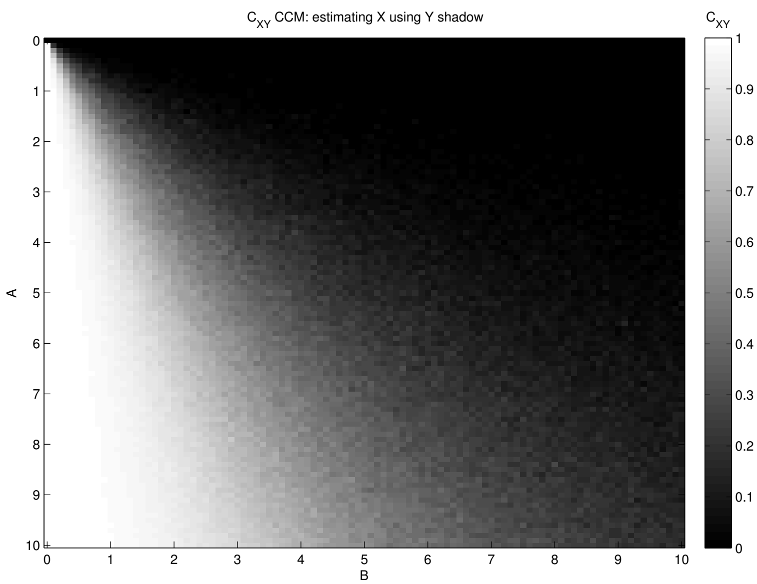
How is CCM useful?

1 Linear Example #1

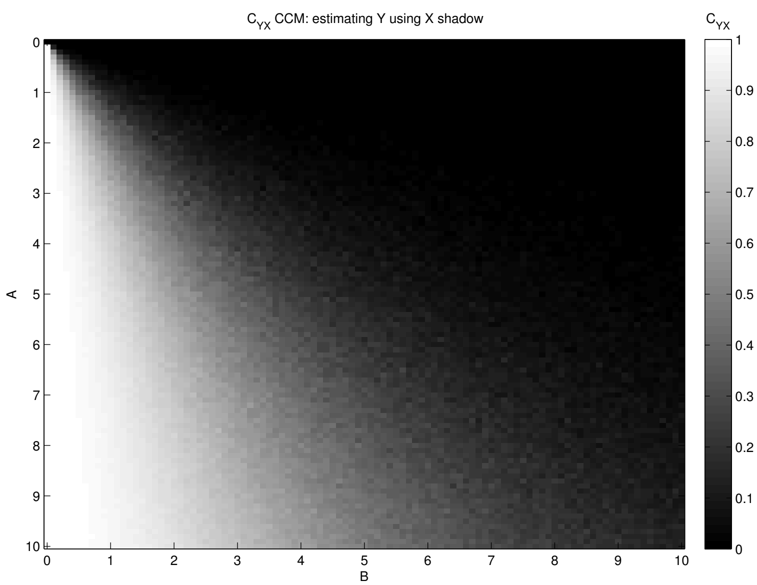
Consider the linear example dynamical system of

$$X_t = \sin(t) \tag{1}$$
$$Y_t = AX_{t-1} + B\eta_t, \tag{2}$$

with $A, B \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0,1)$. Specifically, consider $A, B \in [0, 10]$ step through in increments of 0.1. Figure 19 shows C_{XY} and CYX.



(a) C_{XY}



(b) C_{YX}

Figure 1: Changing A and B. C_{XY} and C_{YX}

Figure 5 shows Δ for this example.

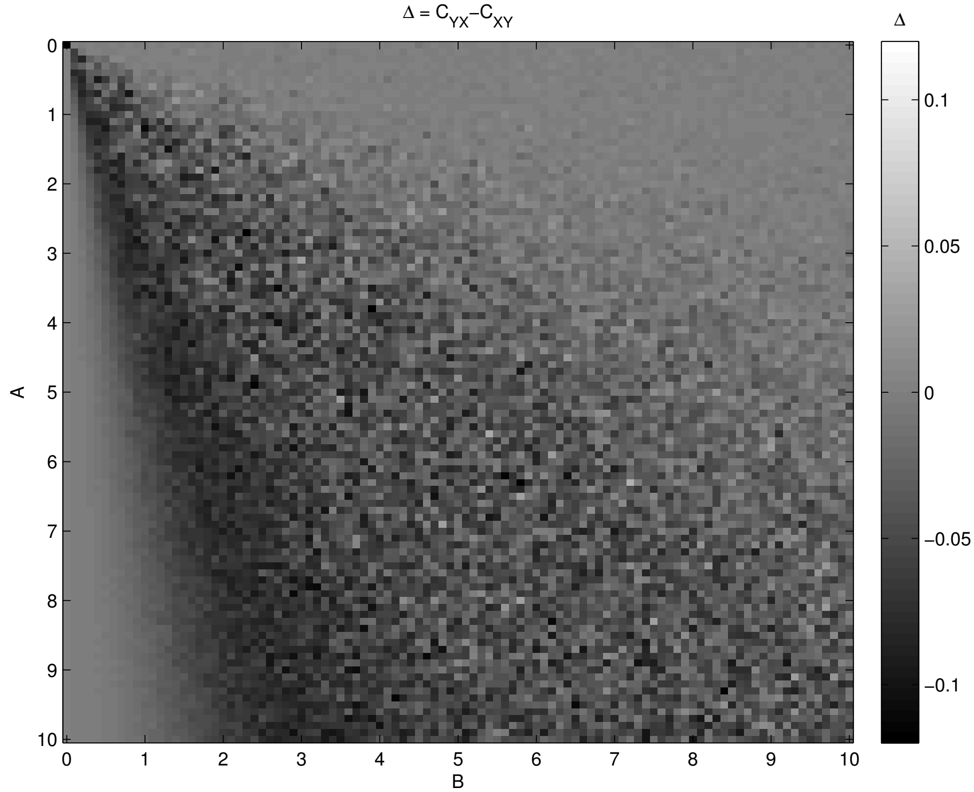


Figure 2: Changing A and B. Δ

Consider the convergence of two specific points in the above plots $(A, B) = (2.6, 2.6)$ and $(A, B) = (3.0, 2.6)$.

