

How is CCM useful?

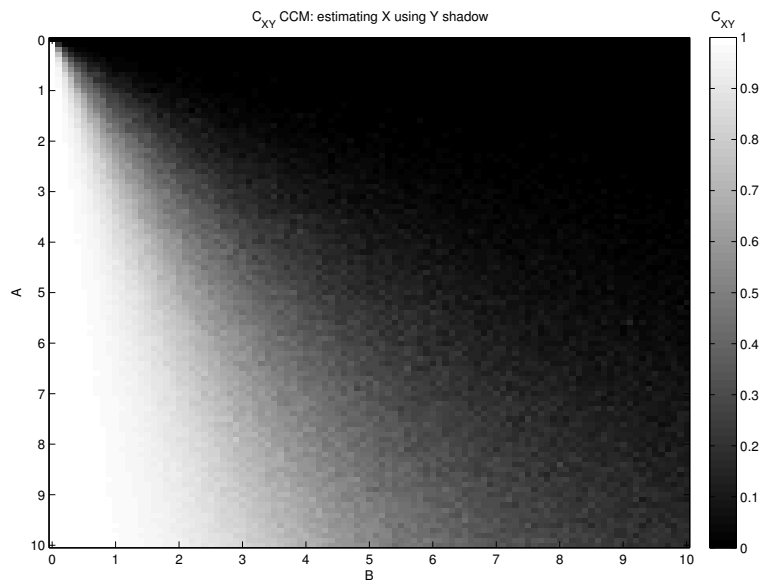
1 Linear Example

Consider the linear example dynamical system of

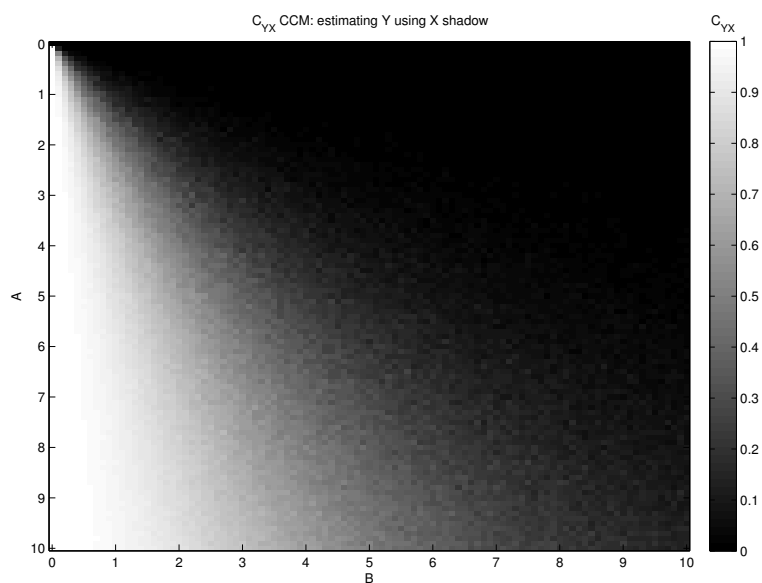
$$X_t = \sin(t) \quad (1)$$

$$Y_t = AX_{t-1} + B\eta_t, \quad (2)$$

with $A, B \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0, 1)$. Specifically, consider $A, B \in [0, 10]$ step through in increments of 0.1. Figure 1 shows C_{XY} and C_{YX} .



(a) C_{XY}



(b) C_{YX}

Figure 1: Changing A and B . C_{XY} and C_{YX}

Figure 3 shows Δ for this example.

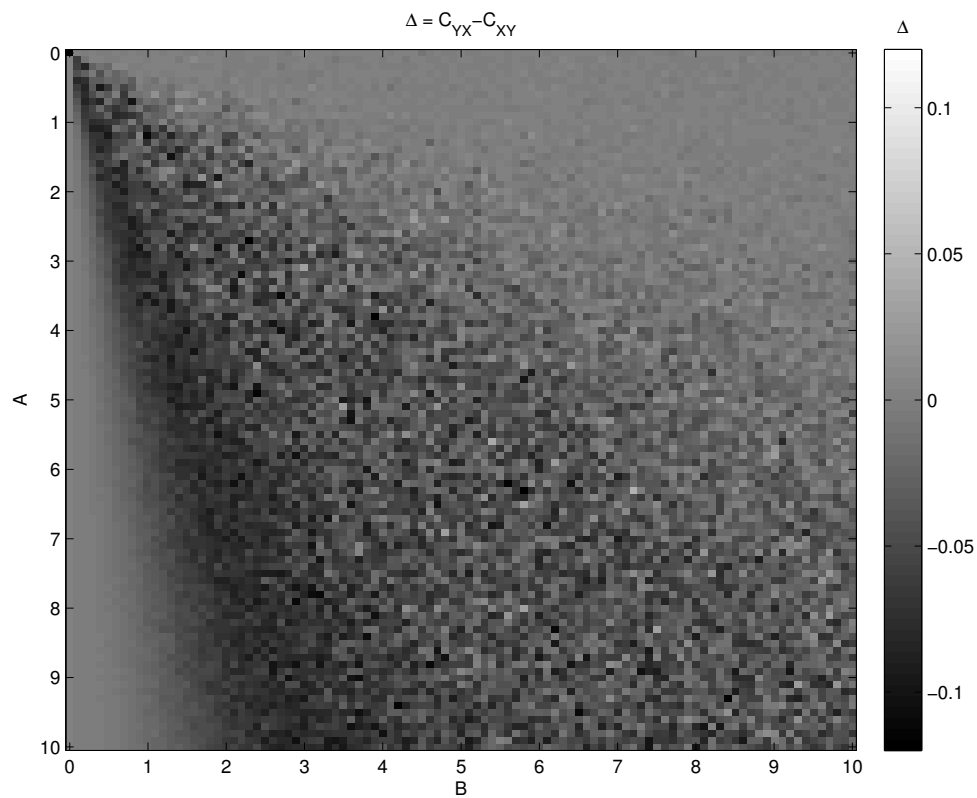


Figure 2: Changing A and B . Δ

Consider the convergence of two specific points in the above plots $(A, B) = (2.6, 2.6)$ and $(A, B) = (3.0, 2.6)$.

