Hyperion 2021:

"Lightening up Dark Matter"

TEAM
KRONOS-42

St. Stephen's
College Delhi

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Abstract
For problem 2, Introducing a mass distribution M(r) and working under the assumptions the dark matter density is spherically and uniformly distributed and with the star assumed to be in our locality we get the required equation.

For problem 3, we perform the previous analysis to a more accurate model and put in known values of the constants to both density distributions to find a relative percentage error of 30% .

For problem 5, we breakdown the observed velocity data into some of components and try to find the fit for the component that gives us the radial velocity due to GC and DM. We assume that the equation for the fit to be a sum of a linear and sinusoidal equation. First in our attempt to remove noise we do a Gaussian regression and get a more filtered data set. We then do a Fourier Transform of the new values to extract the frequencies and amplitudes of the sinusoidal part of our equation and hence get the velocity component due to the exoplanet's influence. To get the linear component, we fit the data with a sum of linear and the above mentioned sinusoidal function to get the slopes of the linear component.

The value of ρ_{DM} is found to be 0.119050 $M_s Pc^{-1}$

Contents

1 Question 1

State and derive Gauss' Law of Gravitation

> Gauss' Law of Gravitation states that:
>
> *The gravitational flux through any closed surface is proportional to the enclosed mass.*

1.1 Mathematical Derivation

For a body of mass M at some position $\mathbf{r}'$ the gravitational field at an arbitrary point $\mathbf{r}$ is given by:

$$\mathbf{g}(\mathbf{r}) = -GM\frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3} \tag{1}$$

Where $\boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$

We can rewrite this in terms of volume mass density as:

$$\mathbf{g}(\mathbf{r}) = -G\iiint_{all\ \ space} \rho(\mathbf{r}')\frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}dV' \tag{2}$$

We could integrate over just the volume of the body, however it is more convenient to integrate over all space. Since the value of ρ is 0 outside the volume of the body this makes no difference to our calculation.
We now find the divergence of this field:

$$\nabla.\mathbf{g} = -G\iiint \rho(\mathbf{r}')\nabla.\left(\frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}\right)dV'$$

Note that since the divergence is over target coordinates while the integral is over source coordinates, we can take the operator inside the integral. We now employ the use of the mathematical identity $\nabla.\left(\frac{\mathbf{r}}{r^3}\right) = 4\pi\delta^3(\mathbf{r})$ where $\delta^3(r)$ is the 3 dimensional Dirac Delta Function.

$$\nabla.\mathbf{g} = -4\pi G\iiint \rho(\mathbf{r}')\delta^3(\mathbf{r} - \mathbf{r}')dV'$$

We use the property of the Delta Function : $\int_{-\infty}^{\infty} f(r)\delta(r-a) = f(a)$

$$\Rightarrow \boxed{\nabla.\mathbf{g} = -4\pi G\rho(\mathbf{r})} \tag{3}$$

This is the **Differential Form of Gauss' Law**

In order to convert to integral form first we integrate over volume:

$$\iiint \nabla.\mathbf{g}\, dV = -4\pi G \iiint \rho(\mathbf{r}) dV$$

Now applying the divergence theorem:

$$\boxed{\oiint_S \mathbf{g}.d\mathbf{a} = -4\pi GM} \tag{4}$$

This is the **Integral form of Gauss' Law**

2 Question 2

Refer to the Fig. 1 given below and consider $a_r(r)$ to be the Dark Matter (DM) contribution to the acceleration of the observed star. Show that the local DM density (ρ_{DM}) in the vicinity of sun is given by:

$$\rho_{DM} = \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)$$

with A and B being the Oort's constants. Further, discuss briefly about the $(A-B)$ term in the expression of dark matter density and state the physical significance of the same.

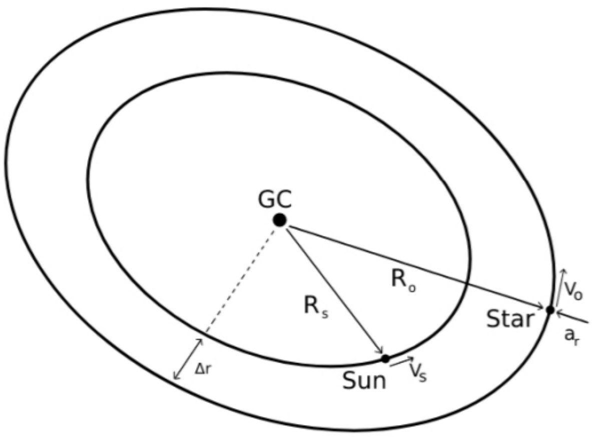


Fig. 1. Diagrammatic representation for the concerned galactic configuration.

We are given that the $a_r(r)$ is the Dark Matter contribution to the acceleration of the observed star. Hence let $M(r)$ be the mass distribution of the Dark Matter in the vicinity of the sun. Then from Newton's Gravitational Law:

$$a_r(r) = -G\frac{M(r)}{r^2}$$

Taking the derivative w.r.t r:

$$\frac{\partial a_r}{\partial r} = 2G\frac{M(r)}{r^3} - G\frac{M'(r)}{r^2} \tag{5}$$

Where $M'(r)$ is the derivative of the mass distribution function w.r.t r.
Assuming that the dark matter density is spherical and uniform we can write it as:

$$\rho_{DM} = \frac{M(r)}{V(r)} = \frac{M(r)}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow M(r) = \frac{4}{3}\pi r^3 \rho_{DM}$$

Noticing the $M'(r)$ term in Eqn. 5 we differentiate the above expression w.r.t r:

$$M'(r) = 4\pi r^2 \rho_{DM} \tag{6}$$

Subsituting in 5:

$$\frac{\partial a_r}{\partial r} = 2G\frac{M(r)}{r^3} - 4\pi G \rho_{DM} \tag{7}$$

In order to describe the first term we turn to the Oort Constants[1] defined as:

$$A = \frac{1}{2}\left(\frac{V_c}{R} - \frac{\partial V_c}{\partial R}\right)_{R_s},\ B = -\frac{1}{2}\left(\frac{V_c}{R} + \frac{\partial V_c}{\partial R}\right)_{R_s} \tag{8}$$

