

# Time Series Leanings

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## 1 Introduction

...TBD...

The definition of probability complements implies  $P(\bar{C}) = 1 - P(C)$  and  $P(\bar{C}|E) = 1 - P(C|E)$ . Applying Eqn. 3 leads to

## 2 Causal Penchant

Define the *causal penchant* as

$$\rho_{EC} := P(E|C) - P(E|\bar{C}) \quad . \quad (1) \quad \text{Thus,}$$

The motivation for this expression is in the interpretation of  $\rho_{EC}$  as a causal indicator; i.e. if  $C$  causes (or *drives*)  $E$ , then  $\rho_{EC} > 0$ , and if  $\rho_{EC} \leq 0$ , then the direction of causal influence in the system is undetermined. If the effect  $E$  is assumed to be recorded in one time series and the cause  $C$  is assumed to be recorded in a different time series, then the direction of causal influence in the system can be determined by comparing various penchants calculated using both time series. The details of these comparisons are discussed in the following sections, but some potential philosophical issues with this definition will be addressed first.

... **Add discussion of Pearl argument that  $P(E|\bar{C})$  is "unobservable".**

Pearl's concerns can be addressed by rewriting Eqn. 1 using the law of total probability, i.e.

$$P(E) = P(E|C)P(C) + P(E|\bar{C})P(\bar{C}) \quad , \quad (2)$$

or Bayes theorem, i.e.

$$P(E|\bar{C}) = P(\bar{C}|E) \frac{P(E)}{P(\bar{C})} \quad . \quad (3)$$

$$\begin{aligned} P(\bar{C}|E) &= 1 - P(C|E) \\ &= 1 - P(E|C) \frac{P(C)}{P(E)} \end{aligned}$$

$$\begin{aligned} P(E|\bar{C}) &= P(\bar{C}|E) \frac{P(E)}{P(\bar{C})} \\ &= \left( 1 - P(E|C) \frac{P(C)}{P(E)} \right) \frac{P(E)}{1 - P(C)} \quad , \end{aligned}$$

and the second term in Eqn. 1 has been written in terms of the of the first term. This expression implies a penchant calculation containing only the conditional probability that is directly observed from the data, i.e.

$$\rho_{EC} = P(E|C) \left( 1 + \frac{P(C)}{1 - P(C)} \right) - \frac{P(E)}{1 - P(C)} \quad (4)$$

This same expression can be derived without Eqn. 3 by using Eqn. 2 to make the appropriate substitution for  $P(E|\bar{C})$  into Eqn. 1. This penchant calculation requires only a single conditional probability be estimated from the data.

The use of either Eqn. 2 or Eqn. 3 may eliminate the concern that  $P(E|\bar{C})$  is fundamentally unobservable. It may also, however, introduce new philosophical concerns in the definition of the penchant. For example, ... **expand this discussion**

It follows from Eqn. 1 that

$$\rho_{EC} \in [1, -1] \quad , \quad (5)$$

but, more importantly for the calculations in the following sections, the penchant is not defined if  $P(C)$  or  $P(\bar{C})$  are zero (because the conditionals in Eqn. 1 would be undefined). Thus, the penchant is not defined if  $P(C) = 0$  or if  $P(C) = 1$ . The former condition is interpreted intuitively as an inability to determine causal influence between two time series using points that do not appear in one of the series, and the latter condition is interpreted intuitively as an inability to determine causal influence between two time series if one of the data series is constant. The use of Bayes theorem in the derivation of Eqn. 4 implies that same conditions apply to  $P(E)$ . It will be seen below that there is no *a priori* assignments of "cause" or "effect" to a given time series when using pendants for causal inference. So, operationally, these conditions of  $P(C)$  and  $P(E)$  only mean that the penchant is undefined between pairs of time series where one series is constant.

The philosophical concerns are perhaps not as important as an answer to the straightforward question of whether or not the penchant is a useful tool for time series causality. The rest of this article will focus on answering that question.

