Astronomy Club, IITK
Hyperion 2022

'Dance of Heavenly Bodies'

Instructions

- You are supposed to type your answers in a `pdf` file and submit the same. Please do not submit handwritten answers as it would be troublesome to evaluate those.
- Your solutions must be properly formatted. Any unconventional notation used must be explicitly stated.
- Attach any relevant code snippets which were utilized in analysis of the problems.
- You must submit all the solutions in English only.
- An introduction summary of not more than 200 words highlighting the basic workflow needs to be included at the beginning of the final report.
- The solutions must be your own and all references must be duly cited. Any form of plagiarism will not be tolerated under any circumstances and would lead to disqualification.
- You must have a valid college ID which would be verified at the time of prize distribution.
- The deadline of submission is **23:59 hrs of 8th December**. After the deadline, a 5 marks penalty will be imposed for every 15 minutes delay. No submission will be entertained post 1 hour of deadline.
- In case of any discrepancy the decision of the Astronomy Club IITK would be final and binding. Be sure that all the submissions would be evaluated with due diligence and utmost fairness.

All the best!
Cosmos Is Within Us

Let the heavenly voyage begin...

Many individuals have found astronomy to be an attractive science for a very long time because of both its observational features and the numerous study opportunities it provides. It would only be fitting to travel far beyond the borders where there have only been a handful on this occasion of concurrent Mars Opposition and Cold Moon - a path stitched through all of the past, present, and forever to the future. With you comes the harbinger of all human knowledge, our friend, and Case Orator **"Hyperion"**.

A Brief Introduction to Hyperion: Hyperion will be our Case Study Orator and will help you explore and analyze the presented problem. For today, Hyperion has chosen the fascinating Heavenly Bodies for us to analyze from his bag of unsolved problems. Do bear in mind that Hyperion is a bit demanding and will demand your neurons to be active throughout.

Humans have observed heavenly bodies such as stars, planets, nebulae, asteroids, and comets for thousands of years, although early cultures thought of these bodies as gods or deities. The movement of these heavenly bodies was important for these early cultures in navigation and distinguishing between seasons. Newton was the first to propose that the same laws as objects on Earth governed these 'celestial figures.' Thus, he gave his theory of universal gravitation which studied the interaction between all objects in the universe. Einstein's theory of relativity, published in 1905 and 1915, superseded Newton's classical mechanics and theory of gravitation. In general relativity, the effects of gravitation are ascribed to space-time curvature instead of a force. Gravitational waves were predicted based on this theory as ripples in space-time, and this has led to a huge leap forward in astronomy and astrophysics.

Now Hyperion needs your help. Can you help Hyperion to find the way through darkness?

Help Hyperion..

1. Two bodies with masses m_1 and m_2 are revolving around each other in a stable manner under the influence of their mutual gravitation. Find out the angular velocity of one of the masses w.r.t. the other one as a function of the distance d between them.

   [5 marks]

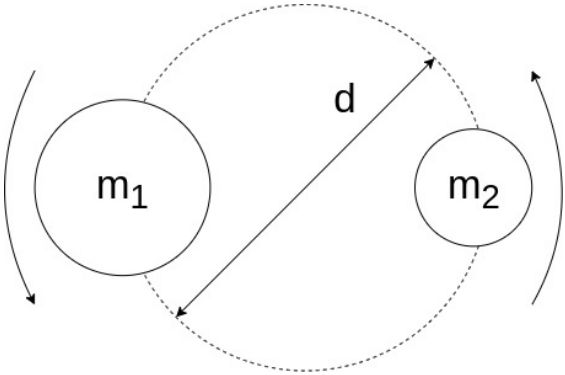

Fig 1: Diagrammatic representation for Q1

2. Assume in the above question $m_1 >> m_2$, with m_2 revolving around m_1 and a drag force $\vec{F}_d = -\beta m_2 \vec{v}_2$ acting on m_2, where $\vec{v}_2$ is the instantaneous velocity of the body with mass m_2. (Calculate the distance R between the two bodies as a function time, given that the initial distance between them is d). Also calculate the total energy of the system as a function of time and plot it. **[15 marks]**

3. Assuming post Newtonian expansion, derive the given relation between the masses of the two bodies in orbit (m_1 and m_2) and the gravitational wave frequency emitted by the same system. (f_{GW} is the frequency of emission and other symbols have their usual meaning)

$$\frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f_{GW}^{-11/3} \dot{f}_{GM}\right]^{3/5}$$

[25 marks]

4. With the help of the above given equation, analyse the provided data to determine the individual masses m_1 and m_2, using any preferred programming language. Note, it should be assumed that $m_1 < m_2$, with the following set bounds:

   - $0 \leq m_1 \leq 60 M_\odot$
   - $0 \leq m_2 \leq 80 M_\odot$

   [25 marks]

5. What event does the obtained data represent? Describe the astronomical system and name all the components in it, clearly stating the mathematical reasoning behind the it. **[10 marks]**

6. If the energy dissipated from the gravitational waves up till this *dance* was $5.37 \times 10^{47} J$, what is the final mass of the object formed after the merger? **[5 marks]**

7. Following question 6, what processes cause the losses of energy both before and immediately after the event? Give possible instrument(s) to obtain the given data set, stating the logistical improvements reduce the sources of error, increase sensitivity, and stating the reasoning behind them, keeping in mind the difference between instrumentation errors and the stochasticity of the event. **[10 marks]**

Link for data files

```
Hyperion would like to remind you that your
  potential is not limited by gravity. ☺
```