ABFT

Example: Algorithm Based Fault Tolerence (ABFT)

Let $e^T = [1, 1, \cdots, 1]$, we define

$$A^c := \begin{pmatrix} A \\ e^T A \end{pmatrix},\ B^r := \begin{pmatrix} B & Be \end{pmatrix},\ C^f := \begin{pmatrix} C & Ce \\ e^T C & e^T Ce \end{pmatrix}.$$

Where A^c is the **column checksum matrix**, B^r is the **row checksum matrix** and C^f is the **full checksum matrix**.

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$$\begin{aligned} A^c \times B^r &= \begin{pmatrix} A \\ e^T A \end{pmatrix} \times \begin{pmatrix} B & Be \end{pmatrix} \\ &= \begin{pmatrix} AB & ABe \\ e^T AB & e^T ABe \end{pmatrix} = \begin{pmatrix} C & Ce \\ e^T C & e^T Ce \end{pmatrix} = C^f \end{aligned}$$

ABFT: Detection

Let us build a small example:

$$A^c = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 2 \\ \color{blue}{5} & \color{blue}{4} & \color{blue}{2} \end{pmatrix},\ B^r = \begin{pmatrix} 1 & 1 & 1 & \color{blue}{3} \\ 2 & 0 & 3 & \color{blue}{5} \\ 0 & 2 & 2 & \color{blue}{4} \end{pmatrix},$$

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$$C^f = A^c \times B^r = \begin{pmatrix} 5 & 1 & 7 & \color{blue}{13} \\ 4 & 3 & 5 & \color{blue}{11} \\ 4 & 6 & 9 & \color{blue}{19} \\ \color{blue}{13} & \color{blue}{9} & \color{blue}{21} & \color{blue}{43} \end{pmatrix}$$

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Everything seems fine. However, a silent error has occurred!

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$$C^f = A^c \times B^r = \begin{pmatrix} 5 & 1 & 7 & 13 \\ 4 & 3 & 5 & 11 \\ 4 & 6 & 9 & 19 \\ 13 & 9 & 21 & 43 \end{pmatrix}$$

Everything seems fine. However, a silent error has occurred!

Indeed, recomputing the checksums we find that:

$$\begin{pmatrix} 5 & + & 1 & + & 7 & = & 13 \\ 4 & + & 3 & + & 5 & = & 12 \\ 4 & + & 6 & + & 9 & = & 19 \\ 13 & + & 10 & + & 21 & = & 44 \end{pmatrix} \text{ Checksums do not match!}$$

ABFT: Correction

$$C^f = A^c \times B^r = \begin{pmatrix} 5 & 1 & 7 & 13 \\ 4 & 3 & 5 & 11 \\ 4 & 6 & 9 & 19 \\ 13 & 9 & 21 & 43 \end{pmatrix}, \begin{pmatrix} 5 & + & 1 & + & 7 & = & 13 \\ 4 & + & 3 & + & 5 & = & 12 \\ 4 & + & 6 & + & 9 & = & 19 \\ 13 & + & 10 & + & 21 & = & 44 \end{pmatrix}$$

Both checksums are affected, giving out the location of the error.

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Both checksums are affected, giving out the location of the error.

We solve:

$$\begin{aligned} 4 + x + 5 &= 11 \\ x &= 11 - 5 - 4 = 2 \end{aligned} \qquad \begin{aligned} 1 + x + 6 &= 9 \\ x &= 9 - 6 - 1 = 2 \end{aligned}$$

ABFT: Correction

$$C^f = A^c \times B^r = \begin{pmatrix} 5 & 1 & 7 & 13 \\ 4 & 3 & 5 & 11 \\ 4 & 6 & 9 & 19 \\ 13 & 9 & 21 & 43 \end{pmatrix}, \begin{pmatrix} 5 & + & 1 & + & 7 & = & 13 \\ 4 & + & 3 & + & 5 & = & 12 \\ 4 & + & 6 & + & 9 & = & 19 \\ 13 & + & 10 & + & 21 & = & 44 \end{pmatrix}$$

Both checksums are affected, giving out the location of the error.

We solve:

$$4 + x + 5 = 11 \qquad\qquad 1 + x + 6 = 9$$

$$x = 11 - 5 - 4 = 2 \qquad\qquad x = 9 - 6 - 1 = 2$$

Recomputing the checksums we find that:

$$\begin{pmatrix} 5 & + & 1 & + & 7 & = & 13 \\ 4 & + & 2 & + & 5 & = & 11 \\ 4 & + & 6 & + & 9 & = & 19 \\ 13 & + & 9 & + & 21 & = & 43 \end{pmatrix}$$ Checksums match ☺