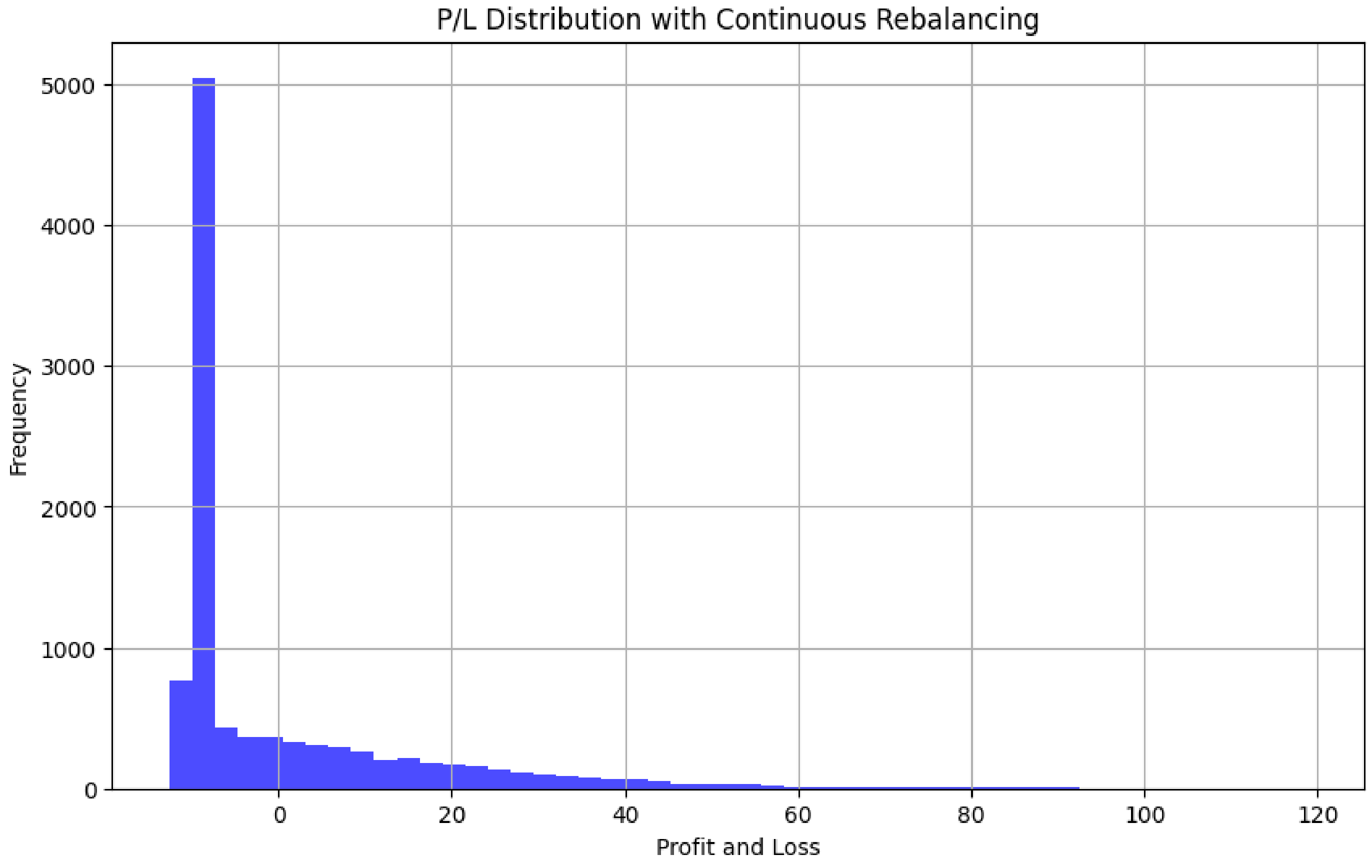
DR 455894 https://github.com/astronaut505/montec