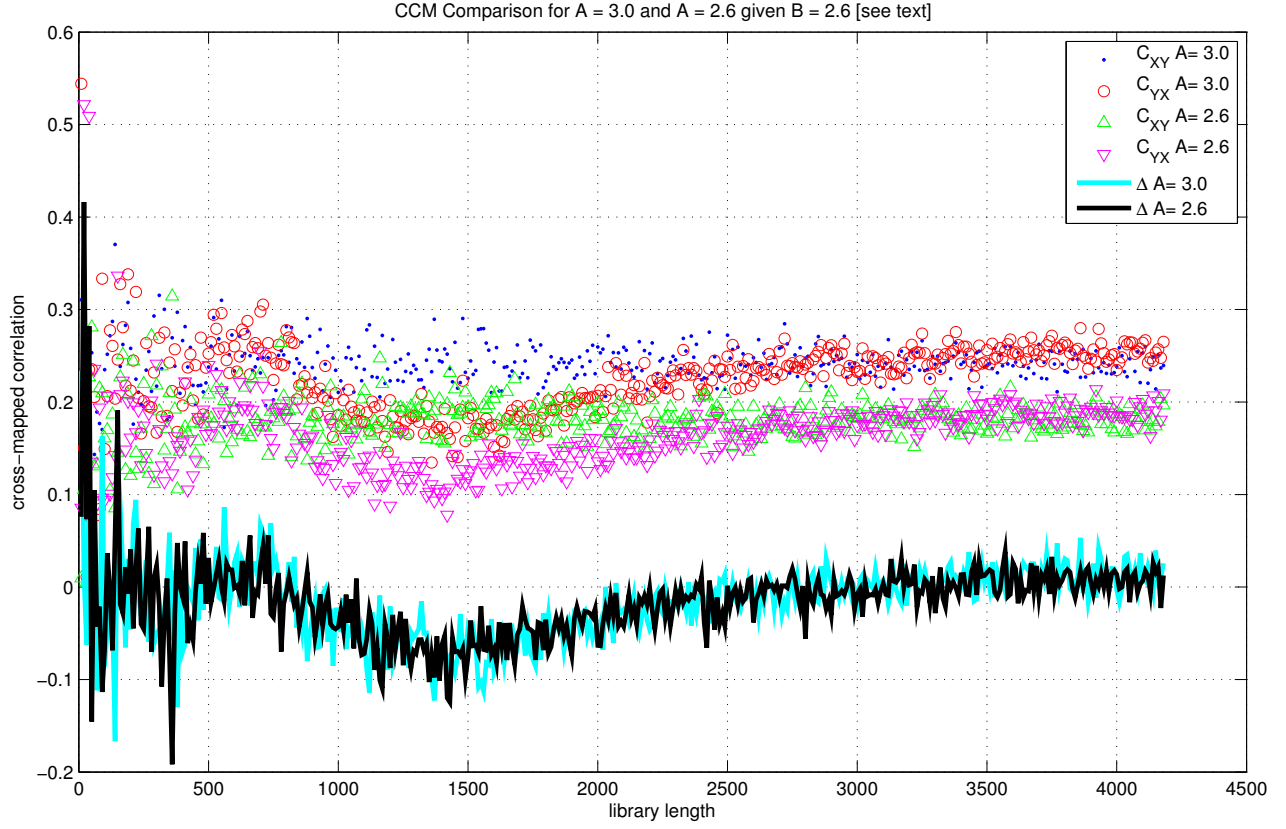


Figure 3:

2 RL Circuit Example

The continuous system is

$$\frac{dI}{dt} = \frac{V(t)}{L} - \frac{R(t)}{L}I, \quad (3)$$

where I is the current at time t , $V(t)$ is the voltage at time t , $R(t)$ is the resistance at time t , and L is the inductance (which is also constant in these examples), and it can be approximated as

$$\dot{I} = \frac{V(t)}{L} - \frac{R(t)}{L}I \Rightarrow I_{t+1} - I_t = \frac{V_t}{L} - \frac{R_t}{L}I_t. \quad (4)$$

Rearranging leads to

$$I_{t+1} = \frac{V_t}{L} + I_t \left(1 - \frac{R_t}{L}\right), \quad (5)$$

$$V_t = L \left(I_{t+1} - I_t \left(1 - \frac{R_t}{L}\right) \right), \quad (6)$$

and

$$R_t = L \left(I_t - I_{t+1} + \frac{V_t}{L} \right). \quad (7)$$

All of the plots of I seen below are produced by using MATLAB's *ode45* to solve Eqn. 3 (i.e. not using the discrete approximation shown). The time series $V(t)$ and $R(t)$ are created by defining values at fixed points and using linear interpolation (i.e. MATLAB's *interp1*) to find the time steps required by the ODE solver (i.e. MATLAB's *ode45*).

3 Changing $V(t)$

Consider the situation where $R(t)$ is constant.

Physical intuition is that V drives I , so we expect to find V CCM causes I ($C_{VI} > C_{IV}$).