### 3 Causal Leaning

Given a pair of times series  $\{\mathbf{X}, \mathbf{Y}\}$ , it is difficult to use the penchant directly for causal inference between the pair. Consider the assignment of  $\mathbf{X}$  as the cause,  $C$ , and  $\mathbf{Y}$  as the effect,  $E$ , i.e.  $\{C, E\} = \{\mathbf{X}, \mathbf{Y}\}$ . If  $\rho_{EC} > 0$ , then the probability that  $\mathbf{X}$  drives  $\mathbf{Y}$  is higher than the probability that it does not, which is stated more sufficiently as  $\mathbf{X}$  has a penchant

to drive  $\mathbf{Y}$  or  $\mathbf{X} \xrightarrow{pen} \mathbf{Y}$ . It is possible, however, that the same penchant could be positive with the opposite cause-effect assignment, i.e.  $\rho_{EC} > 0 \mid \{C, E\} = \{\mathbf{Y}, \mathbf{X}\} \Rightarrow \mathbf{Y} \xrightarrow{pen} \mathbf{X}$ . Even though it is possible that  $\mathbf{X} \xrightarrow{pen} \mathbf{Y}$  and  $\mathbf{Y} \xrightarrow{pen} \mathbf{X}$  are both true, such information does not provide information about the causal relationship within the pair  $\{\mathbf{X}, \mathbf{Y}\}$ .

The *leaning* is meant to address this problem and is defined as

$$\lambda_{EC} := \rho_{EC} - \rho_{CE} \quad . \quad (6)$$

A positive leaning implies the cause  $C$  drives the effect  $E$  more than the effect drives the cause, a negative leaning implies the effect  $E$  drives the cause  $C$  more than the cause drives the effect, and a null leaning (i.e.  $\lambda_{EC} = 0$ ) yields no causal information for the cause-effect pair  $\{C, E\}$ .

Consider again the assignment of  $\{C, E\} = \{\mathbf{X}, \mathbf{Y}\}$ . If  $\lambda_{EC} > 0$ , then  $\mathbf{X}$  has a larger penchant to drive  $\mathbf{Y}$  than  $\mathbf{Y}$  does to drive  $\mathbf{X}$ . More verbosely,  $\lambda_{EC} > 0$  implies the difference between the probability that  $\mathbf{X}$  drives  $\mathbf{Y}$  and the probability that it does not is higher than the difference between the probability that  $\mathbf{Y}$  drives  $\mathbf{X}$  and the probability that it does not. For convenience, this language is boiled down to  $\mathbf{X} \xrightarrow{lean} \mathbf{Y}$ , as in  $\lambda_{EC} > 0 \mid \{C, E\} = \{\mathbf{X}, \mathbf{Y}\} \Rightarrow \mathbf{X} \xrightarrow{lean} \mathbf{Y}$ ,  $\lambda_{EC} < 0 \mid \{C, E\} = \{\mathbf{X}, \mathbf{Y}\} \Rightarrow \mathbf{Y} \xrightarrow{lean} \mathbf{X}$ , and  $\lambda_{EC} = 0 \mid \{C, E\} = \{\mathbf{X}, \mathbf{Y}\} \Rightarrow$  no conclusion.

It follows from Eqn. 6 and the bound for the penchant that  $\lambda_{EC} \in [-2, 2]$ . The leaning is a function of four probabilities,  $P(C)$ ,  $P(E)$ ,  $P(C|E)$  and  $P(E|C)$ . The usefulness of the leaning for causal inference will depend on an effective method for estimating these probabilities from times series data and a more careful definition of the cause-effect assignment within the time series pair. These topics will be discussed with a motivating toy model of a dynamical system for which the penchant and

leaning calculations are simple enough to perform without any computational aid.

