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An approach to modeling bike-sharing systems based on spatial equity concept

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Abstract

Efficient and sustainable mobility is becoming a need for emerging metropolitan areas, essential to grant their attractiveness, quality of life and economic power. Among the possible strategies to adopt, shared mobility systems have proven to be a valuable alternative to accommodate the mobility needs of citizens ensuring at the same time sustainability and transport flexibility. In particular, this paper focuses on bike-sharing systems.
Despite many studies have recognized the importance of planning processes supported by equity principles, only a few have integrated spatial equity concepts in the planning of a bike-sharing system. With this research, we want to propose an original model to determine the number and layout of bike stations, as well as the number of bicycles and racks for each of them. The suggested approach aims at minimizing the costs associated with the system implementation and operation while balancing the level of service for all the users over the territory. A set of illustrative examples and a sensitivity analysis are provided.

Keywords: Bike-sharing system; equitable level of service; bike stations locations; stations dimensioning.

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1. Introduction

Public Bike-Sharing Systems (BSSs) have become exceptionally popular for inner-city transportation in recent years. Traditionally, these systems consist of a set of docking stations dispersed throughout an urban area, and a set of bicycles available to users, offering them a short-term bicycle rental service. Users can take a bicycle from any station, use it for a short journey and, after reaching their destination, leave it at a free dock of the closest station, avoiding the costs and responsibilities related to bicycle ownership.

A BSS can be implemented on the territory in different ways: it can be either station-based, with predetermined bicycle parking racks, or free-floating, with the possibility to drop-off the bicycle almost everywhere, in the immediate vicinity of the user final destination. Additionally, it is characterized by different ownership and management options that can affect the cost of their use (Taylor et al., 2015). BSS is an environmentally friendly transportation alternative when it comes to covering relatively short distances, and it has been recognized as being part of the urban intermodal transportation system, capable of extending accessibility and coverage of transit systems.

The first urban bike-sharing concept has been launched in Amsterdam in 1967 (Fernández, 2011): 50 bicycles were painted white and left in the streets of the city to be freely used. After several unsuccessful attempts to establish these systems, they have started to experience a flourishing expansion phase over the last decades, more precisely since 2007 with the implementation of the Vélib' BSS in Paris, with 10,000 bicycles and 750 automatic stations at the time. From that moment on, various European cities have established similar systems, and their success has progressively spread to North America, Asia, and the rest of the world. By the end of 2016, nearly 2.3 million bicycles were available to the public, with 1.9 million of these located in China alone (Richter, 2018).

This trend of growth is still present. An increasing number of cities are providing their residents and tourists with a bike-sharing service as it involves many advantages: it contributes at alleviating traffic congestion and other problems associated with excessive car use (such as parking, safety, etc.); it reduces noise/air pollution and infrastructural investments compared to other transportation services; finally, it promotes an affordable, convenient and healthier lifestyle in urban areas.

The success of a BSS depends primarily on the ease of finding rental stations within a convenient walking distance from the origin and the destination of urban trips. Consequently, these stations should have a sufficient number of both available bicycles and free parking racks (Avarez-Valdes et al., 2016), so to avoid situations in which the user arrives at the station to rent a bicycle and realizes that none is available (empty station), or those in which he/she does not find a free parking rack to leave the bicycle at the end of the trip (full station). At the same time, appropriate station locations and density can significantly contribute to avoiding expensive redistribution operations among urban areas due to unbalanced demand distributions – temporally and spatially.

In this regard, the design and development of an efficient BSS is a complex task that includes setting up the system while defining the desired level of service that should be granted to the users. Therefore, to implement an efficient BSS, it is necessary to determine the number and location of bike-sharing stations, the number of available bicycles and racks for each station, as well as the potential strategy for redistribution, which will balance the stochastic demand for bicycles and parking spaces.

