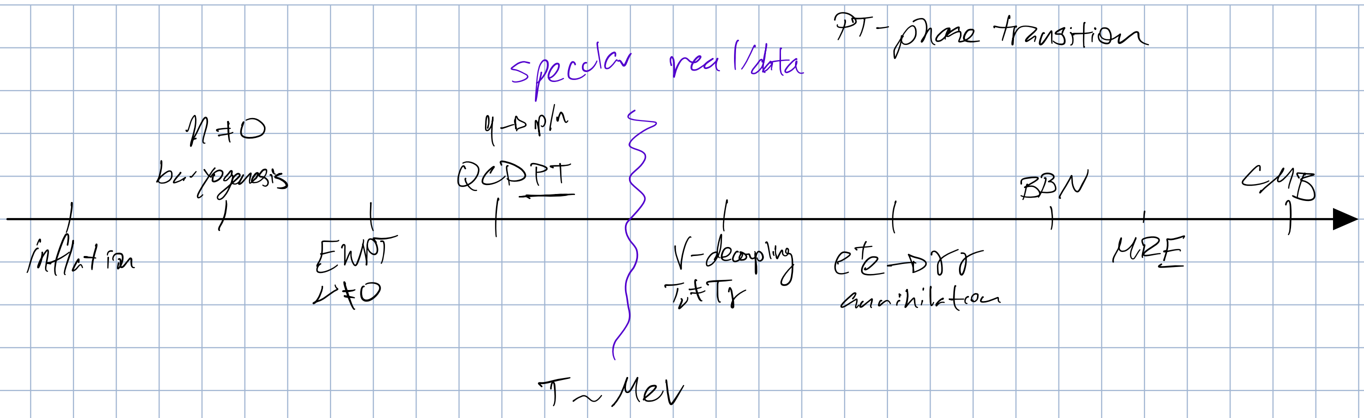


distribution for particle i

$$f_i(|\vec{p}|, t) = \frac{\#\text{ particles}}{\text{volume} \cdot \text{momentum}^3} \longrightarrow \frac{g_i}{\exp\left[\frac{E - \mu}{T}\right] \pm 1}$$

degeneracy

energy fn of $|\vec{p}|$

chemical potential

baryons (−) fermions (+)

equillibrium so there's a temp.

photons: $m_\gamma \cong 0$ $\quad \mu_\gamma = 0$ (no antiphoton)

$$n_\gamma \equiv \int \frac{d^3p}{(2\pi)^3} \frac{g_\gamma}{\exp[E/T] - 1}$$

$\hookrightarrow 2\pi/\hbar$

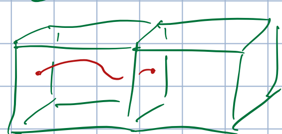
$g_\gamma = 2$ polarization

$E(p) = |\vec{p}|$

$T >> m$

Series of QM boxes w/ boundary conditions

$$e^{ipr} = e^{ip(r+L)}$$



$$d^3p = dp_x\, dp_y\, dp_z$$
$$= (p^2\, dp)(\sin\theta\, d\theta)(d\phi)$$

instead of cartesian, can think of as living on sphere

$$\longrightarrow n_\gamma = g_\gamma \cdot \frac{(4\pi)}{8\pi^3} \int_0^\infty \frac{p^2}{\exp[P/T] - 1}\, dp$$

$y \equiv P/T \quad p = Ty$
$dp = T dy$

$$= \frac{g_\gamma}{2\pi^2} \cdot T^3 \int_0^\infty \frac{y^2 dy}{e^y - 1} = 2\underbrace{\zeta(3)}_{\text{zeta fn}} \cdot \left(\frac{g_\gamma T^3}{2\pi^2}\right)$$

$\zeta(3) \approx 1.3$

$$= \frac{2\zeta(3) T^3}{\pi^2}$$

from now on, $T \equiv T_\gamma$ unless otherwise protons

but usually $E(p) = \sqrt{m^2+p^2} \approx m \xrightarrow[E>>p]{} E(p) = m\left(1+\frac{p^2}{m}\right)^{1/2}$

$E(p) = m\left(1+\frac{p^2}{2m}\right)+\ldots$

$\Delta n_p = n_p - n_{\bar{p}} \overset{\text{baryogenesis}}{=} \frac{1}{2}\,\eta \cdot n_\gamma$

