

# How is CCM useful?

## 1 Example System

The continuous system is

$$\frac{dI}{dt} = \frac{V(t)}{L} - \frac{R(t)}{L}I, \quad (1)$$

where  $I$  is the current at time  $t$ ,  $V(t)$  is the voltage at time  $t$ ,  $R(t)$  is the resistance at time  $t$ , and  $L$  is the inductance (which is also constant in these examples), and it can be approximated as

$$\dot{I} = \frac{V(t)}{L} - \frac{R(t)}{L}I \Rightarrow I_{t+1} - I_t = \frac{V_t}{L} - \frac{R_t}{L}I_t. \quad (2)$$

Rearranging leads to

$$I_{t+1} = \frac{V_t}{L} + I_t \left(1 - \frac{R_t}{L}\right), \quad (3)$$

$$V_t = L \left( I_{t+1} - I_t \left(1 - \frac{R_t}{L}\right) \right), \quad (4)$$

and

$$R_t = L \left( I_t - I_{t+1} + \frac{V_t}{L} \right). \quad (5)$$

All of the plots of  $I$  seen below are produced by using MATLAB's *ode45* to solve Eqn. 1 (i.e. not using the discrete approximation shown). The time series  $V(t)$  and  $R(t)$  are created by defining values at fixed points and using linear interpolation (i.e. MATLAB's *interp1*) to find the time steps required by the ODE solver (i.e. MATLAB's *ode45*).

## 2 Changing $V(t)$

Consider the situation where  $R(t)$  is constant.

**Physical intuition is that  $V$  drives  $I$ , so we expect to find  $V$  CCM causes  $I$  ( $C_{VI} > C_{IV}$ ).**

For this example, the voltage is described by

$$V(t) = A_v \sin(f_v t + \phi_v) + O_v, \quad (6)$$

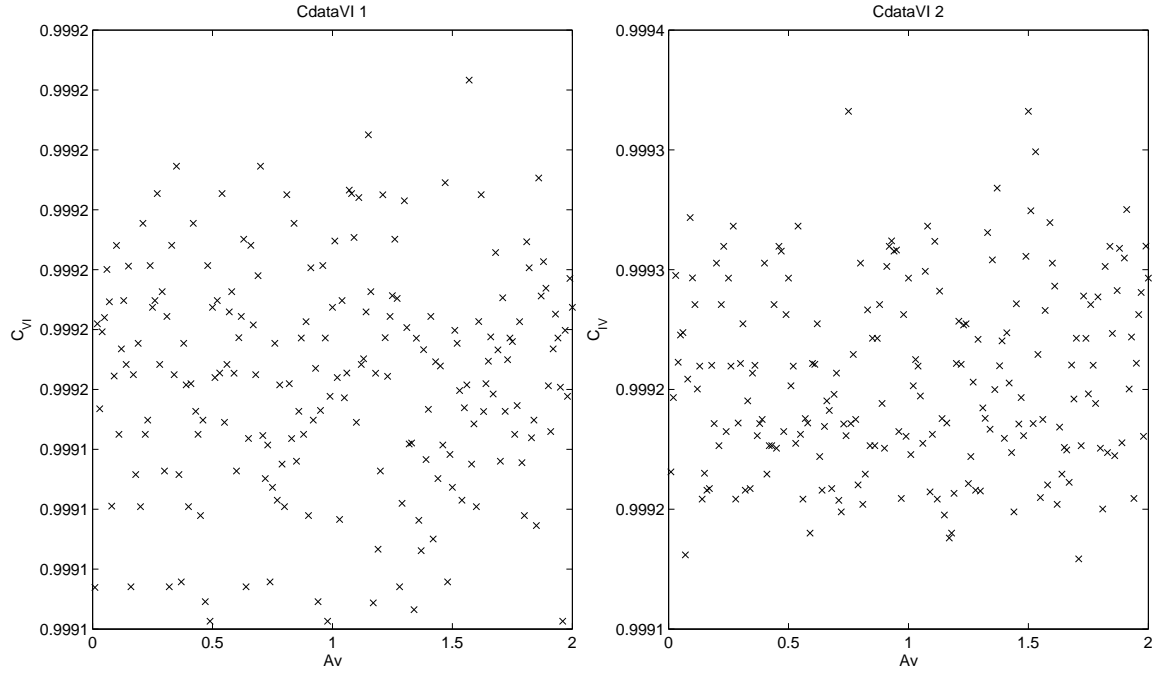
where  $A_v$  is the amplitude,  $f_v$  is the frequency,  $\phi_v$  is the phase, and  $O_v$  is the offset voltage.

### 2.1 Changing $A_v$

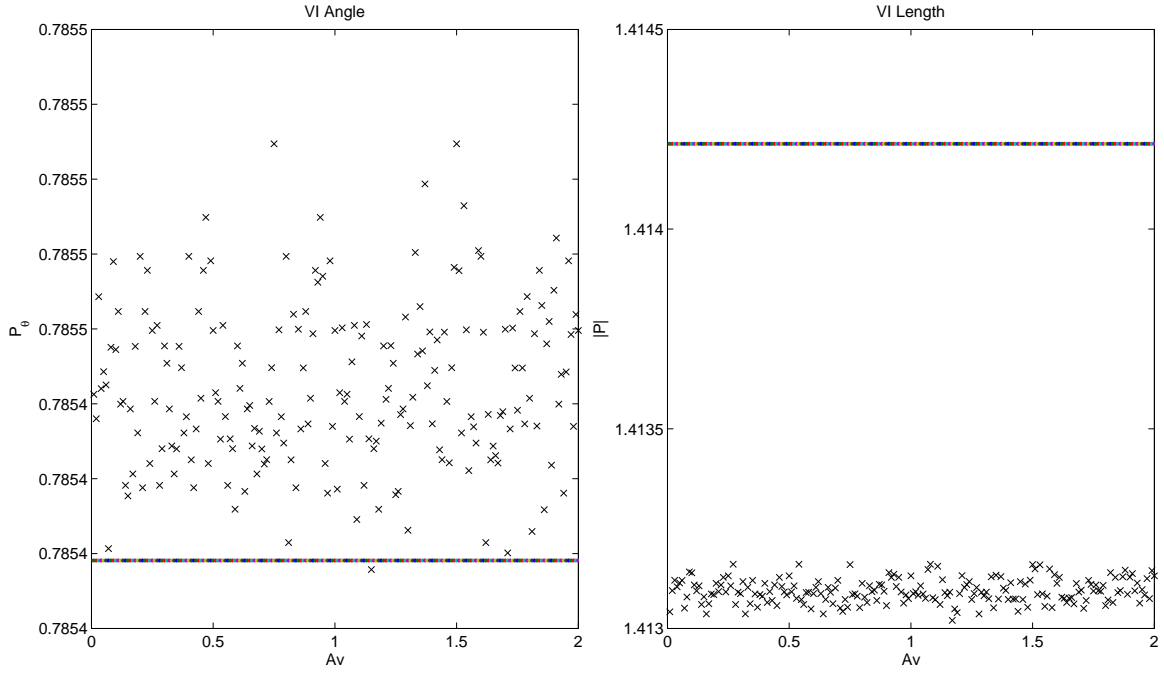
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $A_v \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $V(t)$  and  $I(t)$  are plotted for different  $A_v$  in Figure 1.

Figure 1: Reference plots for changing  $A_v$ .