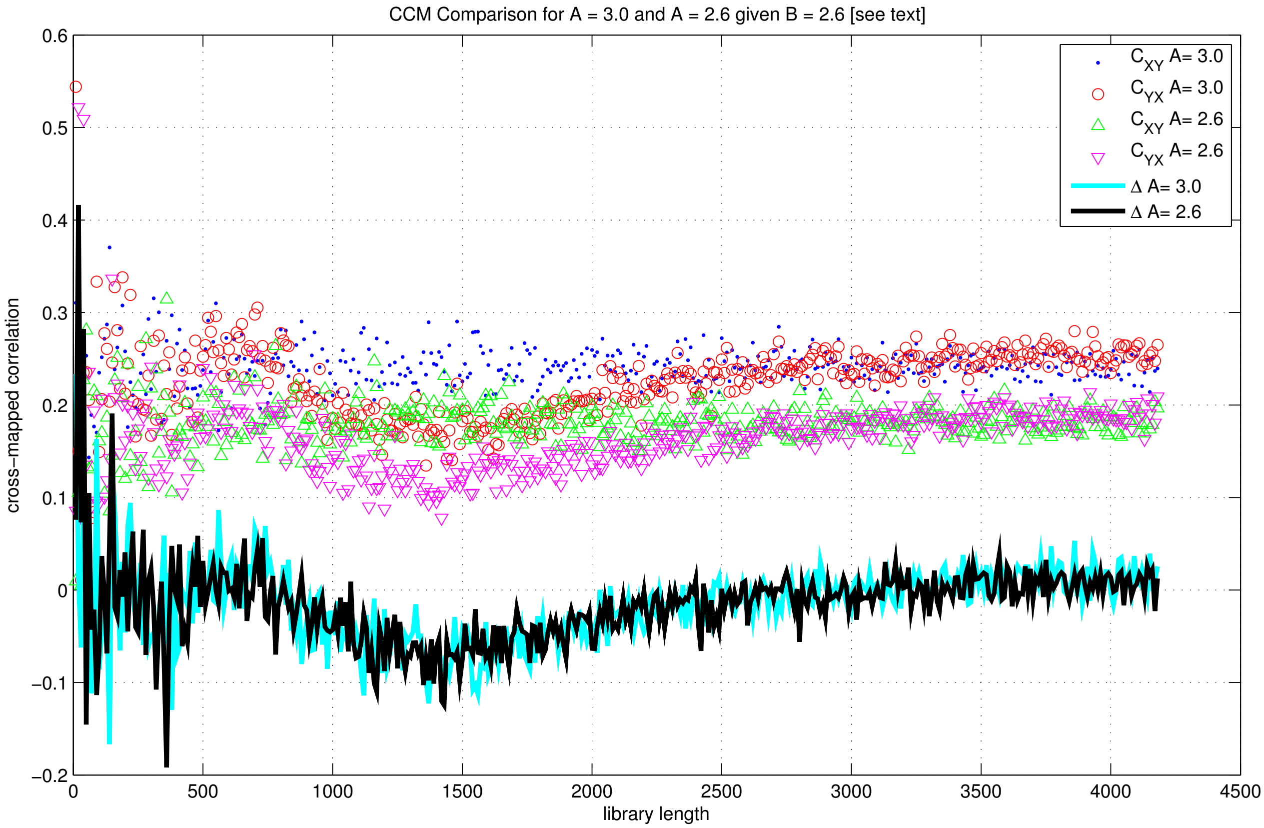


Figure 3:

2 Linear Example #2

Consider the linear example dynamical system of

$$X_t = \sin(t) \tag{3}$$
$$Y_t = AX_{t-1} + B\eta_t \tag{4}$$
$$Z_t = Y_{t-1}, \tag{5}$$

with $A, B \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0, 1)$. Specifically, consider $A, B \in [0, 5]$ step through in increments of 0.1.

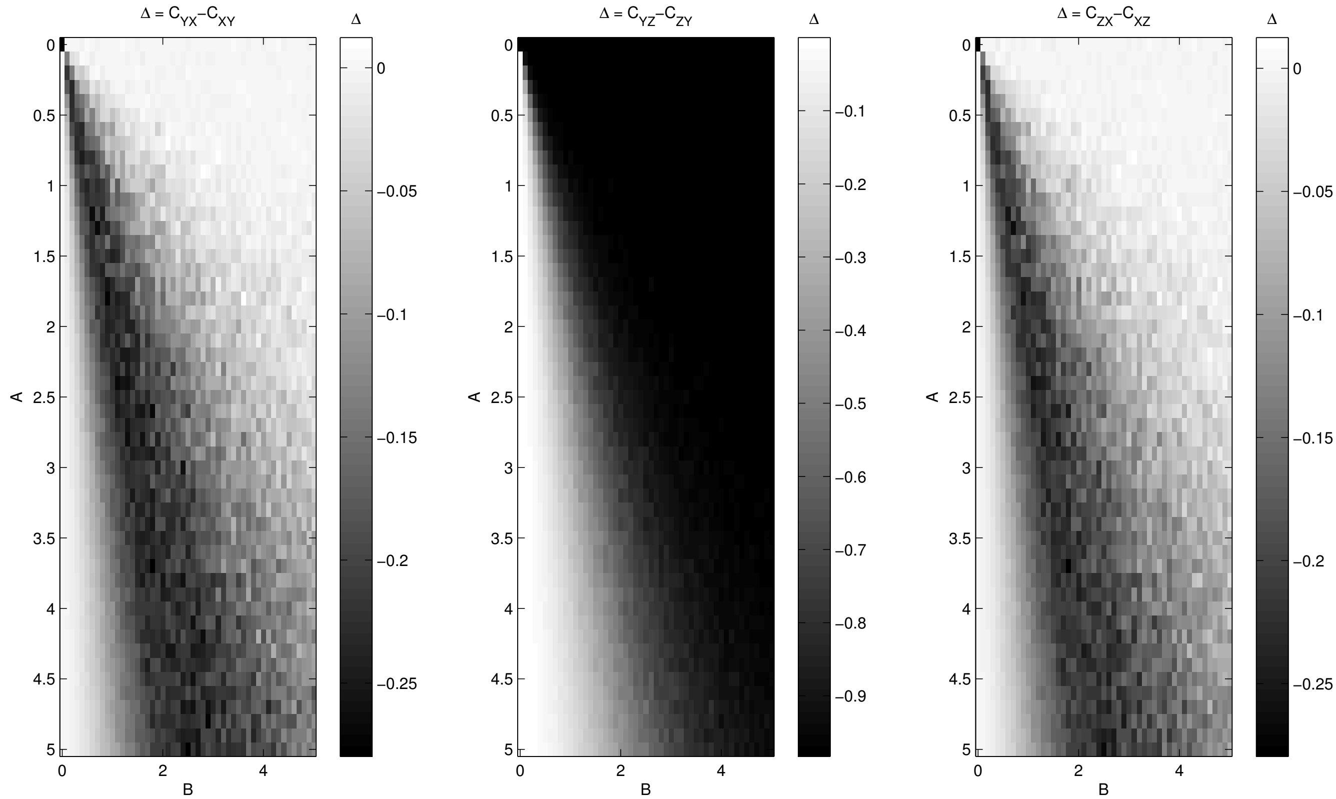


Figure 4:

3 Non-Linear Example

Consider the nonlinear example dynamical system of

$$X_t = \sin(t) \tag{6}$$

$$Y_t = AX_{t-1}\left(1 - BX_{t-1}\right) + C\eta_t, \tag{7}$$

with $A, B, C \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0, 1)$. Specifically, consider $A, B, C \in [0, 5]$ step through in increments of 0.5.