In this paper, a linear programming model has been developed and tested to determine the number and layout of stations, as well as the number of available bicycles and racks for each of them, in order to minimize the costs of setup and operation of a BSS for any defined level of service. The paper has been designed as follows: in the next section, a brief overview of literature in this field is given; a description of the problem is provided in the third section, and its notations and formulation in the fourth, followed by a set of illustrative examples and a sensitivity analysis, with a discussion of the obtained results. Some final observations and directions for further research conclude the paper.

2. Literature review

In general, the approaches to the BSS modeling found in the literature can be grouped into three main categories.

The first approach is based on minimizing costs or maximizing profits. For instance, Lin and Yang (2011) proposed a model for determining optimal station locations and bicycle paths in order to minimize the total costs.

Romero et al. (2012) suggested a bi-level mathematical programming model in order to optimize the BSS station locations while minimizing the total costs and maximizing the number of users of the system. The cost of unmet demand has also been introduced by Lin et al. (2013), alongside the other 'usual' costs related to the implementation and operation of BSS. On the other hand, Sayarshad et al. (2012) modeled the BSS aiming at maximizing the operator profits without taking into account the user costs. The maximization of the net revenue during a predetermined operation period of a BSS is also the main goal of Martinez et al. (2012), who developed a bike-sharing station location model and tested it on the city of Lisbon.

A second possible approach to the BSS modeling is based on the minimization of the total travel time or the total length of the journey. In Chen and Sun (2015), for example, the total travel time of all BSS users is minimized, while setting an upper bound to the total available resources (investment budget) necessary for establishing the system. In addition to the minimization of the total network cycling distance, Guo et al. (2014) aimed also at minimizing the overlapping rate of bike-sharing stations located in different scenic spots.

The third and last approach is based either on maximizing the coverage of user demand or minimizing the unmet demand. García-Palomares et al. (2012) and Frade and Ribeiro (2015) proposed models in which the number and arrangement of BSS stations have been determined by maximizing the users' coverage. In the work of Saharidis et al. (2014), the number and locations of bike-sharing stations are defined with the main goal of minimizing the unmet users' demand, without overcoming the available investment budget.

The importance of an equitable bike-sharing station siting has been also recently highlighted. Some authors focused on measuring the spatial equality of operating BSSs. The study of Hosford and Winters (2018), for instance, assessed the spatial access to BSS in Canadian cities according to the socio-economic characteristics of the residing population. Similarly, Mooney et al. (2019) explored the equity of spatial access to dockless shared bicycles provided by three operators in a pilot program in Seattle. From the perspective of planning and design more equitable systems, Park and Sohn (2017) applied two different location-allocation models to the city of Seoul, showing how the p-median model has an actual advantage in terms of spatial equity. On the other hand, the approach of Conrow et al. (2018) relied on spatial analytics (GIS and spatial optimization) to help site bike stations across an urban region, while preventing the users to travel too far for having access to them.

3. Problem description

Any urban context is usually divided into smaller administrative-functional units (districts). Each district can be represented by a centroid which is the source of the shared-bicycle demand. Figure 1 shows the schematization of four districts, whose centroids are indicated with blue circles. It is assumed that a list of potential locations in which a bicycle station can be built is identified a priori on the territory. In this figure, five possible locations have been marked by small crosses. The acceptable walking distance for a typical user between centroid and bicycle station (and vice versa) is known (R).

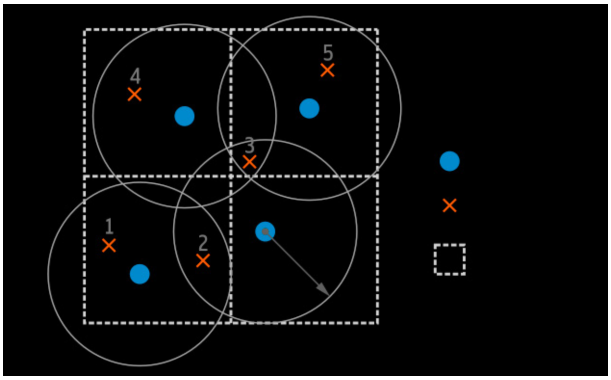

Fig. 1: Schematic representation of districts, centroids, and bike-sharing station potential locations.