The CCM correlations are each plotted in Figure 2 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

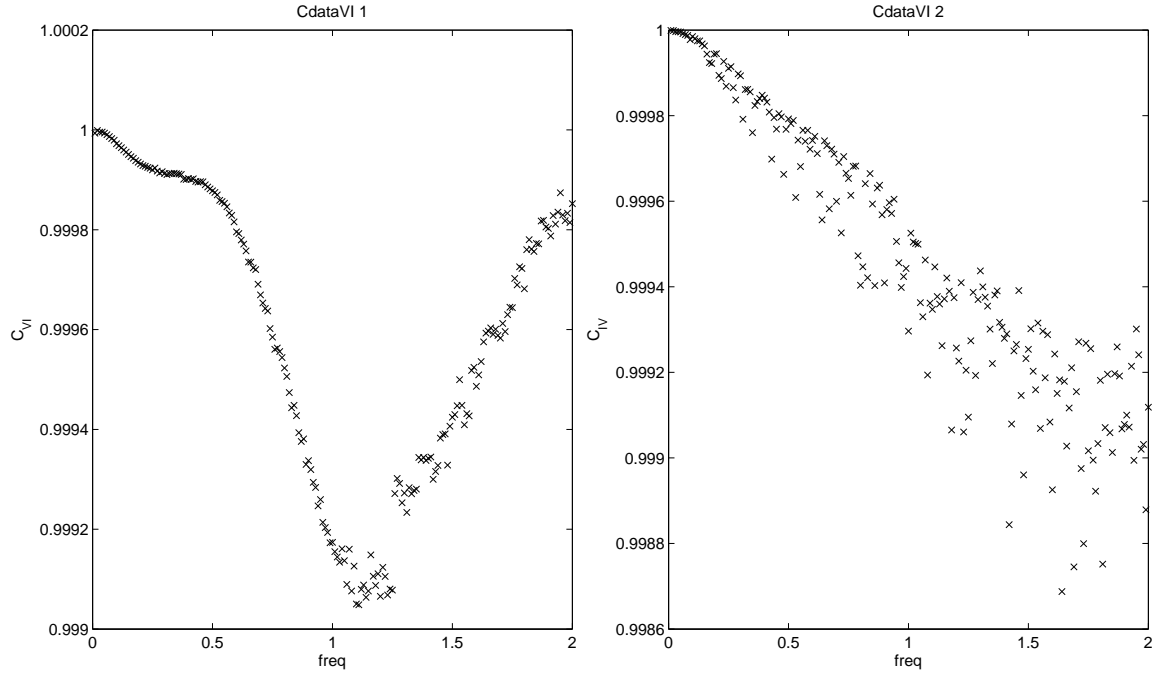
Figure 2: Changing  $A_v$ .

## 2.2 Changing $f_v$

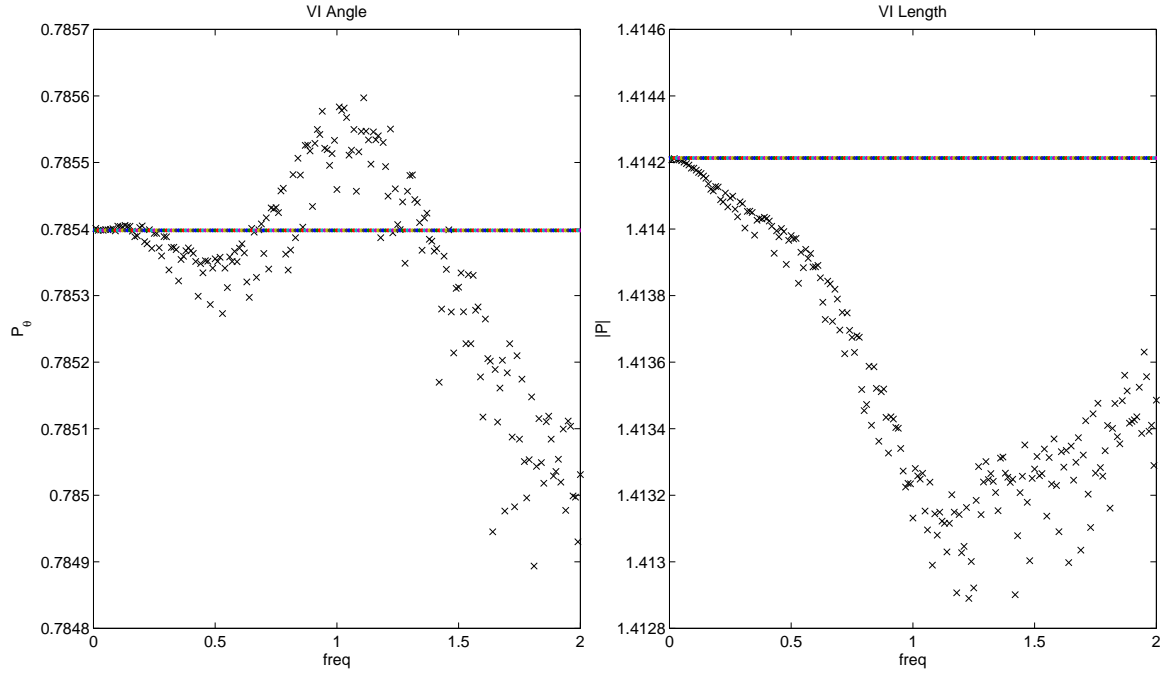
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $f_v \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $V(t)$  and  $I(t)$  are plotted for different  $f_v$  in Figure 3.

Figure 3: Reference plots for changing  $f_v$ .

The CCM correlations are each plotted in Figure 4 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

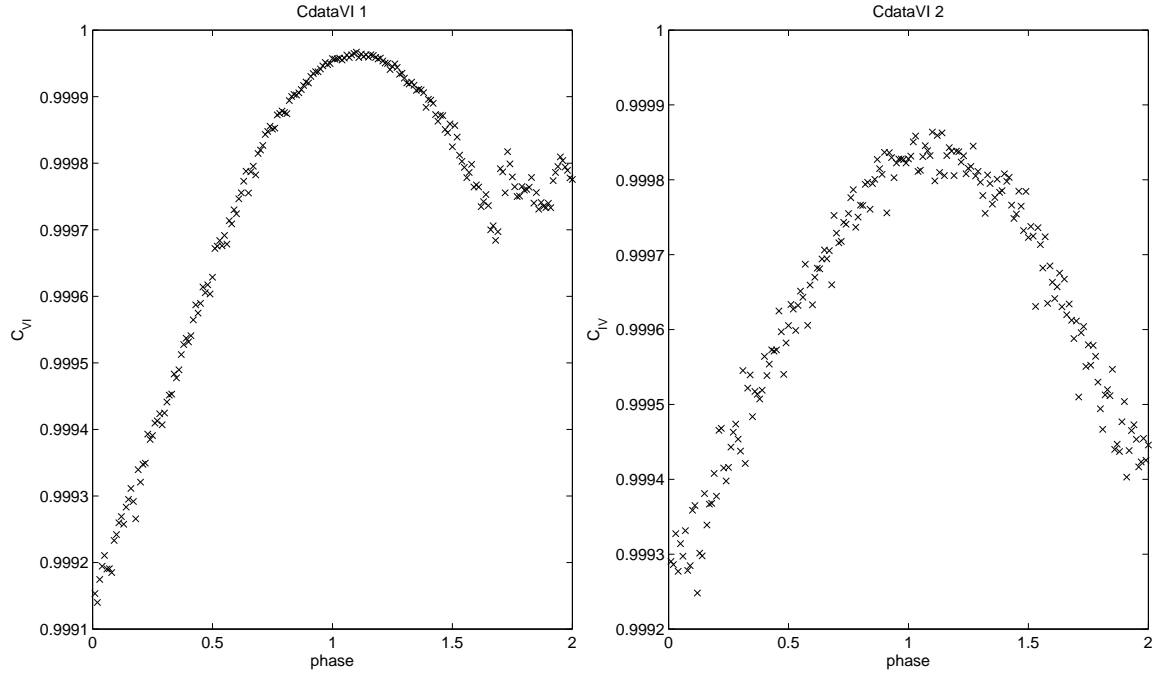
Figure 4: Changing  $f_v$ .

### 2.3 Changing $\phi_v$

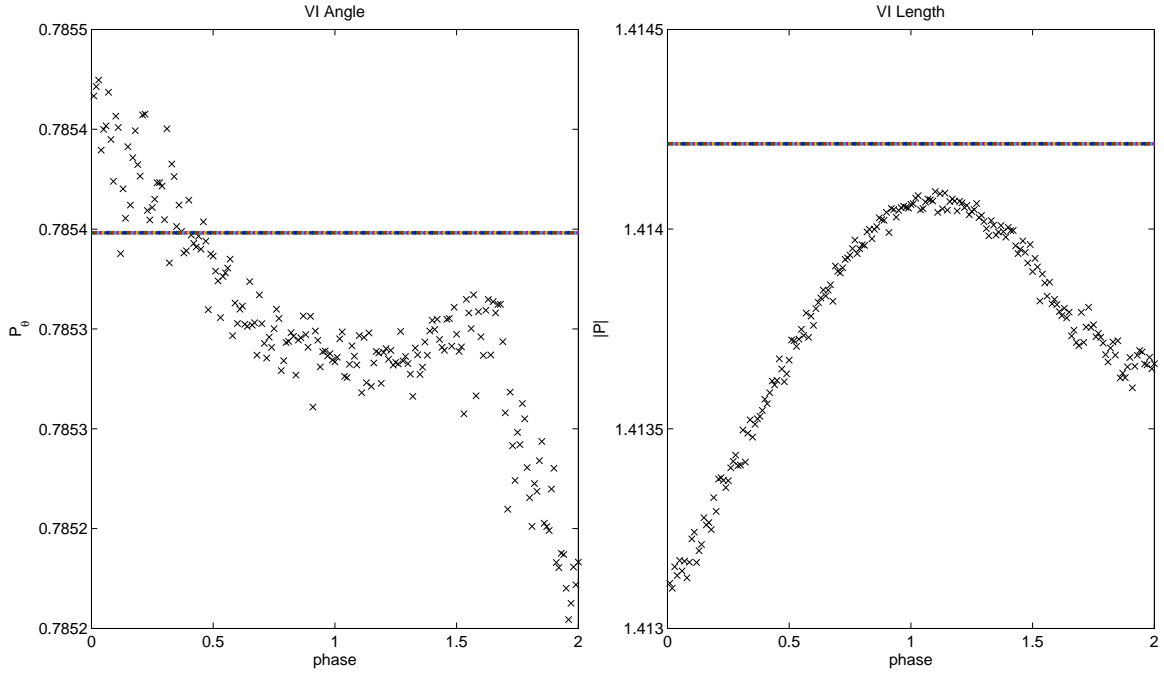
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $\phi_v \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $V(t)$  and  $I(t)$  are plotted for different  $\phi_v$  in Figure 5.

