Time Series Causality Algorithm (TSC1) Notes (

September 18, 2014

1 Example 1

Consider a block sliding down a frictionless ramp. The system is assumed to be conservative, so

$$\frac{mv^2}{2} = mgh = mgd\sin\theta = mgvt\sin^2\theta$$

where v is the velocity of the block, t is the time the block has been in motion, m is the mass of the block, g is the local acceleration due to gravity, h is the height of the block before it is released, and d is the distance the block has traveled. The velocity is then

$$v(t) = 2gt\sin^2\theta \quad .$$

If the angle of the ramp is varied with time as

$$\theta(t) = \sin(t) \quad ,$$

then it is expected that $\theta(t) \to v(t)$, i.e. the changing angle drives the changing magnitude of the velocity.

The angle driving signal and the velocity response signal are seen in Figure 1.

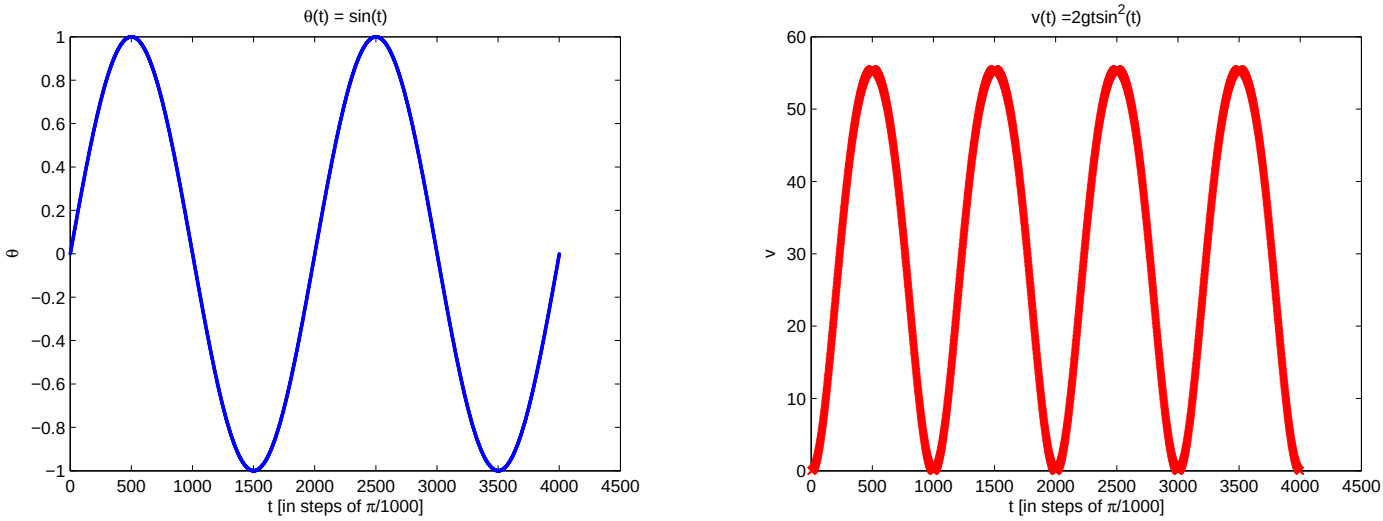

Figure 1

The change in θ is expected to affect v, so a first step is to find the distribution of changes in θ; i.e.

$$\delta\theta(t) \equiv \frac{d\theta}{dt} \approx \frac{\Delta\theta}{\Delta t} = \theta(t) - \theta(t-1) \quad .$$

A histogram of $\{\delta\theta(t) \mid t \geq 0\}$ is seen in Figure 2. A given bin can be picked (e.g. bin 125 in Figure 2), and the values of v that occur immediately following the binned $\delta\theta$ can be plotted to give an idea of how the response signal reacts to the given change in the driving signal. Let $B_i = \{t_k\}$ be the set of times t corresponding to the ith bin of $\delta\theta$ which contains all the $\{\delta\theta(t_k)\}$ that belong to that bin. The plot of the correspondingly binned response signal would be a plot of $\{v(t+1) \mid t \in B_i\}$. Figure 3 is such a plot for B_125.

Conceptually, the response signal is expected to have only a few different values in response to a given change in the driver. However, it might be expected that the actual value of the response might also depend on the previous value of the response signal. For example, a 0.25π change in θ would lead to a different value of $v(t)$ depending on $v(t-1)$. Two ways to approach this problem are to define a change in v (i.e. δv) and then investigate $\{\delta v(t+1) \mid t \in B_i\}$ or to bin v and then investigate $\delta\theta$-binned response values v within a given v-binned set. Both methods are explored below.

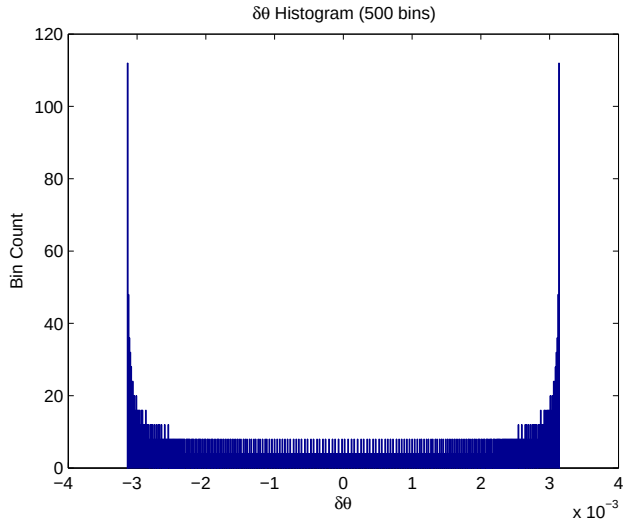

Figure 2

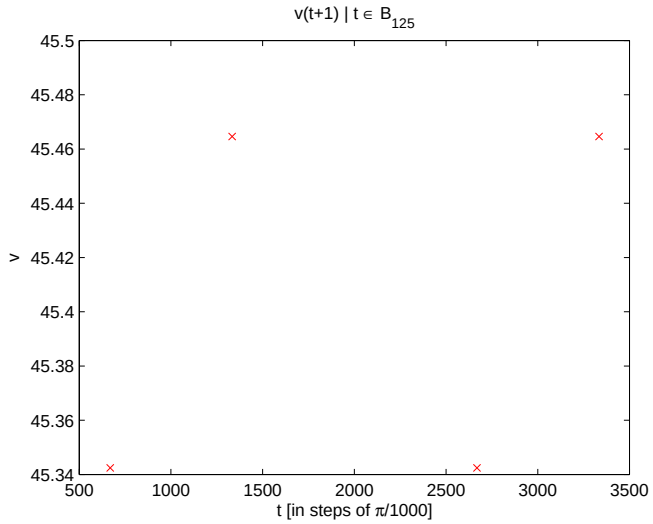

Figure 3