In [1]:
```
#**Question 1:**
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

# Parameters
initial_stock_price = 100
implied_volatility = 0.2
actual_volatility = 0.23
risk_free_rate = 0
dividend_yield = 0
T = 1   # 1 year
n_paths = 10000
n_steps = 1000
dt = T / n_steps

#<--- function to calculate delta of a call option
def option_delta(S, K, T, r, q, sigma):
    d1 = (np.log(S / K) + (r - q + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
    return norm.cdf(d1)

#<--- monte Carlo simulation for stock price paths
np.random.seed(0)
stock_paths = np.zeros((n_paths, n_steps + 1))
stock_paths[:, 0] = initial_stock_price
for t in range(1, n_steps + 1):
    z = np.random.standard_normal(n_paths)
    stock_paths[:, t] = stock_paths[:, t - 1] * np.exp((risk_free_rate - dividend_yield

#<--- delta-hedging strategy with continuous rebalancing
PnL = np.zeros(n_paths)
for i in range(n_paths):
    delta_portfolio = 0
    for t in range(n_steps):
        delta = option_delta(stock_paths[i, t], initial_stock_price, T - t * dt, risk_fr
        stock_price_change = stock_paths[i, t + 1] - stock_paths[i, t]
        PnL[i] += delta_portfolio * stock_price_change
        delta_portfolio = delta  # Update the delta of the portfolio

#<--- calc. expecte value and standard deviation of PnL
expected_PnL = np.mean(PnL)
std_dev_PnL = np.std(PnL)

#<--- results
print(f"Expected Value of P/L: {expected_PnL}")
print(f"Standard Deviation of P/L: {std_dev_PnL}")

#<--- plot a histogram of PnL
plt.figure(figsize=(10, 6))
plt.hist(PnL, bins=50, color='blue', alpha=0.7)
plt.title("P/L Distribution with Continuous Rebalancing")
plt.xlabel("Profit and Loss")
plt.ylabel("Frequency")
plt.grid(True)
plt.show()
```

```
Expected Value of P/L: 0.22158094141167012
Standard Deviation of P/L: 15.778958824008491
```



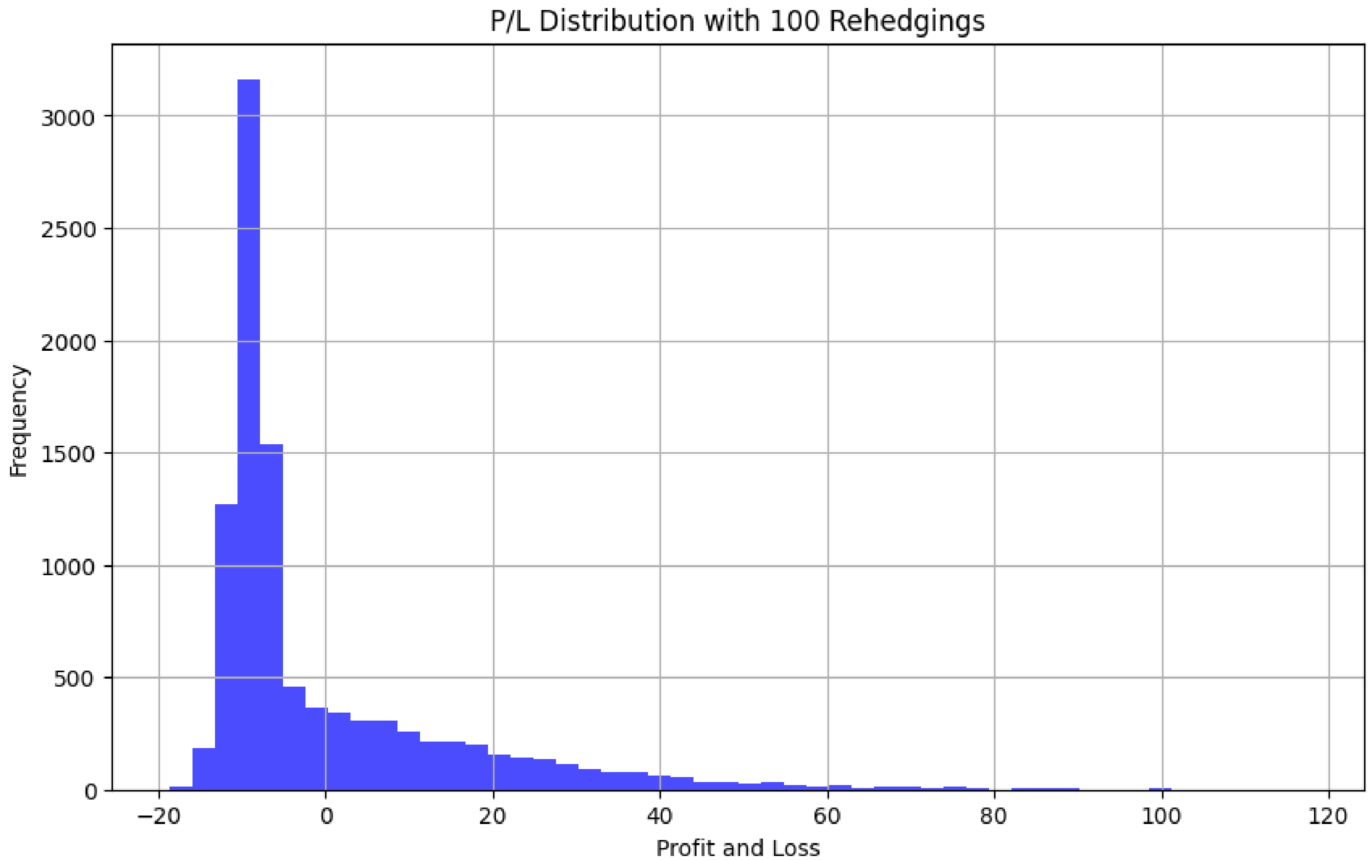
Restults for question 1: Outcomes of Continuous Zero-Delta Rebalancing