Figure 5: Reference plots for changing  $\phi_v$ .

The CCM correlations are each plotted in Figure 16 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

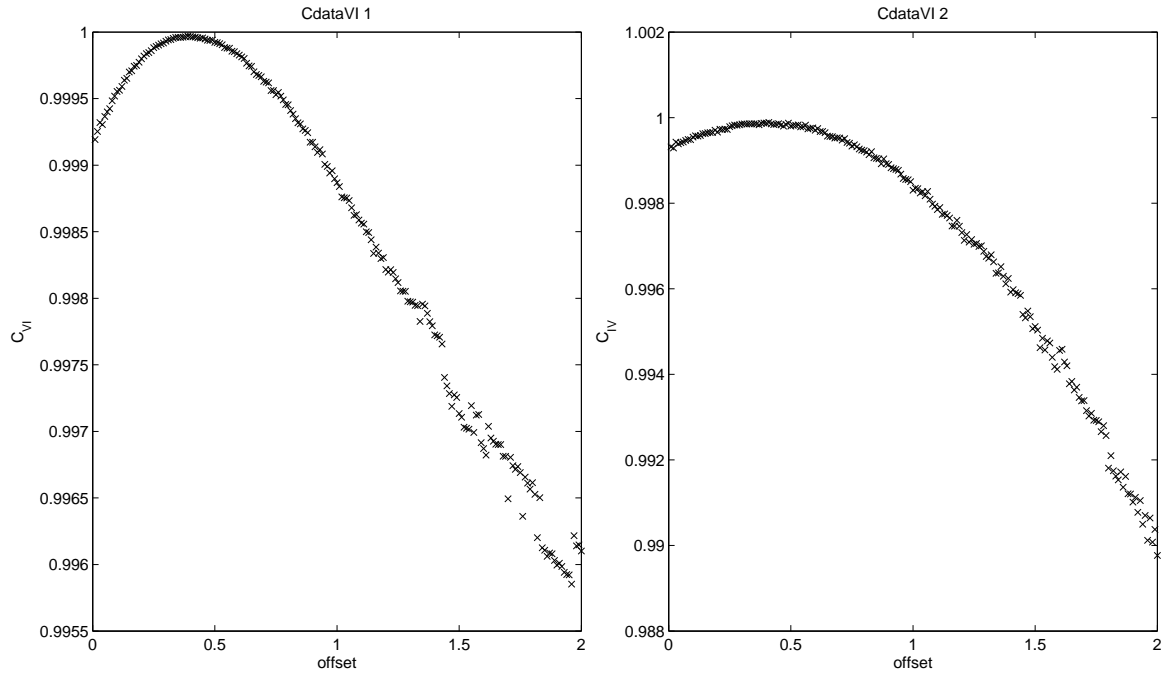
Figure 6: Changing  $\phi_v$ .

## 2.4 Changing $O_v$

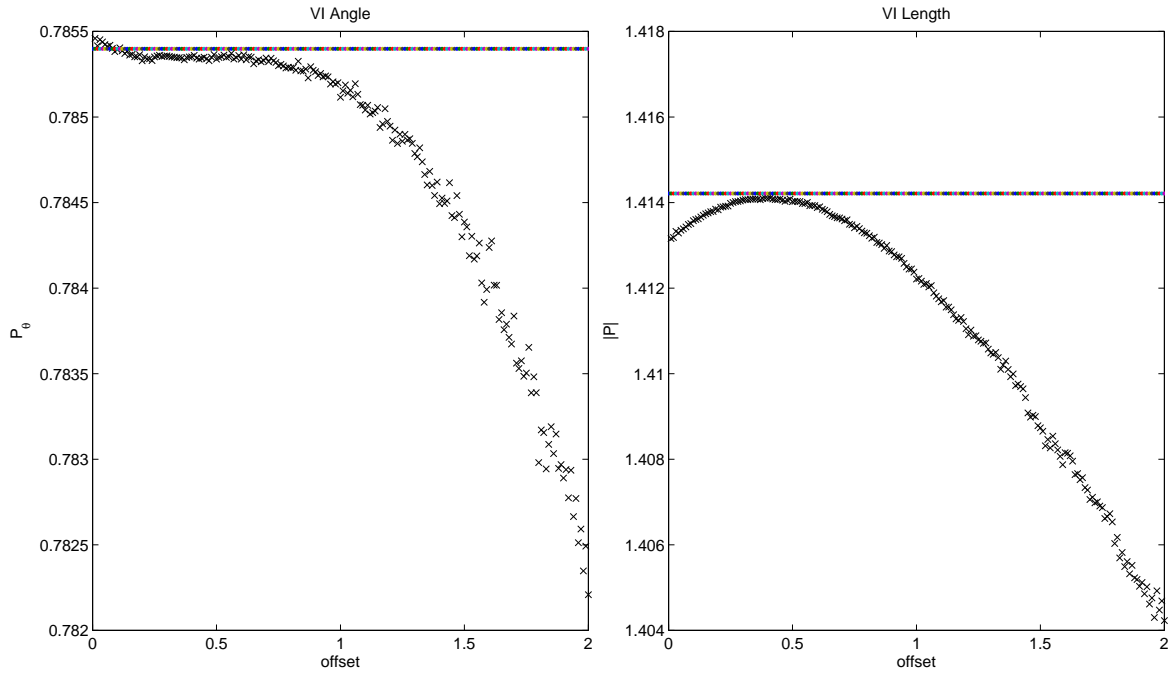
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $O_v \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $V(t)$  and  $I(t)$  are plotted for different  $O_v$  in Figure 7.

Figure 7: Reference plots for changing  $O_v$ .

The CCM correlations are each plotted in Figure 8 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