From Figure 1, it can be noticed that the catchment area expressed with a circle having radius R of a single district can enclose multiple potential locations and, on the other hand, a single location can fall in the catchment area of more than one centroid. The value of the total shared-bicycle demand between each pair of centroids is quantified. Given that the level of demand fluctuates (both according to the time of the day and the day of the week), the proposed model should be applied in relation to a significant demand level (see for example the study Frade and Ribeiro (2014), who explained how to estimate the potential BSS demand when the system has not yet been implemented).

In order to appropriately formulate the problem (that is, determining the optimal number and layout of stations, as well as the number of available bicycles and racks per station while minimizing the overall costs of set up and operation of the BSS), it is necessary to estimate as accurately as possible all the costs involved. In particular, it is important to know the average cost related to the purchasing and maintenance of a bicycle; the average costs to build a BSS station, and the costs associated with any additional rack; as well as the total travel costs, given by the time necessary to walk between centroid and station (and vice versa) plus the actual cycling time between stations.

4. Problem formulation

The model proposed in this paper presents the idea of granting to all users from different districts approximately the same level of service. When we refer to the level of service, we are mainly taking into account two components:

- The number of available bicycles in relation to the expected user demand (while setting a level of allowable tolerance regarding the number of bicycles per unit of demand);
- The walking distances (walking travel times) necessary to reach the bike-station from a given origin, and vice versa from the arrival station to the final destination.

In the following box, the notation used for the problem formulation is provided.

Sets and parameters	
B	set of potential bike-sharing station locations
$k,l \in B$	indices of potential bike-sharing stations
D	set of origin/destination centroids – districts
$i,j,s,t \in D$	indices of centroids - districts
R	maximal acceptable distance between a bike-sharing station and a centroid
d_{ik}, d_{kl}, d_{lj}	distances between nodes i and k, k and l, l and j, respectively
$B_i = \{k \mid d_{ik} \le R\}$; $B_i \subseteq B$	set of potential bike-sharing stations that are on acceptable distance from the centroid (district) i
$D_k = \{i \mid d_{ik} \le R\}$; $D_k \subseteq D$	set of centroids (districts) that are on acceptable distance from the potential bike-sharing station k
f_{ij}	total travel demand of users from node i to node j
c_b	bicycle investment cost at the daily level
c_r	rack investment cost at the daily level
c_w	user's walking cost per distance unit
c_{rb}	riding costs per distance unit
α	allowable tolerance between districts regarding the number of bicycles per unit of demand
β	allowable tolerance between districts regarding walking distances (to and from bike stations) per unit of demand
Decision variables	
x_{iklj}	number of rides between centroids i and j, via bike-sharing stations k and l
b_k	required number of bicycles in the bike-sharing station k
r_k	required number of racks in the bike-sharing station k

The problem formulation can be expressed as follows:

$$\min \sum_{k\in B} c_b b_k + \sum_{k\in B} c_r r_k + \sum_{i\in D}\sum_{k\in B_i}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{\substack{j\in D\\ j\neq i}} x_{iklj}\left[c_w(d_{ik}+d_{lj})+c_{rb}d_{kl}\right] \tag{1}$$

$$\sum_{k\in B_i}\sum_{l\in B_j} x_{iklj} = f_{ij} \quad \forall i\in D, j\in D, \quad i\neq j, \quad k\neq l \tag{2}$$

$$\sum_{i\in D_k}\sum_{\substack{l\in B\\ l\neq k}}\sum_{\substack{j\in D_l\\ j\neq i}} x_{iklj} \le b_k \quad \forall k\in B \tag{3}$$

$$\sum_{j\in D_l}\sum_{\substack{k\in B\\ k\neq l}}\sum_{\substack{i\in D_k\\ i\neq j}} x_{jlki} \le r_l - b_l \quad \forall l\in B \tag{4}$$