The continuous zero-delta rebalancing yields an expected P/L of **$0.22**, suggesting a marginal average gain. However, the strategy entails significant risk, as reflected by a standard deviation of **$15.78**, indicating a wide dispersion of potential outcomes around the mean. Let's check for next strategy.

In [2]:
```
#Question 2 (a):
#<--- parameters for the rehedging strategy
rehedge_steps = 100   # Reducing rehedgings to 100
transaction_cost = 0.001   # 0.1%

#<--- delta-hedging with reduced rehedging (100 steps)
PnL_reduced_100 = np.zeros(n_paths)
for i in range(n_paths):
    delta_portfolio = 0
    for t in range(0, n_steps, n_steps // rehedge_steps):  # Equally spaced rehedgings
        delta = option_delta(stock_paths[i, t], initial_stock_price, T - t * dt, risk_fr
        stock_price_change = stock_paths[i, t + n_steps // rehedge_steps] - stock_paths[
        PnL_reduced_100[i] += delta_portfolio * stock_price_change
        delta_change = delta - delta_portfolio
        transaction_cost_penalty = abs(delta_change * stock_paths[i, t]) * transaction_c
        PnL_reduced_100[i] -= transaction_cost_penalty
        delta_portfolio = delta

#<--- calculating expected value and standard deviation of PnL for reduced rehedging (10
expected_PnL_100 = np.mean(PnL_reduced_100)
std_dev_PnL_100 = np.std(PnL_reduced_100)

#<--- results and plot histogram
print(f"Expected Value of P/L with 100 Rehedgings: {expected_PnL_100}")
print(f"Standard Deviation of P/L with 100 Rehedgings: {std_dev_PnL_100}")

plt.figure(figsize=(10, 6))
plt.hist(PnL_reduced_100, bins=50, color='blue', alpha=0.7)
```

```python
plt.title("P/L Distribution with 100 Rehedgings")
plt.xlabel("Profit and Loss")
plt.ylabel("Frequency")
plt.grid(True)
plt.show()
```

```
Expected Value of P/L with 100 Rehedgings: -0.174465401469347
Standard Deviation of P/L with 100 Rehedgings: 15.700399422620658
```



Results for question 2. (a): Analysis of Delta Hedging with 100 Rehedgings

Reducing rehedgings to 100 steps resulted in an expected P/L of approximately **-$0.17,** indicating an average loss per simulation. The standard deviation of P/L is approximately **$15.70**, suggesting a similar risk profile to continuous rebalancing. The P/L distribution displays a concentration of outcomes around the negative expected value, with fewer instances of large gains or losses. This strategy may not be preferable given the negative expected value, despite a marginally lower standard deviation compared to continuous rebalancing.

In [3]:

```python
#**Question 2 (b):**
# Reducing rehedgings to 10
rehedge_steps = 10

#<--- delta-hedging with reduced rehedging (10 steps)
PnL_reduced_10 = np.zeros(n_paths)
for i in range(n_paths):
    delta_portfolio = 0
    for t in range(0, n_steps, n_steps // rehedge_steps):  # Equally spaced rehedgings
        delta = option_delta(stock_paths[i, t], initial_stock_price, T - t * dt, risk_fr
        stock_price_change = stock_paths[i, t + n_steps // rehedge_steps] - stock_paths[
        PnL_reduced_10[i] += delta_portfolio * stock_price_change
        delta_change = delta - delta_portfolio
        transaction_cost_penalty = abs(delta_change * stock_paths[i, t]) * transaction_c
```