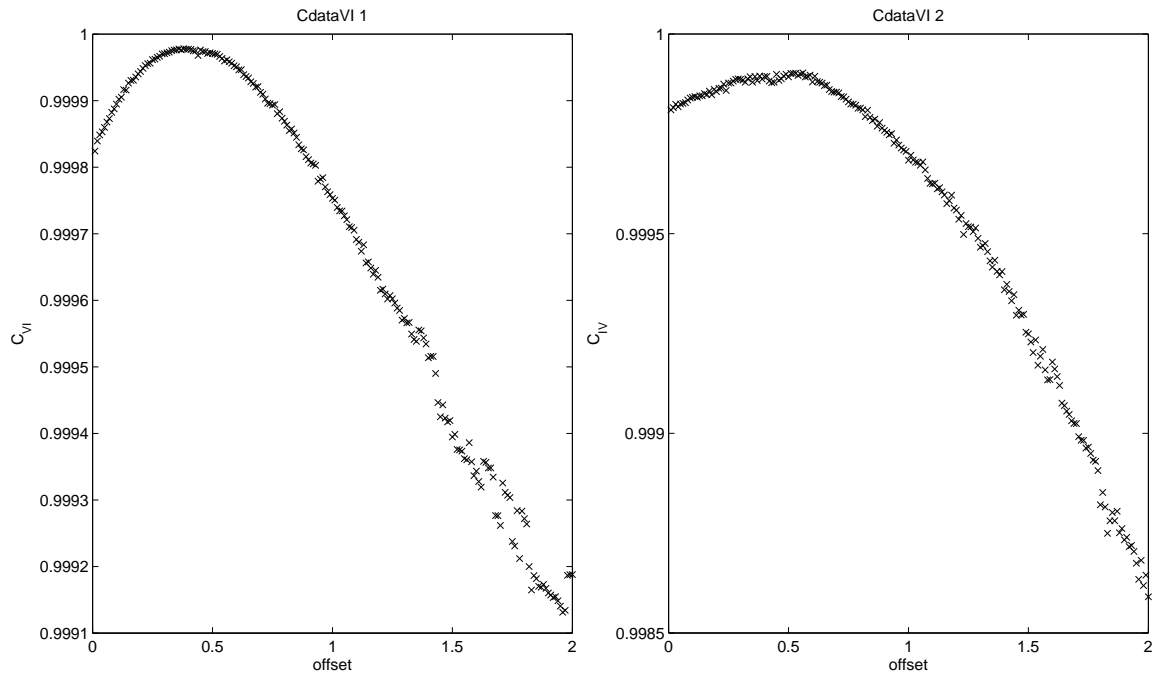
(a)  $C_{VI}$  and  $C_{IV}$



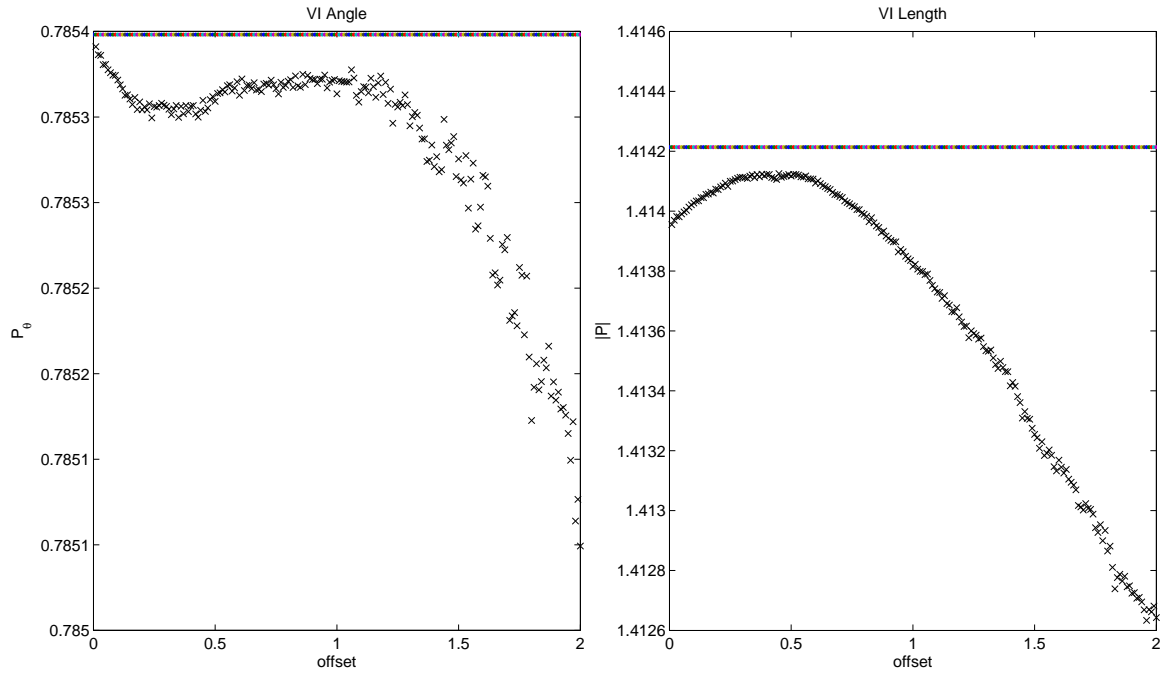
(b)  $P_\theta$  and  $|P|$

Figure 8: Changing  $O_v$ .

Figure 9 shows the effect of increasing the library length from  $2 \times 10^3$  (i.e. `tspan = [0:0.5:1000];`) to  $10^4$  (i.e. `tspan = [0:0.5:5000];`), and Figure 10 extends the above plots to  $O_v \in [0.01, 10.0]$  in steps of 0.05.

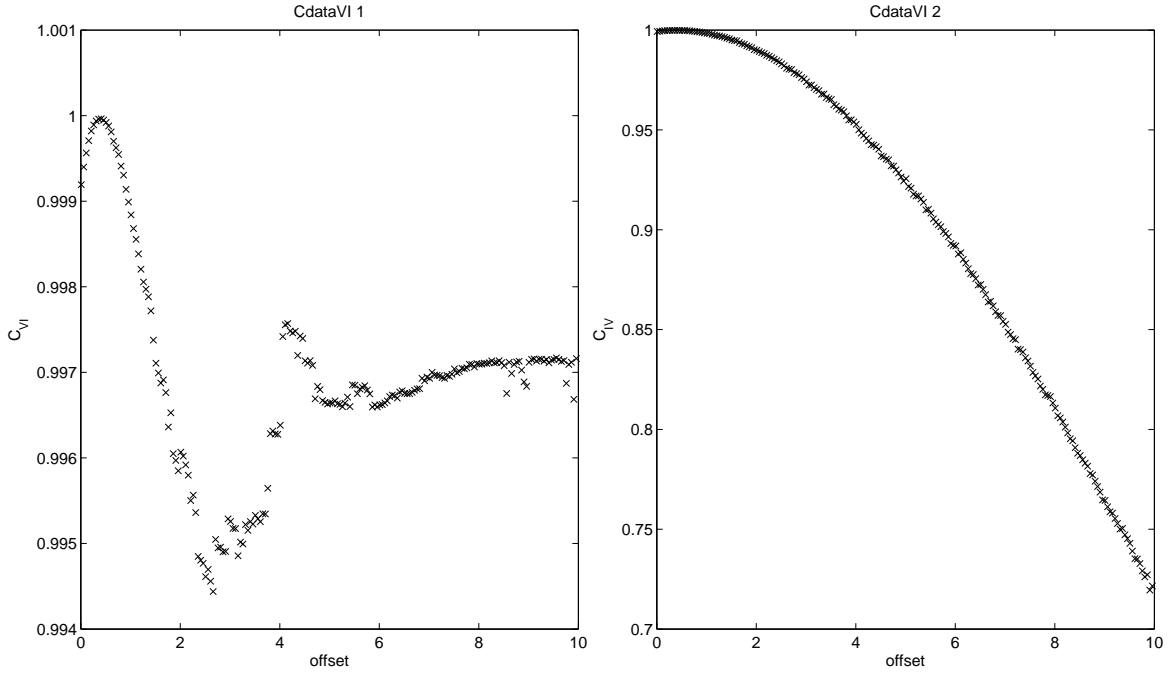


(a)  $C_{VI}$  and  $C_{IV}$

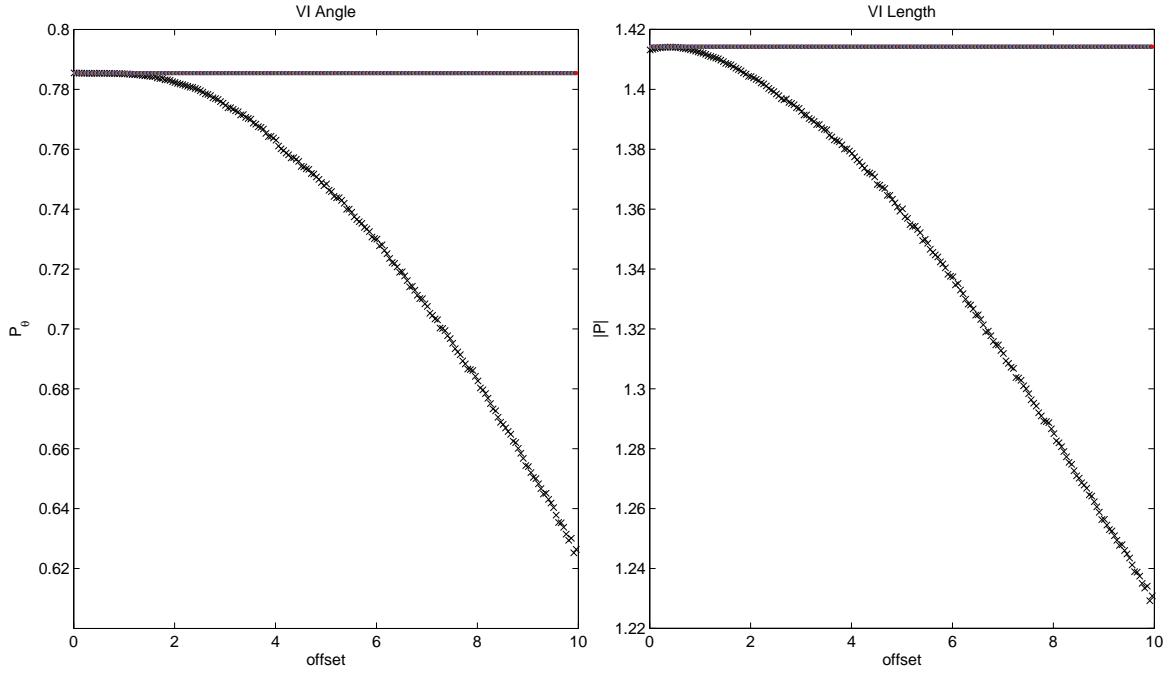


(b)  $P_\theta$  and  $|P|$

Figure 9: Changing  $O_v$  (longer library length).