For this example, the voltage is described by

$$V(t) = A_v \sin(f_v t + \phi_v) + O_v, \quad (8)$$

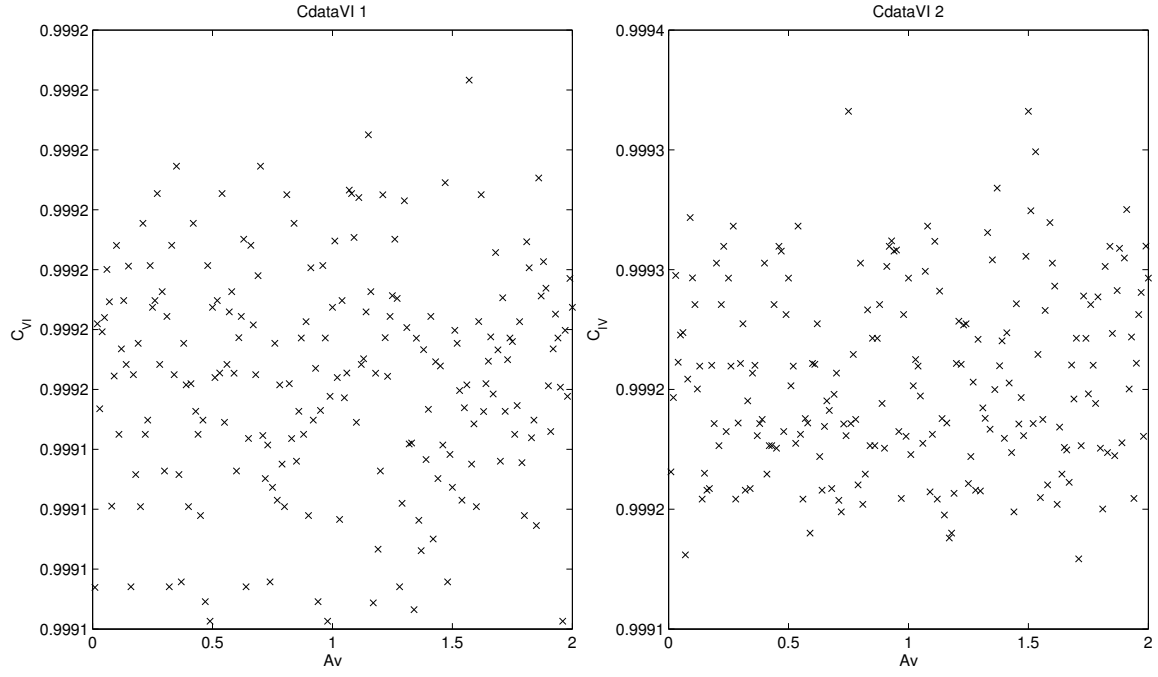
where A_v is the amplitude, f_v is the frequency, ϕ_v is the phase, and O_v is the offset voltage.

3.1 Changing A_v

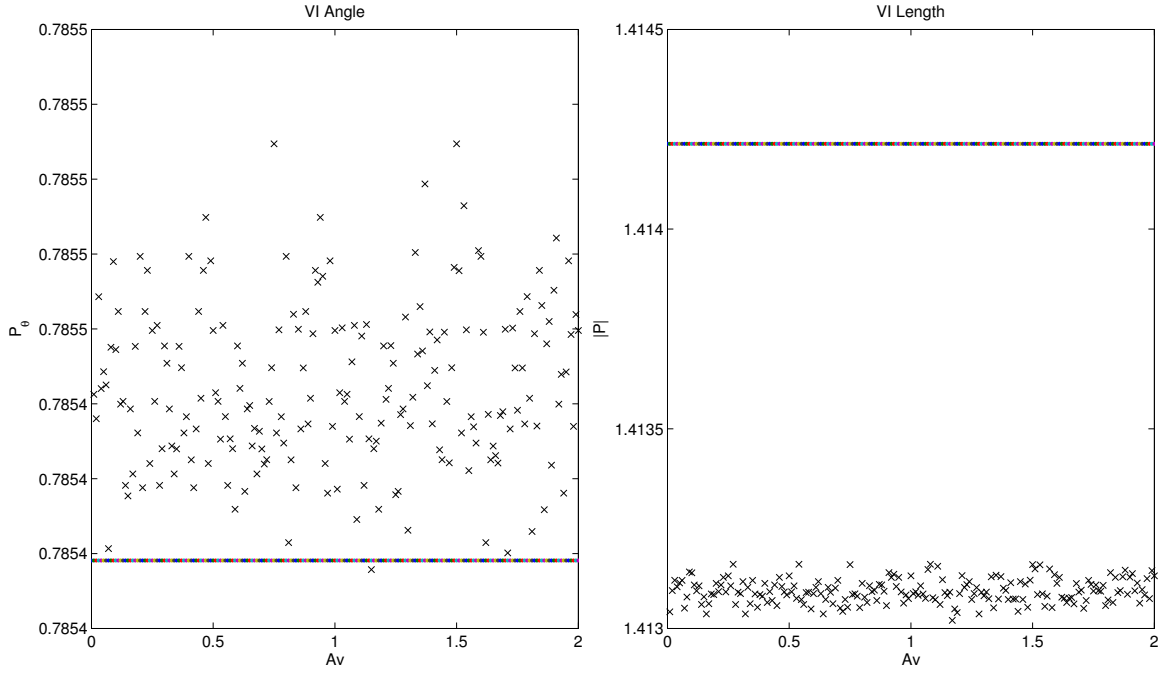
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $A_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both $V(t)$ and $I(t)$ are plotted for different A_v in Figure 4.

Figure 4: Reference plots for changing A_v .

The CCM correlations are each plotted in Figure 5 along with the corresponding PAI elements P_θ and $|P|$.