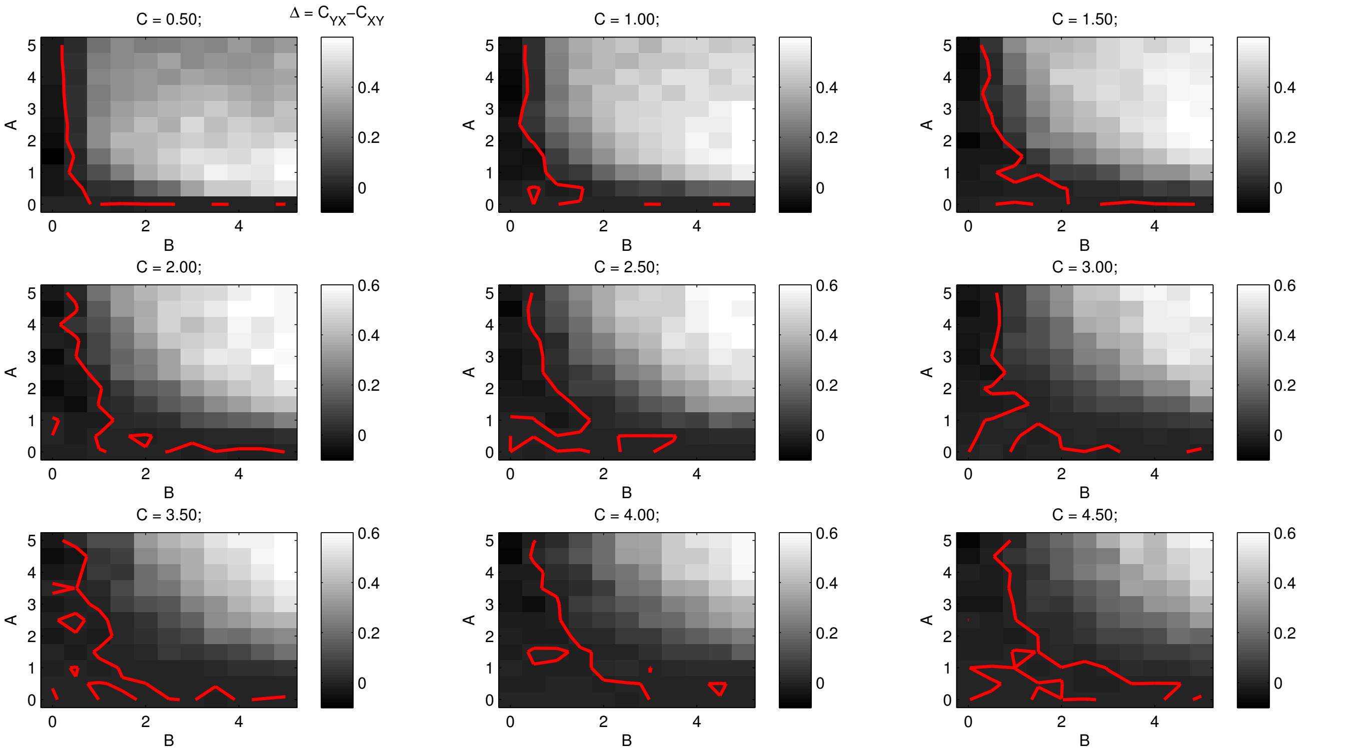


Figure 5:

—

4 RL Circuit Example

The continuous system is

$$\frac{dI}{dt} = \frac{V(t)}{L} - \frac{R(t)}{L} I, \tag{8}$$

where I is the current at time t, $V(t)$ is the voltage at time t, $R(t)$ is the resistance at time t, and L is the inductance (which is also constant in these examples), and it can be approximated as

$$\dot{I} = \frac{V(t)}{L} - \frac{R(t)}{L} I \Rightarrow I_{t+1} - I_t = \frac{V_t}{L} - \frac{R_t}{L} I_t. \tag{9}$$

Rearranging leads to

$$I_{t+1} = \frac{V_t}{L} + I_t \left(1 - \frac{R_t}{L}\right), \tag{10}$$

$$V_t = L\left(I_{t+1} - I_t \left(1 - \frac{R_t}{L}\right)\right), \tag{11}$$

and

$$R_t = L\left(I_t - I_{t+1} + \frac{V_t}{L}\right). \tag{12}$$

All of the plots of I seen below are produced by using MATLAB's *ode45* to solve Eqn. 8 (i.e. not using the discrete approximation shown). The time series $V(t)$ and $R(t)$ are created by defining values at fixed points and using linear interpolation (i.e. MATLAB's *interp1*) to find the time steps required by the ODE solver (i.e. MATLAB's *ode45*).

5 Changing V(t)

Consider the situation where $R(t)$ is constant.

Physical intuition is that V drives I, so we expect to find V CCM causes I ($C_{VI} > C_{IV}$).

For this example, the voltage is described by

$$V(t) = A_v \sin(f_v t + \phi_v) + O_v, \tag{13}$$

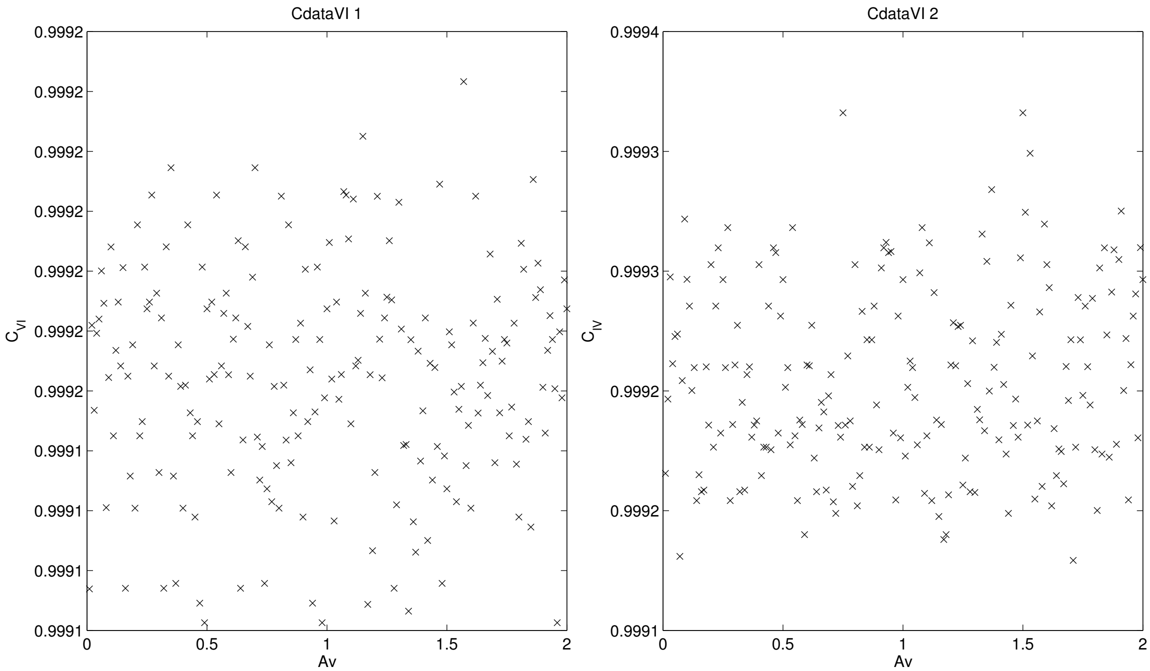
where A_v is the amplitude, f_v is the frequency, ϕ_v is the phase, and O_v is the offset voltage.

5.1 Changing A_v

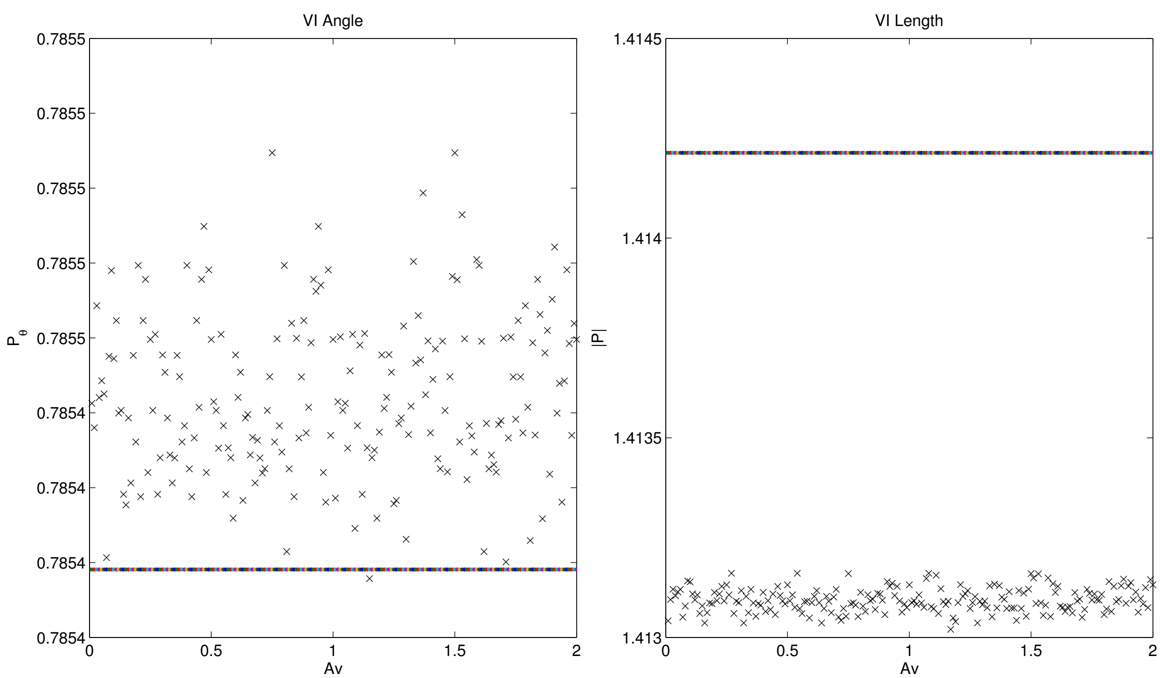
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $A_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both V(t) and I(t) are plotted for different A_v in Figure 6.

