Time Series Leanings

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1 Introduction

...TBD...

2 Causal Penchant

Define the *causal penchant* as

$$\rho_{EC} := P\left(E|C\right) - P\left(E|\bar{C}\right) \quad . \tag{1}$$

The motivation for this expression is in te interpretation of ρ_{EC} as a causal indicator; i.e. if C causes (or *drives*) E, then $\rho_{EC} > 0$, and if $\rho_{EC} \leq 0$, then the direction of causal influence in the system is undetermined. If the effect E is assumed to be recorded in one time series and the cause C is assumed to be recorded in a different time series, then the direction of causal influence in the system can be determined by comparing various penchants calculated using both time series. The details of these comparison is discuss in the following sections, but some potential philosophical issues with this definition will be addressed first.

... **Add discussion of Pearl argument that $P(E|\bar{C})$ is "unobservable".**

Pearl's concerns can be addressed by rewriting Eqn. 1 using the law of total probability, i.e

$$P(E) = P(E|C)P(C) + P(E|\bar{C})P(\bar{C}) \quad , \tag{2}$$

or Bayes theorem, i.e.

$$P(E|\bar{C}) = P(\bar{C}|E)\frac{P(E)}{P(\bar{C})} \quad . \tag{3}$$

The definition of probability complements implies $P(\bar{C}) = 1 - P(C)$ and $P(\bar{C}|E) = 1 - P(C|E)$. Applying Eqn. 3 leads to

$$\begin{aligned} P(\bar{C}|E) &= 1 - P(C|E) \\ &= 1 - P(E|C)\frac{P(C)}{P(E)} \end{aligned}$$

Thus,

$$\begin{aligned} P(E|\bar{C}) &= P(\bar{C}|E)\frac{P(E)}{P(\bar{C})} \\ &= \left(1 - P(E|C)\frac{P(C)}{P(E)}\right)\frac{P(E)}{1-P(C)} \quad , \end{aligned}$$

and the second term in Eqn. 1 has been written in terms of the of the first term. This expression implies a penchant calculation containing only the conditional probability that is directly observed from the data, i.e.

$$\rho_{EC} = P(E|C)\left(1 + \frac{P(C)}{1-P(C)}\right) - \frac{P(E)}{1-P(C)} \tag{4}$$

This same expression can be derived without Eqn. 3 by using Eqn. 2 to make the appropriate substitution for $P(E|\bar{C})$ into Eqn. 1. This penchant calculation requires only a single conditional probability be estimated from the data.

The use of either Eqn. 2 or Eqn. 3 may eliminate the concern that $P(E|\bar{C})$ is fundamentally unobservable. It may also, however, introduce new philosophical concerns in the definition of the penchant. For example, ... **expand this discussion**

It follows from Eqn. 1 that

$$\rho_{EC} \in [1, -1] \quad , \tag{5}$$

but, more importantly for the calculations in the following sections, the penchant is not defined if $P(C)$ or $P(\bar{C})$ are zero (because the conditionals in Eqn. 1 would be undefined). Thus, the penchant is not defined if $P(C) = 0$ or if $P(C) = 1$. The former condition is interpreted intuitively as an inability to determine causal influence between two time series using points that do not appear in one of the series, and the latter condition is interpreted intuitively as an inability to determine causal influence between two time series if one of the data series is constant. The use of Bayes theorem in the derivation of Eqn. 4 implies that same conditions apply to $P(E)$. It will be seen below that there is no *a priori* assignments of "cause" or "effect" to a given time series when using penchants for causal inference. So, operationally, these conditions of $P(C)$ and $P(E)$ only mean that the penchant is undefined between pairs of time series where one series is constant.

The philosophical concerns are perhaps not as important as an answer to the straightforward question of whether or not the penchant is a useful tool for time series causality. The rest of this article will focus on answering that question.

3 Causal Leaning

Given a pair of times series $\{\mathbf{X}, \mathbf{Y}\}$, it is difficult to use the penchant directly for causal inference between the pair. Consider the assignment of $\mathbf{X}$ as the cause, C, and $\mathbf{Y}$ as the effect, E, i.e. $\{C, E\} = \{\mathbf{X}, \mathbf{Y}\}$. If $\rho_{EC} > 0$, then the probability that $\mathbf{X}$ drives $\mathbf{Y}$ is higher than the probability that it does not, which is stated more sufficiently as $\mathbf{X}$ has a penchant to drive $\mathbf{Y}$ or $\mathbf{X} \xrightarrow{pen} \mathbf{Y}$. It is possible, however, that the same penchant could be positive with the opposite cause-effect assignment, i.e. $\rho_{EC} > 0 \mid \{C, E\} = \{\mathbf{Y}, \mathbf{X}\} \Rightarrow \mathbf{Y} \xrightarrow{pen} \mathbf{X}$. Even though it is possible that $\mathbf{X} \xrightarrow{pen} \mathbf{Y}$ and $\mathbf{Y} \xrightarrow{pen} \mathbf{X}$ are both true, such information does not provide information about the causal relationship within the pair $\{\mathbf{X}, \mathbf{Y}\}$.

