Supernova Model Evidence Extractor as Applied to BBH Waveforms

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What is SMEE?

- Written by Josh Logue *et al.* at the University of Glasgow
- Nested Sampling algorithm used to reconstruct waveforms in GW data
- The goal was to distinguish between physical models of SN based on the detected GW signal
- Utilizes principle component analysis (PCA) to reconstruct a signal using provided models

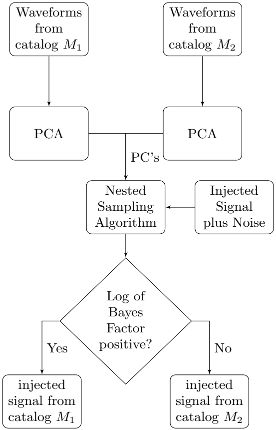


Figure: Graphical representation of SMEE

Principle Component Analysis

- Converts a data set into linearly independent principle components (PCs)
- The original data is now a linear combination of PCs (eigenvectors)
- The first PC holds the most variance in the data and the last holds the least

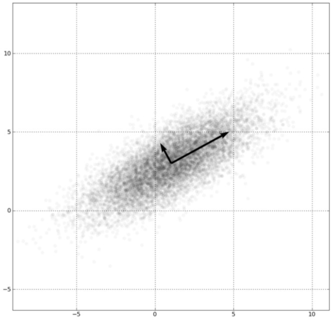


Figure: PCA of multivariate Gaussian data. Source: Wikipedia

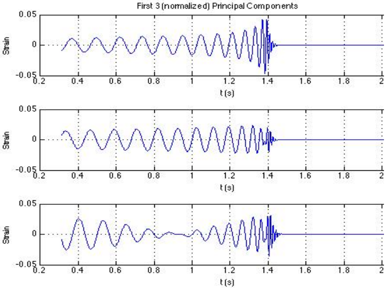
How Does SMEE Work?

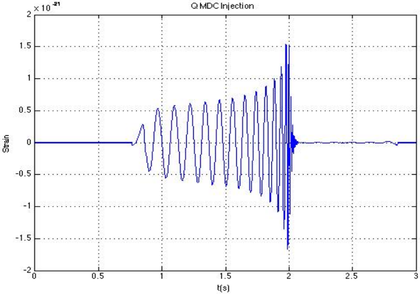
- Calculates principle components (PCs) from catalogue of waveforms that share similar physics
- The PCs will contain the morphology of signals and can accurately reconstruct a signal with a small number of PCs
  - $h_i \approx \sum_{j=1}^{k} U_j \beta_j$
- Model preference is determined by the Bayes factor which is the ratio of the marginalized likelihoods for the two models: $B_{12} = \frac{p(D|M_1)}{p(D|M_2)}$
  - If $B_{12} > 1$, then Model 1 is preferred and if $B_{12} < 1$ Model 2 is preferred.
- The evidence is obtained by using a nested sampling algorithm to calculate: $p(D|M_s) = \int_{\beta_{min}}^{\beta_{max}} p(\beta|M_s) p(D|\beta, M_s) d\beta$

Waveform Catalogues

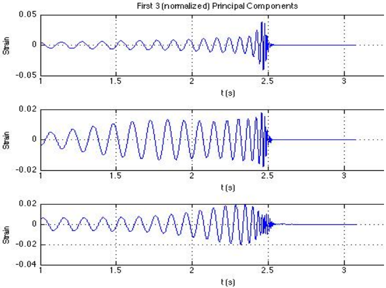
- NR waveforms made at GATech
- Q-series Waveforms
  - 13 waveforms of increasing mass ratio
- HR-series Waveforms
  - 15 waveforms of increasing mass ratio and spin magnitudes
- RO3-series
  - 20 waveforms of increasing mass ratio, spin, and system precession

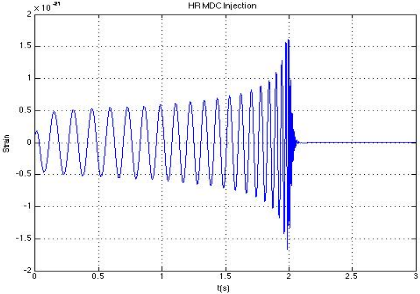
Q-series


First 3 (normalized) Principal Components
Strain
t (s)
0.05
0
-0.05
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