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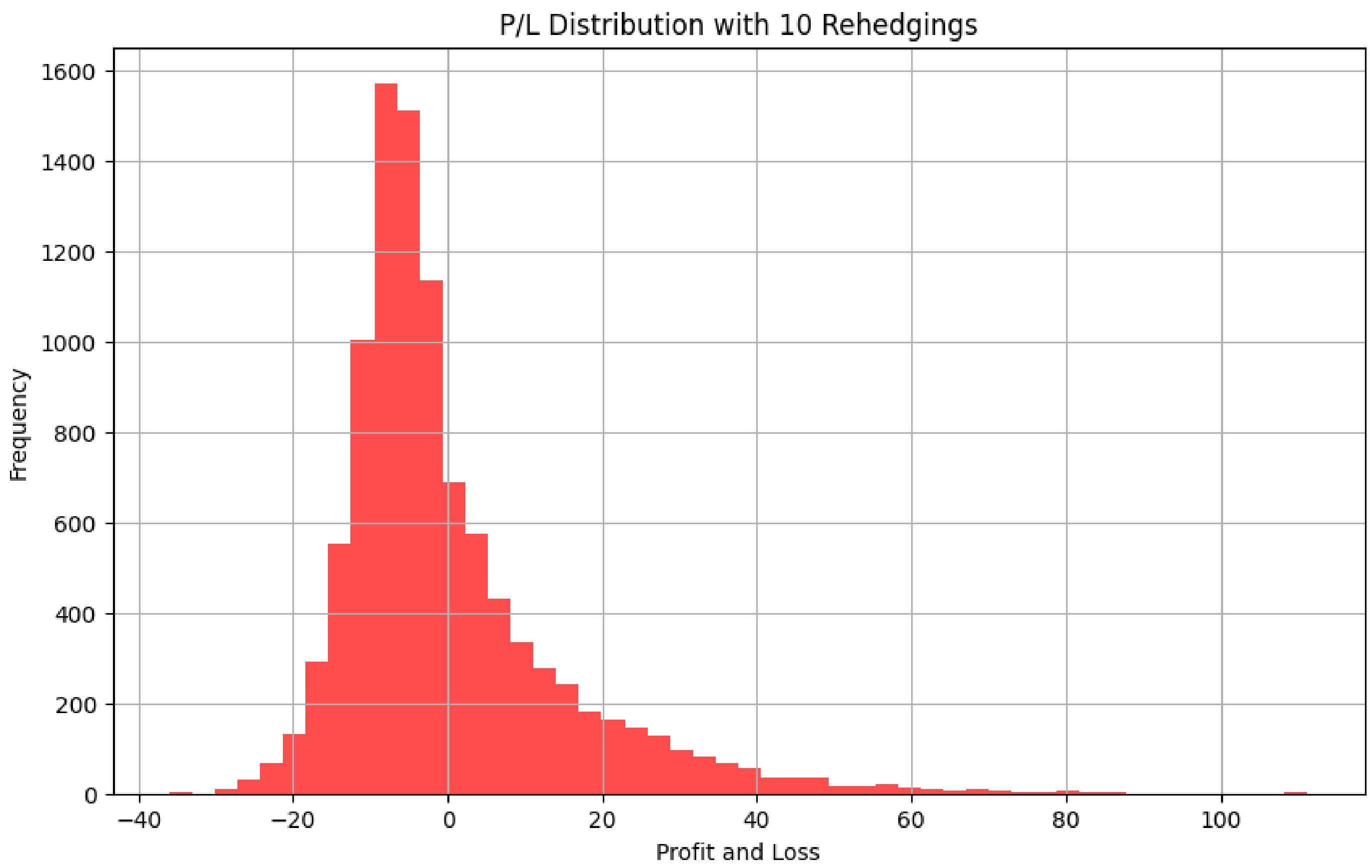
        PnL_reduced_10[i] -= transaction_cost_penalty
        delta_portfolio = delta

#<--- calculating expected value and standard deviation of PnL for reduced rehedging (10
expected_PnL_10 = np.mean(PnL_reduced_10)
std_dev_PnL_10 = np.std(PnL_reduced_10)

#<--- results and plot histogram
print(f"Expected Value of P/L with 10 Rehedgings: {expected_PnL_10}")
print(f"Standard Deviation of P/L with 10 Rehedgings: {std_dev_PnL_10}")

plt.figure(figsize=(10, 6))
plt.hist(PnL_reduced_10, bins=50, color='red', alpha=0.7)
plt.title("P/L Distribution with 10 Rehedgings")
plt.xlabel("Profit and Loss")
plt.ylabel("Frequency")
plt.grid(True)
plt.show()
```