Where V_c is the centripetal velocity. Taking the difference of A and B

$$A - B = \frac{V_s}{R_s}$$

If we consider the acceleration due to dark matter and the equation of centripetal acceleration we can write:

$$-G\frac{M(r)}{r^2} = \frac{v_c^2}{r}$$

Dividing by r

$$-G\frac{M(r)}{r^3} = \frac{v_c^2}{r^2}$$

If we write this for the star in consideration:

$$-G\frac{M(r)}{R_0^3} = \frac{v_0^2}{R_0^2}$$

Now since we are solving for the distribution only in the vicinity of the sun we can say that $\frac{v_0^2}{R_0^2} \approx \frac{v_s^2}{R_s^2} = \left(\frac{v_s}{R_s}\right)^2$. Notice that this is simply $(A-B)^2$.

$$\therefore -G\frac{M(r)}{r^3} = (A-B)^2$$

Substituting this in Eqn. 7

$$\frac{\partial a_r}{\partial r} = 2(A-B)^2 - 4\pi G\rho_{DM}$$

This rearranges to give us

$$\boxed{\rho_{DM} = \frac{1}{4\pi G}\left(2(A-B)^2 - \frac{\partial a_r}{\partial r}\right)} \tag{9}$$

Hence Proved.

The $(A-B)$ term actually represents the ratio of the circular velocity of the sun w.r.t to the Galactic centre and the distance from the Galactic Centre. It is a constant value and further it arises because we are dealing with dark matter only in the vicinity of the sun, if we were to go further the approximation that allowed this term to arise would fail.

3 Question 3

Reconsider your answer to Question 2, can you think of what amount of relative uncertainty (numerical value) your solution has? Perform a numerical analysis of the same and predict the uncertainty in the Dark Matter density achieved from Question 2.

To derive Eqn. 9 we ignored the effect of the Galactic Disk which would have its own contribution to the calculation of Dark Matter density. In order to model its contribution we repeat the process performed in the last section for a **Disk Density Profile**. Hence lets assume that this disk has a surface mass density σ. Hence $M(r) = \pi r^2\sigma$ and $M'(r) = 2\pi r\sigma$.

We now first calculate $\frac{\partial a_r}{\partial r}$ from Eqn. 5:

$$\frac{\partial a_r}{\partial r} = 2G\frac{\pi r^2\sigma}{r^3} - G\frac{2\pi r\sigma}{r^2}$$

$$\Rightarrow \frac{\partial a_r}{\partial r} = 2G\frac{\pi\sigma}{r} - 2G\frac{\pi\sigma}{r}$$

$$\Rightarrow \frac{\partial a_r}{\partial r} = 0$$

Subsituting this in Eqn. 7 and rearranging

$$\therefore \rho'_{DM} = \frac{2G}{4\pi G}\frac{M(r)}{r^3}$$

Substituting $M(r)$:

$$\rho'_{DM} = \frac{1}{2\pi}\frac{\pi r^2 \sigma}{r}$$

$$\Rightarrow \rho'_{DM} = \frac{\sigma}{2r}$$

Since we are dealing with a star close to the sun(vicinity of the sun) we put $r \approx R_s$

$$\boxed{\rho'_{DM} \approx \frac{\sigma}{2R_s}} \tag{10}$$

To calculate the relative uncertainty in our ρ_{DM} we now calculate $\frac{\rho'_{DM}}{\rho_{DM}}$

In order to numerically calculate ρ'_{DM} we use the known values of σ and R_s:
σ is the present day total disc surface density of the Milky way which in the region of $R_s = 8kpc$ is approximately $50M_s\ pc^{-2}$[2], where M_s is the mass of the sun. This gives us:

$$\rho'_{DM} = 0.003125\ M_s/pc^3$$

It is not possible to find the value of ρ_{DM} directly from Eqn. 9 without finding $\frac{\partial a_r}{\partial r}$. However the calculation of ρ'_{DM} was done from our expression and hence can be used to measure its relative uncertainty. Hence we use a previously calculated value of $\rho_{DM} = 0.01056\ M_s/pc^3$[3]

$$\therefore \frac{\rho'_{DM}}{\rho_{DM}} = 0.29592 \approx 0.3$$

Hence the relative uncertainty is approximately 30%.

4 Question 4

What hurdles would Hyperion have to face while making observations and what steps that he can take to reduce the error in the same?

Our primary aim is to measure radial velocities and use them to eventual measure the radial acceleration which in turn can be used to approximate dark matter density. The main problem in this task is that the radial accelerations of stars are typically very small(of the order of $cm\ s^{-1}\ year^{-1}$ as can be seen in our data set), in order to measure this, we need extremely precise values of Radial velocity over many years. Calculating Radial velocities from Doppler shift requires us to measure the spectrum of the light coming from a star which is performed by a **Spectrograph**. The precise measurements that we need to make here will need extremely stable calibration of the spectrograph which is maintained over many years. A method to achieve this is discretization of frequencies performed by a specialized frequency comb called an Astro-Comb which increases the resolution of spectrographs dramatically. This is a well known method to detect wobbles caused by exoplanets and a specialized comb like this(with enough resolution) has been developed by Hao-Li (2008)[4]

The other problem is the gravitational effect of bodies other than the dark matter we are considering and that of the Galactic Centre. The two primary things to be taken into account here are other stars in the vicinity of the target, this can be filtered using data analysis techniques, and the presence of exoplanets around the target star. These create a distinct oscillatory signature in the data. Hence we can either disregard such stars or we can filter out the periodic signal using a Fourier Transform. Alternatively if data about the exoplanet is available it can be used to subtract the unwanted oscillatory signal from the collected data.

Another possible hurdle is the contribution of perspective velocities(motion in the plane of the sky w.r.t to an observer on Earth) which can affect the radial velocity measurement. To account for this we can use data collected by high precision surveys on the motions(specifically perspective velocities) of the target stars.

5 Question 5

Attached is a file named 'vel_data.csv' containing the observed radial velocity (in cm/s) of a star w.r.t. the Galactic Centre (GC) over the course of 4 years. We need to find the radial velocity due to the GC and the DM as a function of time (which should come out as linear) and hence, the radial acceleration by filtering the periodic signal and noise.
You can assume influence due to some 'exoplanets' orbiting the observed star. This also contributes to the observational velocity, and may be expressed as a sinusoidal expression (can you argue why?).

5.1 Method of analysis

5.1.1 Analysis of Data

The Given velocity data is initially plotted as a function of time(shown in Fig. 2) to get an idea of its nature. The data is analysed using Python. We employ 'Pandas'[5] data analysis package import and configure the data into different lists which then is plotted using the 'Matplotlib[6]' library. The Numpy[7] and Scipy[8] libraries are used for various analysis tasks.