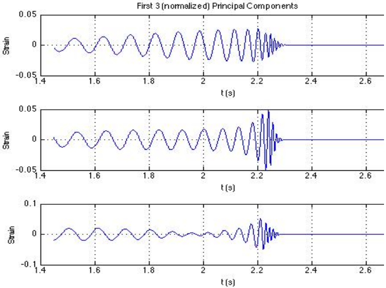

Q MDC Injection
$\times 10^{-21}$
Strain
t(s)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3

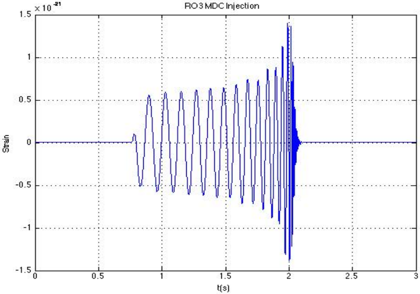
HR-series


First 3 (normalized) Principal Components
Strain
t (s)

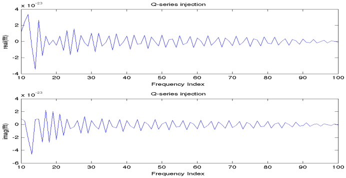

HR MDC Injection
Strain
t(s)

RO3-series

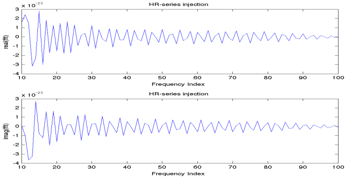

First 3 (normalized) Principal Components
Strain
t (s)
0.05
0
-0.05
0.1
-0.1
1.4
1.6
1.8
2
2.2
2.4
2.6


RO3 MDC Injection
× 10^-21
Strain
t(s)
1.5
1
0.5
0
-0.5
-1
-1.5
2.5
3

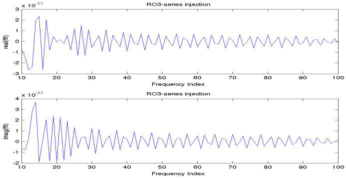
Frequency Content



(a) Frequency content of a Q-series injection



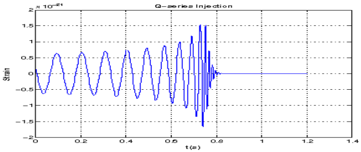
(b) Frequency content of an HR-series injection



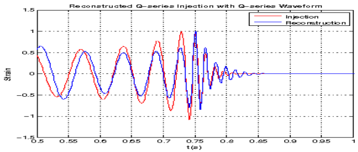
(c) Frequency content of an RO3-series injection

Q-series Reconstruction

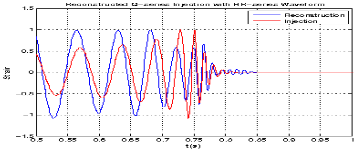
- The plots below show the original MDC injection, along with the reconstruction using different waveforms as described on Slide 4



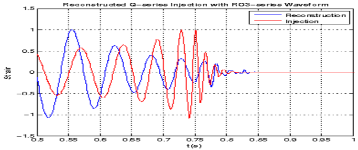
(d) Injection



(e) Bayes Factor: 143.3



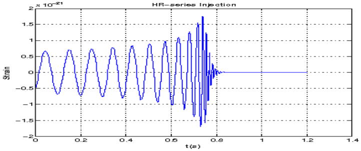
(f) Bayes Factor: 36.8



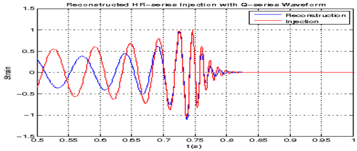
(g) Bayes Factor: -28.3

HR-series Reconstruction

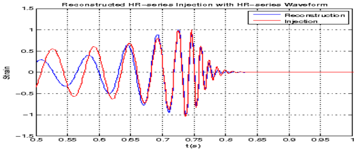
- The plots below show the original MDC injection, along with the reconstruction using different waveforms as described on Slide 4



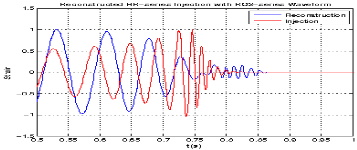
(h) Injection



(i) Bayes Factor: 114.4



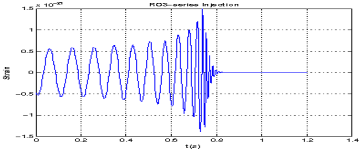
(j) Bayes Factor: 111.6



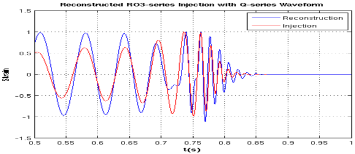
(k) Bayes Factor: -43.9

RO3-series Reconstruction

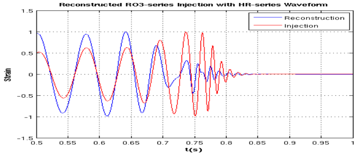
- The plots below show the original MDC injection, along with the reconstruction using different waveforms as described on Slide 4