## 4 Motivating Example

Consider a time series pair  $\bar{\mathbf{T}} = \{\mathbf{X}, \mathbf{Y}\}$  with

$$\begin{aligned}\mathbf{X} &= \{x_t \mid t \in [0, 9]\} \\ &= \{0, 0, 1, 0, 0, 1, 0, 0, 1, 0\} \\ \mathbf{Y} &= \{y_t \mid t \in [0, 9]\} \\ &= \{0, 0, 0, 1, 0, 0, 1, 0, 0, 1\}.\end{aligned}$$

It seems intuitive to say that  $\mathbf{X}$  drives  $\mathbf{Y}$  because  $y_t = x_{t-1}$ . However, to show this result using a leaning calculation requires specification of the cause-effect assignment  $\{C, E\} = \{\mathbf{X}, \mathbf{Y}\}$ . A cause must precede an effect in the cause-effect assignment for consistency with the intuitive definition of causality. It follows that a natural assignment may be  $\{C, E\} = \{x_{t-l}, y_t\}$  where  $l \in [1, 9]$ . This cause-effect assignment will be referred to as the  $l$ -standard assignment.

### 4.1 Defining the pendants

Given  $\bar{\mathbf{T}}$ , one possible pendant that can be defined using the 1-standard assignment is

$$\begin{aligned}\rho_{y_t=1, x_{t-1}=1} &= \kappa \left( 1 + \frac{P(x_{t-1}=1)}{1 - P(x_{t-1}=1)} \right) \\ &\quad - \frac{P(y_t=1)}{1 - P(x_{t-1}=1)},\end{aligned}$$

with  $\kappa = P(y_t=1|x_{t-1}=1)$ . Another pendant defined using this assignment would be the coresponding term with  $\kappa = P(y_t=0|x_{t-1}=0)$ . These two pendants are called the *observed* pendants because  $\kappa$  can be found directly from the time series data.

Equations for the unobserved pendants corresponding to  $\kappa = P(y_t=0|x_{t-1}=1)$  and  $\kappa = P(y_t=1|x_{t-1}=0)$  can be written down. These pendants are defined, but in both cases

$\kappa = 0 \Rightarrow \rho_{y_t x_{t-1}} < 0$ . Thus unobserved pendants imply the effect,  $y_t = 0$  or  $1$  (for this toy model) is most likely not caused by the postulated cause,  $x_{t-1} = 1$  or  $0$ , respectively. Using these unobserved pendants to define leanings becomes a comparison of how unlikely postulated causes are to cause given effects. Such comparisons are not as easily interpreted in the intuitive framework of causality, and as such, are not explored as tools for causal inference in this article.

### 4.2 Finding the pendants from the data

The probabilities in the pendant calculations can be estimated from the time series data with frequency counts, e.g.

$$P(y_t=1|x_{t-1}=1) = \frac{n_{EC}}{n_C} = \frac{3}{3} = 1,$$

where  $n_{EC}$  is the number of times  $y_t = 1$  and  $x_{t-1} = 1$  appears in  $\bar{\mathbf{T}}$ , and  $n_C$  is related to the number of times the assumed cause,  $x_{t-1} = 1$ , has appeared in  $\bar{\mathbf{T}}$  and is defined in more detail below.

Estimating the other two probabilities in this pendant calculation using frequency counts from  $\bar{\mathbf{T}}$  is slightly more subtle. The underlying assumption that the assumed cause must precede the assumed effect must be considered when defining the frequency counts. This concern is addressed by shifting  $\mathbf{X}$  and  $\mathbf{Y}$  into  $\tilde{\mathbf{X}}$  and  $\tilde{\mathbf{Y}}$  such that, for any given  $t$ ,  $\tilde{\mathbf{X}}_t$  precedes  $\tilde{\mathbf{Y}}_t$ , and defining

$$P(y_t=1) = \frac{n_E}{L} = \frac{3}{9} \quad (7)$$

and

$$P(x_{t-1}=1) = \frac{n_C}{L} = \frac{3}{9}, \quad (8)$$

where  $n_C$  is the number of times  $\tilde{x}_t = 1$ ,  $n_E$  is the number of times  $\tilde{y}_t = 1$ , and  $L$  is the library length of  $\tilde{\mathbf{X}}$  and  $\tilde{\mathbf{Y}}$  (which are assumed

to be the same length). For this example, those subsets are

$$\begin{aligned}\tilde{\mathbf{X}} &= \{0, 0, 1, 0, 0, 1, 0, 0, 1\} \\ \tilde{\mathbf{Y}} &= \{0, 0, 1, 1, 0, 0, 1, 0, 0, 1\}\end{aligned}$$

which are both shorter than their counterparts above by a single value because the penchants are being calculated using the 1-standard cause-effect assignment. It follows that  $\tilde{x}_t = x_{t-1}$  and  $\tilde{y}_t = y_t$ .