(a)  $C_{VI}$  and  $C_{IV}$



(c)  $P_\theta$  and  $|P|$

Figure 10: Changing  $O_v$  (larger domain for  $O_v$ ).

### 3 Changing $R(t)$

Consider the situation where  $V(t)$  is constant.

**Physical intuition is that  $R$  drives  $I$ , so we expect to find  $R$  CCM causes  $I$  ( $C_{RI} > C_{IR}$ ).**

For this example, the voltage is described by

$$R(t) = A_r \sin(f_r t + \phi_r) + O_r, \quad (7)$$

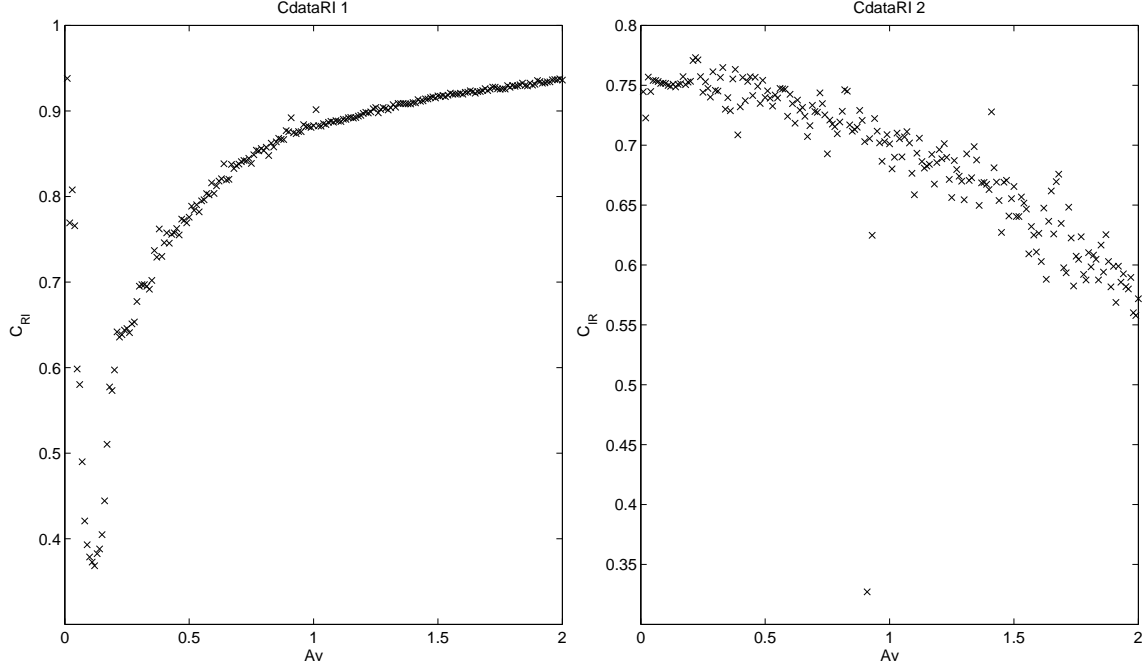
where  $A_r$  is the amplitude,  $f_r$  is the frequency,  $\phi_r$  is the phase, and  $O_r$  is the offset voltage.

### 3.1 Changing $A_r$

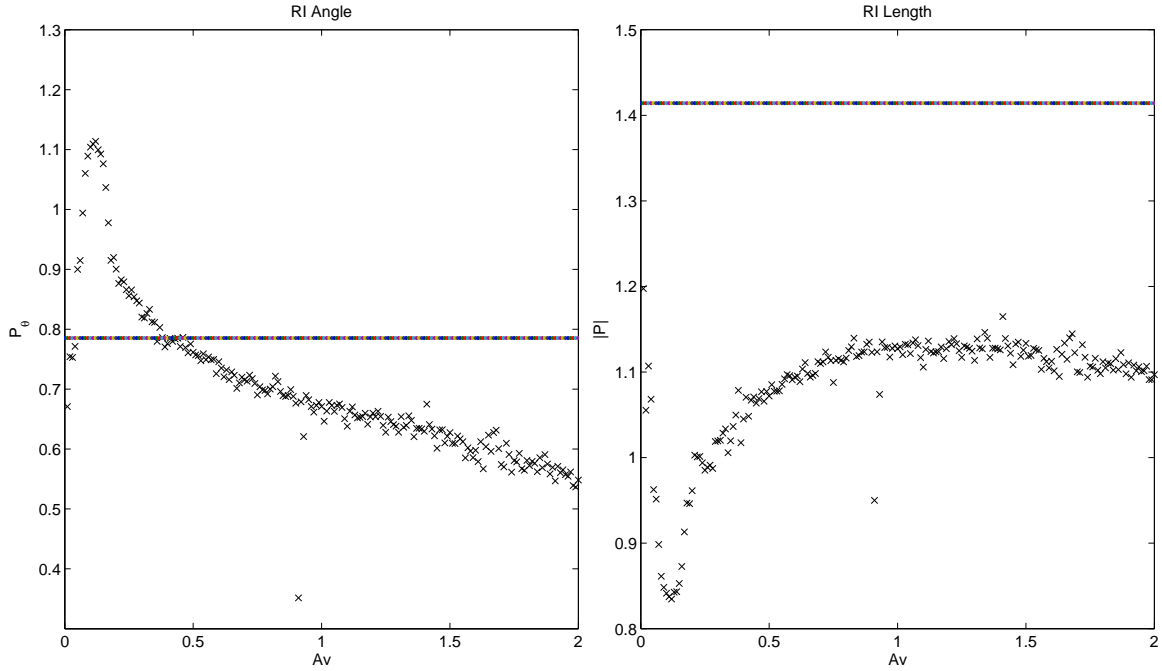
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $A_r \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $R(t)$  and  $I(t)$  are plotted for different  $A_r$  in Figure 11.

Figure 11: Reference plots for changing  $A_r$ .

The CCM correlations are each plotted in Figure 12 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

Figure 12: Changing  $A_r$ .