The *leaning* is meant to address this problem and is defined as

$$\lambda_{EC} := \rho_{EC} - \rho_{CE} \quad . \tag{6}$$

A positive leaning implies the cause C drives the effect E more than the effect drives the cause, a negative leaning implies the effect E drives the cause C more than the cause drives the effect, and a null leaning (i.e. $\lambda_{EC} = 0$) yields no causal information for the cause-effect pair $\{C, E\}$.

Consider again the assignment of $\{C, E\} = \{\mathbf{X}, \mathbf{Y}\}$. If $\lambda_{EC} > 0$, then $\mathbf{X}$ has a larger penchant to drive $\mathbf{Y}$ than $\mathbf{Y}$ does to drive $\mathbf{X}$. More verbosely, $\lambda_{EC} > 0$ implies the difference between the probability that $\mathbf{X}$ drives $\mathbf{Y}$ and the probability that it does not is higher than the difference between the probability that $\mathbf{Y}$ drives $\mathbf{X}$ and the probability that it does not. For convenience, this language is boiled down to $\mathbf{X} \xrightarrow{lean} \mathbf{Y}$, as in $\lambda_{EC} > 0 \mid \{C, E\} = \{\mathbf{X}, \mathbf{Y}\} \Rightarrow \mathbf{X} \xrightarrow{lean} \mathbf{Y}$, $\lambda_{EC} < 0 \mid \{C, E\} = \{\mathbf{X}, \mathbf{Y}\} \Rightarrow \mathbf{Y} \xrightarrow{lean} \mathbf{X}$, and $\lambda_{EC} = 0 \mid \{C, E\} = \{\mathbf{X}, \mathbf{Y}\} \Rightarrow$ no conclusion.

It follows from Eqn. 6 and the bound for the penchant that $\lambda_{EC} \in [-2, 2]$. The leaning is a function of four probabilities, $P(C)$, $P(E)$, $P(C|E)$ and $P(E|C)$. The usefulness of the leaning for causal inference will depend on an effective method for estimating these probabilities from times series data and a more careful definition of the cause-effect assignment within the time series pair. These topics will be discussed with a motivating toy model of a dynamical system for which the penchant and leaning calculations are simple enough to perform without any computational aid.

4 Motivating Example

Consider a time series pair $\bar{\mathbf{T}} = \{\mathbf{X}, \mathbf{Y}\}$ with

$$\begin{aligned} \mathbf{X} &= \{x_t \mid t \in [0,9]\} \\ &= \{0,0,1,0,0,1,0,0,1,0\} \\ \mathbf{Y} &= \{y_t \mid t \in [0,9]\} \\ &= \{0,0,0,1,0,0,1,0,0,1\}\,. \end{aligned}$$

It seems intuitive to say that $\mathbf{X}$ drives $\mathbf{Y}$ because $y_t = x_{t-1}$. However, to show this result using a leaning calculation requires specification of the cause-effect assignment $\{C, E\} = \{\mathbf{X}, \mathbf{Y}\}$. A cause must precede an effect in the cause-effect assignment for consistency with the intuitive definition of causality. It follows that a natural assignment may be $\{C, E\} = \{x_{t-l}, y_t\}$ where $l \in [1, 9]$. This cause-effect assignment will be referred to as the l-standard assignment.

4.1 Defining the penchants

Given $\bar{\mathbf{T}}$, one possible penchant that can be defined using the 1-standard assignment is

$$\begin{aligned} \rho_{y_t=1,x_{t-1}=1} &= \kappa\left(1 + \frac{P\left(x_{t-1}=1\right)}{1-P\left(x_{t-1}=1\right)}\right) \\ &\quad - \frac{P\left(y_t=1\right)}{1-P\left(x_{t-1}=1\right)}\ , \end{aligned}$$

with $\kappa = P\left(y_t = 1 | x_{t-1} = 1\right)$. Another penchant defined using this assignment would be the cooresponding term with $\kappa = P\left(y_t = 0 | x_{t-1} = 0\right)$. These two penchants are called the *observed* penchants because κ can be found directly from the time series data.

