1 Ray-Circle Collisions

Suppose (in 2 dimensions) we have a circle with the equation $(x-a)^2+(y-b)^2 = r^2$ and a ray characterized by an origin (x_0, y_0) and a direction, assumed to be normalized, $(\delta x, \delta y)$. Can we numerically find the points of intersection of the ray with the circle?

Suppose we have some scalar $c > 0$. Then what we want is some point along this ray to also be a solution to the circle equation. A point along the ray is given by $(x_0, y_0) + c(\delta x, \delta y)$, so we need to find c values such that

$$(x_0 + c\delta x - a)^2 + (y_0 + c\delta y - b)^2 = r^2$$

The only unknown here is c, so lets rearrange to find it.

$$(x_0^2 + 2x_0c\delta x - 2x_0a + c^2\delta x^2 - 2c\delta xa + a^2) + (y_0^2 + 2y_0c\delta y - 2y_0b + c^2\delta y^2 - 2c\delta yb + b^2) = r^2$$

$$(2x_0c\delta x - 2c\delta xa + 2y_0c\delta y - 2c\delta yb) + (c^2\delta x^2 + c^2\delta y^2) + x_0^2 - 2x_0a + a^2 + y_0^2 - 2y_0b + b^2 = r^2$$

$$c(2x_0\delta x - 2\delta xa + 2y_0\delta y - 2\delta yb) + c^2(\delta x^2 + \delta y^2) + (x_0^2 - 2x_0a + a^2 + y_0^2 - 2y_0b + b^2 - r^2) = 0$$

Now we have a $\delta x^2 + \delta y^2$, but because the direction vector is normalized this is term is equal to one. Also, this is a quadratic, which should make sense. If the ray doesn't touch the circle, then there are no real roots. If the ray is tangent, then there is one root, and if the ray passes through the circle, then there are two roots.

$$c(2x_0\delta x - 2\delta xa + 2y_0\delta y - 2\delta yb) + c^2 + (x_0^2 - 2x_0a + a^2 + y_0^2 - 2y_0b + b^2 - r^2) = 0$$

And, fortunately for us, we can use the quadratic equation to find the two roots, and we can make the computer do this operation.

2 Ray-Sphere Collisions

This should be mostly the same as our previous section. Here, the notable items are the equation for a sphere $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$, the origin of the ray (x_0, y_0, z_0), and the direction vector $(\delta x, \delta y, \delta z)$. Again we can plug in some scaled version of the ray using a scalar d as a solution to the sphere equation.

$$(x_0 + d\delta x - a)^2 + (y_0 + d\delta y - b)^2 + (z_0 + d\delta z - c)^2 = r^2$$

And already it's nearly exactly the same as before, so with a guess we can just skip ahead to our solution.

$$d(2x_0\delta x - 2\delta xa + 2y_0\delta y - 2\delta yb + 2z_0\delta z - 2\delta zc) + d^2(\delta x^2 + \delta y^2 + \delta z^2) + (x_0^2 - 2x_0a + a^2 + y_0^2 - 2y_0b + b^2 + z_0^2 - 2z_0c + c^2 - r^2) = 0$$