### 4.3 Mean observed leaning for $\bar{\mathbf{T}}$

The two observed penchants in this example that assume  $\mathbf{X}$  causes  $\mathbf{Y}$  (i.e. using the 1-standard assignment) are found from the data to be

$$\rho_{y_t=1, x_{t-1}=1} = 1 \quad (9)$$

and

$$\rho_{y_t=0, x_{t-1}=0} = 1 \quad (10)$$

The complements of these observed penchants are found using the complementary 1-standard assignment of  $\{C, E\} = \{y_{t-1}, x_t\}$  and are found from the data to be

$$\rho_{x_t=1, y_{t-1}=0} = \frac{3}{7} \quad (11)$$

$$\rho_{x_t=0, y_{t-1}=1} = \frac{3}{7} \quad (12)$$

and

$$\rho_{x_t=0, y_{t-1}=0} = -\frac{3}{7} \quad (13)$$

The *mean observed penchant* is the algebraic mean of the observed penchants, i.e.

$$\begin{aligned}\langle \rho_{y_t, x_{t-1}} \rangle &= \frac{1}{2} (\rho_{y_t=1, x_{t-1}=1} + \rho_{y_t=0, x_{t-1}=0}) \\ &= 1\end{aligned}$$

and

$$\begin{aligned}\langle \rho_{x_t, y_{t-1}} \rangle &= \frac{1}{3} (\rho_{x_t=1, y_{t-1}=0} \\ &\quad + \rho_{x_t=0, y_{t-1}=1} + \rho_{x_t=0, y_{t-1}=0}) \\ &= \frac{1}{7}.\end{aligned}$$

The *mean observed leaning* follows from the definition of the mean observed penchants as

$$\langle \lambda_{y_t, x_{t-1}} \rangle = \langle \rho_{y_t, x_{t-1}} \rangle - \langle \rho_{x_t, y_{t-1}} \rangle \quad (14)$$

$$= \frac{6}{7} \quad (15)$$

The positive leaning implies the probability that  $x_{t-1}$  drives  $y_t$  is higher than the probability that  $y_{t-1}$  drives  $x_t$ ; i.e.  $\mathbf{X} \xrightarrow{\text{lean}} \mathbf{Y}$  given the 1-standard cause-effect assignment. This result is expected and agrees with the intuitive definition of causality in this example.

### 4.4 Unobserved penchants

The *unobserved* penchants (using the 1-standard assignment from the beginning of the subsection) for this example are

$$\rho_{y_t=1, x_{t-1}=0} = -1 \quad (16)$$

$$\rho_{y_t=0, x_{t-1}=1} = -1 \quad (17)$$

and their complements are

$$\rho_{x_t=1, y_{t-1}=1} = -\frac{3}{7} \quad (18)$$

These values can be incorporated into the averaging calculation to yield a *mean total penchant*; i.e.

$$\begin{aligned}\langle \langle \rho_{y_t, x_{t-1}} \rangle \rangle &= \frac{1}{4} (\rho_{y_t=1, x_{t-1}=1} + \rho_{y_t=0, x_{t-1}=0} \\ &\quad + \rho_{y_t=1, x_{t-1}=0} + \rho_{y_t=0, x_{t-1}=1}) \\ &= 0\end{aligned}$$

and

$$\begin{aligned}\langle \langle \rho_{x_t, y_{t-1}} \rangle \rangle &= \frac{1}{4} (\rho_{x_t=1, y_{t-1}=1} + \rho_{x_t=0, y_{t-1}=0} \\ &\quad + \rho_{x_t=1, y_{t-1}=0} + \rho_{x_t=0, y_{t-1}=1}) \\ &= 0.\end{aligned}$$

Thus, the *mean total leaning* (defined analogous to Eqn. 14) would be  $\langle \langle \lambda_{y_t, x_{t-1}} \rangle \rangle = \langle \langle \rho_{y_t, x_{t-1}} \rangle \rangle - \langle \langle \rho_{x_t, y_{t-1}} \rangle \rangle = 0$  and would not be useful for casual inference in this example.