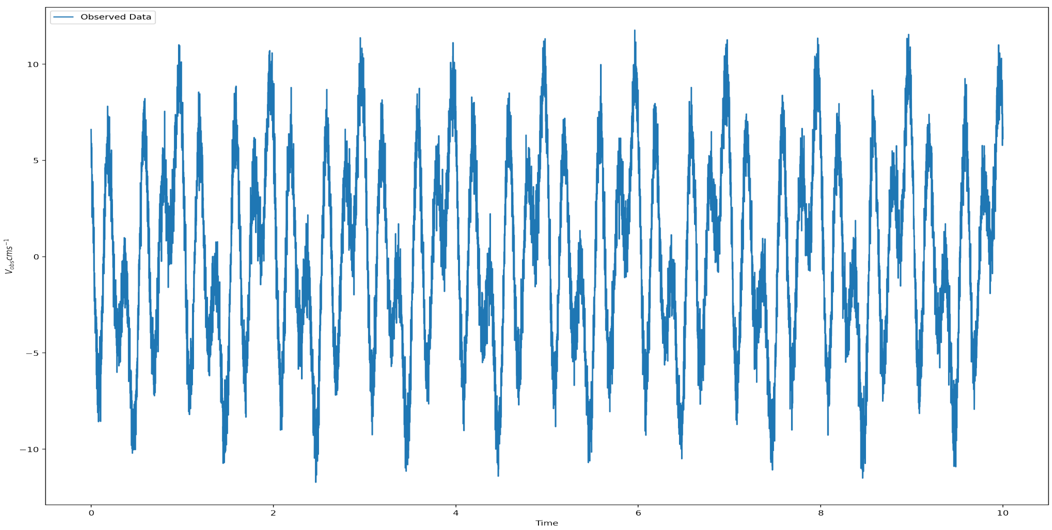


Fig. 2. Given Observational Data plotted

We can express the total observational velocity (V_{obs}) as sum of all the influences. This includes the Velocity due to Galactic centre(V_{GC}) and Dark matter (V_{DM}), Contribution due to exoplanets(if present) (V_{EP}) and Noise (V_{Noise})) The expression can be written as,

$$V_{obs} = V_{GC} + V_{DM} + V_{EP} + V_{Noise} \tag{11}$$

Our aim is to extract the velocity due to the GC and DM while filtering out the EP and Noise influence.

Our analysis began with the assumption that the observed behaviour can be represented as a sum of analytical functions, which can describe the velocity fluctuations which contribute in the equation above. The distinct oscillatory signature visible in Fig. 2 suggests an exoplanet influence that will have to be accounted for; hence a function that can fit this data(once filtered from noise) must be the sum of a linear function $g(x)$ which is expected for the GC and DM velocities and the periodic function $h(x)$ which encodes the motion of the exoplanet(s) and any other stellar influence.

$$f(x) = g(x) + h(x) \tag{12}$$

The contribution of the linear function is considerably small compared to the observational data obtained due to large contribution of Exoplanet fluctuations. Our main goal here is to find the analytical function (2) and hence decompose it into both a linear and nonlinear components.

5.1.2 Determination of $f(x)$

Now to proceed with our analysis, we currently have a data set which has all the contributions added up, as mentioned above we have to find the linear and non-linear components. We first aim to filter noise from our data set using a Gaussian Regression fit to the observed data.

5.1.3 Implementation of Gaussian regression[9]

To fit the data with a Gaussian regression model, we used the Sci-kit package available for python. We initialised the model with a Radial Basis Function (RBF) kernel with an optimizer value of 10. The model is then fit with observational data (Code in Appendix A.1). This fit helps us extract some noise and produce a much more filtered data set. The resulting model along with the original data is plotted below:

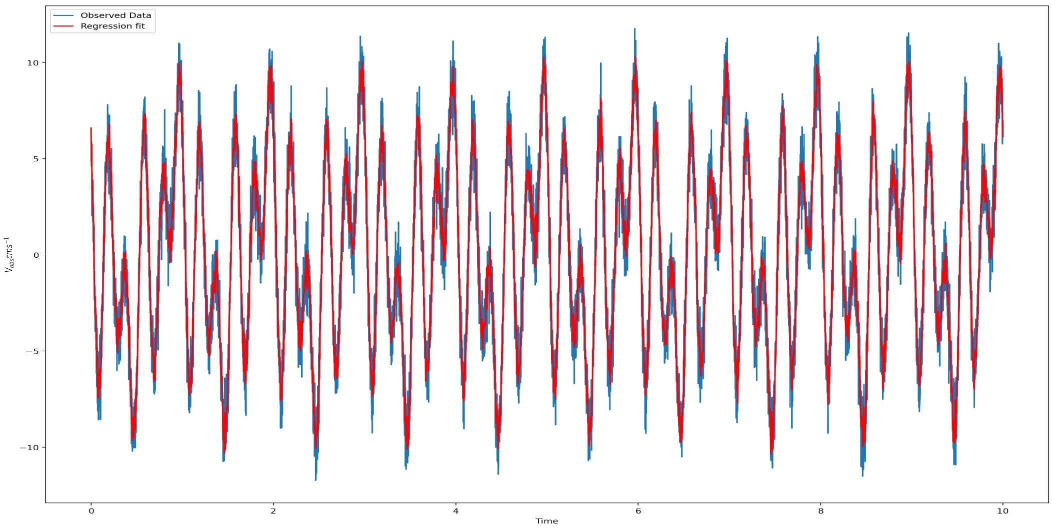


Fig. 3. A Gaussian Regression Fit

Now we can also plot the filtered out noise to demonstrate the use of this method:

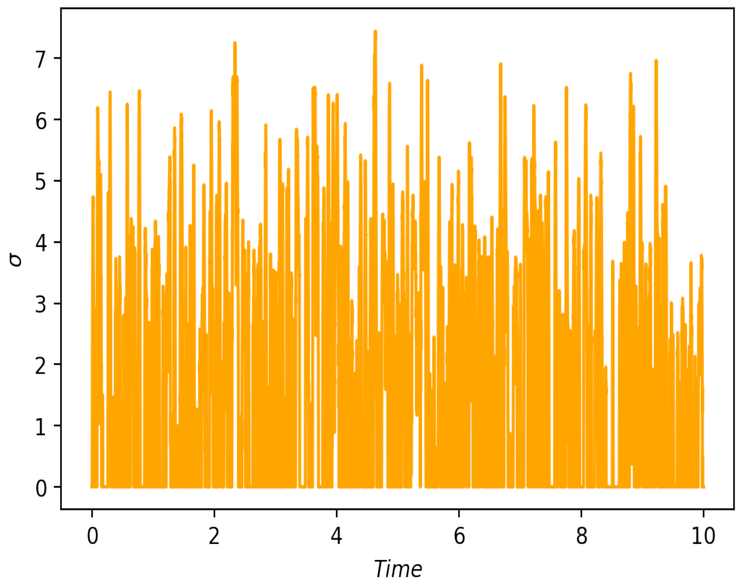


Fig. 4. Extracted Noise, σ: Standard Deviation

5.1.4 Fourier analysis[10]

After we have established a reduced noise model of our data, our goal next is to find the functions that we need. We first use Fourier Analysis to extract the frequencies of the periodic part of our function. Hence we perform a fast Fourier transform on the (filtered) data which can be used to analyse oscillatory behaviour. We employ the use of the 'Scipy' library to compute a Fast Fourier Transform(FFT) of the data in the Frequency Domain. Having done this we now need to convert back to the time domain and hence need a sampling rate. We obtain this as the reciprocal of the time unit we currently have($= 0.001001$) which gives us $F_s = 999.000999001\ year^{-1}$ (Code in Appendix A.2).

After performing a Fast fourier transform on the data, we obtained 3 peaks which are shown below,

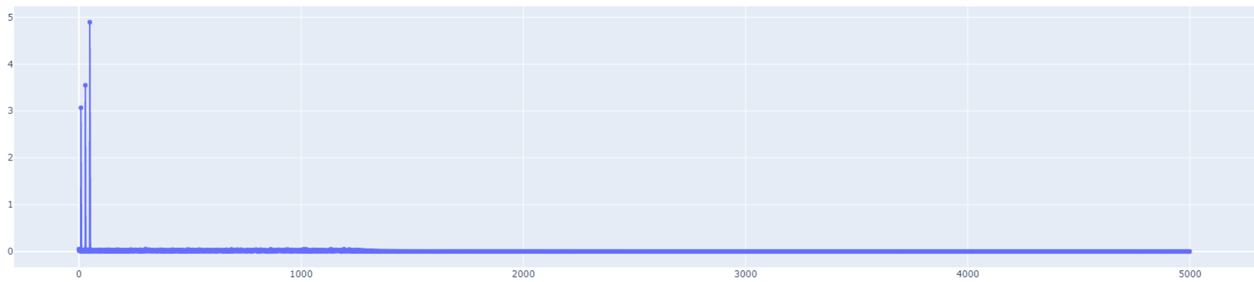


Fig. 5. Peaks obtained after FFT

The 3 peaks found correspond to oscillatory behaviour at 3 distinct frequencies and hence our periodic function can be written as the sum of 3 Cosine functions with respective amplitudes and frequencies. A cosine function is chosen since the observational data starts from a point which is not the origin. We can hence write our periodic function as:

$$h(x) = ACos(\omega_1 x) + BCos(\omega_2 x) + CCos(\omega_3 x) \tag{13}$$

(*Where A,B,C are amplitudes of the function and ω_1,ω_2,ω_3 are the corresponding frequency terms, (Note ;- $\omega = 2\pi f$)*)

Now the values for the unknowns in this equation can be obtained by individually looking at the peaks and the corresponding amplitudes and frequencies from the graph. Note that the frequencies are f values and must be multiplied by 2π to obtain $omega$ values. The corresponding values of the obtained constants are as follows,

A	3.067455
B	3.548172
C	4.893752
f_1	0.999000999000 $year^{-1}$
f_2	2.997002997003 $year^{-1}$
f_3	4.995004995005 $year^{-1}$

Using these values we can plot the function $g(x)$ along with the original observational data by substituting them. Which will will result in,