Figure 6: Reference plots for changing A_v.

The CCM correlations are each plotted in Figure 7 along with the corresponding PAI elements P_θ and $|P|$.



(a) C_{VI} and C_{IV}



(b) P_θ and $|P|$

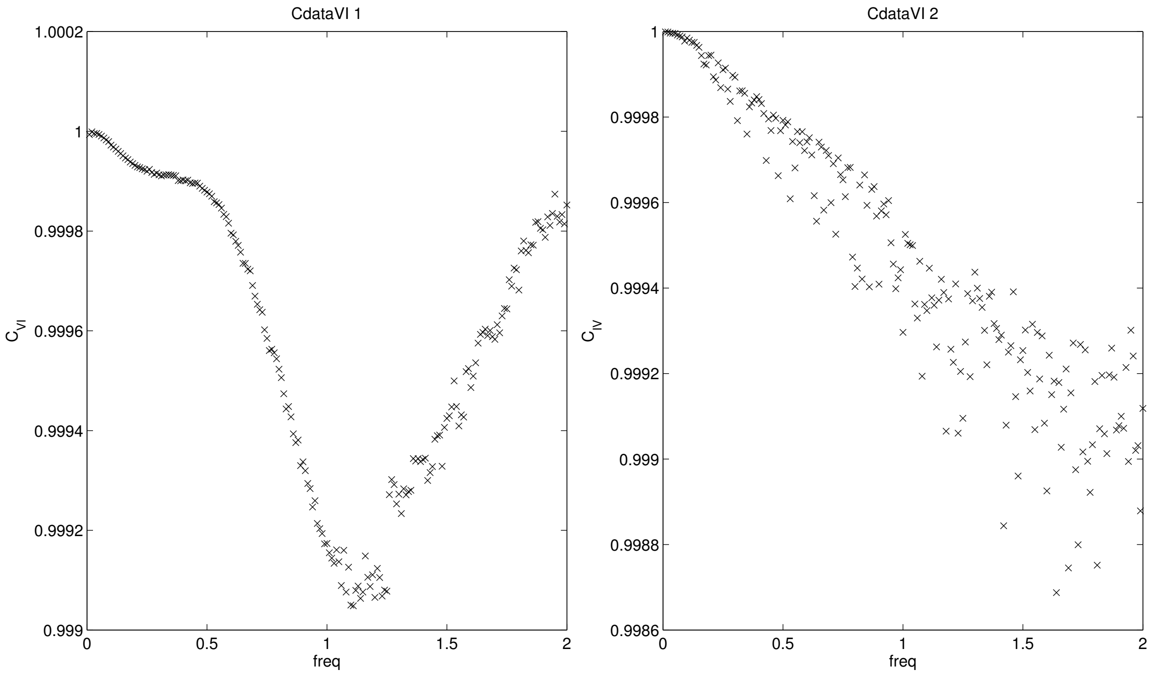
Figure 7: Changing A_v.

5.2 Changing f_v

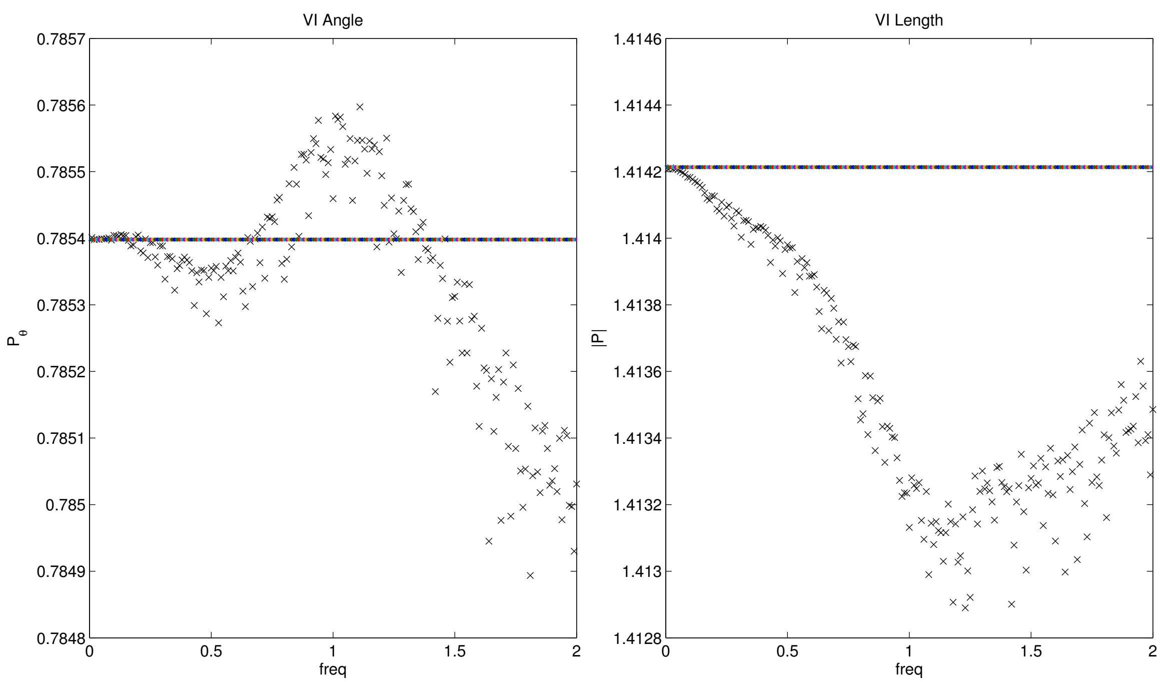
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $f_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both V(t) and I(t) are plotted for different f_v in Figure 8.

Figure 8: Reference plots for changing f_v.

The CCM correlations are each plotted in Figure 9 along with the corresponding PAI elements P_θ and $|P|$.



(a) C_{VI} and C_{IV}



(b) P_θ and $|P|$

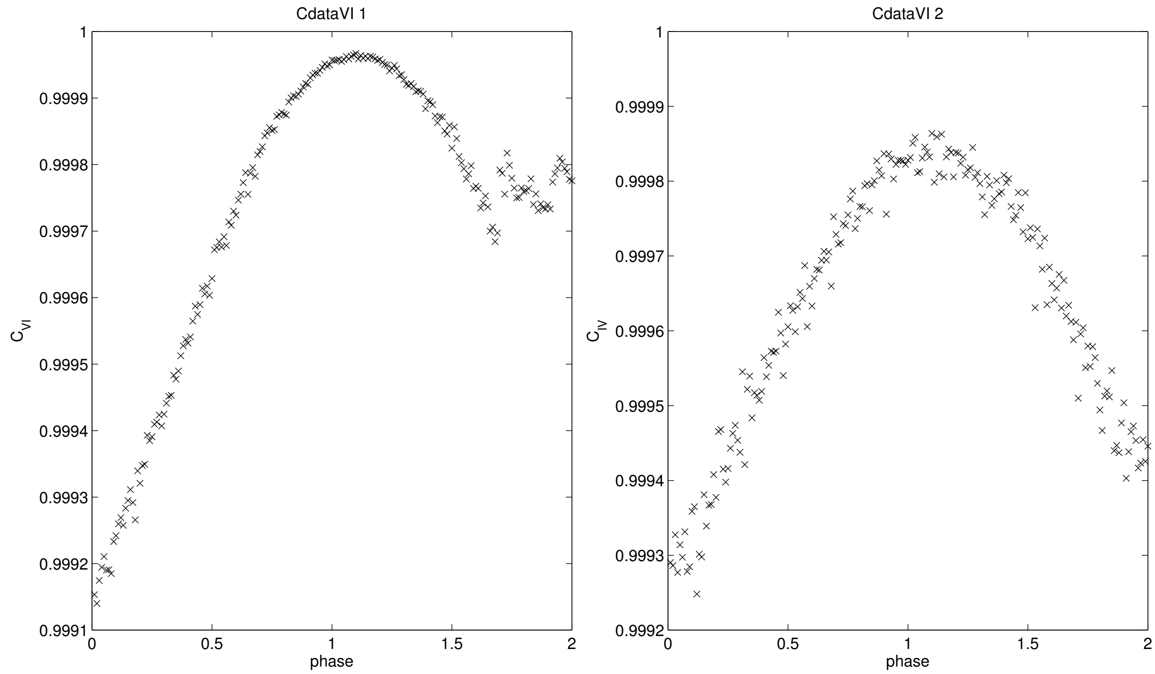
Figure 9: Changing f_v.

