Supernova Model Evidence Extractor as Applied to BBH Waveforms

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October 7, 2013

What is SMEE?

- Written by Josh Logue *et al.* at the University of Glasgow
- Nested Sampling algorithm used to reconstruct waveforms in GW data
- The goal was to distinguish between physical models of SN based on GW signal measured
- Utilizes principle component analysis (PCA) to reconstruct signal

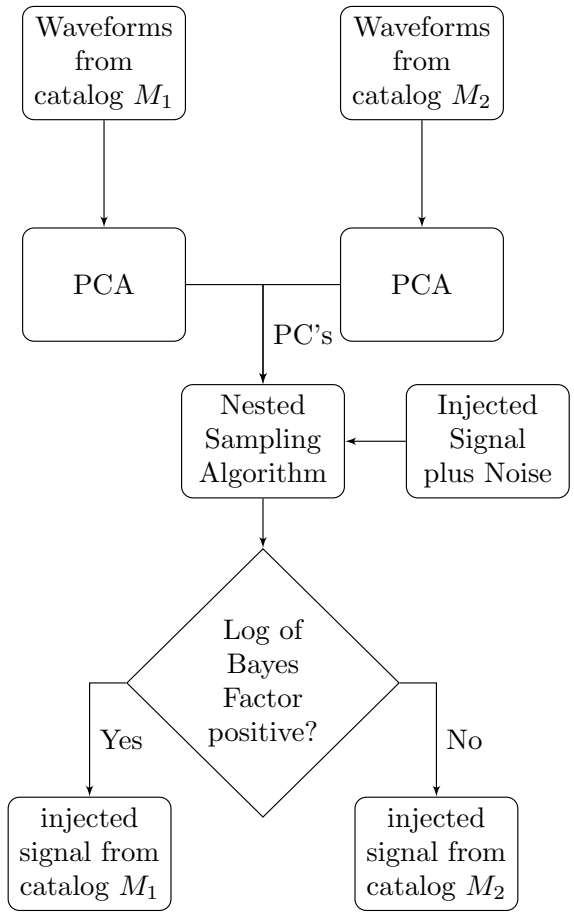


Figure: Graphical representation of SMEE

Principle Component Analysis

- Converts a data set into linearly independent principle components (PCs)
- The original data is now a linear combination of PCs (eigenvectors)
- The first PC holds the most variance in the data and the last holds the least

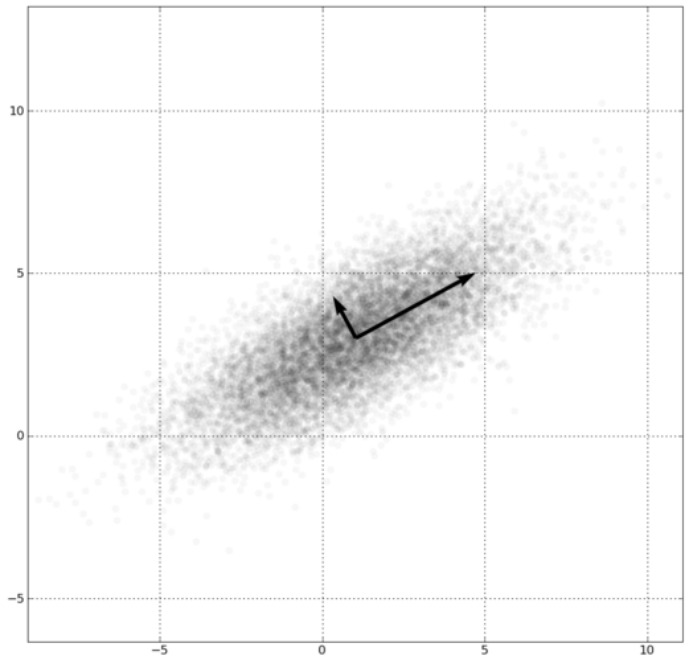


Figure: PCA of multivariate gaussian data. Source: Wikipedia

How Does SMEE Work?