$m_p = 0.938$ GeV $\quad (<\Lambda_{QCD})$

$\mu_p \neq 0 \longrightarrow \mu_p = -\mu_{\bar{p}}$ $\quad$ same # of p & $\bar{p}$ $\rightarrow \mu \to 0$

momentum

$$\Delta n_p = g_p \int \frac{d^3p}{(2\pi)^3} \cdot \left(\underbrace{f_p}_{\text{proton}}(p) - \underbrace{f_{\bar{p}}}_{\text{antiproton}}(p)\right)$$

$$= \frac{g_p\,(4\pi)}{8\pi^3} \cdot \int_0^\infty p^2\left(\frac{1}{\exp\left[E-\mu/T\right]+1} - \frac{1}{\exp\left[E+\mu/T\right]+1}\right)dp$$

$T << p \rightarrow E \approx m_p$, $\exp[\,]$ dominates, ignore $+1$

$$= \frac{g_p}{2\pi^2} \cdot \int_0^\infty p^2\left(\exp\left[\frac{-(m+\frac{p^2}{2m}-\mu_p)}{T}\right] - \underbrace{\cancel{\exp\left[\frac{-(m+\frac{p^2}{2m}+\mu_p)}{T}\right]}}_{<<1}\right)dp$$

$$\Delta n_p = \frac{g_p}{2\pi^2} \cdot e^{-(m-\mu)/T} \cdot \int_0^\infty p^2 e^{-p^2/2mT}\,dp$$

$$= \frac{g_p}{2\pi^2} \cdot e^{-(m-\mu)/T}\,(2mT)^{3/2} \int_0^\infty x^2 \underbrace{e^{-x^2}}_{\text{Gauss}}\,dx \qquad x^2 \equiv \frac{p^2}{2mT}$$

$$= g_p\left(\frac{mT}{2\pi}\right)^{3/2} \cdot e^{-(m-\mu)/T} = \frac{1}{2}\,\eta \cdot n_\gamma$$

$$\Delta n_p \approx n_p = \frac{1}{2}\,\eta \cdot n_\gamma = \frac{1}{2}\,\eta \cdot \left(\frac{2\,\zeta(3)\,T_\gamma^3}{\pi^2}\right) = g_p\left(\frac{m_pT}{2\pi}\right)^{3/2} e^{-(m_p-\mu_p)/T}$$

$\longrightarrow \quad \equiv \mu_p(T)$

b/c $E \approx m \rightarrow \rho = m \cdot n \cong m \cdot \Delta n$

ν – decoupling

$n_\gamma \to n_\nu$
+1 instead of −1

weak force

$\nu e^- \leftrightarrow \nu \bar{e}$ equilibrium for ν $T > T_{decoupling} \approx 1\,MeV$

$$R_{\nu e} = G_F^2 \cdot T^5 = H(T) = 1.66 \sqrt{g_*(T)}\, \frac{T^2}{m_{pl}} \qquad G_N = \frac{1}{m_{pl}^2}$$

inverse time

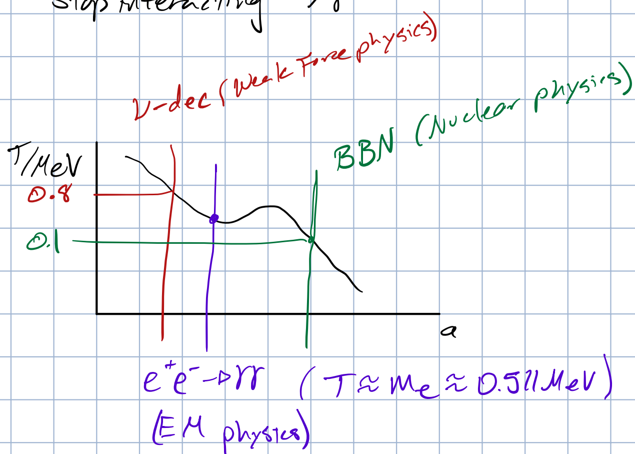
$$R_{\nu e} = n \cdot v \cdot \sigma$$
$$\underbrace{(1/cm^3)\cdot(cm/s)}_{flux}\cdot(cm^2)$$

$$\sigma = G_F^2 T^2$$

$$H = R \to T_{dec} \approx \left(\frac{g_*}{G_F^2 m_{pl}}\right)^{1/3} \approx 1\,MeV$$

