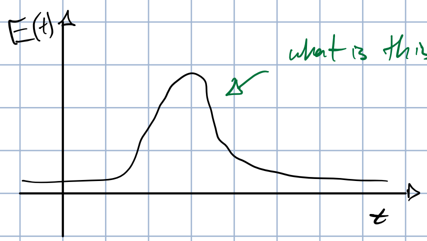
$\vec{a}_1 \quad E_0 \, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$ $\qquad \frac{\omega}{k} = c$



what is this signal's frequency

$$E(t) = \int_{-\infty}^{\infty} \hat{E}(\omega)\, e^{-i\omega t}\, d\omega$$

$$\hat{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t)\, e^{i\omega t}\, dt$$

assuming $E(t) \in \mathbb{R} \rightarrow \hat{E}(\omega)^* = \hat{E}(-\omega)$

squish one, spreads other

$t \rightarrow \omega$ delta fⁿ

poynting vector

$$\frac{dW}{dt\,dA} = \frac{c}{4\pi} E^2(r)$$

what's total energy per unit area

$$\frac{dW}{dA} = \frac{c}{4\pi} \underbrace{\int_{-\infty}^{\infty} E^2(t)\, dt}_{\text{area under curve}}$$

$$\int_{-\infty}^{+\infty} E^2(t)\, dt = 2\pi \int_{-\infty}^{\infty} |\hat{E}(\omega)|^2 \, d\omega \qquad \text{Parseval's Thm}$$

no ~~$\frac{1}{4\pi}$~~ b/c 2 contributions from $0 \rightarrow \infty$ integral

$$\frac{dW}{dA} = c \int_0^{\infty} |\hat{E}(\omega)|^2 \, d\omega$$

energy per area per frequency $\rightarrow \quad \frac{dW}{dA\,d\omega} = c\, |\hat{E}(\omega)|^2$

sometimes more convenient to solve for scalar potential + vector potential vs Electric field + Magnetic Field

how do we go from $6 \rightarrow 4$?

well B_x, B_y, B_z + E_x, E_y, E_z aren't indpt. they are related by Faraday's law + induction

2.5 (skip 2.4)

$\nabla \cdot \vec{B} = 0$ & $\nabla \times \vec{E} = -\frac{1}{c} \partial_t \vec{B}$

$\nabla \cdot \vec{B} = 0 \rightarrow \nabla \cdot (\nabla \times \vec{A}) = 0$

any magnetic field can be written as curl of vector $\rightarrow$ $\vec{B} = \nabla \times \vec{A}$

$\nabla \times E + \frac{1}{c} \partial_t (\nabla \times \vec{A}) = 0$ curl of $\vec{E}$ & curl of $\vec{A}$

$\nabla \times (\nabla \cdot \Phi) = 0$
curl of scalar's gradient is zero

$\nabla \times [\vec{E} + \frac{1}{c} \partial_t (\vec{A})] = 0$

$\rightarrow \vec{E} + \frac{1}{c} \partial_t (\vec{A}) = -\nabla \phi$ curl = scalar potential gradient

$\vec{A}$ vector potential
ϕ scalar potential

OG Max Goi: $\nabla \cdot \vec{E} = 4\pi\rho$ $\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \partial_t (\vec{E})$

$$\nabla^2 \phi - \frac{1}{c^2} \partial_t^2 (\phi) + \frac{1}{c} \partial_t \left(\nabla \cdot \vec{A} + \frac{1}{c} \partial_t (\phi) \right) = -4\pi\rho$$

$\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 A$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \partial_t^2 (\vec{A}) - \nabla \left(\nabla \cdot \vec{A} + \frac{1}{c} \partial_t (\phi) \right) = -\frac{4\pi}{c} \vec{J}$$

$\vec{A} \rightarrow \vec{A} + \nabla \vec{\Psi}$ & $\phi \rightarrow \phi - \frac{1}{c} \frac{\partial \Psi}{\partial t}$ still gives same $\vec{E}$ & $\vec{B}$.
(gauge transformation)

$\rightarrow$ $\vec{A}$ & ϕ are not unique

pick Lorentz gauge $\rightarrow$ $\nabla \cdot \vec{A} + \frac{1}{c} \partial_t (\phi) = 0$

$$\left(\nabla^2 - \frac{1}{c^2} \partial_t^2 \right) \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix} = -4\pi \begin{pmatrix} \rho \\ \vec{J}/c \end{pmatrix}$$

$\delta(x) = 0$ if $x \neq 0$ $\int_{-\epsilon}^{\epsilon} \delta(x)\,dx = 1$ $\int f(x')\,\delta(x - x')\,dx' = f(x)$

$\mathcal{L}_x F(x) = f(x)$ & $\mathcal{L}_x G(x) = \delta(x)$

$\rightarrow F(x) = \int G(x - x')\, f(x)\, dx'$

"convolution"