Equations for the unobserved penchants corresponding to $\kappa = P\left(y_t = 0 | x_{t-1} = 1\right)$ and $\kappa = P\left(y_t = 1 | x_{t-1} = 0\right)$ can be written down. These penchants are defined, but in both cases $\kappa = 0 \Rightarrow \rho_{y_t x_{t-1}} < 0$. Thus unobserved penchants imply the effect, $y_t = 0$ or 1 (for this toy model) is most likely not caused by the postulated cause, $x_{t-1} = 1$ or 0, respectively. Using these unobserved penchants to define leanings becomes a comparison of how unlikely postulated causes are to cause given effects. Such comparisons are not as easily interpreted in the intuitive framework of causality, and as such, are not explored as tools for causal inference in this article.

4.2 Finding the penchants from the data

The probabilities in the penchant calculations can be estimated from the time series data with frequency counts, e.g.

$$P\left(y_t = 1 | x_{t-1} = 1\right) = \frac{n_{EC}}{n_C} = \frac{3}{3} = 1\ ,$$

where n_{EC} is the number of times $y_t = 1$ and $x_{t-1} = 1$ appears in $\bar{\mathbf{T}}$, and n_C is related to the number of times the assumed cause, $x_{t-1} = 1$, has appeared in $\bar{\mathbf{T}}$ and is defined in more detail below.

Estimating the other two probabilities in this penchant calculation using frequency counts from $\bar{\mathbf{T}}$ is slightly more subtle. The underlying assumption that the assumed cause must precede the assumed effect must be considered when defining the frequency counts. This concern is addressed by shifting $\mathbf{X}$ and $\mathbf{Y}$ into $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ such that, for any given t, $\tilde{\mathbf{X}}_t$ precedes $\tilde{\mathbf{Y}}_t$, and defining

$$P\left(y_t = 1\right) = \frac{n_E}{L} = \frac{3}{9} \tag{7}$$

and

$$P\left(x_{t-1} = 1\right) = \frac{n_C}{L} = \frac{3}{9}\ , \tag{8}$$

where n_C is the number of times $\tilde{x}_t = 1$, n_E is the number of times $\tilde{y}_t = 1$, and L is the library length of $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ (which are assumed to be the same length). For this example, those subsets are

$$
\begin{aligned}
\tilde{\mathbf{X}} &= \{0,0,1,0,0,1,0,0,1\} \\
\tilde{\mathbf{Y}} &= \{0,0,1,0,0,1,0,0,1\}
\end{aligned}
$$

which are both shorter than there counterparts above by a single value because the penchants are being calculated using the 1-standard cause-effect assignment. It follows that $\tilde{x}_t = x_{t-1}$ and $\tilde{y}_t = y_t$.

4.3 Mean observed leaning for $\bar{\mathbf{T}}$

The two observed penchants in this example that assume $\mathbf{X}$ causes $\mathbf{Y}$ (i.e. using the 1-standard assignment) are found from the data to be

$$\rho_{y_t=1,x_{t-1}=1} = 1 \tag{9}$$

and

$$\rho_{y_t=0,x_{t-1}=0} = 1 \ . \tag{10}$$

The complements of these observed penchants are found using the complementary 1-standard assignment of $\{C,E\} = \{y_{t-1}, x_t\}$ and are found from the data to be

$$
\begin{aligned}
\rho_{x_t=1,y_{t-1}=0} &= \frac{3}{7} & (11)\\
\rho_{x_t=0,y_{t-1}=1} &= \frac{3}{7} \ . & (12)
\end{aligned}
$$

and

$$\rho_{x_t=0,y_{t-1}=0} = -\frac{3}{7} \ . \tag{13}$$

The *mean observed penchant* is the algebraic mean of the observed penchants, i.e.

$$
\begin{aligned}
\langle \rho_{y_t,x_{t-1}} \rangle &= \frac{1}{2}\left(\rho_{y_t=1,x_{t-1}=1} + \rho_{y_t=0,x_{t-1}=0}\right) \\
&= 1
\end{aligned}
$$

and

$$
\begin{aligned}
\langle \rho_{x_t,y_{t-1}} \rangle &= \frac{1}{3}\big(\rho_{x_t=1,y_{t-1}=0} \\
&\quad +\rho_{x_t=0,y_{t-1}=1} + \rho_{x_t=0,y_{t-1}=0}\big) \\
&= \frac{1}{7} \ .
\end{aligned}
$$

The *mean observed leaning* follows from the definition of the mean observed penchants as

$$
\begin{aligned}
\langle \lambda_{y_t,x_{t-1}} \rangle &= \langle \rho_{y_t,x_{t-1}} \rangle - \langle \rho_{x_t,y_{t-1}} \rangle & (14)\\
&= \frac{6}{7} \ . & (15)
\end{aligned}
$$

The positive leaning implies the probability that x_{t-1} drives y_t is higher than the probability that y_{t-1} drives x_t; i.e. $\mathbf{X} \xrightarrow{lean} \mathbf{Y}$ given the 1-standard cause-effect assignment. This result is expected and agrees with the intuitive definition of causality in this example.

