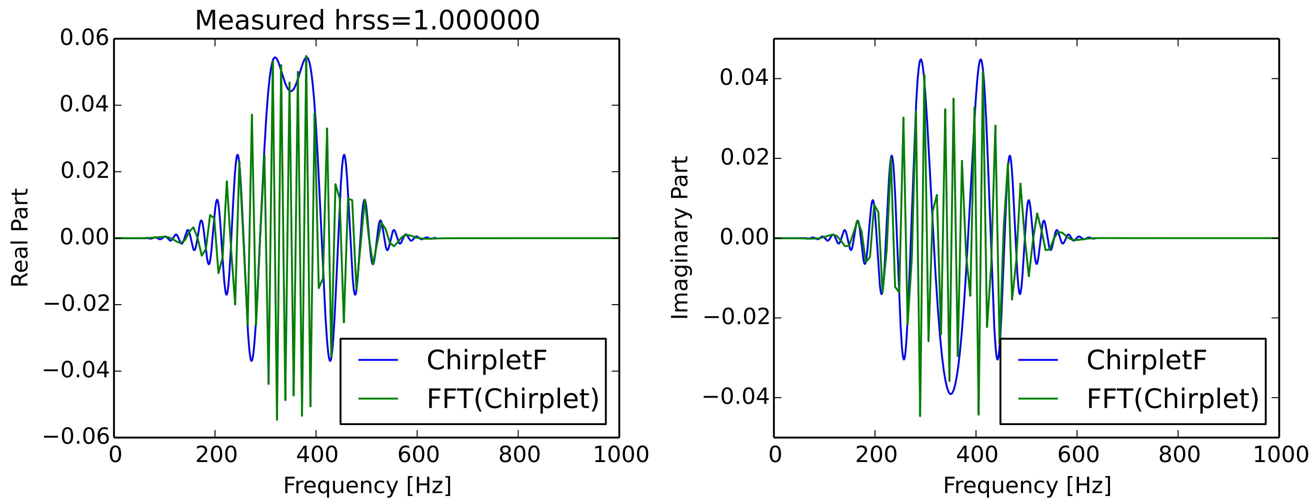

Figure 3: Plus-polarised chirplet with $h_{\mathrm{rss}} = 1$, $\alpha = \pi/4$, $Q = 50$, $f_0 = 350\,\mathrm{Hz}$, $\mathcal{D} = -5000\,\mathrm{Hz\,s}^{-1}$.. Left panel shows the real part of the Fourier transform, right panel shows the imaginary part of the Fourier transform; blue trace is the analytic Fourier transform, using `SimBurstChirpletF()`, green trace is the FFT of the time-domain chirplet `SimBurstChirplet()`.