5.3 Changing ϕ_v

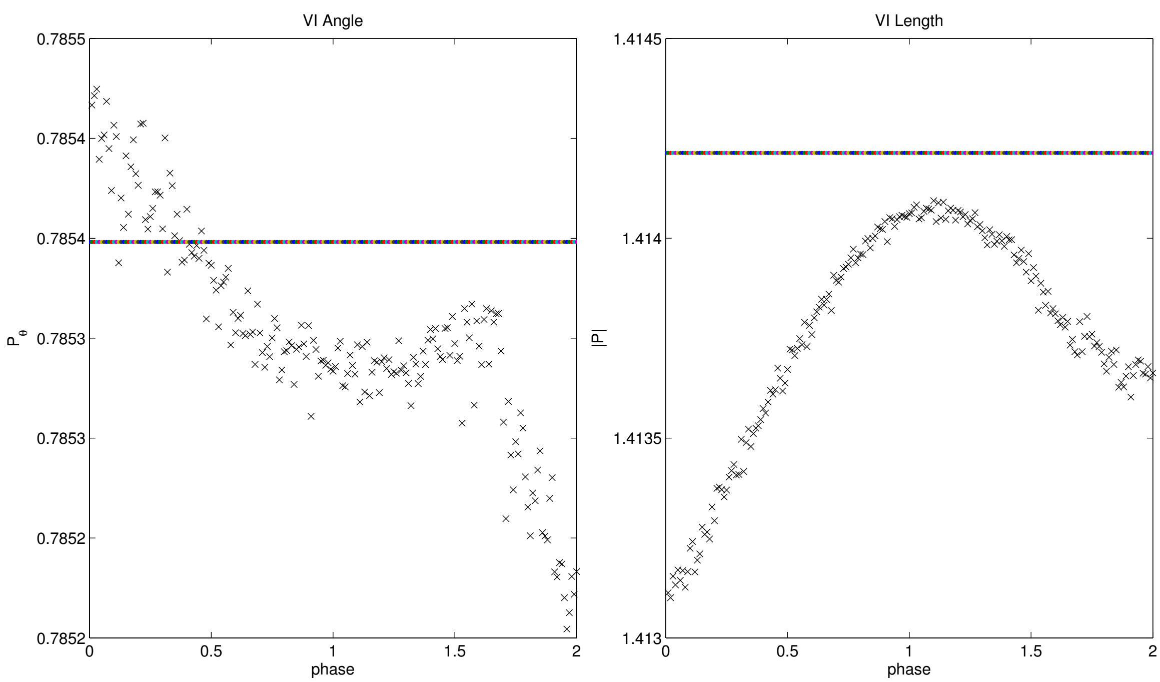
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $\phi_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both V(t) and I(t) are plotted for different ϕ_v in Figure 10.

Figure 10: Reference plots for changing ϕ_v.

The CCM correlations are each plotted in Figure 11 along with the corresponding PAI elements P_θ and $|P|$.



(a) C_{VI} and C_{IV}



(b) P_θ and $|P|$

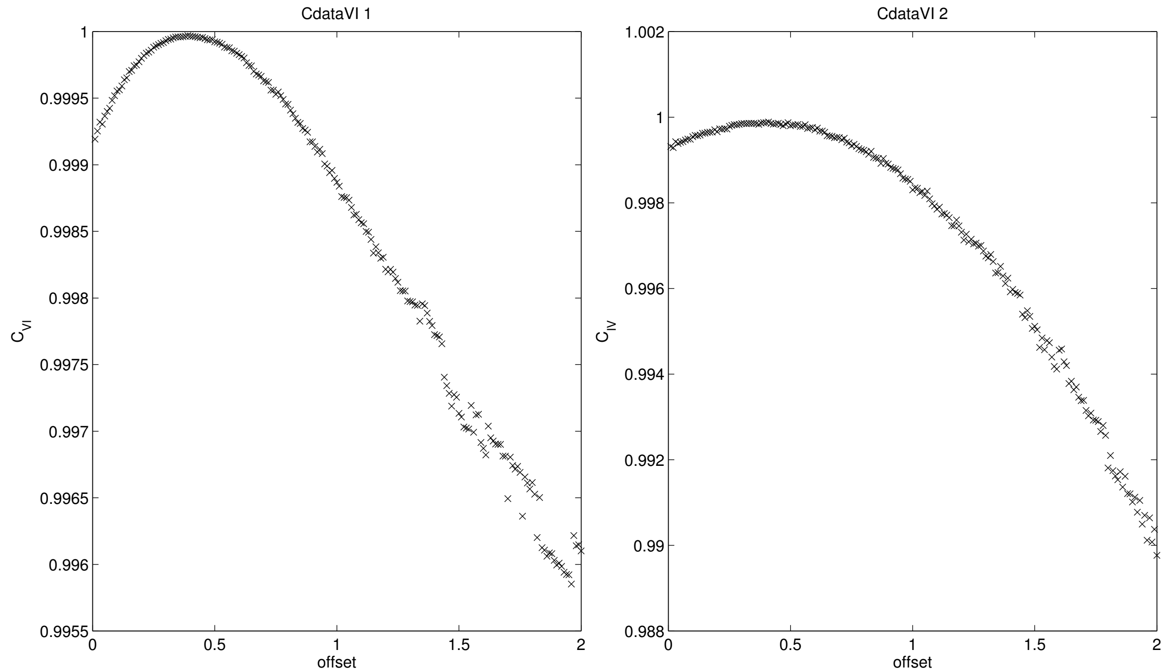
Figure 11: Changing ϕ_v.