4.4 Unobserved penchants

The *unobserved* penchants (using the 1-standard assignment from the beginning of the subsection) for this example are

$$
\begin{aligned}
\rho_{y_t=1,x_{t-1}=0} &= -1 & (16)\\
\rho_{y_t=0,x_{t-1}=1} &= -1 & (17)
\end{aligned}
$$

and their complements are

$$\rho_{x_t=1,y_{t-1}=1} = -\frac{3}{7} \tag{18}$$

These values can incorporated into the averaging calculation to yield a *mean total penchant*; i.e.

$$
\begin{aligned}
\langle\langle \rho_{y_t,x_{t-1}} \rangle\rangle &= \frac{1}{4}\big(\rho_{y_t=1,x_{t-1}=1} + \rho_{y_t=0,x_{t-1}=0} \\
&\quad \rho_{y_t=1,x_{t-1}=0} + \rho_{y_t=0,x_{t-1}=1}\big) \\
&= 0
\end{aligned}
$$

and

$$
\begin{aligned}
\langle\langle \rho_{x_t,y_{t-1}} \rangle\rangle &= \frac{1}{4}\big(\rho_{x_t=1,y_{t-1}=1} + \rho_{x_t=0,y_{t-1}=0} \\
&\quad \rho_{x_t=1,y_{t-1}=0} + \rho_{x_t=0,y_{t-1}=1}\big) \\
&= 0 \ .
\end{aligned}
$$

Thus, the *mean total leaning* (defined analogous to Eqn. 14) would be $\langle\langle \lambda_{y_t,x_{t-1}} \rangle\rangle = \langle\langle \rho_{y_t,x_{t-1}} \rangle\rangle - \langle\langle \rho_{x_t,y_{t-1}} \rangle\rangle = 0$ and would not be useful for casual inference in this example.

4.5 Cause-effect assignment independence

It may be argued that the causal inference above was a little disingenuous in that the assumed cause-effect relationship was known to be correct. It can be shown, however, that causal inference is independent of the assumed cause-effect relationship. For example, consider the 1-standard cause-effect assignment $\{C, E\} = \{y_{t-l}, x_t\}$. The mean observed leaning would be

$$\langle \lambda_{x_t,y_{t-1}} \rangle = \langle \rho_{x_t,y_{t-1}} \rangle - \langle \rho_{y_t,x_{t-1}} \rangle \quad (19)$$
$$= -\frac{6}{7} \ , \quad (20)$$

which implies $\mathbf{X} \xrightarrow{lean} \mathbf{Y}$, as expected for this example.

In general, $\lambda_{AB} := \rho_{AB} - \rho_{BA} \Rightarrow -\lambda_{AB} = \rho_{BA} - \rho_{AB} := \lambda_{BA}$. Thus, the causal inference is independent of which times series is initially assumed to be the cause (or effect).

4.6 Weighted Mean Observed Leaning

The *weighted mean observed penchant* is defined similarly to the mean observed penchant but each term is weighted by the number of times that penchant appears in the data; e.g.

$$\begin{aligned}\langle \rho_{y_t,x_{t-1}} \rangle_w &= \frac{1}{L-l} \big(n_{y_t=1,x_{t-1}=1} \rho_{y_t=1,x_{t-1}=1} \\ &\quad + n_{y_t=0,x_{t-1}=0} \rho_{y_t=0,x_{t-1}=0} \big) \\ &= 1\end{aligned}$$

and

$$\begin{aligned}\langle \rho_{x_t,y_{t-1}} \rangle_w &= \frac{1}{L-l} \big(n_{x_t=1,y_{t-1}=0} \rho_{x_t=1,y_{t-1}=0} \\ &\quad + n_{x_t=0,y_{t-1}=1} \rho_{x_t=0,y_{t-1}=1} \\ &\quad + n_{x_t=0,y_{t-1}=0} \rho_{x_t=0,y_{t-1}=0} \big) \\ &= \frac{3}{63} \ ,\end{aligned}$$

where n_{ab} is the number of times the assumed cause a appears with the assumed effect b in the data, L is the library length of the times series data, and l is the lag used in the l-standard cause-effect assignment under which these penchants are being calculated.

The *weighted mean observed leaning* follows naturally as

$$\begin{aligned}\langle \lambda_{y_t,x_{t-1}} \rangle_w &= \langle \rho_{y_t,x_{t-1}} \rangle_w - \langle \rho_{x_t,y_{t-1}} \rangle_w \\ &= \frac{60}{63} \ .\end{aligned}$$

For this example, $\langle \lambda_{y_t,x_{t-1}} \rangle_w \Rightarrow \mathbf{X} \xrightarrow{lean} \mathbf{Y}$ as expected.