(a) C_{VI} and C_{IV}



(b) P_θ and $|P|$

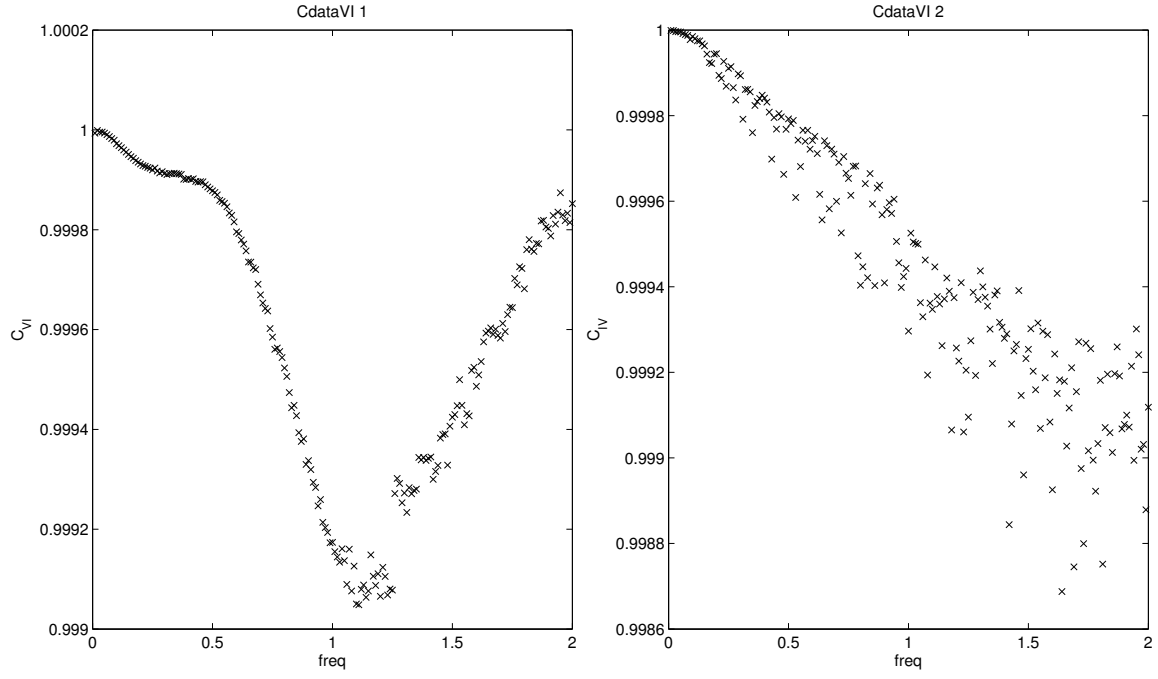
Figure 5: Changing A_v .

3.2 Changing f_v

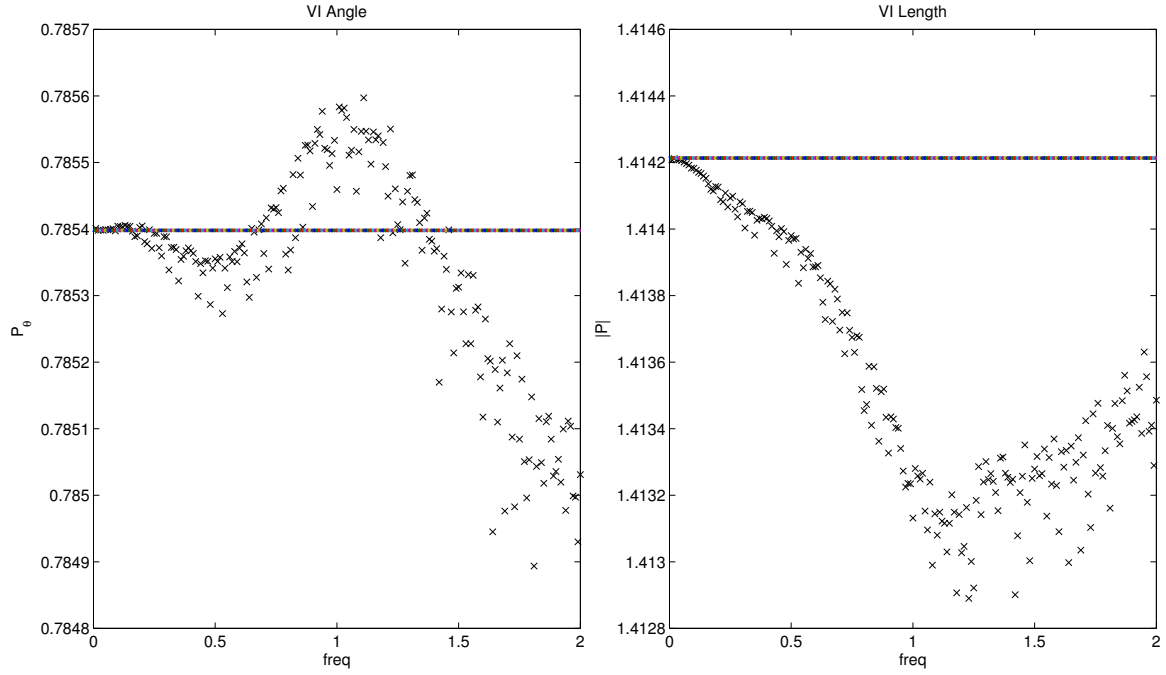
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $f_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both $V(t)$ and $I(t)$ are plotted for different f_v in Figure 6.

Figure 6: Reference plots for changing f_v .