$$\left|\frac{1}{\sum_j f_{ij}}\sum_{k\in B_i} b_k - \frac{1}{\sum_i f_{ji}}\sum_{k\in Bj} b_k\right| \le \alpha \quad \forall i\in D, j\in D, \quad i\neq j \tag{5}$$

$$\left|\frac{1}{\sum_j (f_{ij}+f_{ji})}\Big(\sum_{k\in B_i}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{\substack{j\in D\\ j\neq i}} d_{ik}x_{iklj} + \sum_{\substack{j\in D\\ j\neq i}}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{k\in B_i} d_{ik}x_{jlki}\Big) - \frac{1}{\sum_t (f_{st}+f_{ts})}\Big(\sum_{k\in B_s}\sum_{\substack{l\in B_t\\ l\neq k}}\sum_{\substack{t\in D\\ t\neq s}} d_{sk}x_{sklt} + \sum_{\substack{t\in D\\ t\neq s}}\sum_{\substack{l\in B_t\\ l\neq k}}\sum_{k\in B_s} d_{sk}x_{tlks}\Big)\right| \le \beta \tag{6}$$

$$\forall i\in D, s\in D, \quad i\neq s$$

$$x_{iklj}, b_k, r_k \in N_0 \quad \forall i\in D, j\in D, k\in B, l\in B \tag{7}$$

Constraint (5), which is expressed by absolute values, could be linearized by using the following transformation giving two sets of constraints:

$$\frac{1}{\sum_j f_{ij}}\sum_{k\in B_i} b_k - \frac{1}{\sum_i f_{ji}}\sum_{k\in Bj} b_k \le \alpha \quad \forall i,j \quad i\neq j \tag{5a}$$

$$\frac{1}{\sum_i f_{ji}}\sum_{k\in B_j} b_k - \frac{1}{\sum_j f_{ij}}\sum_{k\in Bi} b_k \le \alpha \quad \forall i,j \quad i\neq j \tag{5b}$$

Similarly, constraint (6) is transformed into two following sets of constraints:

$$\frac{1}{\sum_j (f_{ij}+f_{ji})}\Big(\sum_{k\in B_i}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{\substack{j\in D\\ j\neq i}} d_{ik}x_{iklj} + \sum_{\substack{j\in D\\ j\neq i}}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{k\in B_i} d_{ik}x_{jlki}\Big) - \frac{1}{\sum_t (f_{st}+f_{ts})}\Big(\sum_{k\in B_s}\sum_{\substack{l\in B_t\\ l\neq k}}\sum_{\substack{t\in D\\ t\neq s}} d_{sk}x_{sklt} + \sum_{\substack{t\in D\\ t\neq s}}\sum_{\substack{l\in B_t\\ l\neq k}}\sum_{k\in B_s} d_{sk}x_{tlks}\Big) \le \beta \tag{6a}$$

$$\forall i\in D, s\in D, \quad i\neq s$$

$$\frac{1}{\sum_t (f_{st}+f_{ts})}\Big(\sum_{k\in B_s}\sum_{\substack{l\in B_t\\ l\neq k}}\sum_{\substack{t\in D\\ t\neq s}} d_{sk}x_{sklt} + \sum_{\substack{t\in D\\ t\neq s}}\sum_{\substack{l\in B_t\\ l\neq k}}\sum_{k\in B_s} d_{sk}x_{tlks}\Big) - \frac{1}{\sum_j (f_{ij}+f_{ji})}\Big(\sum_{k\in B_i}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{\substack{j\in D\\ j\neq i}} d_{ik}x_{iklj} + \sum_{\substack{j\in D\\ j\neq i}}\sum_{\substack{l\in B_j\\ l\neq k}}\sum_{k\in B_i} d_{ik}x_{jlki}\Big) \le \beta \tag{6b}$$

$$\forall i\in D, s\in D, \quad i\neq s$$

The objective function (Eq. 1) tries to minimize the total daily costs that derive from the implementation of a new bike-sharing system. The first two terms in the objective function represent the costs of daily operating and maintaining a BSS, while the third term is related to the daily costs of the system usage by users.

