Seperable: $\frac{dy}{dx} = P(x)\,Q(y)$

$\rightarrow \frac{dy}{Q(y)} = P(x)\,dx$

① Get y & $\frac{dy}{dx}$ on one side
② Find Missing piece

Linear: $\frac{dy}{dx} = P(x)\cdot y + Q(x)$

$\rightarrow \mu(x)\frac{dy}{dx} + \mu(x)P(x)\cdot y = Q(x)\,\mu(x)$

eg

treat as result of product rule
implicit differentiation

$R(x)\cdot y = S'(x)$

$\boxed{R(x)}\cdot\frac{dy}{dx} + R'(x)\cdot y = S'(x)$

$\frac{dy}{dx} + \frac{1}{R(x)}R'(x)y = S'(x)$

$\mu(x)\frac{dy}{dx} + \frac{\mu(x)}{R(x)}R'(x)y = S'(x)$

$\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)\,Q(x)$

to get what we want,
need f^n whose $\frac{d}{dx}$
is $R'(x)$ & itself

$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)\,e^{f(x)}$

Needs:

repeats itself $\rightarrow e^{f(x)}$, $\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)\,e^{f(x)}$

$\frac{dy}{dx}$ is derivative w/ implicit differentiation

$P(x) = f'(x) \rightarrow f(x) = \int f'(x)\,dx = \int P(x)\,dx$

$e^{f(x)}$ $\quad \mu(x) = e^{\int P(x)dx}$

$\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)\,Q(x)$

$\frac{d}{dx}\left[\mu(x)\cdot y\right] = \mu(x)\,Q(x)$

$D_x\left[\mu(x)\cdot y\right] = \mu(x)\,Q(x)$

$\int D_x\left[\mu(x)\cdot y\right]dx = \mu(x)\,y = \int \mu(x)\,Q(x)\,dx$

$x^3 y = 2x$ $\quad D_x$

$x^3\frac{dy}{dx} + 3x^2 y = 2$

$\frac{dy}{dx} + \frac{3}{x}y = \frac{2}{x^3}$

$\mu(x) = e^{\int \frac{3}{x}dx} = e^{3\ln(x)} = e^{\ln(x^3)} = x^3$

$\mu(x)\frac{dy}{dx} + \mu(x)\frac{3}{x}y = \mu(x)\frac{2}{x^3}$

only thing here, no x f^n

$\frac{dy}{dx} + P(x)y = Q(x)$

$x^3\frac{dy}{dx} + 3x^2y = 2$

seperable | linear

$\frac{dy}{dx} + y = 2 \qquad y(0) = 0$

$\frac{dy}{dx} + y = 2$

$P(x) = 1$
$Q(x) = 2$

$\frac{dy}{dx} = 2 - y$

$\mu(x) = e^{\int P(x)dx} = e^{\int 1 dx} = e^x$

don't need $+C$ b/c can divide out everywhere

$\int \frac{1}{2-y}dy = \int 1 dx$

$\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)Q(x)$

$-\ln(2-y) = x + C$

$e^x\frac{dy}{dx} + e^x \cdot 1 y = e^x 2$

$\ln(2-y) = -x - C$

$\frac{d}{dx}[e^x y] = 2e^x$

$2 - y = e^{-x-C} = e^{-x} \cdot e^{-C}$

integrate

$2 - y = C_0 e^{-x}$

$e^x y = 2e^x + C$

$2 - 0 = C_0 e^0$

$e^0 \cdot 0 = 2e^0 + C$
$0 = 2 + C$
$C = -2$

$2 = C$

$e^x y = 2e^x - 2$

$y = 2 - 2e^{-x}$

$2 - y = 2e^{-x}$

$y(x) = 2 - 2e^{-x}$