Formula Sheet

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3.8 Poisson Probability Distributions

Poisson Formula: $p(y) = (\frac{\lambda^y}{y!})e^{-\lambda}$

Expected Value: $E(Y) = \lambda$

Variance: $\sigma^2(Y) = \lambda$

Standard Deviation: $\sigma(Y) = \sqrt{\lambda}$

3.11 Chebyshev's Theorem

$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$ OR $P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$

- Lower Interval Bound = $\mu - k\sigma$
- Upper Interval Bound = $\mu + k\sigma$

4.2 Probability Distribution for a Continuous Random Variable

Distribution Function: $F(y) = \int_{-\infty}^{\infty} f(y)dy$

Properties

$F(y) = P(Y \leq y) \; for \; -\infty < y < \infty$

$F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$

$F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$

$F(y)$ is a nondecreasing function of y $[y_1 < y_2;\ F(y_1) \leq F(y_2)]$

Probability Density Function: $f(y) = \frac{dF(y)}{dy} = F'(y)$

Properties

$f(y) \geq 0$ for all $y, -\infty < y < \infty$

$$\int_{-\infty}^{\infty} f(y)dy = 1$$

Probability of Y in an Interval [a, b]: $P(a \leq Y \leq b) = \int_{a}^{b} f(y)dy$

4.3 Expected Values for Continuous Random Variables

Expected Value of a Continuous Random Variable:

$$E(Y) = \int_{-\infty}^{\infty} yf(y)dy$$

Expected Value of a Function of a Continuous Random Variable:

$$E[g(y)] = \int_{-\infty}^{\infty} g(y)f(y)dy$$

Properties

$E(c) = c$

$E[cg(Y)] = cE[g(Y)]$

$E[g_1(Y) + g_2(Y) + ... + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + ... + E[g_k(Y)]$

4.4 The Uniform Probability Distribution

Uniform Distribution: $f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, \ \theta_1 \le y \le \theta_2 \\ 0, \text{elsewhere} \end{cases}$

Probability of Y in an Interval [a, b]: $P(a \le Y \le b) = \frac{b-a}{d-c}$

Expected Value for a Uniform Distribution: $E(Y) = \frac{\theta_1 + \theta_2}{2}$

Variance for a Uniform Distribution: $\sigma^2(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$

Standard Deviation for a Uniform Distribution: $\sigma(Y) = \sqrt{\frac{(\theta_2 - \theta_1)^2}{12}}$

5.2 Bivariate and Multivariate Probability Distributions

Bivariate Probability Function:

$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2), \quad -\infty < y_1 < \infty, -\infty < y_2 < \infty$

Properties

$p(y_1, y_2) \geq 0 \; for \; all \; y_1, \; y_2$

$\sum p(y_1, y_2) = 1, \; the \; sum \; is \; over \; all \; values \; (y_1, y_2)$

Bivariate Distribution Function:

$F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2), \; -\infty < y_1 < \infty, -\infty < y_2 < \infty$

Properties

$F(-\infty, \infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$

$F(\infty, \infty) = 1$

If $y_1^* \geq y_1 \; and \; y_2^* \geq y_2, \; then$

$$F(y_1^*, y_2^*) - F(y_1^*, y_2) - F(y_1, y_2^*) + F(y_1, y_2) \geq 0$$

Y_1 and Y_2 are jointly continuous random variables if there exists a nonnegative

function $f(y_1, y_2)$ such that, $F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 dt_1$ for all

$-\infty < y_1 < \infty, -\infty < y_2 < \infty$

Properties

$f(y_1, y_2) \geq 0 \; for \; all \; y_1, \; y_2$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y_1, y_2)dy_1 dy_2 = 1$$

5.3 Marginal and Conditional Probability Distributions

Marginal Probability Function:

$$p_1(y_1) = \sum_{all\, y_2} p(y_1, y_2) \qquad \& \qquad p_2(y_2) = \sum_{all\, y_1} p(y_1, y_2)$$

Marginal Density Function:

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)dy_2 \qquad \& \qquad f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)dy_1$$

Conditional Discrete Probability Function:

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1=y_1|Y_2=y_2)}{P(Y_2=y_2)} = \frac{p(y_1,y_2)}{p_2(y_2)},$$

provided that $p_2(y_2) > 0$

Conditional Distribution Function: $F(y_1|y_2) = P(Y_1 \leq y_1|Y_2 = y_2)$

Conditional Density Function:

$f(y_1|y_2) = \frac{f(y_1,y_2)}{f_2(y_2)}$, provided that $f_2(y_2) > 0$

$f(y_2|y_1) = \frac{f(y_1,y_2)}{f_1(y_1)}$, provided that $f_1(y_1) > 0$

5.4 Independent Random Variables

Distribution Function Independence:

Y_1 and Y_2 are *independent* if and only if, $F(y_1, y_2) = F_1(y_1)F_2(y_2)$ for all (y_1, y_2)

Joint Probability Function Independence:

Y_1 and Y_2 are *independent* if and only if, $p(y_1, y_2) = p_1(y_1)p_2(y_2)$ for all (y_1, y_2)

Joint Density Function Independence:

Y_1 and Y_2 are *independent* if and only if, $f(y_1, y_2) = f_1(y_1)f_2(y_2)$ for all (y_1, y_2)

Joint Density Function Independence Using Functions of y_1 and y_2:

Y_1 and Y_2 are *independent* if and only if, $f(y_1, y_2) = g(y_1)h(y_2)$ where $g(y_1)$ is a nonnegative function of y_1 alone and $h(y_2)$ is a nonnegative function of y_2 alone