Astronomy Club, IITK
Hyperion 2021

"Lightening up Dark Matter"

Instructions

- You are supposed to type your answers in a `pdf` file and submit the same. Please do not submit handwritten answers as it would be troublesome to evaluate those.
- Your solutions must be properly formatted. Any unconventional notation used must be explicitly stated.
- Attach any relevant code snippets which were utilized in analysis of the problems.
- You must submit all the solutions in English only.
- An introduction summary of not more than 200 words highlighting the basic workflow needs to be included at the beginning of the final report.
- The solutions must be your own and all references must be duly cited. Any form of plagiarism will not be tolerated under any circumstances and would lead to disqualification.
- You must have a valid college ID which would be verified at the time of prize distribution.
- The deadline of submission is **23:59 hrs of 9th October**. After the deadline, a 5 marks penalty will be imposed for every 15 minutes delay. No submission will be entertained post 1 hour of deadline.
- In case of any discrepancy the decision of the Astronomy Club IITK would be final and binding. Be sure that all the submissions would be evaluated with due diligence and utmost fairness.

All the best!
Cosmos Is Within Us

Let the cosmic journey begin...

Astronomy, for a very long time, has been an intriguing field for a lot of people not only due to the observational aspects, but also at the same time due to the vast research prospects it offers. On the occasion of Autumn Astronomy Day 2021, it would only be appropriate to venture far beyond the frontiers where there have been only a few- a trail knitted across all the past, the present, and forever to the future. With you comes the harbinger of all human knowledge, our friend, and Case Orator **"Hyperion"**.

A Brief Introduction to Hyperion: Hyperion will be our Case Study Orator and will help you explore and analyze the presented problem. For today, Hyperion has chosen the Dark Matter dilemma for us to analyze from his bag of unsolved problems. Do bear in mind that Hyperion is a bit demanding and will demand your neurons to be active throughout.

Dark matter makes up roughly 85% of the observed matter in the universe. It influences various phenomena such as gravitational lensing and structure formation that existing theories of gravity cannot explain. Unlike ordinary matter, dark matter does not interact with electromagnetic radiation and only weakly interacts with baryonic matter and radiation. Several researchers are still working on deep-driven conceptualization of what exactly could be the possibilities around DM, and among one of them is "Quantitative Analysis of Dark Matter Density."

Now Hyperion needs your help. Can you help Hyperion to find the way through darkness?

Help Hyperion..

1. State and derive the Gauss' Law of Gravitation. **[5 marks]**

2. Refer to the Figure 1 given below and consider $a_r(r)$ to be the Dark Matter (DM) contribution to the acceleration of the observed star. Show that the local DM density (ρ_{DM}) in the vicinity of sun is given by: **[20 marks]**

$$\rho_{DM} = \frac{1}{4\pi G}\left[2(A-B)^2 - \frac{\partial a_r}{\partial r}\right] \tag{1}$$

with A & B being the Oort's constants. Further, discuss briefly about the $(A - B)$ term in the expression of dark matter density and state the physical significance of the same.

Note: You can assume spherical DM Density profile for Question 2

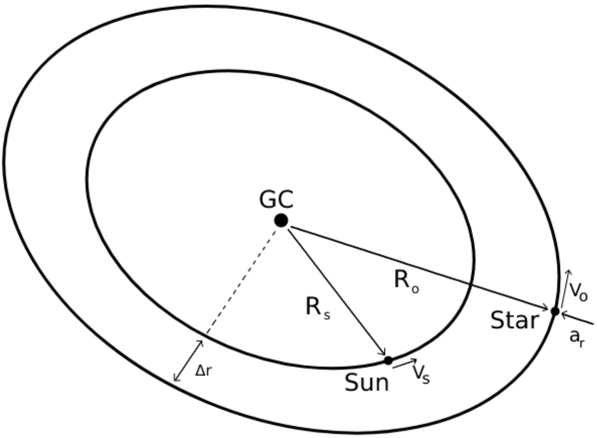

Fig 1: Diagrammatic representation for the concerned galactic configuration.

Researchers are looking for novel and convenient ways to model the density profile of DM; because it is challenging to measure DM densities using conventional methods in astrometry. For instance, we might assume that measuring galactic rotation curves using stellar paths and trajectories might give us a hint about the true nature of DM. However, these methods are mostly irrelevant for predictions, as they rely heavily on the assumption of equilibria.

While discussing about uncertainties that lie in the estimation of DM density, Hyperion would also like you to :

3. Reconsider your answer to Question 2, can you think of what amount of relative uncertainty (numerical value) your solution has? Perform a numerical analysis of the same and predict the uncertainty in the Dark Matter density achieved from Question 2. **[20 marks]**

Now that we have established a theoretical foundation, let us now apply our understanding to help Hyperion and perform data analysis to find the density of DM in the locality of our star. To accomplish this we must have precise data of the radial velocities of the neighbouring stars which is best accomplished by using Doppler shift.

4. What hurdles would Hyperion have to face while making observations and what steps that he can take to reduce the error in the same?

[10 marks]

5. Attached below is a file named `'vel_data.csv'` (in the link provided) containing the observed radial velocity (in cm/s) of a star w.r.t. the Galactic Centre (GC) over the course of 4 years. We need to find the radial velocity due to the GC and the DM as a function of time (which should come out as linear) and hence, the radial acceleration by filtering the periodic signal and noise. **[25 marks]**

   You can assume influence due to some 'exoplanets' orbiting the observed star. This also contributes to the observational velocity, and may be expressed as a sinusoidal expression (can you argue why?).

   (**Hint:** Th're art as many cradles as th're art jewels in hunter's belly)

6. Similarly, we have also provided you with acceleration data in the `'acc_data.csv'` (in the link provided) (Star number, Acceleration in $cms^{-1}year^{-1}$) file in which we have calculated the acceleration in Question 4 of multiple stars. Find the average acceleration by plotting a probability distribution curve and thereby using a Gaussian fit over the data. **[15 marks]**

   Note: For the sake of this question,the observational data was taken for stars within a radius of r = 3kpc. Hence, $r_{avg} \cong 3$ kpc is a suitable approximation.

7. Using the above obtained $\frac{\partial a_r}{\partial r}$, find out the DM density using the equation you proved in Question 2. **[5 marks]**

Link for data files

`Hyperion wishes you a future not as dark as dark matter ☺`