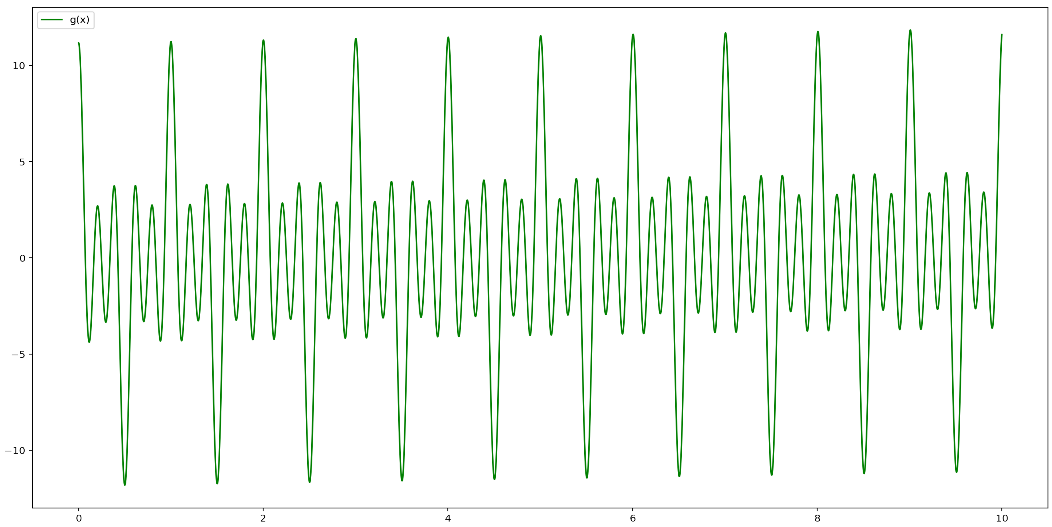


Fig. 6. $g(x)$ plotted seperately

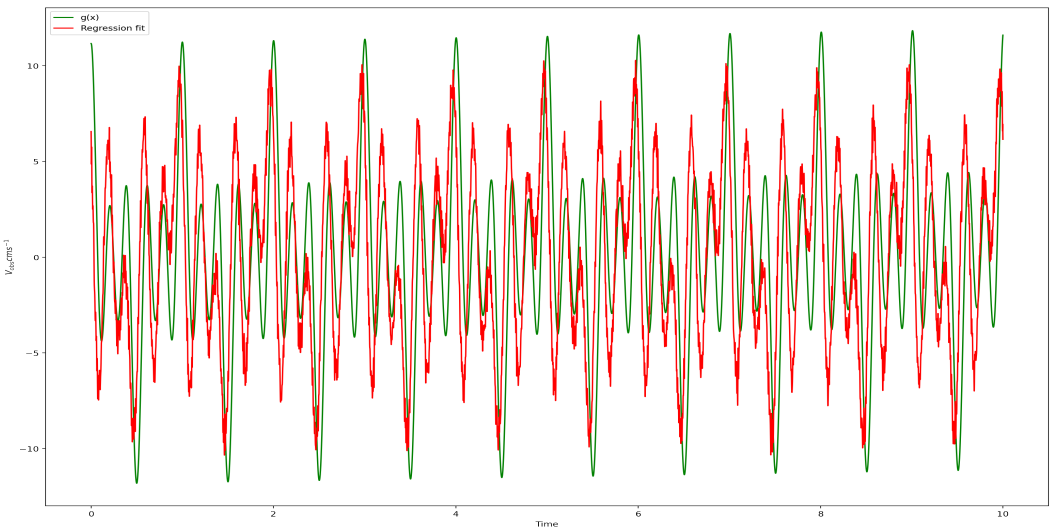


Fig. 7. With the modified observational data

From here we can see that the Cosine functions give us a very good approximate of the underlying function. But still it is not perfectly matching, This is because what we have here is a pure sinusoidal function, which doesn't have any added influences that we have mentioned above.

5.1.5 Adding a linear function

The non-linear periodic function , $h(x)$ is the most contributing term in the original function $f(x)$. The next term is a linear function $g(x)$ that we have to find out. We can write a general form of $g(x)$ as,

$$h(x) = mx + c \tag{14}$$

Where m is the slope and c is the intercept

We have to now get the linear term coefficients, For this we can take the total function $f(x)$ as our parent function and since we already know the Non-linear function $g(x)$ which is in the form of Eqn.(13).Now we do a curve fit with the already determined $g(x)$ and $h(x)$. This was done using a normal curve fit using 'Scipy'. The equation that we are fitting is of the form:

$$f(x) = A\cos(2\pi f_1 x) + B\cos(2\pi f_2 x) + C\cos(2\pi f_3 x) + mx + c \tag{15}$$

By fitting this with modified data set, we can compute the value of m which is slope of the linear increase. This slope will be our final acceleration value which is contributed the Galactic center and the Dark matter vicinity

Fitting the final equation Eqn 15 and plotting with actual data yields the following plot.

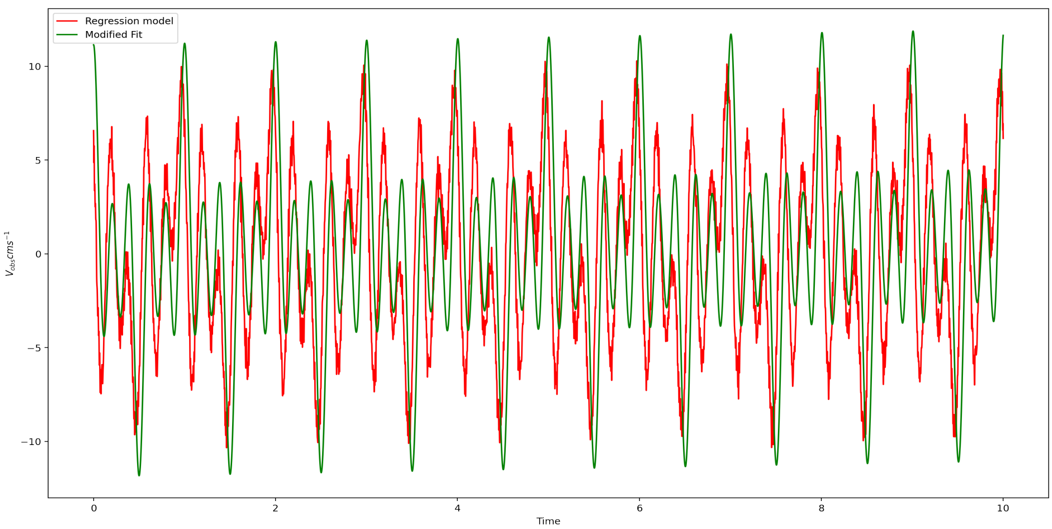


Fig. 8. Final curve fit with a linear function

From the following Curve fit, we obtained the following values for slope(m) and intercept (c) from this fit,

m	0.08133786
c	-0.36489605

The intercept value is not considered since it is not significant for our analysis, Using these values we can plot the linear function.

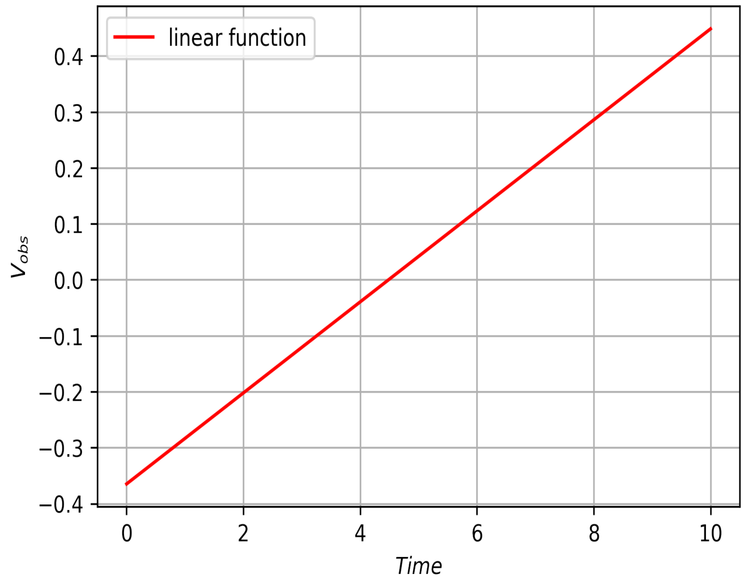


Fig. 9. Linear function

The final acceleration due to the Galactic Center and Dark matter is given by the slope i.e.

$$\boxed{a_r = 0.08133786\ cm\ s^{-1} year^{-1}} \tag{16}$$

The sinusoidal model of the exoplanetary influence exists because of the orbital nature of its motion, i.e. as the planet orbits a star the star slightly wobbles from its position moving away and close to us periodically(since the orbit and hence extent of influence of the planet is periodic). Due to the Doppler the planet moves away from us its spectrum is red-shifted, and as it moves towards us it is blue-shifted, hence using the spectrum we can easily measure both positive and negative velocities of the planet. Hence as it wobbles away and toward us due to the exoplanet we see a signature periodic velocity change which can be modelled as a sinusoidal function.[11]