Conceptually, the weighted mean observed penchant is preferred to the mean penchant because it accounts for the frequency of observed cause-effect pairs within the data, which is assumed to be a predictor of causal influence. For example, given some pair $\{\mathbf{A}, \mathbf{B}\}$, if it is known that a_{t-1} causes b_t and both $b_t = 0 | a_{t-1} = 0$ and $b_t = 0 | a_{t-1} = 1$ are observed in the data, then comparison of the frequencies with which the pair occur would be used to determine which of the two pairs represents the true cause-effect relationship and which pair represents, e.g., the effects of noise in the system.

For this example, the weighted mean observed leaning provides the same causal inference as the mean observed leaning. The weighted mean calculation, however, can be made computationally less expensive than the mean calculation, which will be of practical concern for empirical data sets.

4.7 Algorithm

Most empirical data sets make observed leaning calculations either tedious, or prohibitively difficult, to do without a computational aid. Consider the following algorithm to calculate the observed penchants of a time series pair

$\bar{\mathbf{T}} = \{\mathbf{X}, \mathbf{Y}\} = \{\{x_t\}, \{y_t\}\}$ for some time step $t \in [0, L]$ where L is the library length of both times series[1]:

1. Initialize three counters, e.g. $N_{EC} = N_C = N_E = 0$
2. Initialize two iterators, e.g. $m = l$ and $n = l$
3. If $y_m = y_n \pm \delta_y$, then increment one of the counters, e.g. $N_E = N_E + 1$
4. If $x_{m-l} = x_{n-l} \pm \delta_x$, then increment another of the counters, e.g. $N_C = N_C + 1$
5. If both $y_m = y_n \pm \delta_y$ and $x_{m-l} = x_{n-l} \pm \delta_x$, then increment the last counters, e.g. $N_{EC} = N_{EC} + 1$
6. Increment one of the iterators, e.g. $n = n + 1$
7. Repeat steps 3-6 $\forall\ n \in [l, L]$
8. If $N_C = 0$, issue a warning and skip to the final step
9. Calculate the estimated $P(E|C)$, e.g. $P_{EC} = \frac{N_{EC}}{N_C}$
10. Calculate the estimated $P(C)$, e.g. $P_C = \frac{N_C}{L-l}$
11. Calculate the estimated $P(E)$, e.g. $P_E = \frac{N_E}{L-l}$
12. Calculated and record the observed penchant, i.e. $\rho_{EC} = P_{EC}\left[1 + P_C\left(1 - P_C\right)^{-1}\right] - P_E\left(1 - P_C\right)^{-1}$
13. Repeat step 1
14. Repeat steps 3-13 $\forall\ m \in [l, L]$

[1] The library length is assumed to be the same for both time series as a simplifying assumption. The algorithm would not need to be altered significantly to account for slightly different library lengths. However, if the two time series have dramatically different library lengths (e.g. $L_y = L_x/2$), then the algorithm would need to change; e.g. by sampling shorter time series from the longer one and running the equal-length leaning algorithm using the shorter time series and each of the sampled time series.

This algorithm will be labeled $\text{PEN}(\mathbf{X}, \mathbf{Y}, l, \delta_x, \delta_y)$, and it calculates the observed penchant $\rho_{y_t, x_{t-l}}\ \forall\ t \in [0, L]$. There are $L - l$ penchants returned by this algorithm, and the arithmetic mean of this set of penchants corresponds to the weighted mean observed penchant calculation discussed in Section 4.6.

$\text{PEN}(\mathbf{X}, \mathbf{Y}, l, \delta_x, \delta_y)$ depends on five parameters, the time series pair, $\{\mathbf{X}, \mathbf{Y}\}$, the lag l used to define the l-standard cause-effect assignment and the tolerances, δ_x and δ_y, used to determine if two data points in the time series should be considered equal. The lag is part of the definition of the penchant, but the tolerances have been explicitly added to the algorithm to deal with noise in the data. The effect of these tolerance on the causal inference for a given times series set will be explored in the following sections.

Assume there exists a single-valued algorithm that returns the arithmetic mean of a set of numbers, i.e. $\text{MEAN}(\cdot)$, then the weighted mean observed leaning algorithm, $\text{LEAN}(\mathbf{X}, \mathbf{Y}, l, \delta_x, \delta_y)$, is defined as the difference between $\text{MEAN}(\text{PEN}(\mathbf{X}, \mathbf{Y}, l, \delta_x, \delta_y))$ and $\text{MEAN}(\text{PEN}(\mathbf{Y}, \mathbf{X}, l, \delta_x, \delta_y))$, in that order.

A MATLAB implementation of $\text{LEAN}(\mathbf{X}, \mathbf{Y}, l, \delta_x, \delta_y)$ is used in all of the results shown below and is available at *github*\ ….