```
Expected Value of P/L with 10 Rehedgings: 0.04076704350260095
Standard Deviation of P/L with 10 Rehedgings: 14.3895961547026
```



Results for question 2. (b): Evaluation of Delta Hedging with 10 Rehedgings

When the hedging strategy is adjusted to only 10 rehedgings, the expected P/L shows a slight average gain of approximately **$0.04**. This contrasts with the strategy involving 100 rehedgings, which indicated a small average loss. Moreover, the standard deviation of P/L reduces to about **$14.39**, depicting a decrease in risk compared to the 100 rehedging steps.

The histogram for the 10 rehedgings strategy presents a more centralized distribution of outcomes around the small positive expected value, with a reduced frequency of extreme results. This approach suggests a more consistent performance with lower variability, which could be considered a more efficient strategy given the reduced risk and the shift from a negative to a positive expected return.

In [4]:

```
#**Question 2 (c):**
#<--- Trigger rehedging strategy
delta_trigger = 0.05  # Rebalance if delta changes by 5 percentage points

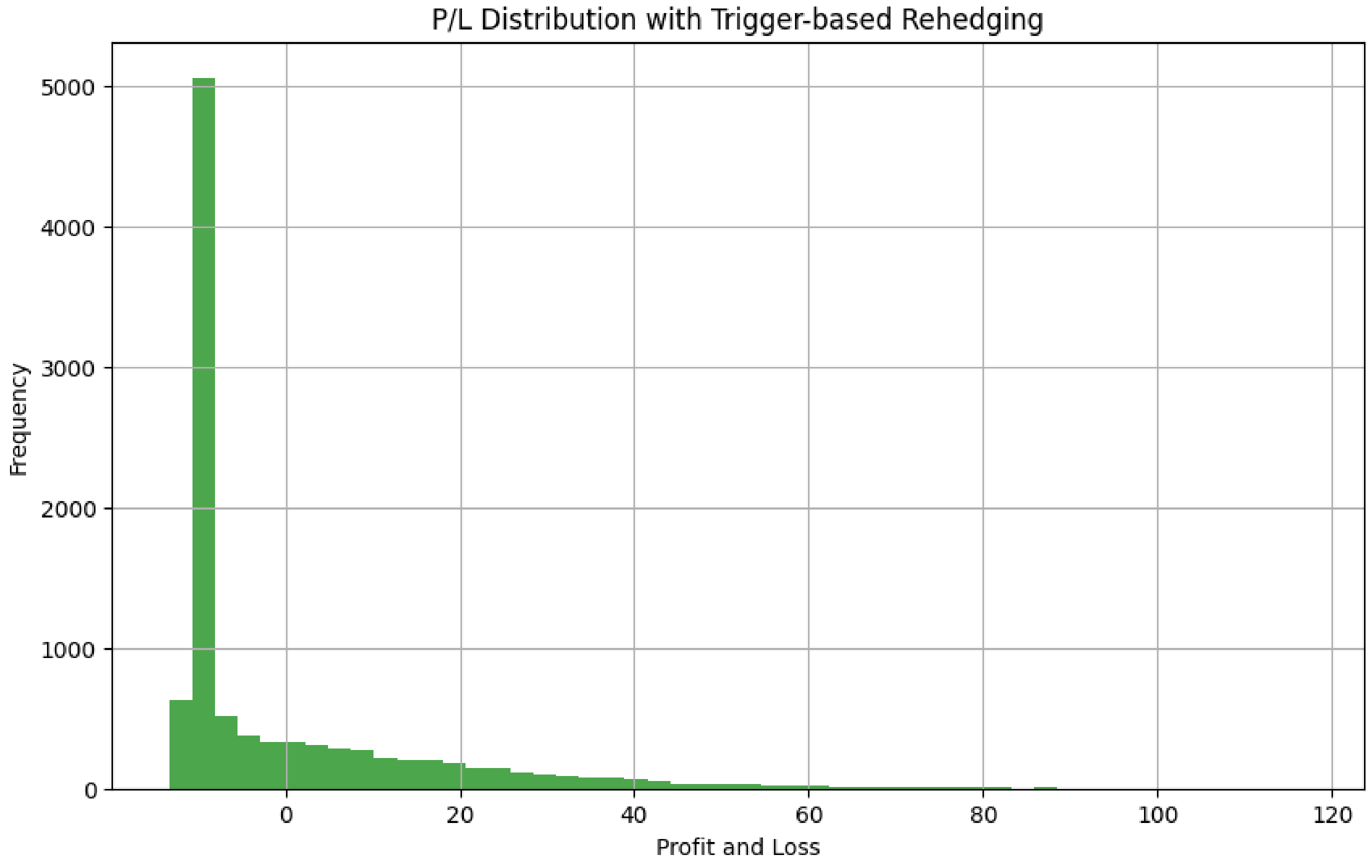
#<--- delta-hedging with trigger-based rehedging
PnL_trigger_based = np.zeros(n_paths)
for i in range(n_paths):
    delta_portfolio = 0
    last_delta = 0
    for t in range(n_steps):
        delta = option_delta(stock_paths[i, t], initial_stock_price, T - t * dt, risk_fr
        if t == 0 or abs(delta - last_delta) >= delta_trigger:  # Check for trigger
            delta_change = delta - delta_portfolio
            transaction_cost_penalty = abs(delta_change * stock_paths[i, t]) * transacti
            PnL_trigger_based[i] -= transaction_cost_penalty
            delta_portfolio = delta
            last_delta = delta
        stock_price_change = stock_paths[i, t + 1] - stock_paths[i, t]
        PnL_trigger_based[i] += delta_portfolio * stock_price_change