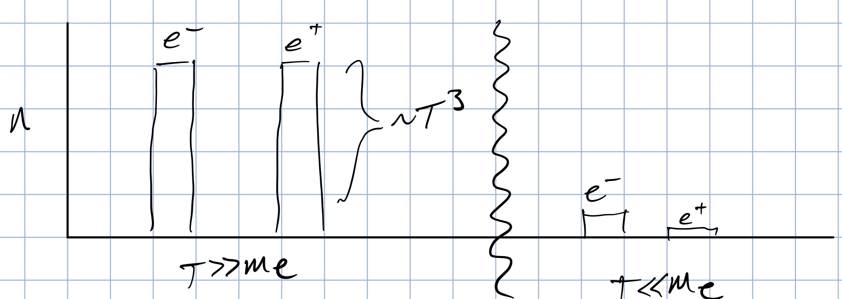
$T_\gamma < T_{dec} \to T_\nu$ can be different

stop interacting w/ γ



Electron - Positron Annihilation $(e \bar{e}^+ \to \gamma\gamma)$ (stops being impt)

$$\Delta n_e = n_{e^-} - n_{e^+} = \frac{1}{2} \eta \cdot n_\gamma$$



$e^+ \to$ dead
e^- alive b/c $\eta \neq 0$

Before: $T \gtrsim m_e$ (right before $e^- e^+ \to \gamma\gamma$)

$$S(T) = \frac{\rho(T) + P(T)}{T}$$ entropy density nolabel $\to$ photons

$$= \frac{2\pi^2}{45} \cdot g_{*,s}(T) \cdot T^3$$

temps can be different, indiv. temp

$$g_{*,s}(T) = \sum_{bosons} g_k \left(\frac{T_k}{T}\right)^3 + \frac{7}{8}\sum_{fermions} g_k \left(\frac{T_k}{T}\right)^3$$

total temp

with $T \gtrsim m_e$ $\to$ all T are same, just weighted sum

cast: $\underbrace{\text{photons}}_{2\text{ pol.}}$, e^-, e^+, $\underbrace{3\nu_i, 3\bar{\nu}_i}_{1\text{ pol.}}$

$$g^i_{*,s} = \underset{\gamma}{2} + \frac{7}{8}\left(\underset{e^{\pm}\cdot 2\text{pol.}}{2\cdot 2} + \underset{\nu_i \cdot \bar{\nu} \cdot \text{pol.}}{3\cdot 2\cdot 1}\right)$$

$$= 2 + \frac{7}{8}\cdot 10 = 2 + \frac{70}{8} = \frac{16}{8} + \frac{70}{8} = \frac{86}{8} = \frac{43}{4}$$

initial entropy: $S_i(T) = \frac{2\pi^2}{45} \cdot g^i_{*,s} \cdot T^3$

After: $T \lesssim m_e$

Cast: $\underset{T_\gamma}{\text{photons}}$, $\underbrace{3\nu_i, 3\bar{\nu}_i}_{T_\gamma}$ (lost all e^+e^-)

$$g^f_{*,s} = 2 + \frac{7}{8}(3\cdot 2\cdot 1)\left(\frac{T_\nu}{T_\gamma}\right)^3 = 2 + \frac{56}{8}\left(\frac{T_\nu}{T_\gamma}\right)^3 = 2 + \frac{21}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3$$

final entropy: $S_f(T) = \frac{2\pi^2}{45} g^f_{*,s}(T) \cdot T_\gamma^3$

transfer is basically instantaneously (no need for scale factor)

$S_i(T) = S_f(T)$, $T = T_\nu$

all ν same T_γ b/c weak force & $\underset{\text{scatter rate}}{R} \gg H$

$$\to g^i_{*,s}\, T_\nu^3 = g^f_{*,s}\, T_\gamma^3$$

$$\frac{43}{4} T_\nu^3 = \left(2 + \frac{21}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3\right) T_\gamma^3$$

$$\longrightarrow \quad T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$