The CCM correlations are each plotted in Figure 7 along with the corresponding PAI elements P_θ and $|P|$.



(a) C_{VI} and C_{IV}



(b) P_θ and $|P|$

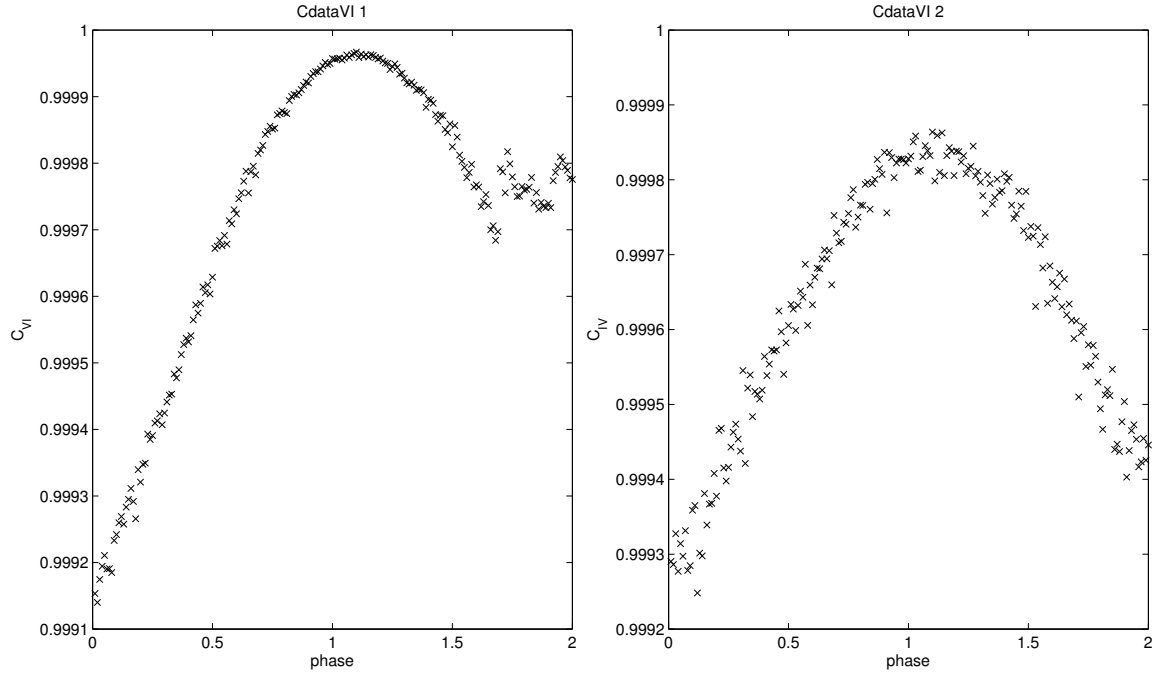
Figure 7: Changing f_v .

3.3 Changing ϕ_v

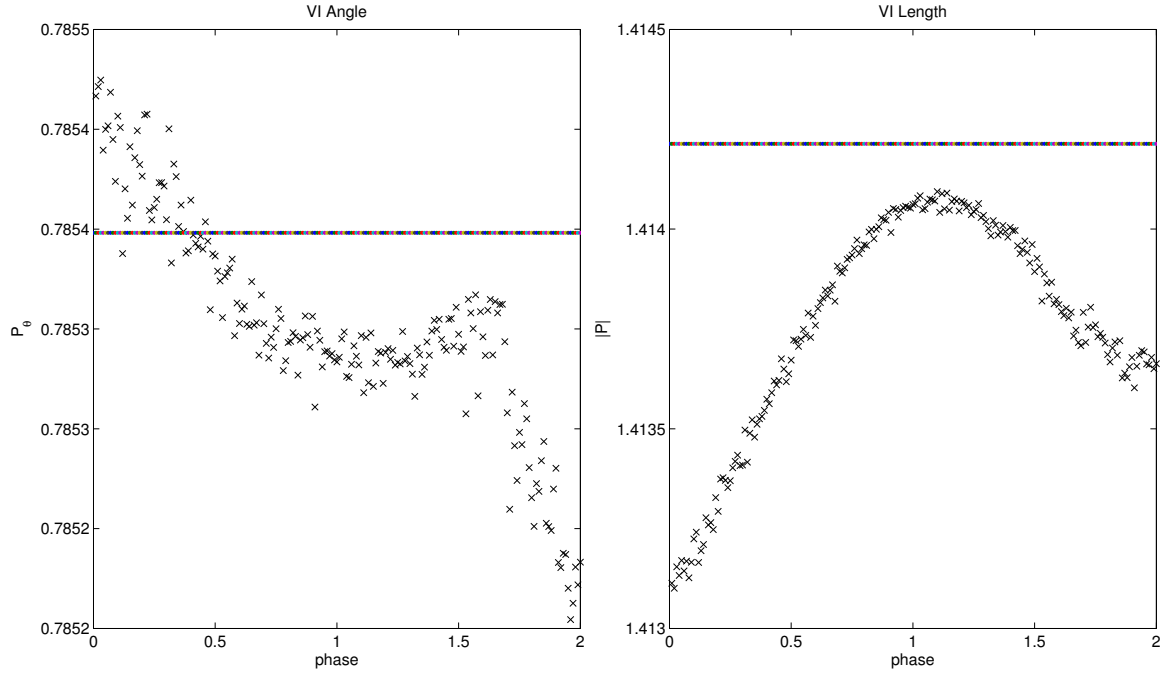
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $\phi_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both $V(t)$ and $I(t)$ are plotted for different ϕ_v in Figure 8.

Figure 8: Reference plots for changing ϕ_v .