5 Simple Example Systems

5.1 Impulse with Noisy Response Linear Example

Consider the linear example dynamical system of

$$X_t = \{0,2,0,0,2,0,0,2,0,0\} \quad (21)$$
$$Y_t = X_{t-1} + B\eta_t, \quad (22)$$

with $B \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0,1)$. Specifically, consider $B \in [0,2]$ in increments of 0.02. The response system Y is just a lagged version of the driving signal with varying levels of standard Gaussian noise applied at each time step.

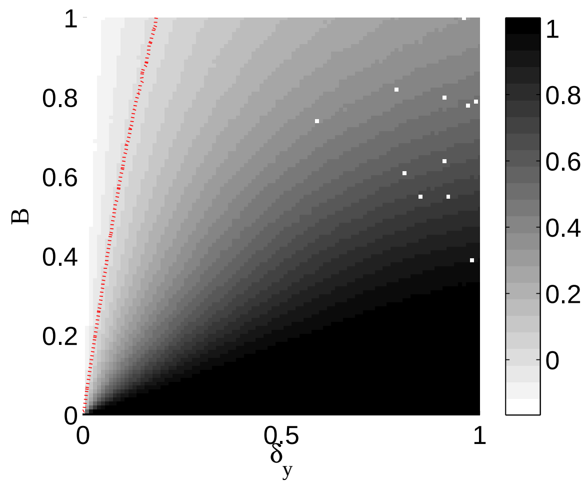

Figure 1: (Color available online.) Leaning as a function of both the noise and the y-tolerance. The red dashed line is the zero contour. See the text for am explanation of the missing data for large δ_y.

5.2 Cyclic Linear Example

Consider the linear example dynamical system of

$$X_t = \sin(t) \quad (23)$$
$$Y_t = X_{t-1} + B\eta_t, \quad (24)$$

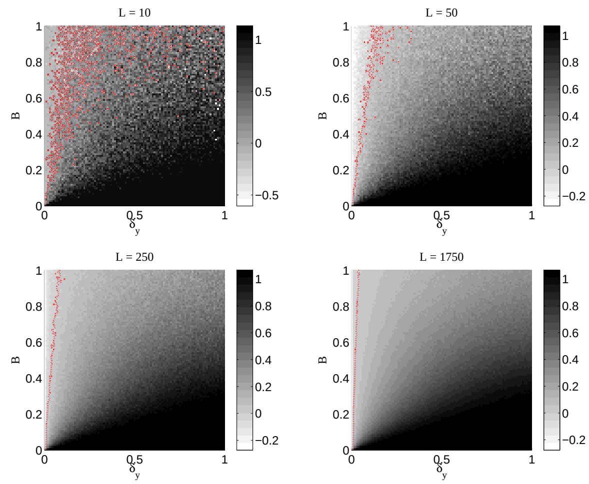

Figure 2: (Color available online.) Leaning as a function of both the noise and the y-tolerance for different library lengths. The red dashed line is the zero contour. See the text for am explanation of the missing data for large δ_y.

with $B \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0,1)$. Specifically, consider $B \in [0,2]$ in increments of 0.02. The response system Y is just a lagged version of the driving signal with varying levels of standard Gaussian noise applied at each time step.

5.3 Non-Linear Example

Consider the non-linear dynamical system of

$$X_t = \sin(t) \quad (25)$$
$$Y_t = AX_{t-1}(1 - BX_{t-1}) + C\eta_t, \quad (26)$$

with $A, B, C \in \mathbb{R} \geq 0$ and $\eta_t \sim \mathcal{N}(0,1)$. Specifically, consider $A, B, C \in [0,5]$ in increments of 0.5.

5.4 RL Circuit Example

Both of the previous examples included a noise term, η_t. Consider a series circuit containing a resistor, inductor, and time varying voltage

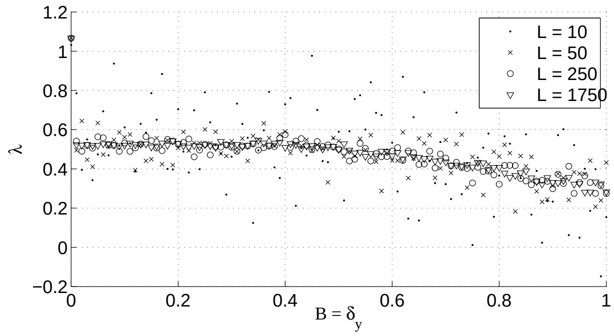

Figure 3: (Color available online.) The leaning agrees with intuition for most noise levels when the y-tolerance is set to the noise level (i.e. $B = \delta_y$).