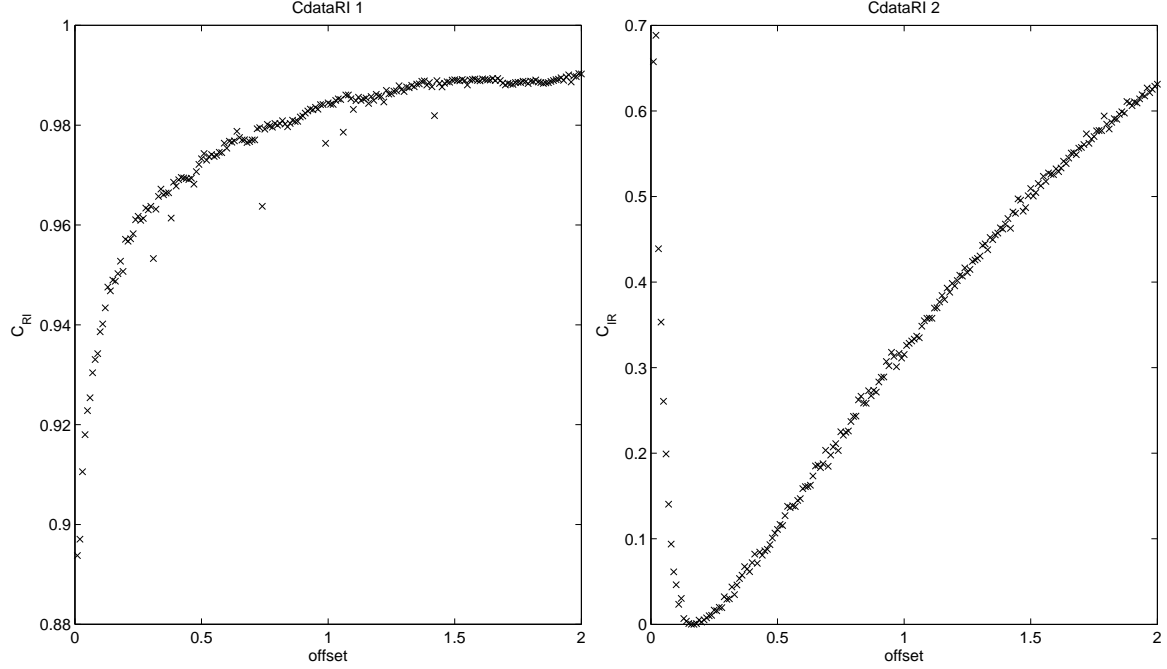


### 3.2 Changing $O_r$

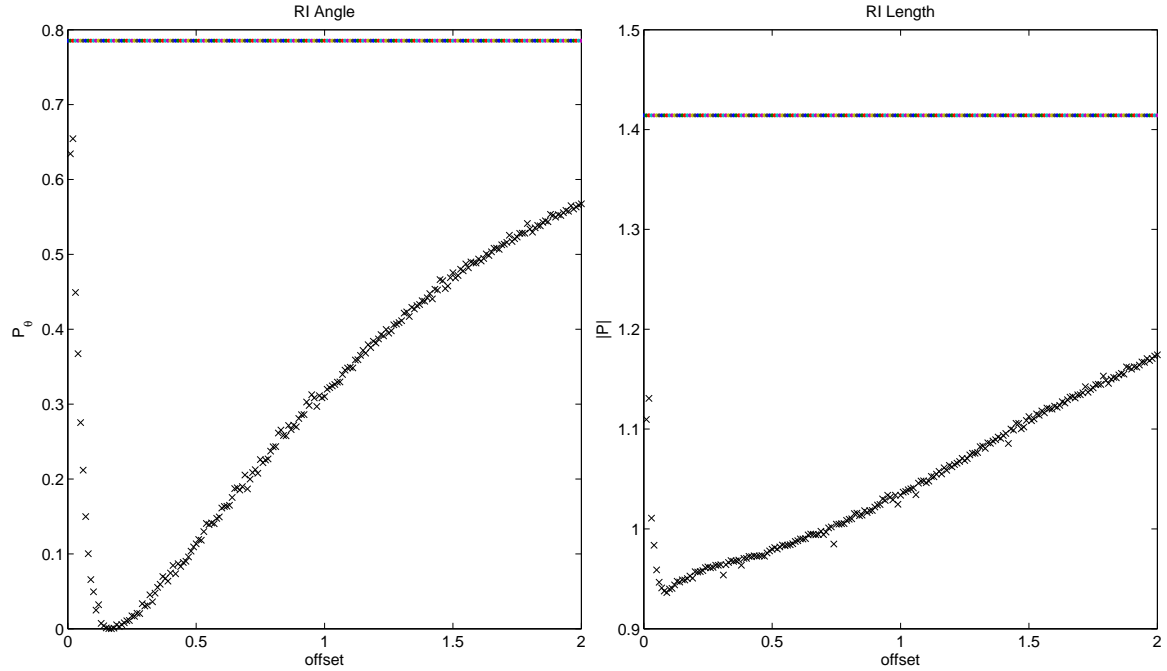
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $O_r \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $R(t)$  and  $I(t)$  are plotted for different  $f_v$  in Figure 13.

Figure 13: Reference plots for changing  $O_r$ .

The CCM correlations are each plotted in Figure 14 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

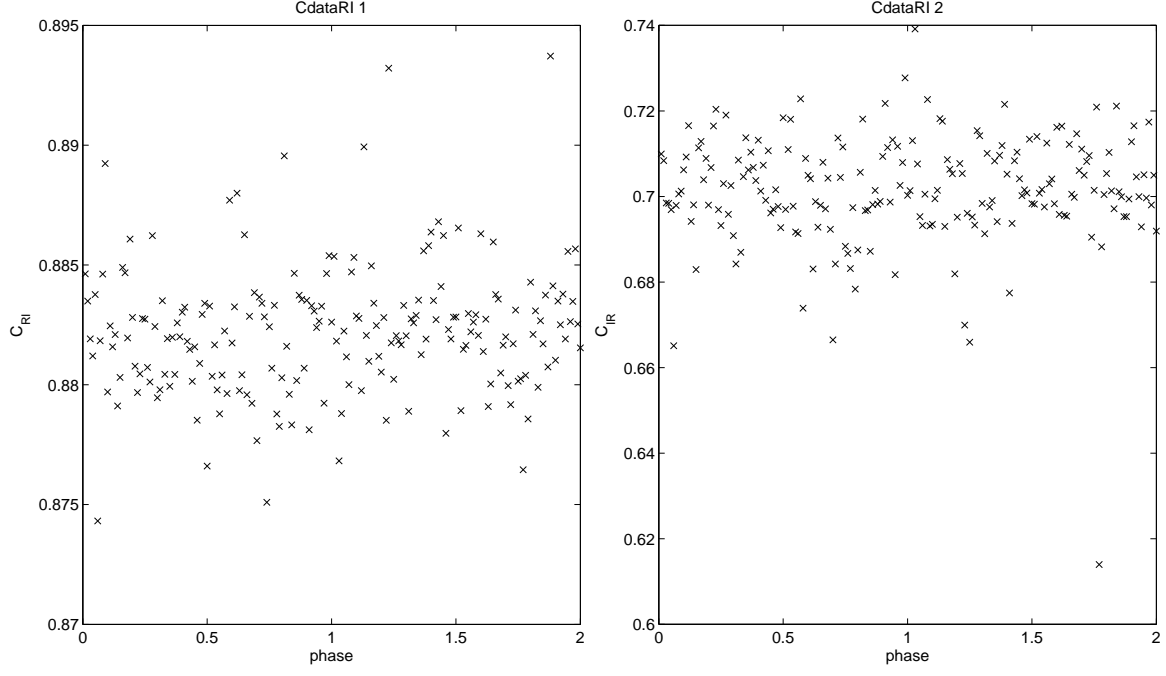
Figure 14: Changing  $O_r$ .

### 3.3 Changing $\phi_r$

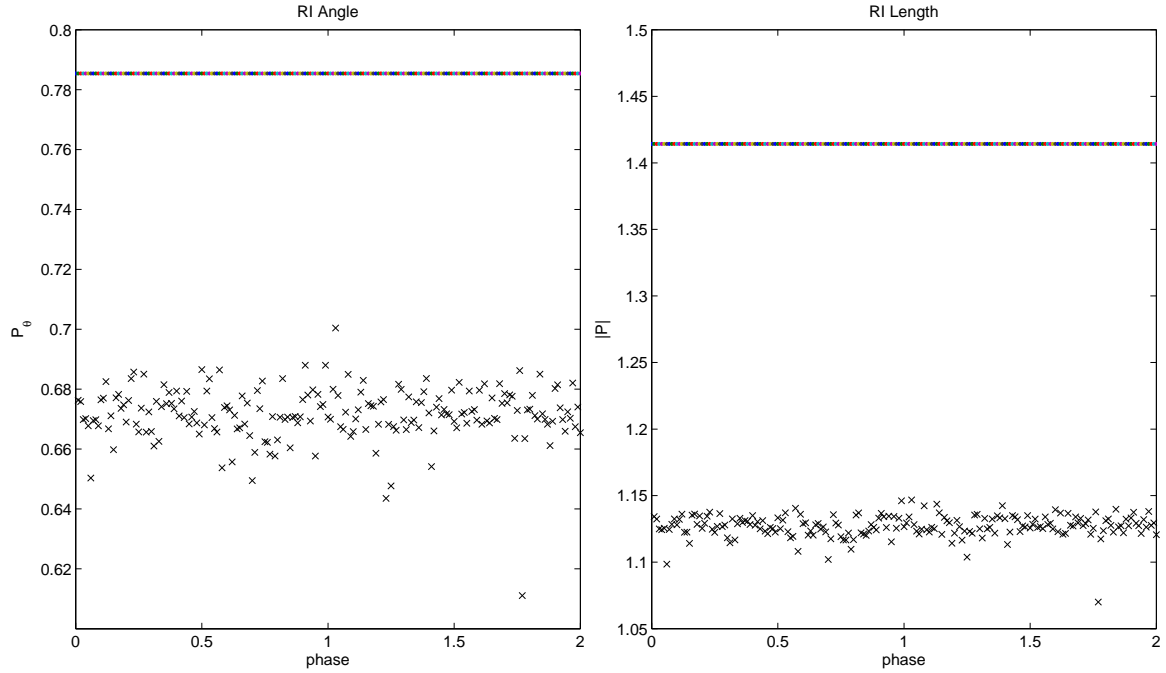
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $\phi_r \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $R(t)$  and  $I(t)$  are plotted for different  $\phi_r$  in Figure 15.

