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Probability and Applied Statistics Formula Sheet

Definition 1.1 - Mean

This finds the mean of a sample:

Mean: $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

- Population Mean: μ

Definition 1.2 - Variance

Variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2$

- Population Variance: σ^2

Definition 1.3 – Standard Deviation

Standard Deviation: $s = \sqrt{s^2}$

- Population Standard Deviation: $\sigma = \sqrt{\sigma^2}$

Definition 2.7 – Permutation

Permutation Equation: $P_r^n = \frac{n!}{(n-r)!}$

Definition 2.8 - Combination

Combination Equation: $\binom{n}{r} = C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$

Definition 2.9 – Conditional Probability

Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Definition 2.10 – Independence/Dependence

Events A and B are independent if:

$$P(A|B) = P(A), P(B|A) = P(B),$$

OR $P(A \cap B) = P(A)P(B)$

Otherwise, events are dependent

(Theorem 2.5 - The Multiplicative Law of Probability)

If events are dependent,

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

If events are independent,

$$P(A \cap B) = P(A)P(B)$$

If events are mutually exclusive,

$P(A \cap B) = 0$ AND

$P(A \cup B) = P(A) + P(B)$

(Theorem 2.6 – The Additive Law of Probability)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(Theorem 2.7 – Complement)

$$P(A) = 1 - P(\overline{A})$$

(Theorem 2.8 – Theorem of Total Probability)

Theorem of Total Probability: $P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$

(Theorem 2.9 – Bayes' Rule)

Bayes' Rule: $P\left(B_j \middle| A\right) = \frac{P(A \cap B_j)}{P(A)} = \frac{P\left(A \middle| B_j\right)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$

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Definition 3.4 – Discrete Random Variable

When Y is a discrete random variable,

Expected Value: $E(Y) = \sum_{y} yp(y)$

Variance: $V(Y) = E[\left(y - E(Y)\right)^{2}]$

Standard Deviation: $S(Y) = \sqrt{V(Y)}$

Definition 3.7 – Binomial Distribution

Binomial Distributions: $p(y) = \binom{n}{y} p^{y} q^{n-y}$

Expected Value: $E(Y) = np$

Variance: $V(Y) = npq.$

Standard Deviation: $\sqrt{npq}$

Definition 3.8 – Geometric Probability Distribution

Geometric Distributions: $p(y) = q^{y-1}p$

Expected Value: $E(Y) = \frac{1}{p}$

Variance: $V(Y) = \frac{1-p}{p^{2}}$

Standard Deviation: $\sqrt{\frac{1-p}{p^{2}}}$

Defintion 3.9 – Negative Binomial Distribution

$p(y) = \binom{y-1}{r-1} p^{r} q^{y-r}$

Expected Value: $E(Y) = \frac{r}{p}$

Variance: $V(Y) = \frac{r(1-p)}{p^{2}}$

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Definition 3.10 – Hypergeometric Distribution

Hypergeometric Distributions: $p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$

Expected Value: $E(Y) = \frac{nr}{N}$

Variance: $V(Y) = n(\frac{r}{n})(\frac{N-r}{N})(\frac{N-n}{N-1})$

Standard Deviation: $S(Y) = \sqrt{n(\frac{r}{n})(\frac{N-r}{N})(\frac{N-n}{N-1})}$