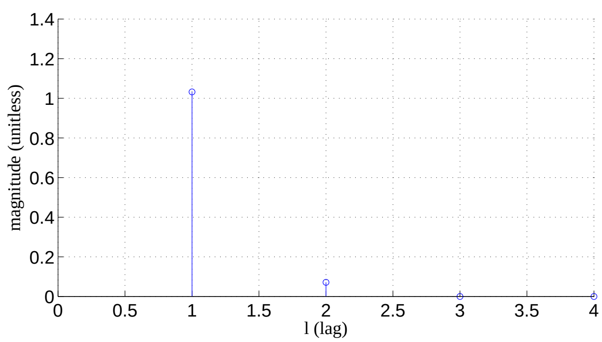

Figure 4: (Color available online.) Different l-standard cause-effect assignments lead to different leanings.

source related by

$$\frac{dI}{dt} = \frac{V(t)}{L} - \frac{R}{L} I, \tag{27}$$

where I is the current at time t, $V(t) = \sin(\Omega t)$ is the voltage at time t, R is the resistance, and L is the inductance. Eqn. 27 was solved using the *ode45* integration function in MATLAB. The time series $V(t)$ is created by defining values at fixed points and using linear interpolation to find the time steps required by the ODE solver.

Consider the situation where $L = 10$ Henries and $R = 5$ Ohms are constant. Physical intuition is that V drives I, and so we expect

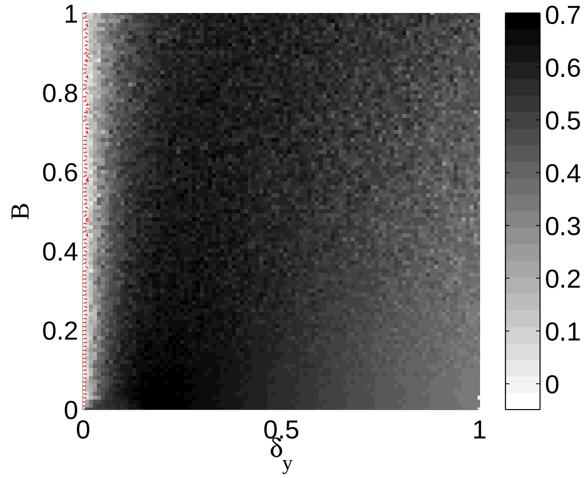

Figure 5: (Color available online.) Leaning as a function of both the noise and the y-tolerance. The red dashed line is the zero contour. See the text for am explanation of the missing data for large δ_y.

to find that V CCM causes I (i.e., $C_{VI} > C_{IV}$ or $\Delta = C_{VI} - C_{IV} > 0$).

6 Empirical Data

7 Conclusion

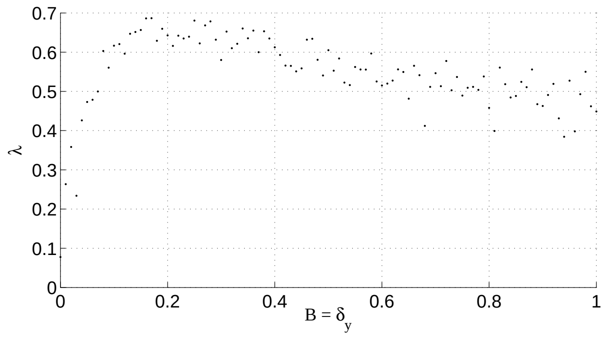

Figure 6: (Color available online.) The leaning agrees with intuition for most noise levels when the y-tolerance is set to the noise level (i.e. $B = \delta_y$).

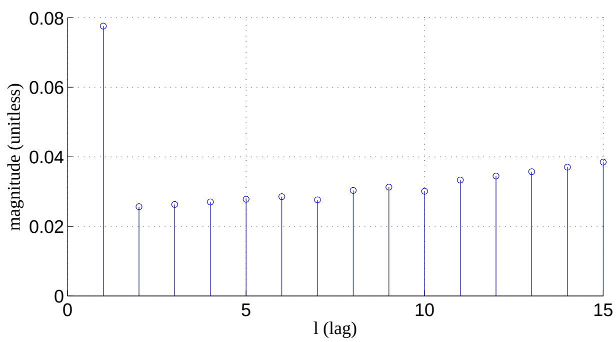

Figure 7: (Color available online.) Different l-standard cause-effect assignments lead to different leanings.

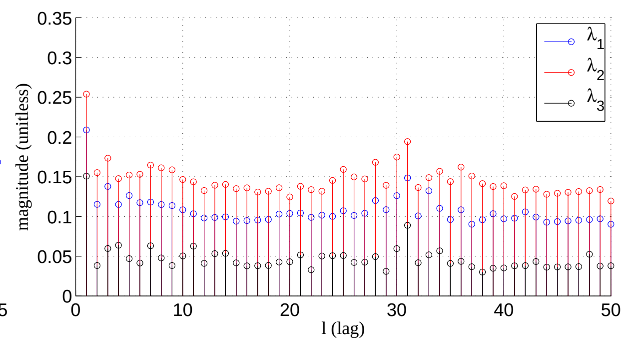

Figure 10: (Color available online.) Different l-standard cause-effect assignments lead to different leanings.