The CCM correlations are each plotted in Figure 9 along with the corresponding PAI elements P_θ and $|P|$.



(a) C_{VI} and C_{IV}



(b) P_θ and $|P|$

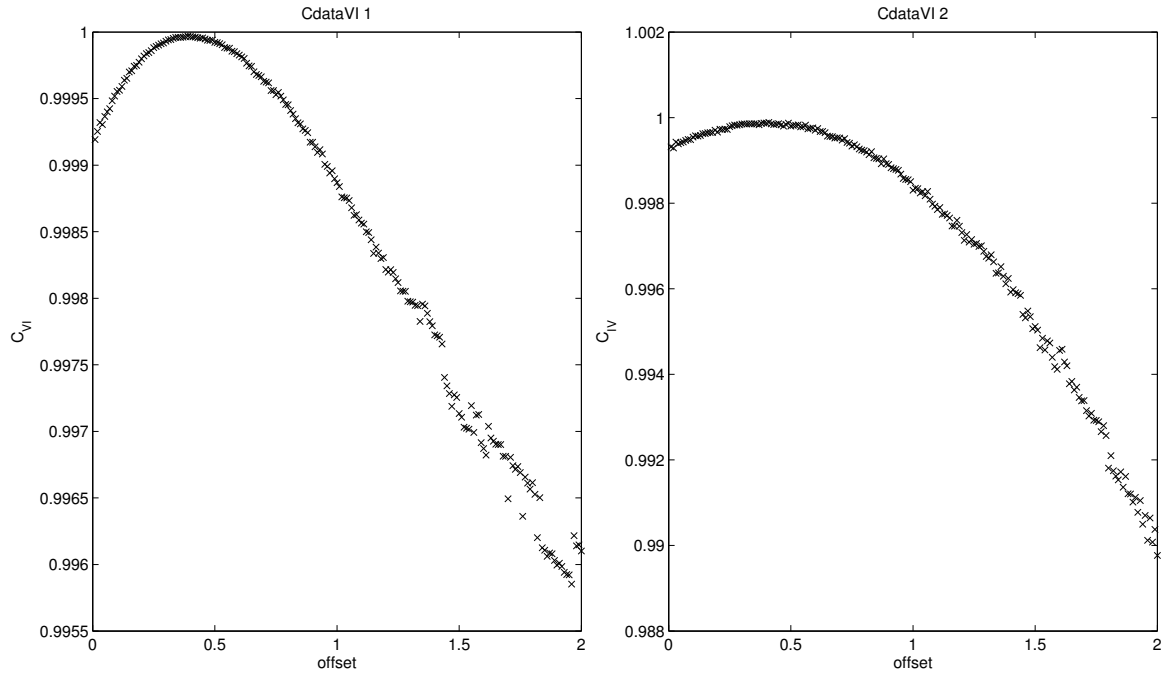
Figure 9: Changing ϕ_v .

3.4 Changing O_v

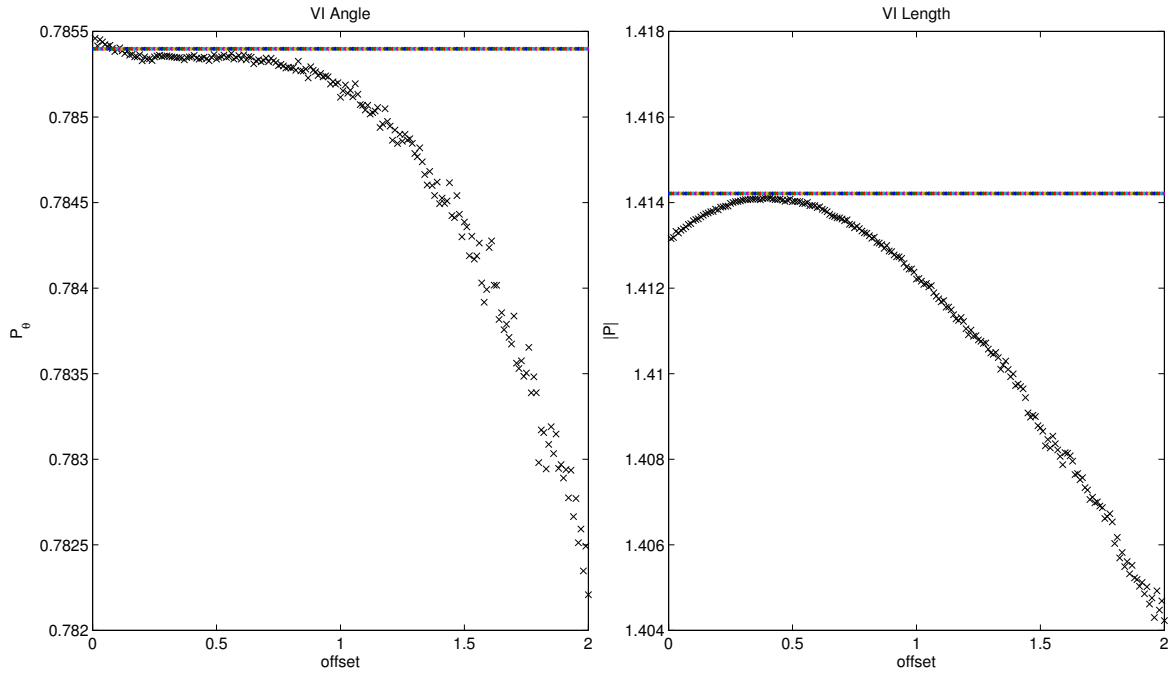
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $O_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both $V(t)$ and $I(t)$ are plotted for different O_v in Figure 10.

Figure 10: Reference plots for changing O_v .