Figure 15: Reference plots for changing  $\phi_r$ .

The CCM correlations are each plotted in Figure ?? along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

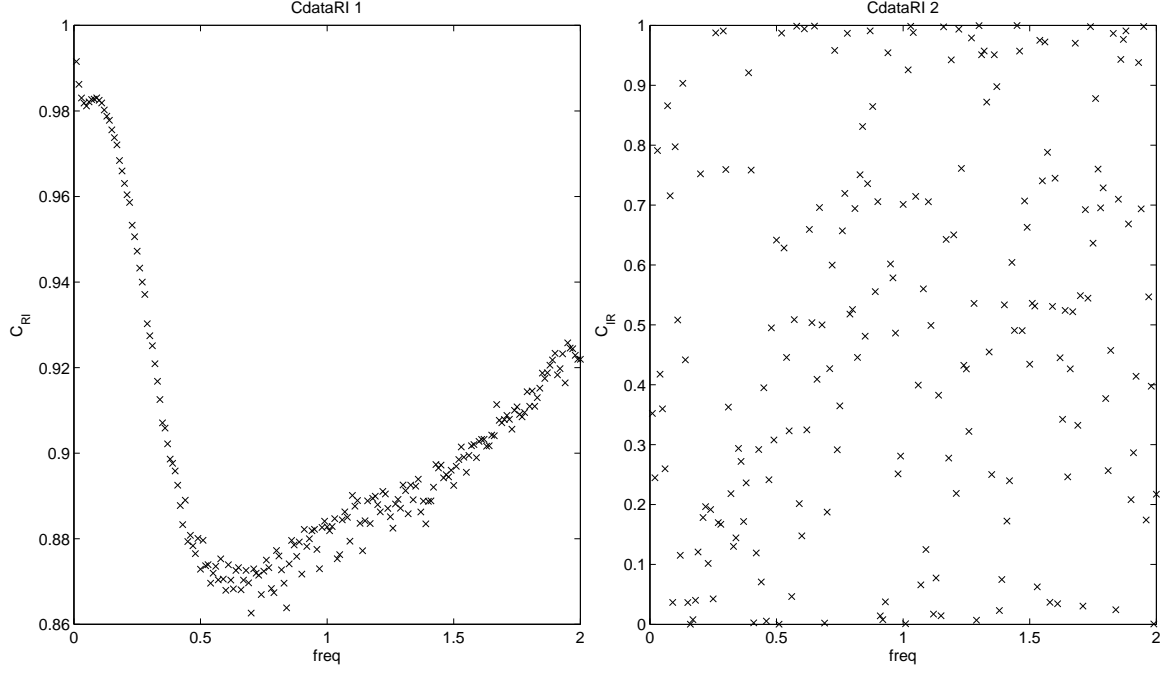
Figure 16: Changing  $\phi_r$ .

### 3.4 Changing $f_r$

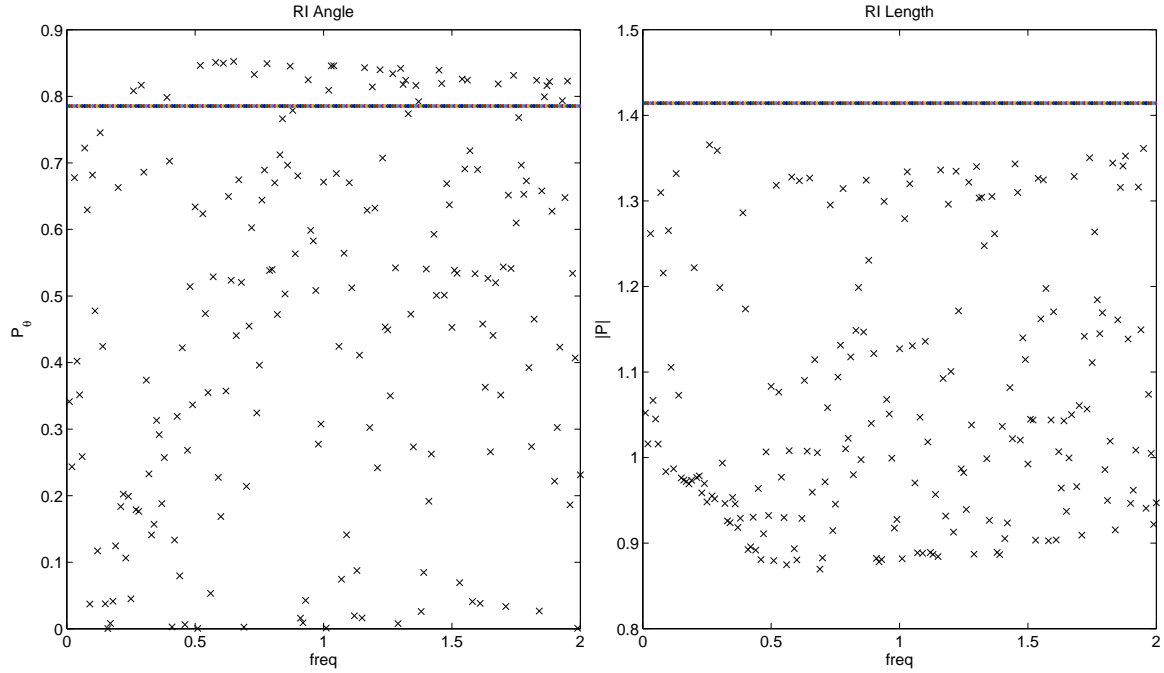
Consider evaluating the CCM correlations  $C_{VI}$  and  $C_{IV}$  for each  $f_r \in [0.01, 2.0]$  in steps of 0.01. For reference, both  $R(t)$  and  $I(t)$  are plotted for different  $f_r$  in Figure 17.

Figure 17: Reference plots for changing  $f_r$ .

The CCM correlations are each plotted in Figure 18 along with the corresponding PAI elements  $P_\theta$  and  $|P|$ .