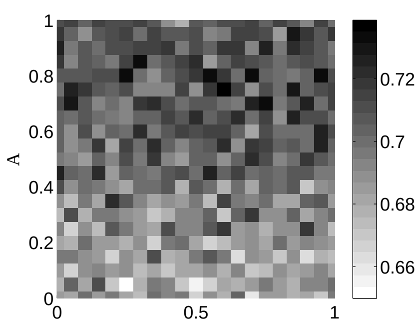

Figure 9: (Color available online.) The leaning agrees with intuition for most noise levels when the y-tolerance is set to the noise level (i.e. $B = \delta_y$).

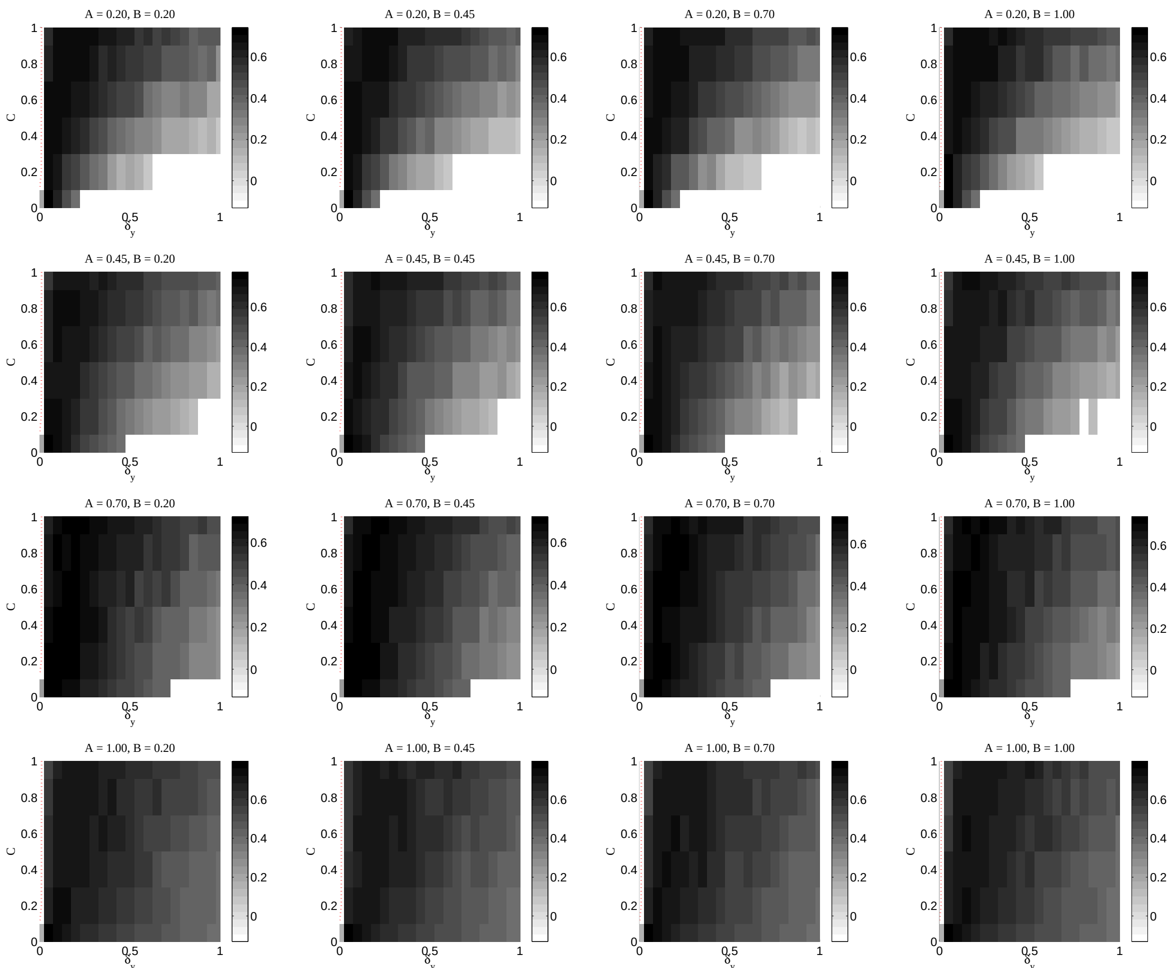

Figure 8: (Color available online.) Leaning as a function of both the noise and the y-tolerance for values of A and B. The red dashed line is the zero contour. See the text for am explanation of the missing data for large δ_y.