The first constraint (Eq. 2) explains that the total number of rides between stations should be equal to the total demand for riding between any pair of nodes. The number of available bicycles, as well as empty racks for each station, should be enough to satisfy all the expected demand (Eq. 3 and 4).

The following two constraints (Eq. 5 and 6) refer to the BSS level of service (as previously defined) and can be considered as spatial equity constraints. No two districts have a difference in the number of bicycles per one demanded ride larger than the allowed tolerance (Eq. 5). No two districts have a difference in walking distances (to and from bike stations) per one demanded ride larger than it is allowed (Eq. 6).

The last constraint (Eq. 7) specifies the decision variables as non-negative integers.

5. Numerical examples generation and sensitivity analysis of spatial equity impact on the BSS

The proposed model has been tested on an illustrative case study represented by an area of approximately 3 km^2 with 12 districts (centroids) and 48 potential bike-sharing stations' locations. Once defined the district borders, the coordinates of centroids and potential stations have been randomly generated resulting in 30 instances. The user demand between each pair of centroids has been randomly generated in the range of 0 to 4 bicycle requests in the considered time interval, with a total demand equal to 164, which has been later used in all the instances. It is supposed that the density of stations locations is greater in the central districts, and lower in the peripheral ones. Euclidean distances have been used in the calculations of travel paths between the nodes.

The implementation costs of a BSS depend on several aspects such as the quality and the type of bikes, the technologies used for the racks, the size of the system, etc. The same applies to user costs that depend on salaries and system usage fares. For this reason, since this is a test case, all costs are set for illustrative purposes only. The bicycle investment cost at the daily level is $c_b = 0.02$, while the cost of purchasing and maintenance of a bicycle rack at the daily level is equal to $c_r = 0.05$. The user walking cost for distance unit is $c_w = 1.8$, and the riding cost (actual cycling travel time between stations) is $c_{rb} = 0.1$. According to Kabra et al. (2016), who stated that the most of the bike-share usage comes from areas within 300 m from the stations, the maximum acceptable distance between a bike-sharing station and a centroid is set to $R = 0.3$ km.

Regarding the values of the allowable tolerances α and β (equity constraints, Eq. 5 and 6) four possible scenarios have been tested. We have set two values for α (0.7 and 1) and β (0.1 and 0.2). Each combination of these tolerances corresponds to one of the four scenarios. For each scenario, 30 instances have been solved using CPLEX 12.6 software, on a 64-bit 2.30 GHz Intel Core i7 - 4712MQ with 6 GB of RAM. Each instance has 2382 variables and 732 constraints. Also, all instances have been solved in the case without equity constraints (Eq. (5) and (6) have not been considered) in order to compare the obtained results.

It is important to note that for both scenarios in which $\beta = 0.2$, all 30 instances resulted feasible (although with a greater number of instances the problem could be infeasible for certain configurations). This has not happened for $\beta = 0.1$, as the problem is further constrained. In particular:

- for $\beta = 0.1$ and $\alpha = 0.7$, the problem resulted infeasible for 7 instances; additionally, 9 out of the 23 remaining instances have been solved with a gap between 0.01 and 0.14 when a maximum solving time was set to 600 s;
- for $\beta = 0.1$ and $\alpha = 1$, the problem resulted infeasible for the same 7 instances; additionally, 6 out of the 23 remaining instances have been solved with a gap between 0.04 and 0.10 when a maximum solving time was set to 600 s. In this scenario, α is bigger compared to the previous one, so the problem resulted slightly less constrained.

The obtained results have been summarized in Table 1.