(a)  $C_{VI}$  and  $C_{IV}$

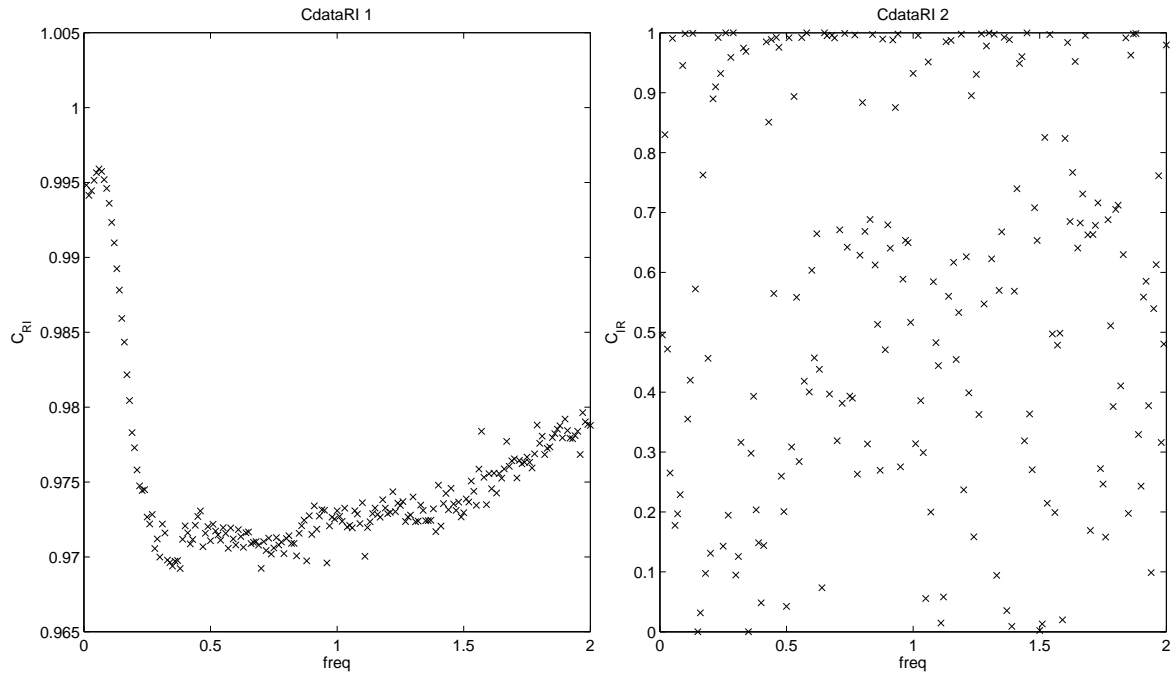


(b)  $P_\theta$  and  $|P|$

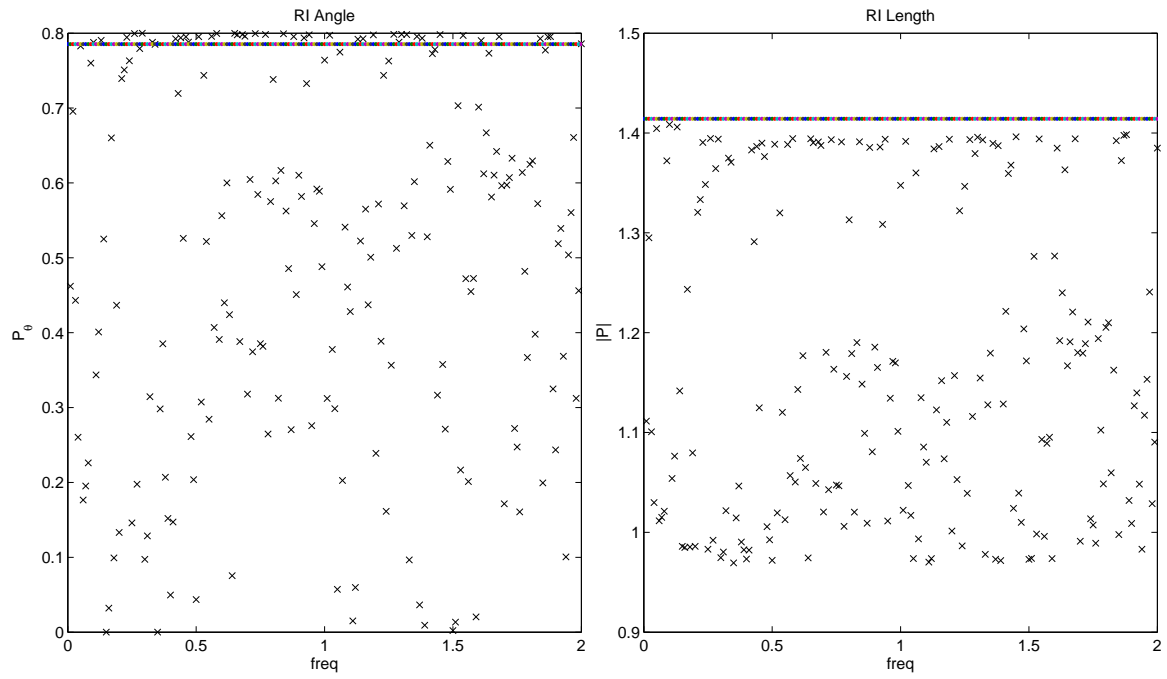
Figure 18: Changing  $f_r$ .

Figure 19 shows the effect of increasing the library length from  $2 \times 10^3$  (i.e. `tspan = [0:0.5:1000];`) to  $10^4$  (i.e. `tspan =`

$[0:0.5:5000];$ ), and Figure 20 extends the above plots to  $f_r \in [0.01, 10.0]$  in steps of 0.05.

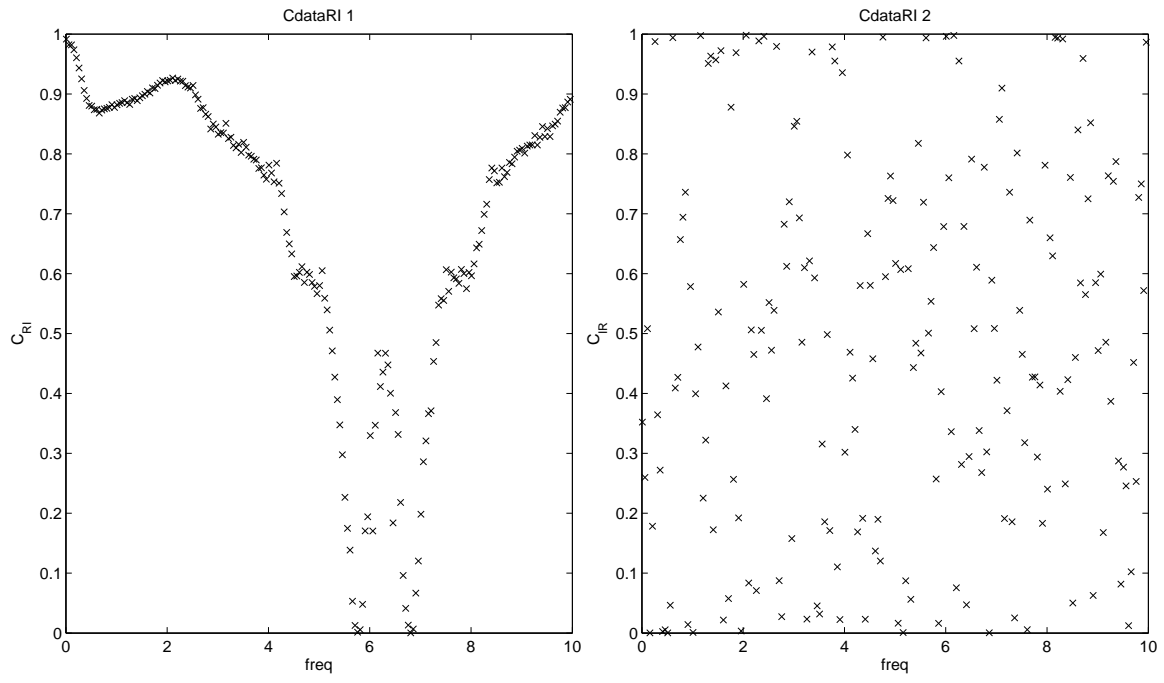


(a)  $C_{VI}$  and  $C_{IV}$

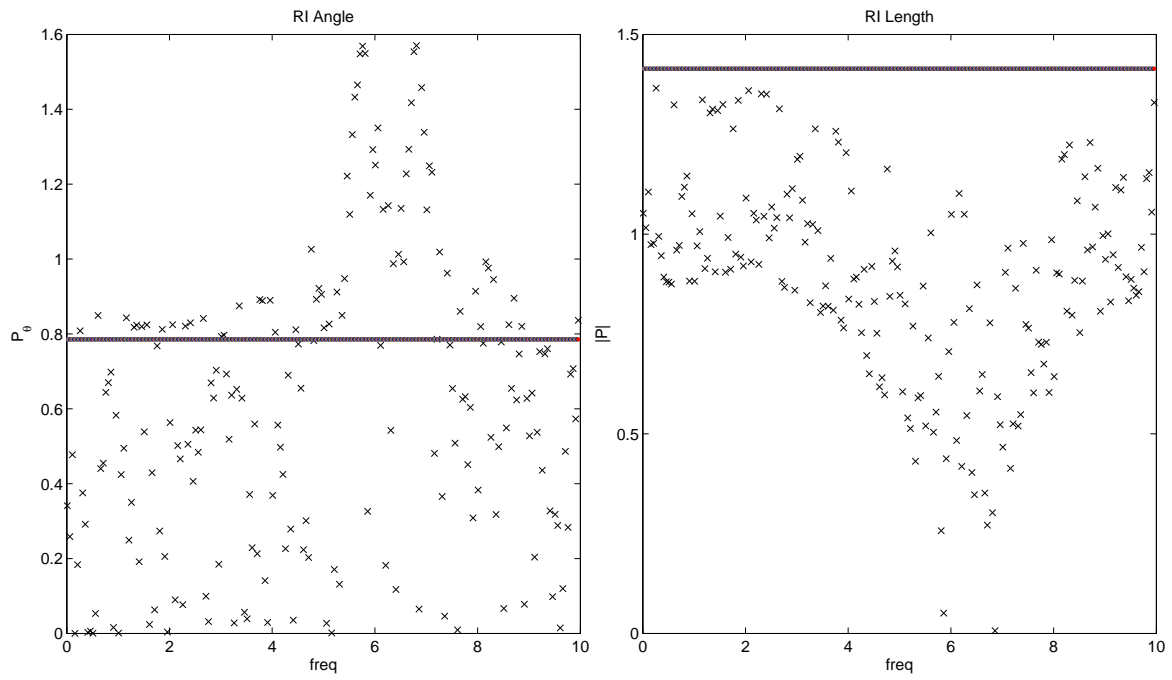


(b)  $P_\theta$  and  $|P|$

Figure 19: Changing  $f_r$  (longer library length).



(a)  $C_{VI}$  and  $C_{IV}$



(b)  $P_\theta$  and  $|P|$

Figure 20: Changing  $f_r$  (larger domain for  $f_r$ ).