#### 4.5 Cause-effect assignment independence

It may be argued that the causal inference above was a little disingenuous in that the assumed cause-effect relationship was known to be correct. It can be shown, however, that causal inference is independent of the assumed cause-effect relationship. For example, consider the 1-standard cause-effect assignment  $\{C, E\} = \{y_{t-l}, x_t\}$ . The mean observed leaning would be

$$\begin{aligned} \langle \lambda_{x_t, y_{t-1}} \rangle &= \langle \rho_{x_t, y_{t-1}} \rangle - \langle \rho_{y_t, x_{t-1}} \rangle \quad (19) \\ &= -\frac{6}{7}, \quad (20) \end{aligned}$$

which implies  $\mathbf{X} \xrightarrow{\text{lean}} \mathbf{Y}$ , as expected for this example.

In general,  $\lambda_{AB} := \rho_{AB} - \rho_{BA} \Rightarrow -\lambda_{AB} = \rho_{BA} - \rho_{AB} := \lambda_{BA}$ . Thus, the causal inference is independent of which times series is initially assumed to be the cause (or effect).

#### 4.6 Weighted Mean Observed Leaning

The *weighted mean observed penchant* is defined similarly to the mean observed penchant but each term is weighted by the number of times that penchant appears in the data; e.g.

$$\begin{aligned} \langle \rho_{y_t, x_{t-1}} \rangle_w &= \frac{1}{L-l} (n_{y_t=1, x_{t-1}=1} \rho_{y_t=1, x_{t-1}=1} \\ &\quad + n_{y_t=0, x_{t-1}=0} \rho_{y_t=0, x_{t-1}=0}) \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} \langle \rho_{x_t, y_{t-1}} \rangle_w &= \frac{1}{L-l} (n_{x_t=1, y_{t-1}=0} \rho_{x_t=1, y_{t-1}=0} \\ &\quad + n_{x_t=0, y_{t-1}=1} \rho_{x_t=0, y_{t-1}=1} \\ &\quad + n_{x_t=0, y_{t-1}=0} \rho_{x_t=0, y_{t-1}=0}) \\ &= \frac{3}{63}, \end{aligned}$$

where  $n_{ab}$  is the number of times the assumed cause  $a$  appears with the assumed effect  $b$  in the data,  $L$  is the library length of the times series data, and  $l$  is the lag used in the  $l$ -standard cause-effect assignment under which these penchants are being calculated.

The *weighted mean observed leaning* follows naturally as

$$\begin{aligned} \langle \lambda_{y_t, x_{t-1}} \rangle_w &= \langle \rho_{y_t, x_{t-1}} \rangle_w - \langle \rho_{x_t, y_{t-1}} \rangle_w \\ &= \frac{60}{63}. \end{aligned}$$

For this example,  $\langle \lambda_{y_t, x_{t-1}} \rangle_w \Rightarrow \mathbf{X} \xrightarrow{\text{lean}} \mathbf{Y}$  as expected.

Conceptually, the weighted mean observed penchant is preferred to the mean penchant because it accounts for the frequency of observed cause-effect pairs within the data, which is assumed to be a predictor of causal influence. For example, given some pair  $\{\mathbf{A}, \mathbf{B}\}$ , if it is known that  $a_{t-1}$  causes  $b_t$  and both  $b_t = 0 | a_{t-1} = 0$  and  $b_t = 0 | a_{t-1} = 1$  are observed in the data, then comparison of the frequencies with which the pair occur would be used to determine which of the two pairs represents the true cause-effect relationship and which pair represents, e.g., the effects of noise in the system.

For this example, the weighted mean observed leaning provides the same causal inference as the mean observed leaning. The weighted mean calculation, however, can be made computationally less expensive than the mean calculation, which will be of practical concern for empirical data sets.