- Calculates principle components (PCs) from catalogue of waveforms that share similar physics
- In principle the PCs will contain the morphology of signals and can accurately reconstruct a signal with a small number of PCs
  - $h_i \approx \sum_{j=1}^{k} U_j \beta_j$
- Model preference is determined by the Bayes factor which is the ratio of the marginalized likelihoods for the two models: $B_{12} = \frac{p(D|M_1)}{p(D|M_2)}$
  - If $B_{12} > 1$, then Model 1 is preferred and if $B_{12} < 1$ Model 2 is preferred.
- The evidence is obtained by using a nested sampling algorithm to calculate: $p(D|M_s) = \int_{\beta_{min}}^{\beta_{max}} p(\beta|M_s) p(D|\beta, M_s) d\beta$

Principle Component Analysis

- $\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{V}^T$
  - $\mathbf{M}$ is an n x m matrix containing the data
  - $\mathbf{U}$ and $\mathbf{V}$ are matrices of the eigenvectors of $\mathbf{M}\mathbf{M}^T$ and $\mathbf{M}^T\mathbf{M}$, respectively
  - $\mathbf{S}$ is a diagonal matrix containing the square roots of the eigenvalues
- Step #1: Calculate the covariance matrix $\mathbf{C}$, of $\mathbf{M}$
- Step #2: Calculate the eigenvalues ($\mathbf{S}^2$) and eigenvectors ($\mathbf{V}$) of $\mathbf{C}$
- Step #3: Organize $\mathbf{S}$ in descending order of eigenvalues along with the corresponding eigenvectors in $\mathbf{V}$

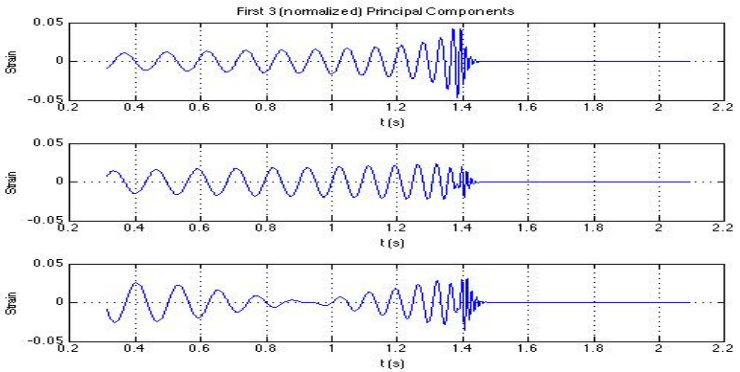
Principle Component Analysis

- Step #4: Compute the eigenvectors of the real covariance matrix **U** (the PCs)
  - $\mathbf{U} = \mathbf{M} \times \mathbf{V}$
- Step #5: Calculate the β values by projecting **M** onto **U**
  - $\beta = \mathbf{M} \cdot \mathbf{U}$
- The reconstructed waveform is thus given by $\mathbf{D} = (\beta \cdot \mathbf{U}^T)^T$

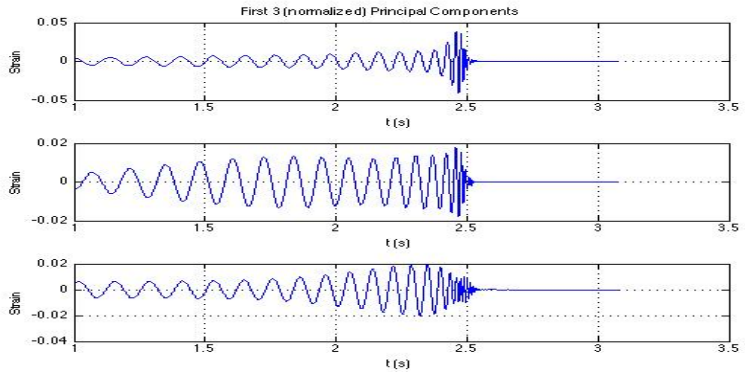
Waveform Catalogues

- NR waveforms made at GATech
- Q-series Waveforms
  - 13 waveforms of increasing mass ratio
- HR-series Waveforms
  - 15 waveforms of increasing mass ratio and spin magnitudes
- RO3-series
  - 20 waveforms of increasing mass ratio, spin, and system precession