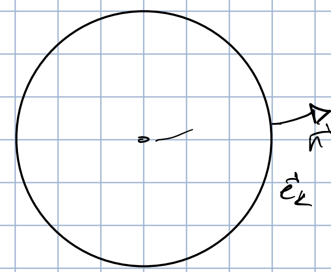
What is green's fn of laplacian? G ∇^2

$$\nabla^2 G = -\delta(\vec r - \vec r\,')$$

For sphere at $\vec r\,'$ → $\int \nabla^2 G \, d\vec r\,' = -1$

$$\int_{sphere} \nabla(\nabla G)\, d\vec r\,' = \int_{\Sigma} \vec n\,' \cdot \nabla G \cdot d\Sigma$$

derivative of G @ radius



$$\rightarrow \int \frac{\partial G}{\partial r}\, d\Sigma = 4\pi r^2 \frac{\partial G}{\partial r} = -1$$

$$\frac{\partial G}{\partial r} = -\frac{1}{4\pi r}$$

$$\rightarrow G = \frac{1}{4\pi r}$$

$\nabla \vec E = 4\pi\rho$ $\vec E = -\nabla\phi$

$\nabla^2\phi = -4\pi\rho$ $\rho(\vec x) = q\,\delta(\vec x - \vec x\,')$

$$\rightarrow \phi = \int \cancel{\frac{1}{4\pi(\vec r - r')}}\ \cancel{4\pi}\ q\ \delta(x - x')\, dr'$$

$$\phi = q/r$$

$$\phi(\vec r, t) = \int \frac{[\rho]\, d^3 r'}{|\vec r - r'|}$$

$$\vec A(\vec r, t) = \int \frac{[J]\, d^3 r'}{|\vec r - r'|}$$

retardation $[Q] \equiv Q(\vec r\,', \underbrace{t - \frac{1}{c}|\vec r - r'|})$

→ want retardation time

where was everything 1 unit time ago?

2 contributions
1. related to velocity
2. related to acceleration

Moving Charges

charge q @ position $\vec{r}_0(t)$, $\vec{u}(t) = \dot{\vec{r}}_0(t)$

be careful for when exactly taking time derivative

$$\rho(\vec{r},t) = q\,\delta(\vec{r}-\vec{r}_0(t))$$
$$\vec{J}(\vec{r},t) = q\,\vec{u}(t)\,\delta(\vec{r}-\vec{r}_0(t))$$

localize charge

Green's fn

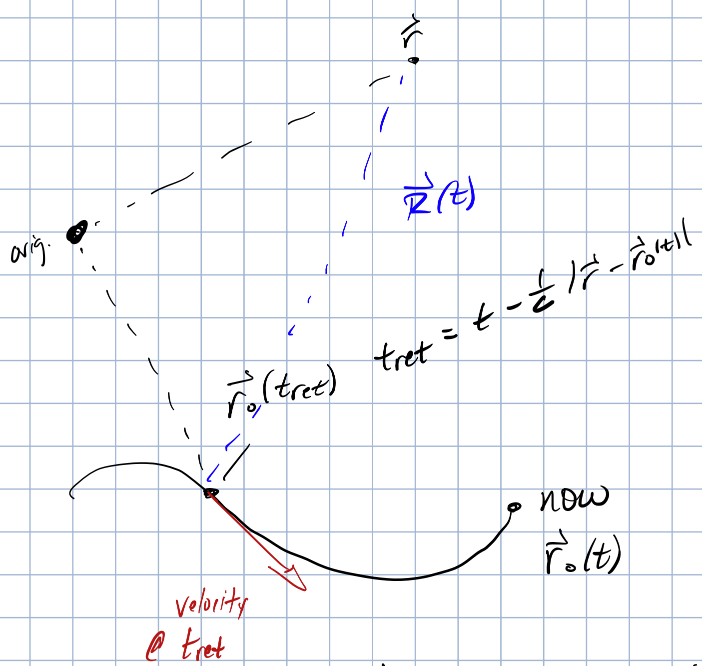
$$\phi(\vec{r},t) = \int d^3\vec{r}\,' \int dt'\, \frac{\rho(\vec{r}\,',t')}{|\vec{r}-\vec{r}\,'|}\,\delta\left(t'-t+\frac{1}{c}|\vec{r}-\vec{r}\,'|\right)$$

only evaluate @ t_{ret} only evaluate @ $\vec{r}\,' = \vec{r}_0(t')$

$$= q\int d^3\vec{r}\,' \int \frac{dt'}{|\vec{r}-\vec{r}_0(t')|}\,\delta\left(t'-t+\frac{1}{c}|\vec{r}-\vec{r}\,'|\right)\delta(\vec{r}\,'-\vec{r}_0(t'))$$

$$= q\int dt'\,\frac{1}{|\vec{r}-\vec{r}_0(t')|}\,\delta\left(t'-t+\frac{1}{c}|\vec{r}-\vec{r}_0(t')|\right)$$

" integrals are like delta fns they smell fear "



w/ constant velocity, where does $\vec{E}$ point to?   current location of charge.