#### 4.7 Noise tolerances

If the example time series was the expected time series but the data points were measured in the presence of noise, then the result may be

the noisy time series pair  $\bar{\mathbf{T}}' = \{\mathbf{X}', \mathbf{Y}'\}$  with

$$\begin{aligned}\mathbf{X}' &= \{x'_t \mid t \in [0, 9]\} \\ &= \{0, 0, 1.1, 0, 0, 1, -0.1, 0, 0.9, 0\} \\ \mathbf{Y}' &= \{y'_t \mid t \in [0, 9]\} \\ &= \{0, -0.2, 0.1, 1.2, 0, 0.1, 0.9, -0.1, 0, 1\}.\end{aligned}$$

The previous time series pair,  $\bar{\mathbf{T}}$  had only five observed penchants, but  $\bar{\mathbf{T}}'$  necessarily has more due to the noise. It can be seen in the time series definitions that  $x'_t = x_t \pm 0.1 := x_t \pm \delta_x$  and  $y'_t = x_t \pm 0.2 := x_t \pm \delta_y$ . The weighted mean observed leaning for  $\bar{\mathbf{T}}'$  is  $\langle \lambda_{y'_t, x'_{t-1}} \rangle_w \approx 0.19$ .

The effects of noise on the leaning calculations can be addressed by using the tolerances  $\delta_x$  and  $\delta_y$  in the probability estimations from the data. For example, the penchant calculation in Eqn. 9 relied on estimating  $P(y_t = 1 | x_{t-1} = 1)$  from the data, but if, instead, the data is known to be noisy, then the relevant probability estimate may be  $P(y_t \in [1 - \delta_y, 1 + \delta_y] | x_{t-1} \in [1 - \delta_x, 1 + \delta_x])$ .

If the tolerances,  $\delta_x$  and  $\delta_y$ , are made large enough, then the noisy system (i.e.  $\bar{\mathbf{T}}'$ ) weighted mean observed leaning,  $\langle \lambda_{y'_t \pm \delta_y, x'_{t-1} \pm \delta_x} \rangle_w$ , can, at least in the simple examples considered here, be made equal to the noiseless system (i.e.  $\bar{\mathbf{T}}$ ) weighted mean observed leaning, i.e.  $\langle \lambda_{y'_t \pm \delta_y, x'_{t-1} \pm \delta_x} \rangle_w = \langle \lambda_{y_t, x_{t-1}} \rangle_w$ .

The noise in this example did not affect the causal inference, i.e.  $\langle \lambda_{y'_t, x'_{t-1}} \rangle_w \Rightarrow \mathbf{X} \xrightarrow{\text{lean}} \mathbf{Y}$  as expected. However, it will be shown in the following sections that noise in the time series can lead to counterintuitive causal inferences. Thus, the tolerances described here will be used with all leaning calculations with the tolerances set to the estimated (or known if possible) noise levels for the individual time series.

## 4.8 Stationarity Dependence

## 5 Simple Example Systems

### 5.1 Impulse with Noisy Response Linear Example

Consider the linear example dynamical system of

$$X_t = \{0, 2, 0, 0, 2, 0, 0, 2, 0, 0\} \quad (21)$$

$$Y_t = X_{t-1} + B\eta_t, \quad (22)$$

with  $B \in \mathbb{R} \geq 0$  and  $\eta_t \sim \mathcal{N}(0, 1)$ . Specifically, consider  $B \in [0, 2]$  in increments of 0.02. The response system  $Y$  is just a lagged version of the driving signal with varying levels of standard Gaussian noise applied at each time step.

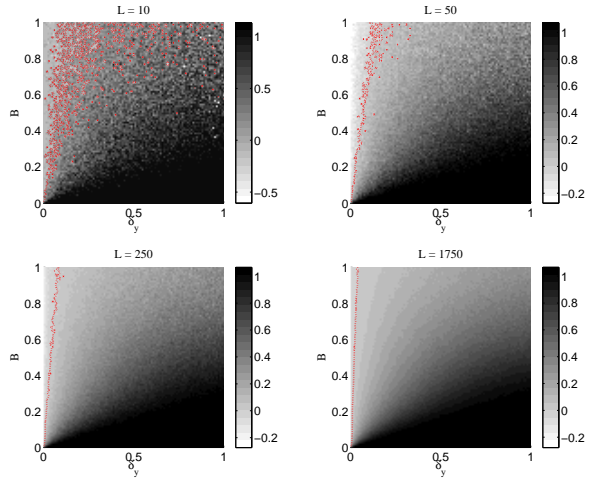


Figure 1: (Color available online.) Leaning as a function of both the noise and the y-tolerance for different library lengths. The red dashed line is the zero contour. See the text for an explanation of the missing data for large  $\delta_y$ .