#<--- calculating expected value and standard deviation of PnL for trigger-based rehedgi
expected_PnL_trigger = np.mean(PnL_trigger_based)
std_dev_PnL_trigger = np.std(PnL_trigger_based)

#<--- display results and plot histogram
print(f"Expected Value of P/L with Trigger-based Rehedging: {expected_PnL_trigger}")
print(f"Standard Deviation of P/L with Trigger-based Rehedging: {std_dev_PnL_trigger}")

plt.figure(figsize=(10, 6))
plt.hist(PnL_trigger_based, bins=50, color='green', alpha=0.7)
plt.title("P/L Distribution with Trigger-based Rehedging")
plt.xlabel("Profit and Loss")
plt.ylabel("Frequency")
plt.grid(True)
plt.show()
```

```
Expected Value of P/L with Trigger-based Rehedging: -0.25019406704507635
Standard Deviation of P/L with Trigger-based Rehedging: 15.727085914326965
```



Results for question 2. (c): Insights on Trigger-based Rehedging Strategy

The trigger-based rehedging strategy, which involves portfolio rebalancing when delta changes by 5 percentage points, has an expected P/L of approximately **-$0.25**. This reflects a small average loss for each simulation, indicating that this method might not effectively capitalize on the volatility spread.

Moreover, the standard deviation of the P/L is about **$15.73**, which is comparable to the continuous rebalancing strategy, suggesting a similar risk level. The histogram displays a tight clustering of outcomes near zero but with a noticeable lean towards losses, confirming the negative expected value.

This trigger-based approach, despite potentially reducing transaction costs, does not seem to enhance the strategy's performance in terms of risk-adjusted returns. The negative expected value combined with a high standard deviation points to an inefficient balance, as the reduction in transaction costs does not appear to compensate adequately for the risks involved.

Summary

Based on the analysis of the P/L distributons from the histograms and the expected values and standard deviations for each of the hedging strategies, we can draw the following conclusions to determine the superior outcome:

- **Continuous Rebalancing (Question 1)**: This strategy showed a slight average gain **(Expected P/L: $0.22)** but came with a high level of risk **(Standard Deviation: $15.78)**.

- **100 Rehedgings (2a)**: The strategy resulted in an average loss **(Expected P/L: -$0.17)** with a standard deviation **($15.70)** slightly lower than continuous rebalancing but still substantial.

- **10 Rehedgings (2b)**: This method yielded a small average gain **(Expected P/L: $0.04)** and **presented the lowest standard deviation ($14.39)** among the strategies, indicating the least variability in P/L.
- **Trigger-based Rehedging (2c)**: This approach showed an average loss **(Expected P/L: -$0.25)** and a **standard deviation ($15.73)** comparable to continuous rebalancing.

Considering the goal of achieving the highest expected P/L with the lowest standard deviation, the strategy with **10 rehedgings (2b)** stands out as the most efficient. It not only reverses the loss seen in 100 rehedgings and trigger-based strategies to a profit but also reduces the risk compared to the other strategies. Therefore, based on the given data, reducing the number of rehedgings to 10 provides a **superior outcome**.