5.4 Changing O_v

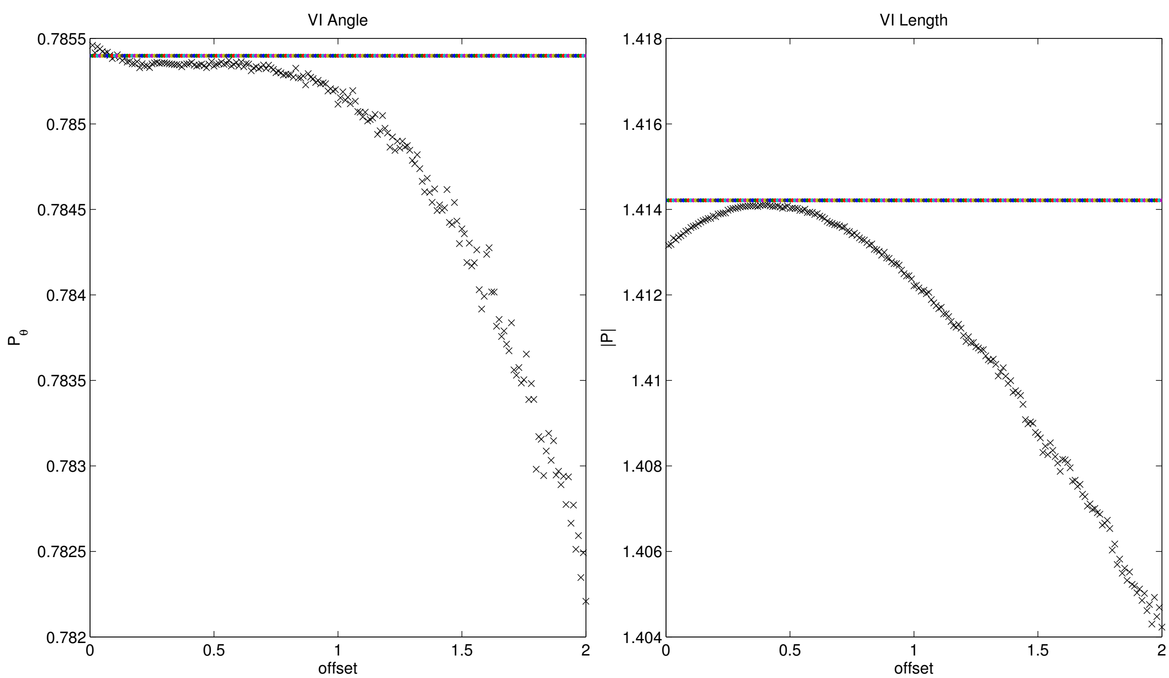
Consider evaluating the CCM correlations C_{VI} and C_{IV} for each $O_v \in [0.01, 2.0]$ in steps of 0.01. For reference, both V(t) and I(t) are plotted for different O_v in Figure 12.

Figure 12: Reference plots for changing O_v.

The CCM correlations are each plotted in Figure 13 along with the corresponding PAI elements P_θ and $|P|$.



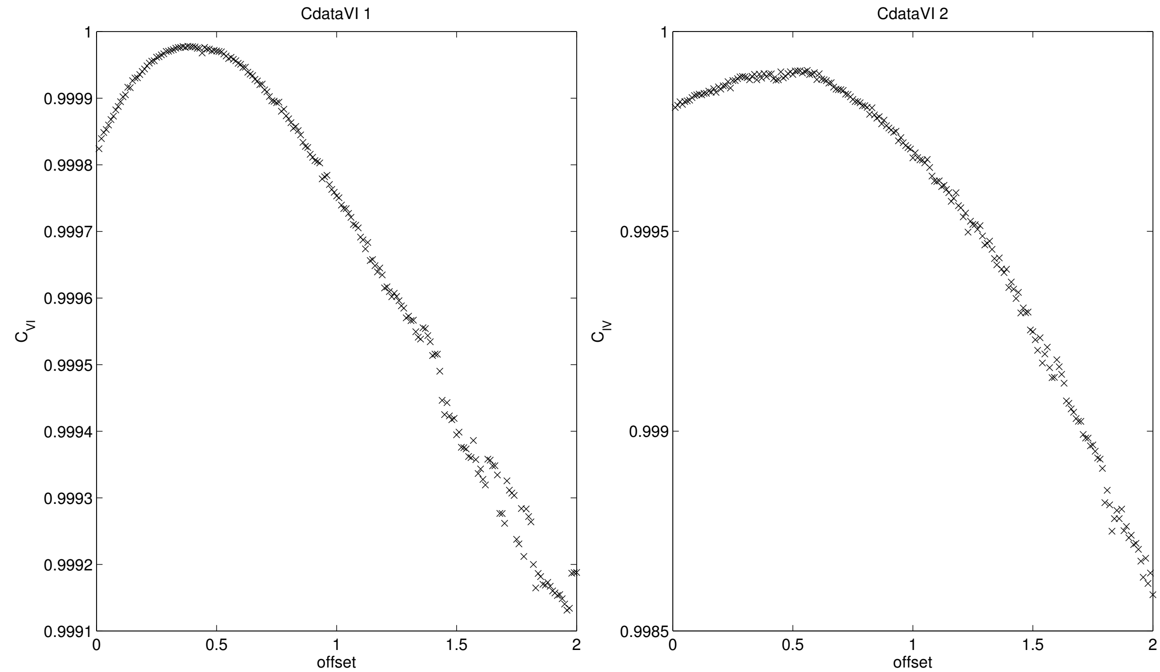
(a) C_{VI} and C_{IV}



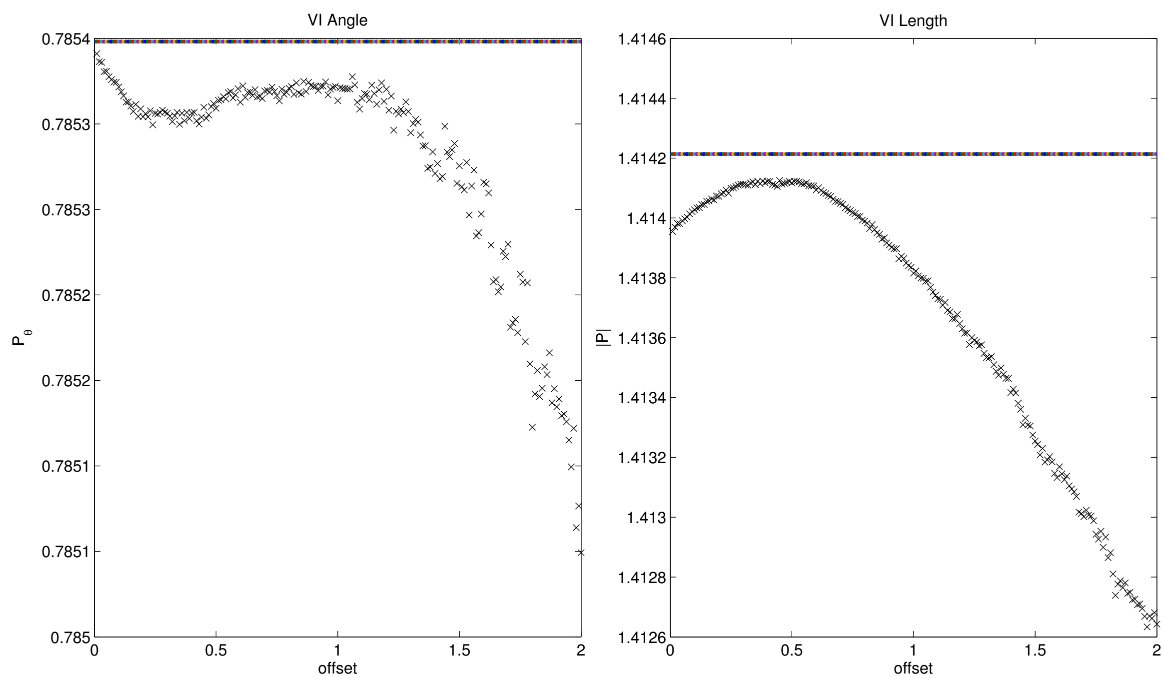
(b) P_θ and $|P|$

Figure 13: Changing O_v.

Figure 14 shows the effect of increasing the library length from 2×10^3 (i.e. `tspan = [0:0.5:1000];`) to 10^4 (i.e. `tspan = [0:0.5:5000];`), and Figure 15 extends the above plots to $O_v \in [0.01, 10.0]$ in steps of 0.05.