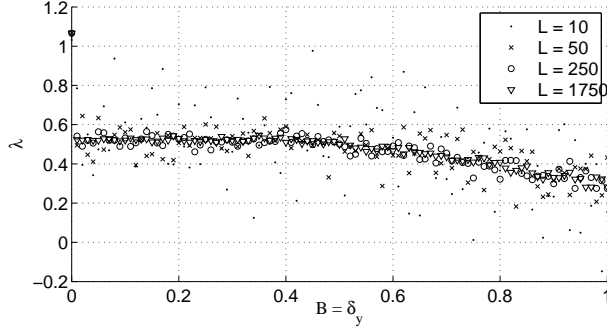


Figure 2: (Color available online.) The leaning agrees with intuition for most noise levels when the y-tolerance is set to the noise level (i.e.  $B = \delta_y$ ).

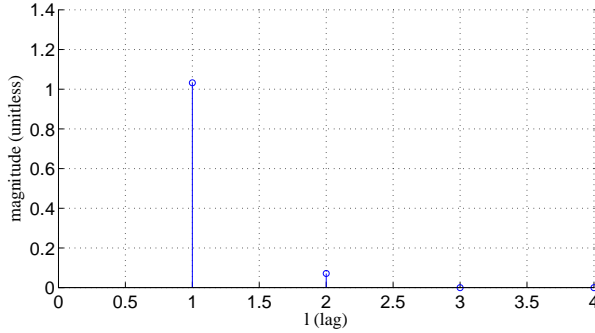


Figure 3: (Color available online.) Different  $l$ -standard cause-effect assignments lead to different leanings.

## 5.2 Cyclic Linear Example

Consider the linear example dynamical system of

$$X_t = \sin(t) \quad (23)$$

$$Y_t = X_{t-1} + B\eta_t, \quad (24)$$

with  $B \in \mathbb{R} \geq 0$  and  $\eta_t \sim \mathcal{N}(0, 1)$ . Specifically, consider  $B \in [0, 2]$  in increments of 0.02. The response system  $Y$  is just a lagged version of the driving signal with varying levels of standard Gaussian noise applied at each time step.

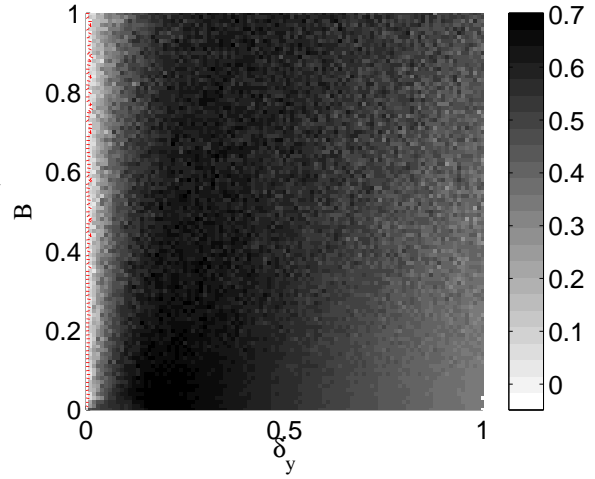


Figure 4: (Color available online.) Leaning as a function of both the noise and the y-tolerance. The red dashed line is the zero contour. See the text for an explanation of the missing data for large  $\delta_y$ .