6 Question 6

Code for this section can be found in Appendix

Given acceleration data we first plot the data as a function of star number to get an idea of the distribution:

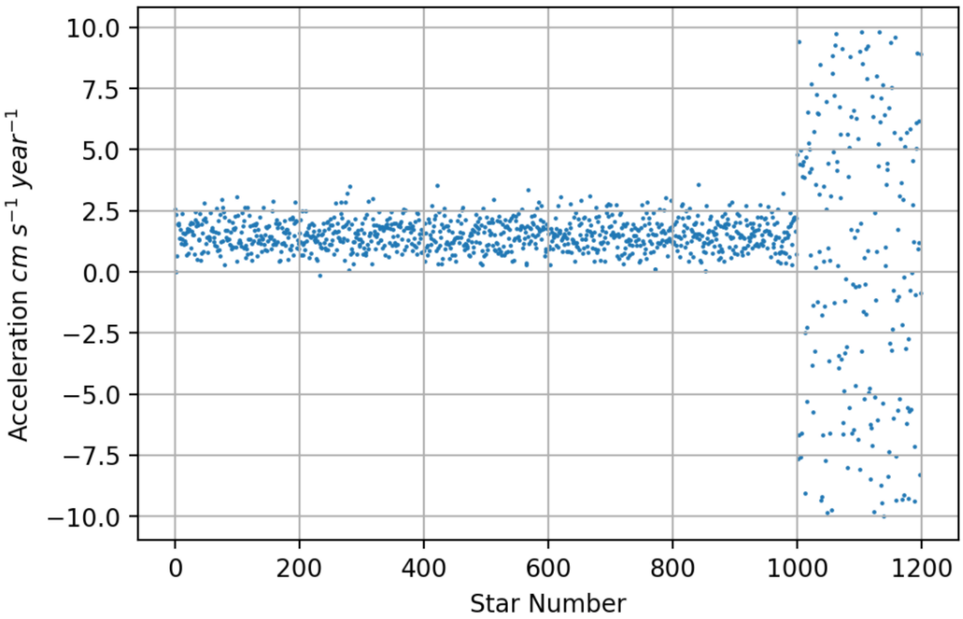
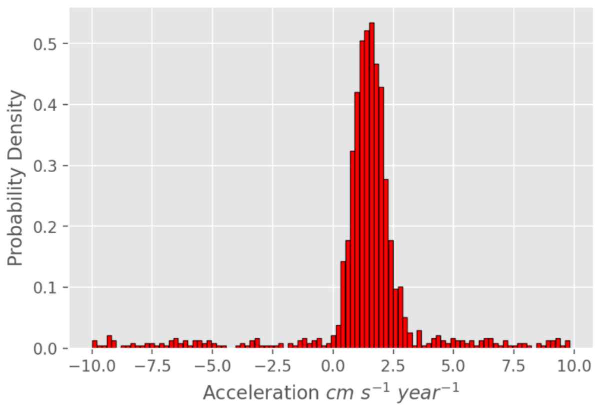
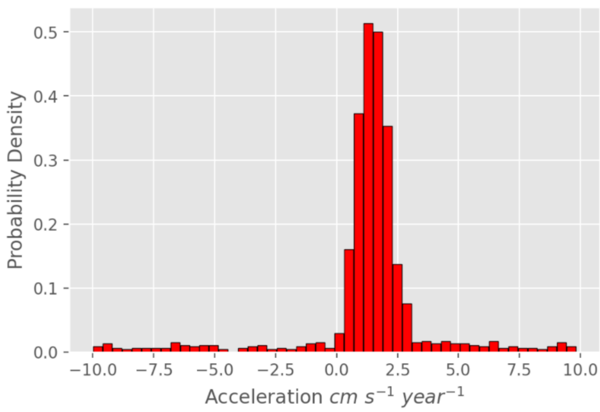


Fig. 10. Acceleration Distribution

We can see that it appears to be dense in a region and quite scattered in the complete limits(characteristic of a random distrbution) we hence plot a histogram of this data:



(a) 100 Bins



(b) 50 Bins

Fig. 11. Probability Distribution of the Data

We see the familiar Gaussian type distribution and hence try to fit a Gaussian Curve to this data:

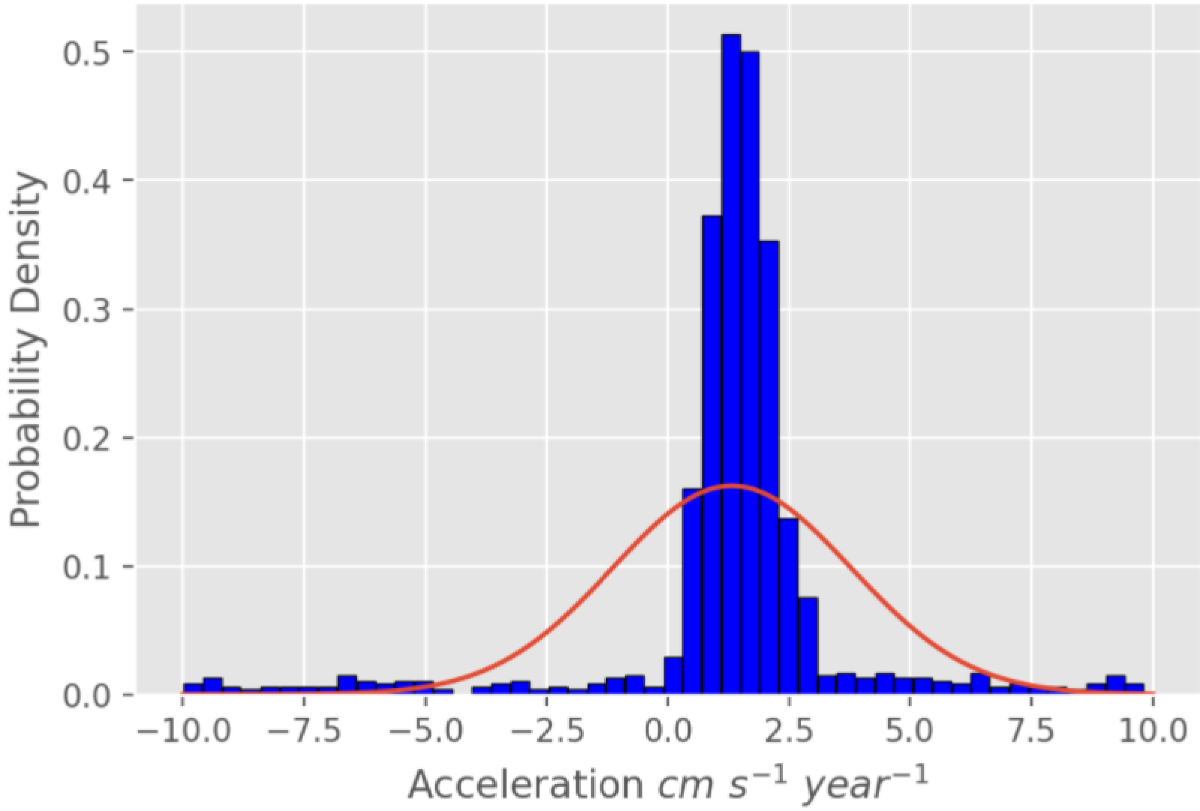


Fig. 12. Gaussian Fit

This gives us:

Mean	1.32429075868251
σ	2.456796290759117

Hence

$$\boxed{a_{mean} = 1.324291\ cm\ s^{-1}\ year^{-1}} \tag{17}$$

6.1 Alternate Distribution

We notice that the last 200 data points contain outlines that differ quite visibly from the first 1000 data points as seen in Fig. 10 and may be the reason for the high standard deviation found. We hence try to produce a distribution disregarding those points, the histogram we found is shown below:

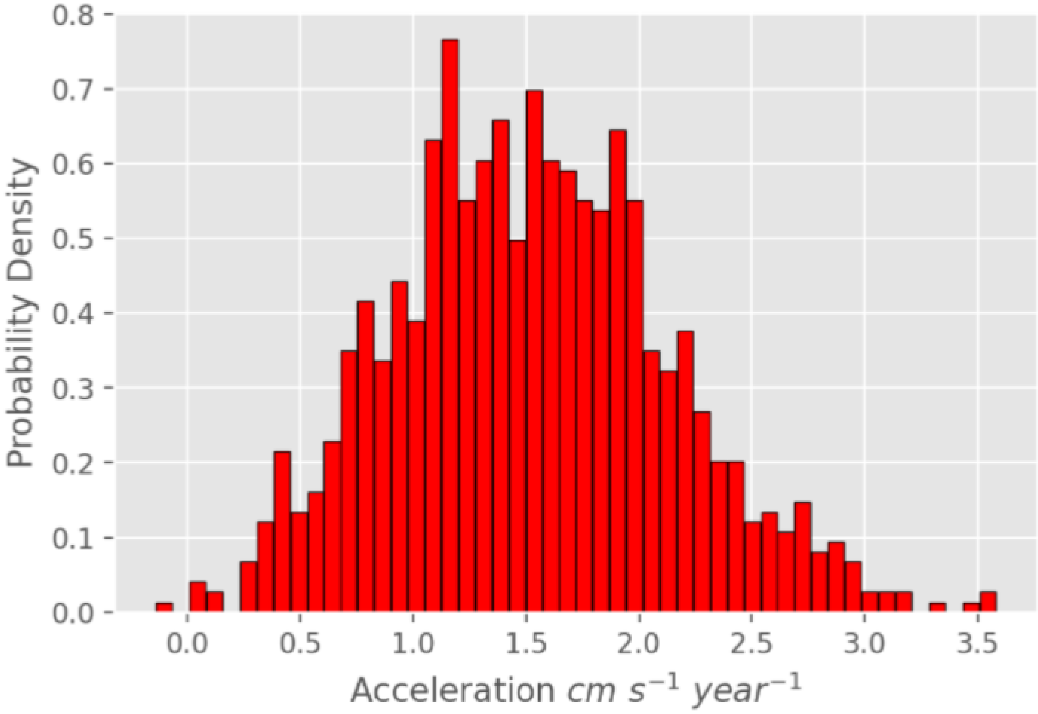


Fig. 13. Probability Density of modified data

Noticeably this data section too has a shape that resembles a normal distribution: we hence fit a Gaussian Curve to this modified data:

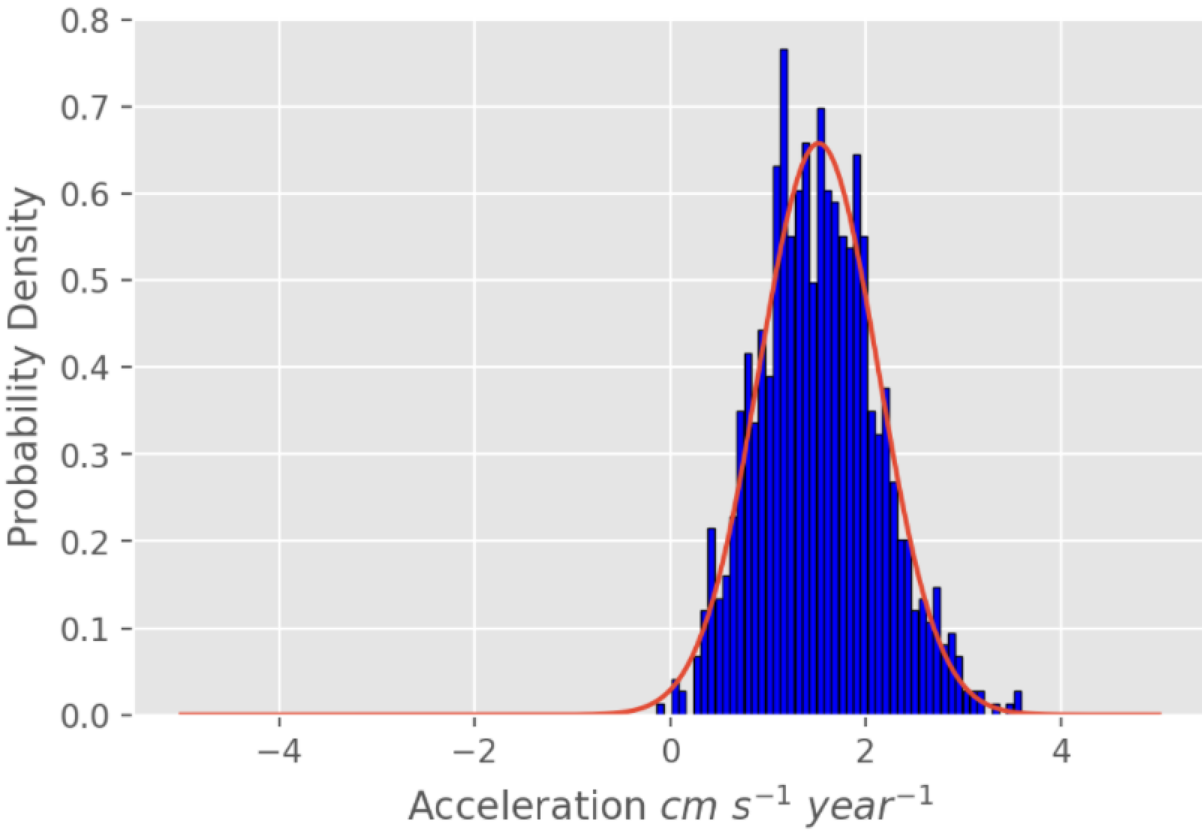


Fig. 14. Gaussian Fit to modified data

This gives us:

Mean	1.5195369669058731
σ	0.6066079911767505

Which has a noticeably lower standard deviation than the full data set.

Hence

$$\boxed{a_{mean} = 1.519537\ cm\ s^{-1}\ year^{-1}} \tag{18}$$

7 Question 7

We use this mean acceleration(for full data set) Δa_r and $\Delta r = 3kpc$ to approximate $\frac{\partial a_r}{\partial r} \approx -\frac{\Delta a_r}{\Delta r}$(the negative sign is used since the derivative must come out to be negative). This gives us:

$$\boxed{\frac{\partial a_r}{\partial r} \approx -0.441430\ cm\ s^{-1}\ year^{-1}\ kpc^{-1}} \tag{19}$$

We use $A - B \approx 27.2\ km\ s^{-1}\ kpc^{-1}$ and use Eqn. 9 to calculate ρ_{DM} as,

$$\boxed{\rho_{DM} \approx 0.107272\ M_s\ pc^{-3}} \tag{20}$$

Where M_s : Mass of the Sun

7.1 Calculation for Section of Data Set

We also perform this calculation for the section of the dataset described in Sec. 6.1, with $\Delta a_r = 1.519537$ and the same $\Delta r = 3kpc$ to find:

$$\boxed{\frac{\partial a_r}{\partial r} \approx -0.50651\ cm\ s^{-1}\ year^{-1}\ kpc^{-1}} \tag{21}$$

Hence we find:

$$\boxed{\rho_{DM} \approx 0.119050\ M_s\ pc^{-3}} \tag{22}$$

8 References

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A Appendix I: Question 5 Code

A.1 Gaussian regression - Python code

```
#Importing  required libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
#Importing SkLearn libraries
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, ConstantKernel as C

data = pd.read_csv('vel_data.csv') #Reading Data
print(data)
df = pd.DataFrame(data)

A = df.Time #Generating Lists
B = df.obv
plt.figure(figsize=(17,12))
#plt.plot(A,B)


# Reshaping the lists into two dimensional lists
A = np.atleast_2d(A).T
B = np.atleast_2d(B).T


C_new = B.ravel()

#Creating a RBF kernel
kernel = C(1.0, (1e-3, 1e3)) * RBF(10, (1e-2, 1e2))

#Gaussian model
gp = GaussianProcessRegressor(kernel=kernel, n_restarts_optimizer=10)

#Fitting the gaussian model with the observational Data
gp.fit(A,B)

#Generating a prediction from the obtained data and extracting Noise
y_pred, sigma = gp.predict(A, return_std=True)
```

A.2 Fast Fourier Transform - Python Code[10]

```
    #Importing Required Libraries
import plotly.graph_objects as go
import scipy.fftpack #FFT library
from scipy.signal import find_peaks



#code for finding the frequency\approx
Fs = 999.000999001 #Sampling Rate
L = 10000 # number of data points
#print(L)
Y = scipy.fftpack.fft(y_pred[:,0]) #Performing a Fast Fourier transform
P2 = np.abs(Y/L)
P1 = np.zeros(int(L/2))
P1[0:int(L/2)] = P2[0:int(L/2)];
P1[1:int(L/2)-2] = 2*P1[1:int(L/2)-2]
print(P1.shape)
P1 = np.delete(P1,0) #Deleting 0th element since it will result in abnormal peaks

f = np.linspace(0.0, 10000,10000)


#Here plot the peaks of the FFT
indices = find_peaks(f)[0]
fig = go.Figure()
fig.add_trace(go.Scatter(
    y=P1,
    mode='lines+markers',
    name='Original Plot'
))

fig.add_trace(go.Scatter(
    x=f,
    y=[z[j] for j in indices],
    mode='markers',
    marker=dict(
        size=8,
        color='red',
        symbol='cross'
    ),
    name='Detected Peaks'
))

fig.show()


print(u[9],u[29],u[49]) #Printing frequencies of 3 peaks that are detected
```

A.3 Functional fitting - Python code

```
from scipy.optimize import curve_fit # Importing curve fit package

def func(x, a, b): # Defining a function with periodic and linear terms
  return a*x + b + 3.067455*np.cos(0.99900*2*np.pi*x)
  +3.548172*np.cos(2*np.pi*2.99700299*x)
  +4.893752*np.cos(4.99500*2*np.pi*x) #Defining the parent function f(x)

pars, cov = curve_fit(func, time, vels) #solving for parameters
```

B Appendix II :Question 6 Code

B.1 Gaussian distribution

```
#Importing required libraries
import matplotlib.mlab as mlab
import numpy as np
import statistics
from scipy.stats import norm
import matplotlib.pyplot as plt


d = pd.read_csv('acc_data.csv', header=None) # Importing data
df = pd.DataFrame(d) # Creating a dataframe

#Assigning variables to the dataframe

N = df[0]
acc = df[1]
#Fitting a Gaussian function to the histogram and plotting alogside it
(mu, sigma) = norm.fit(acc) #Mean and starndard deviation
n,bins,patches=plt.hist(acc,100,density = True,facecolor='blue',align='mid',
edgecolor = 'black')
plt.style.use('ggplot')
y = norm.pdf(bins,mu,sigma)
plt.plot(bins,y,'g--',linewidth=3)
```