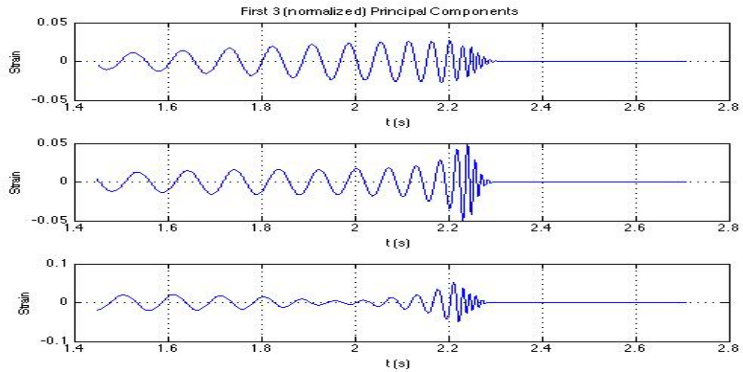
Principle Components



(a) First 3 Q-series principle components



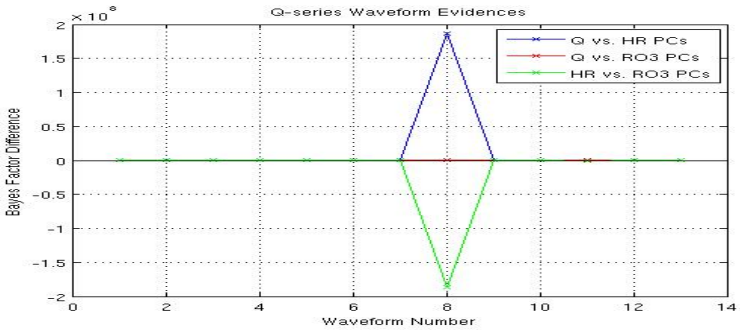
(b) First 3 HR-series principle components



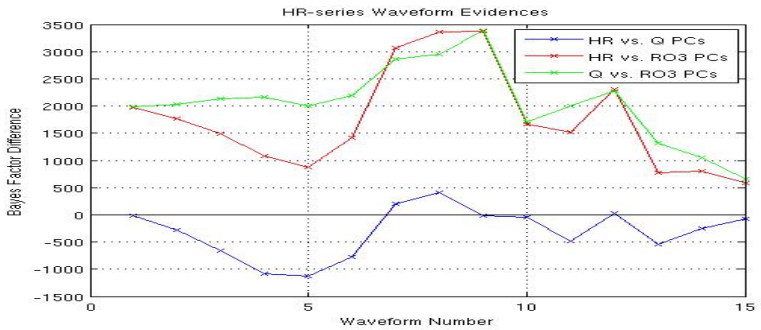
(c) First 3 RO3-series principle components

Preliminary Results

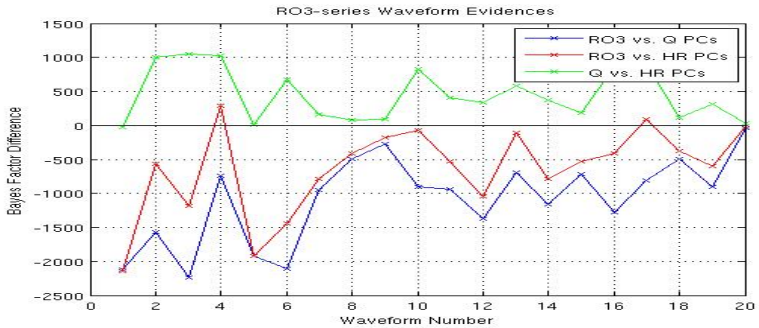
- The plots below show the results of SMEE. They are labeled as M_1 vs. M_2 for each catalogue ($log(B_1) - log(B_2)$)



(d) Q-series Evidences



(e) HR-series Evidences



(f) RO3-series Evidences

The Next Step

- Investigate why SMEE's reconstruction of the HR and RO3-series waveforms are so off
- Expand to using more sophisticated BBH catalogues from GATech