The CCM correlations are each plotted in Figure 11 along with the corresponding PAI elements P_θ and $|P|$.



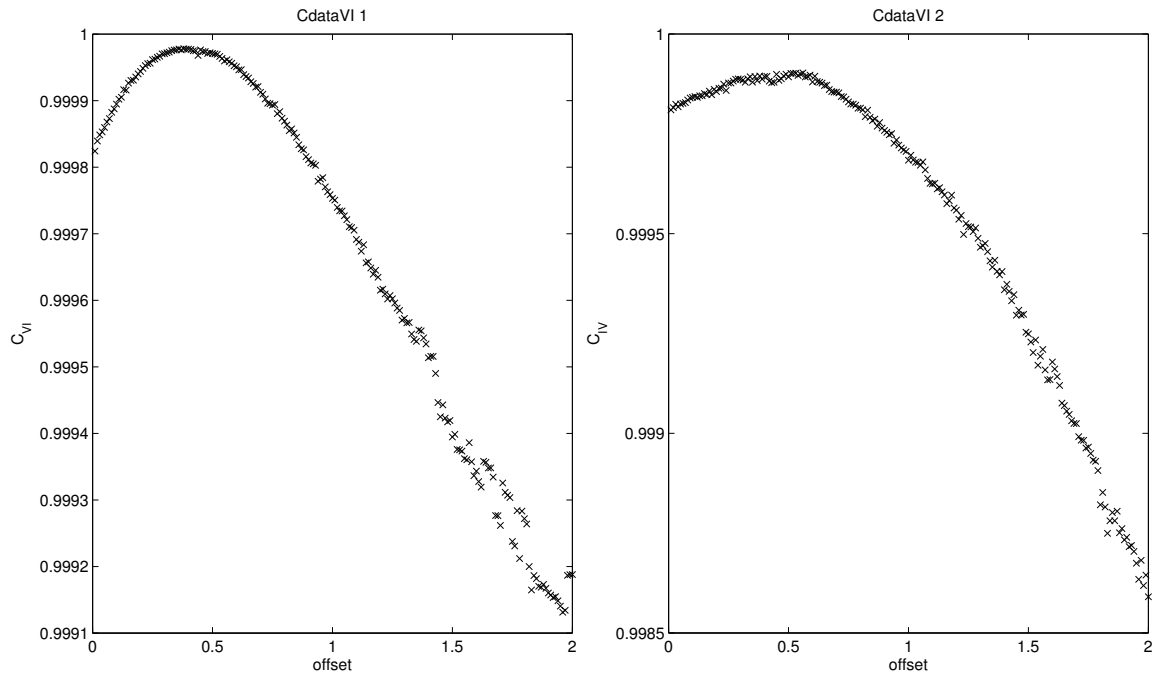
(a) C_{VI} and C_{IV}



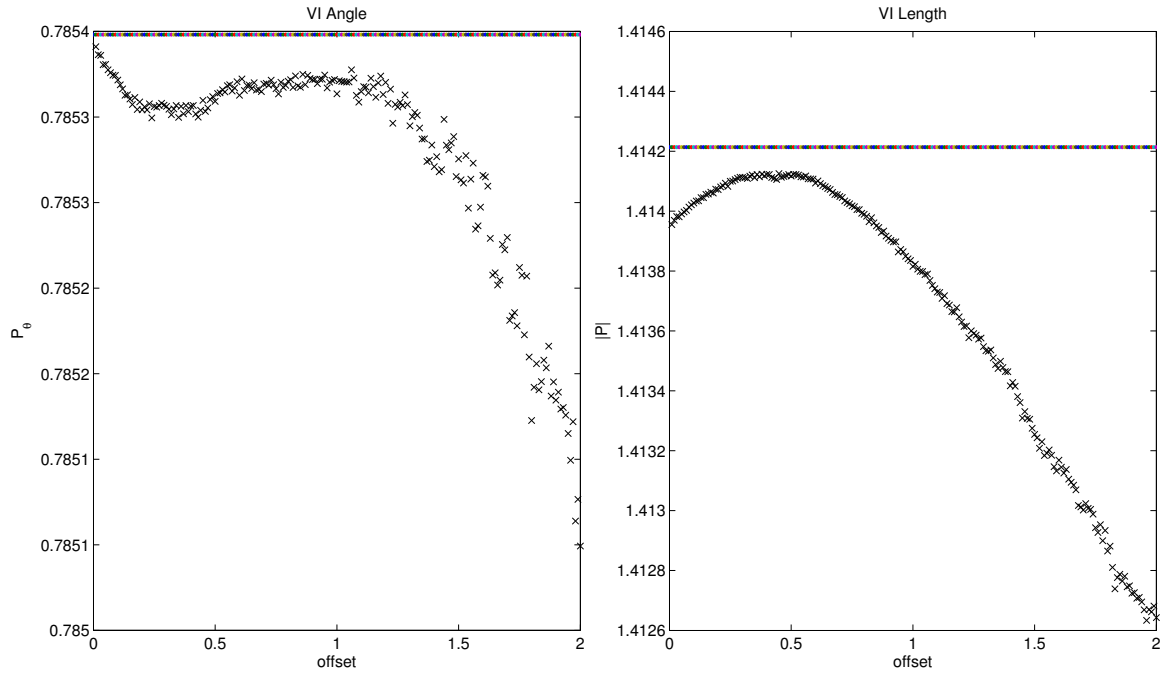
(b) P_θ and $|P|$

Figure 11: Changing O_v .

Figure 12 shows the effect of increasing the library length from 2×10^3 (i.e. `tspan = [0:0.5:1000];`) to 10^4 (i.e. `tspan = [0:0.5:5000];`), and Figure 13 extends the above plots to $O_v \in [0.01, 10.0]$ in steps of 0.05.