## 5.3 Non-Linear Example

Consider the non-linear dynamical system of

$$X_t = \sin(t) \quad (25)$$

$$Y_t = AX_{t-1}(1 - BX_{t-1}) + C\eta_t, \quad (26)$$

with  $A, B, C \in \mathbb{R} \geq 0$  and  $\eta_t \sim \mathcal{N}(0, 1)$ . Specifically, consider  $A, B, C \in [0, 5]$  in increments of 0.5.

## 5.4 RL Circuit Example

Both of the previous examples included a noise term,  $\eta_t$ . Consider a series circuit containing a resistor, inductor, and time varying voltage source related by

$$\frac{dI}{dt} = \frac{V(t)}{L} - \frac{R}{L}I, \quad (27)$$

where  $I$  is the current at time  $t$ ,  $V(t) = \sin(\Omega t)$  is the voltage at time  $t$ ,  $R$  is the resistance, and  $L$  is the inductance. Eqn. 27 was solved

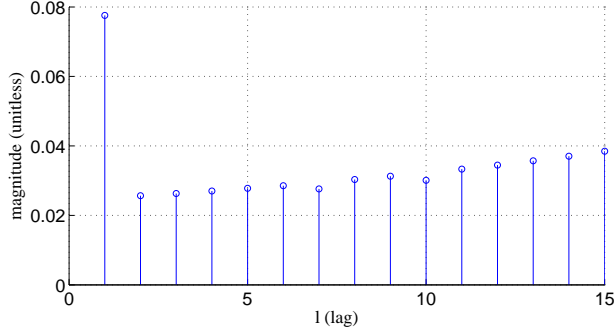


Figure 5: (Color available online.) Different  $l$ -standard cause-effect assignments lead to different leanings.

using the *ode45* integration function in MATLAB. The time series  $V(t)$  is created by defining values at fixed points and using linear interpolation to find the time steps required by the ODE solver.

Consider the situation where  $L = 10$  Henries and  $R = 5$  Ohms are constant. Physical intuition is that  $V$  drives  $I$ , and so we expect to find that  $V$  CCM causes  $I$  (i.e.,  $C_{VI} > C_{IV}$  or  $\Delta = C_{VI} - C_{IV} > 0$ ).

## 6 Empirical Data

## 7 Conclusion

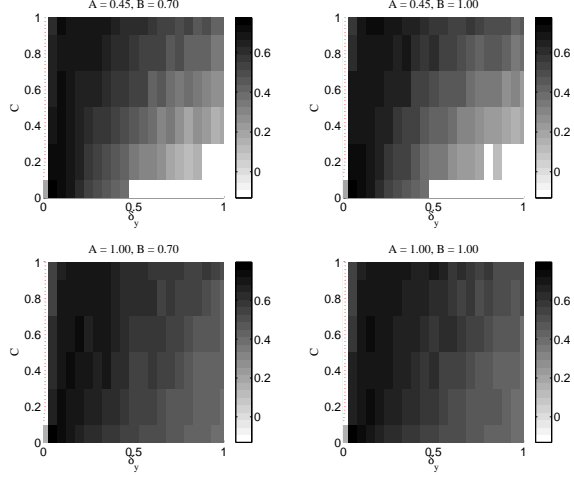


Figure 6: (Color available online.) Leaning as a function of both the noise and the  $y$ -tolerance for values of  $A$  and  $B$ . The red dashed line is the zero contour. See the text for an explanation of the missing data for large  $\delta_y$ .

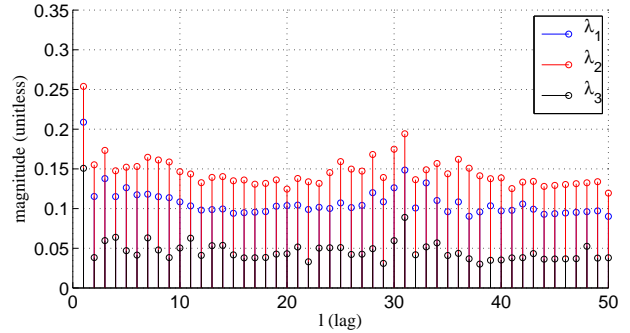


Figure 7: (Color available online.) Different  $l$ -standard cause-effect assignments lead to different leanings.