Table 1 allows to have an overview of the impacts of the equality constraints on three different aspects, namely: the total bike-sharing costs (objective function values); the journey lengths (considering walking and riding distance components); the bike-sharing station layout (taking into account opened stations, number of bicycles and number of racks). Generally, it seems that the spatial equity constraints have an influence on the selection of the potential station locations and their size, even if the observed instances are small size problems. Without equity constraints, the total required number of bicycles is 164, that is, equal to the total demand. On the other hand, when the equity constraints are considered, a greater number of both bicycles and racks is required, and more stations are active. This happens to meet the required tolerance α (more bicycles per unit of demand and, consequently, more racks) and β (more open stations are needed to distribute evenly the walking distances among districts). Comparing the results with and without equality constraints, the same considerations have been done for the BSS station layout. If the riding distances are comparable, the walking distances are lower when the equality concept is not embedded. The total costs of the system depend on the number of bicycles and racks, and on the journey length: consequently, the objective function values result lower in the scenario without equality constraints. Looking at the configurations with equality constraints, additional remarks can be done: the lower is α, the higher are the total cost values, the number of open stations, bicycles, and racks. On the other hand, no big differences can be detected when looking at the riding distances: they basically stay the same for the two selected α values. However, walking distances slightly increase in case of equality constraints. In particular, the lower is β, the higher are the mean walking distances.

Table 1. Summary of results.

		with equality constraints				without equality constraints
		$\alpha = 0.7$ $\beta = 0.1$	$\alpha = 0.7$ $\beta = 0.2$	$\alpha = 1.0$ $\beta = 0.1$	$\alpha = 1.0$ $\beta = 0.2$	
objective function values	mean	102.01	96.26	100.67	94.60	91.20
	min	85.47	84.42	84.16	83.00	79.10
	max	118.38	110.66	117.66	109.80	109.58
	st.dev.	9.64	7.55	9.88	7.55	7.68
walking distances	mean	37.91	33.82	37.50	33.45	32.15
	min	28.55	27.49	27.91	27.12	25.87
	max	46.86	42.65	46.85	42.02	41.88
	st.dev.	5.62	4.27	5.76	4.16	4.12
riding distances	mean	131.88	136.45	131.70	136.77	136.58
	min	111.62	115.25	109.67	115.21	113.80
	max	144.95	147.99	146.18	148.60	149.22
	st.dev.	8.86	8.15	9.64	8.42	8.16
number of opened stations	mean	23.65	17.30	22.96	15.90	12.93
	min	15.00	13.00	15.00	12.00	12.00
	max	31.00	23.00	33.00	20.00	15.00
	st.dev.	4.47	2.51	4.84	1.95	0.83
number of bicycles	mean	176.87	193.23	168.61	178.77	164.00
	min	164.00	164.00	164.00	164.00	164.00
	max	202.00	259.00	185.00	232.00	164.00
	st.dev.	12.75	28.93	6.75	22.23	0.00
number of racks	mean	340.87	357.23	332.61	342.77	328.00
	min	328.00	328.00	328.00	328.00	328.00
	max	366.00	423.00	349.00	396.00	328.00
	st.dev.	12.75	28.93	6.75	22.23	0.00

6. Conclusions

This paper presents an original model for the dimensioning of a public BSS: on one hand, it determines the optimal number and layout of bike-sharing stations on the observed territory; on the other hand, it determines the capacity (number of parking racks) and the number of bicycles for each station according to the expected user demand. The proposed linear mathematical problem aims at minimizing the setup and operating costs of a new BSS, with a set of constraints that reflects the idea of a balanced and equitable level of service to be granted to all the users of the system. This concept has been tested on a set of small size illustrative examples, followed by a sensitivity analysis in order to validate the proposed approach. It has been shown that the suggested model can serve as a good basis for the optimal and more equitable development of a public BSS. Of course, future research needs to be directed in several directions. The functionality of the model should be checked on a network of larger dimensions, closer to a real case study; more accurate values of the input parameters can be selected; different equitable constraints may be embedded; a heuristic approach may also be developed, as it can be quite difficult to expect a real dimension problem to be solved optimally.

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