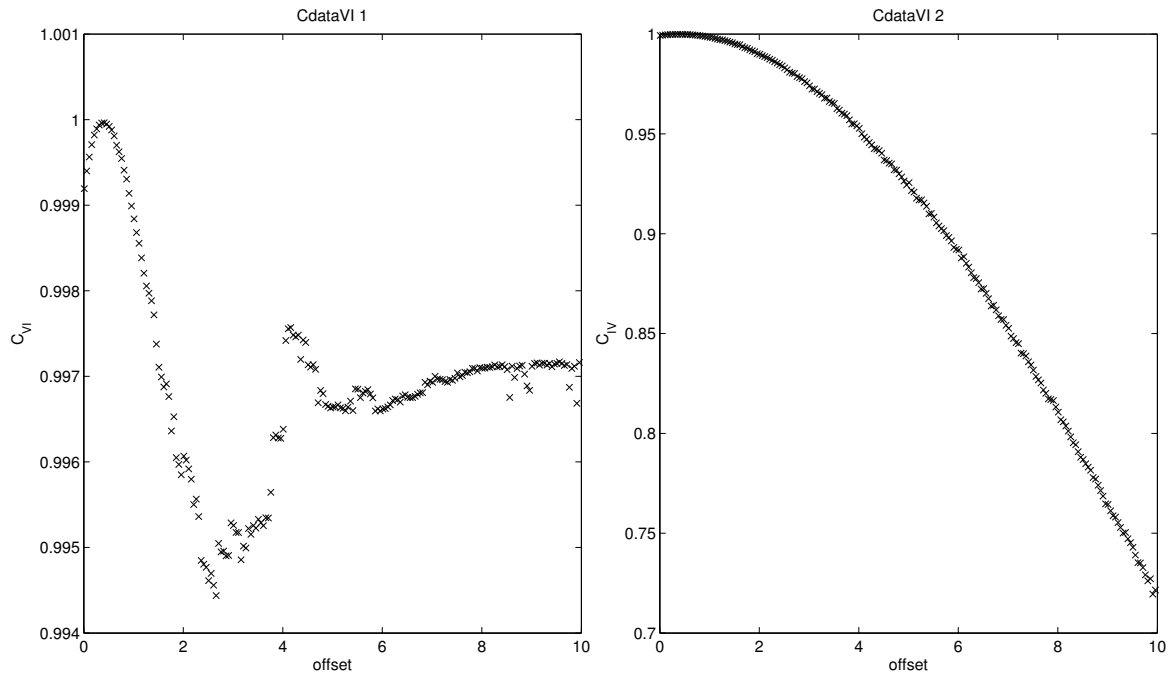


(a) C_{VI} and C_{IV}

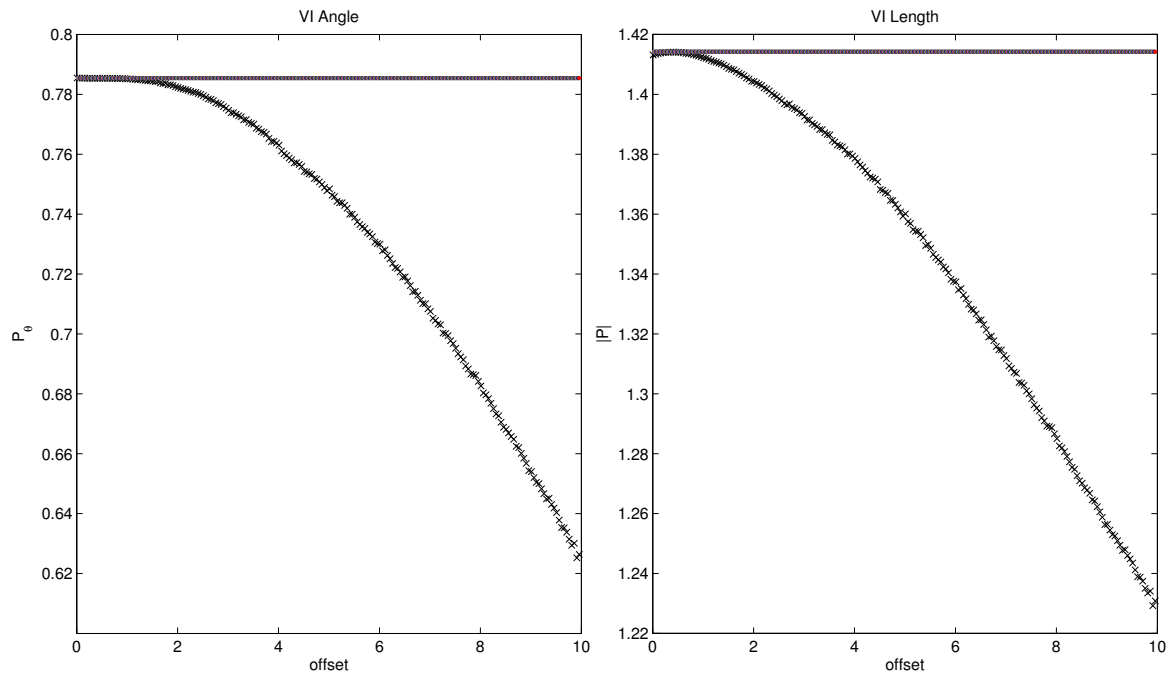


(b) P_θ and $|P|$

Figure 12: Changing O_v (longer library length).



(a) C_{VI} and C_{IV}



(c) P_θ and $|P|$

Figure 13: Changing O_v (larger domain for O_v).