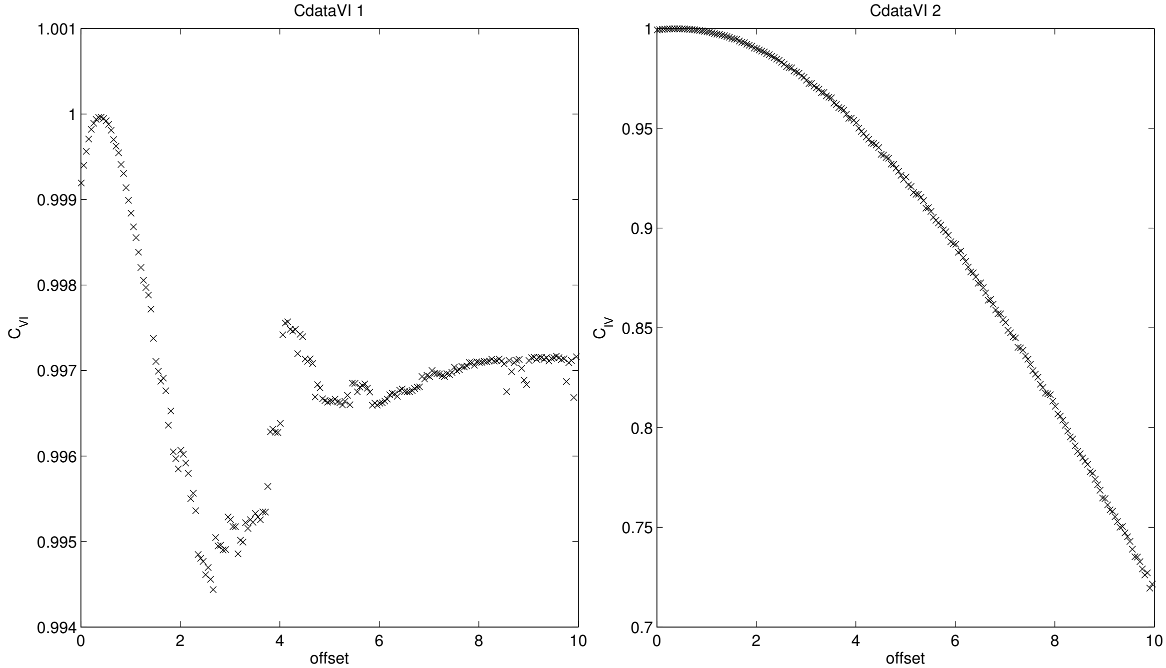


(a) C_{VI} and C_{IV}

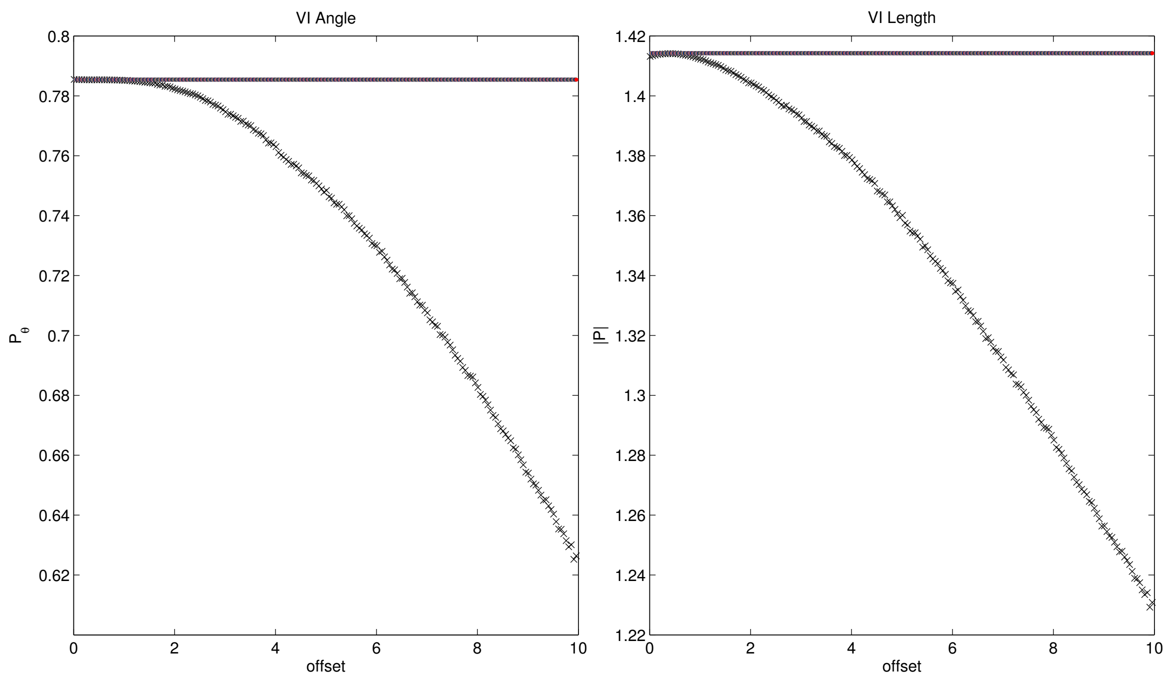


(b) P_θ and $|P|$

Figure 14: Changing O_v (longer library length).



(a) C_{VI} and C_{IV}



(c) P_θ and $|P|$

Figure 15: Changing O_v (larger domain for O_v).

—

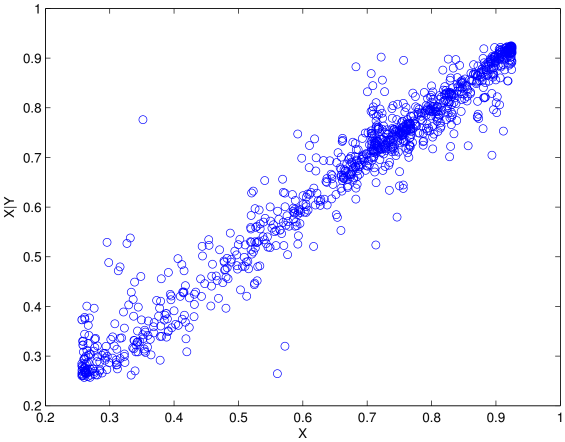
6 PAI

Consider the system (Sugihara Figure 3 C and D)

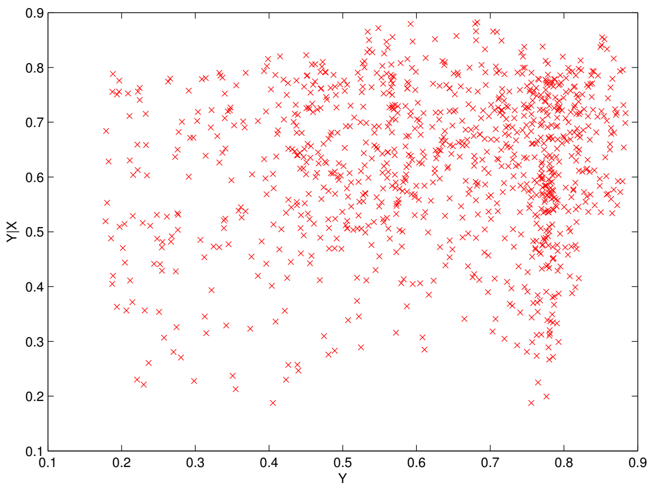
$$X_{t+1} = X_t \left(r_x - r_x X_t - \beta_{xy} Y_t \right) \tag{14}$$

$$Y_{t+1} = Y_t \left(r_y - r_y Y_t - \beta_{yx} X_t \right), \tag{15}$$

with $r_y = r_y = 3.7$, $X_0 = 0.2$, $Y_0 = 0.4$, $\beta_{xy} = 0$, and $\beta_{yx} = 0.32$. Plots of the correlation between X and $X|Y$, as well as, Y and $Y|X$ are shown below.