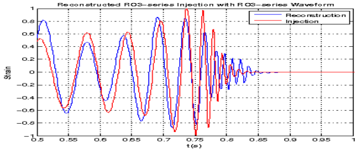
(l) Injection



(m) Bayes Factor: 80.2

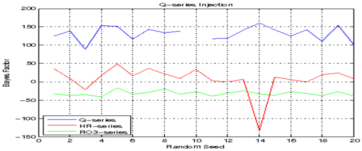


(n) Bayes Factor: -20.5

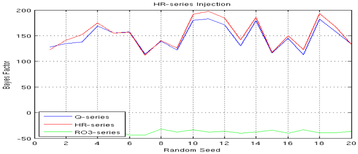


(o) Bayes Factor: -1.4

Gathered Results: SNR=20, PCs=6



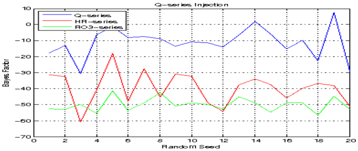
(p) Q-series injection



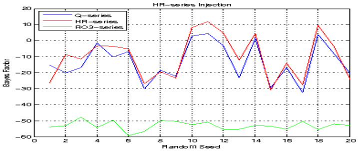
(q) HR-series injection

(r) RO3-series injection

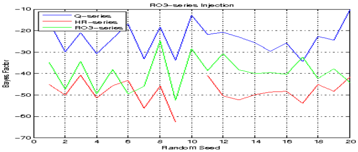
Gathered Results: SNR=10, PCs=8



(s) Q-series injection

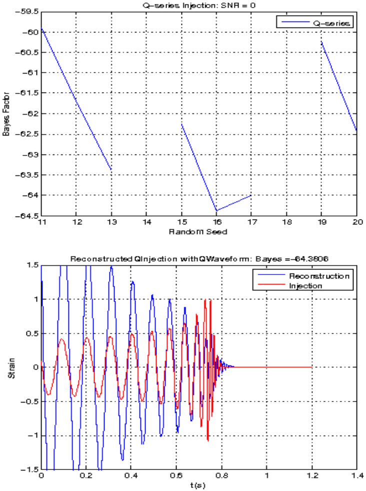


(t) HR-series injection



(u) RO3-series injection

Gathered Results: SNR=0, PCs=8



- The End

Principle Component Analysis

- $\mathbf{M} = \mathbf{USV}^T$
  - $\mathbf{M}$ is an m x n matrix containing the data:

$$\mathbf{M_{m,n}} = \begin{bmatrix} wf_{1,1} & wf_{2,1} & \cdots & wf_{n,1} \\ wf_{1,2} & wf_{2,2} & \cdots & wf_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ wf_{1,m} & wf_{2,m} & \cdots & wf_{n,m} \end{bmatrix}$$

  - $\mathbf{U}$ and $\mathbf{V}$ are matrices of the eigenvectors of $\mathbf{MM}^T$ and $\mathbf{M}^T\mathbf{M}$, respectively
  - $\mathbf{S}$ is a diagonal matrix containing the square roots of the eigenvalues of $\mathbf{U}$ of $\mathbf{V}$
- Step #1: Calculate the covariance matrix $\mathbf{C}$, of $\mathbf{M}$
- Step #2: Calculate the eigenvalues ($\mathbf{S}^2$) and eigenvectors ($\mathbf{V}$) of $\mathbf{C}$
- Step #3: Organize $\mathbf{S}$ in descending order of eigenvalues along with the corresponding eigenvectors in $\mathbf{V}$

Principle Component Analysis

- Step #4: Compute the eigenvectors of the real covariance matrix $\mathbf{U}$ (the PCs)
  - $\mathbf{U} = \mathbf{M} \times \mathbf{V}$
- Step #5: Calculate the β values by projecting $\mathbf{M}$ onto $\mathbf{U}$
  - $\beta = \mathbf{M} \cdot \mathbf{U}$
- The reconstructed waveform is thus given by $\mathbf{D} = (\beta \cdot \mathbf{U}^T)^T$