(a) C_{XY}



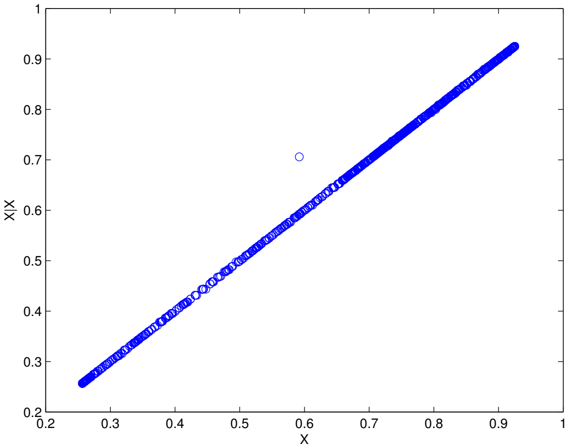
(b) C_{YX}

Figure 16: Changing A and B. C_{XY} and C_{YX}

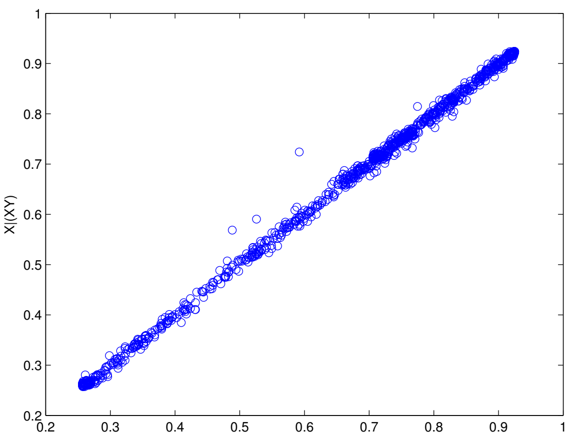
This leads to

C_{XY}	C_{YX}	$\Delta = C_{YX} - C_{XY}$
0.973921	0.110698	-0.8632

But, these are not the only correlations that can be tested:

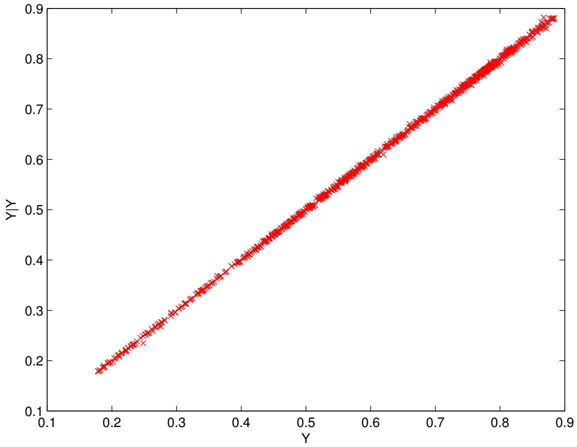


(a) C_{XX}

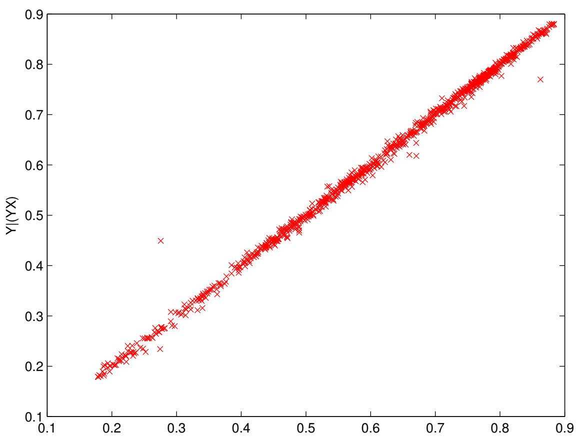


(b) $C_{X(XY)}$

Figure 17: Changing A and B. C_{XY} and C_{YX}



(c) C_{XX}



(d) $C_{X(XY)}$

Figure 18: Changing A and B. C_{XY} and C_{YX}

This leads to

C_{XX}	$C_{X(XY)}$	C_{YY}	$C_{Y(YX)}$	$\Delta = C_{Y(YX)} - C_{X(XY)}$
0.999841	0.998989	0.999908	0.998693	-2.9548e-04

Consider a comparison of PAI and CCM given the linear example system from above.

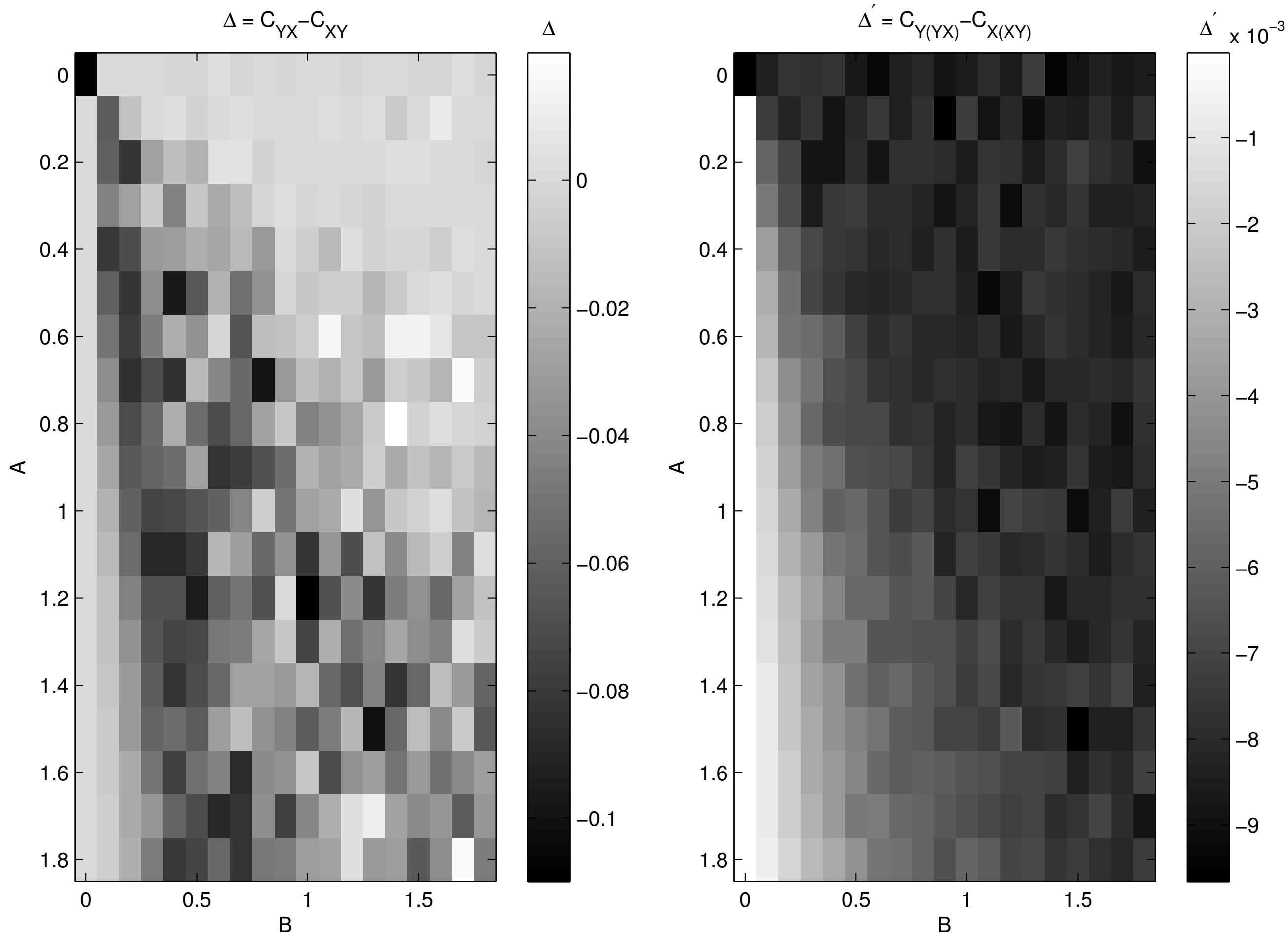


Figure 19: Changing A and B.