Combined Peano scan and VQ approach to image compression

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ABSTRACT

In order to achieve high compression ratios, while maintaining visual quality, a new approach combining Peano scanning and vector quantization is proposed. The Peano scan technique clusters highly correlated pixels, and a vector quantization scheme is developed to exploit interblock correlation.

An efficient scan technique serves as a useful preprocessing unit in image compression. In this work, Peano curves are used locally to transform the original image into one containing clustered bands of correlated data. The Peano scan results in a much lower mean absolute difference between neighbouring blocks than the Raster scan (almost 50% reduction) i.e. better reordering/clustering is achieved. A codebook is formed using the LBG algorithm. The clustered image is subdivided into blocks of 4*4 and vector quantized using the codebook. The pattern of indices of codevectors chosen for successive blocks is exploited. Due to clustering achieved, frequently the same codevector is used to encode a series of successive blocks. Transmission of the same codevector index for each block is inefficient. A method of assigning additional bits to encode the repeat pattern is proposed. This causes an overall reduction in bit-rate. Another technique employs dynamic partitioning of the codebook into a large passive part and a smaller active part. If the mean squared difference between successive blocks is below a predetermined threshold, only the active part is searched. This leads to lowering of bit-rate and search time.

The combined approach suggested exploits interblock correlation in an image, using an efficient scan technique. It leads to lower bit-rates than conventional VQ methods, at comparable visual quality.

1. INTRODUCTION

Digital images require a large amount of data for their representation. Often digitization is done for 256 grey levels, requiring eight bits per pixel for encoding. For an image of size 256*256 pixels or 512*512 pixels, coded at eight bits per pixel, a large amount of data is generated, requiring considerable storage memory. Transmission of digital images, especially video, requires high bit-rates and consequently channels with large bandwidths. Current research is toward integrating different forms of information over one channel, for eg. combining data, voice and video signals over ISDN lines. Bandwidth is always a constraint and therefore the principal goal in the design of an image encoding system is to minimize the bit-rate, subject to some image quality constraints.

Image information is particularly suited for compression due to considerable spatial and temporal redundancy. A variety of techniques for image compression have been proposed and implemented. These include transform coding, wavelet decomposition, vector quantization etc. In this paper, a new direction for compression is proposed. The key idea is to use an efficient scanning and reordering technique so that the original image is transformed into a highly correlated one. The Peano scan was found to perform very well. Then, an efficient vector quantization scheme is proposed to exploit the correlation achieved by scanning.

2. PEANO SCAN AND REORDERING

The incorporation of an efficient scan strategy into an encoding scheme results in clustering highly correlated blocks in an image[1] . The motivation is that clustering leads to higher performance of the compression algorithm.

In this approach, the Peano scan is used locally on two-dimensional image data. It transforms the original data into another two-dimensional set wherein correlated pixels are clustered. Digital images of size M*N pixels are sub divided into blocks of m*m. The Peano scan is applied to each block of m*m. This is shown in Fig. 1 for a block of 8*8 pixels. The m^2 pixels thus obtained are laid out in a line. These one-dimensional Peano scanned lines corresponding to the original sub-blocks are ordered with respect to the interblock correlation existing in the original image (Fig. 2). In this manner, the original image is transformed into an image of size $\frac{M}{m} * Nm$. The new image contains clustered bands of image data. Fig. 3 shows an original 256*256 image transformed to 32*2048 by Peano scanning blocks of 8*8. (The 32*2048 is shown as a 256*256 image, for practical reasons).

The reason for clustering achieved is due to the properties of the Peano curve. It can be easily inferred that the Peano curve passes through every element in the digital space, in such a manner that points close on the curve are actually close in space. Conversely, points close in space are likely to be close on the Peano curve. Thus, the curve acts as an excellent transform mechanism from a line to n-dimensional space, while maintaining the correlation inherent in the n-dimensional space.

The Peano scan is more effective than the Raster scan in clustering. The mean absolute difference (MAD) between neighbouring blocks for the Peano scanned image is much lower than that for the Raster scanned one. Fig. 4. illustrates this result for a set of test images. Sorek and Zeevi[2] showed that for on-line visual data compression along a one dimensional scan, the Peano scan is closer to the optimum. The image with clustered bands of data, is then vector quantized.

3. VECTOR QUANTIZATION

A vector quantizer is a system for mapping a sequence of continuous or discrete vectors into a digital sequence suitable for communication over a digital channel[3]. A fundamental result of Shannon's rate distortion theory is that better performance can always be achieved by coding vectors instead of scalars.

A good metric for system performance is the average squared error between the vectors chosen for encoding and the set of original input vectors. For a set of vectors of dimension k, it can be shown that the kth dimensional Euclidean centroid would minimize the average distortion. A set of such centroids can be used to partition the space into regions with a centroid each. All the input vectors in a particular region are coded in terms of the centroid of that region. Each centroid is called a codevector and a set of codevectors form a codebook. Design of a good codebook is important for performance of the image compression algorithm.

Lloyd's algorithm for optimal PCM design can be extended for vectors. The algorithm has the following steps:

Step 1. Given: A training sequence and an initial decoder.

Step 2. Encode the training sequence into a sequence of channel symbols using the given decoder minimum distortion rule. If the average distortion is small enough, quit.

Step 3. Replace the old reproduction codeword of the decoder for each channel symbol ν by the centroid of all training vectors which mapped into ν in step 2. Go to step 2.

Each step must either reduce the distortion or leave it unchanged. The algorithm is stopped when the distortion falls below a defined threshold. Linde, Buzo and Gray[4] have developed this algorithm for vector quantizers, training vectors and general distortion measures and the algorithm is often called the LBG algorithm.

In this paper the LBG algorithm with successive splitting of codevectors has been used to form a codebook of predefined size.

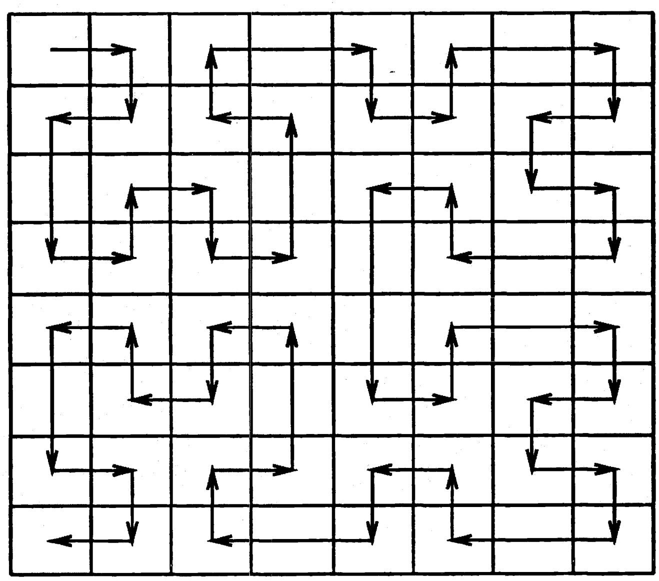

Fig. 1: Peano scan of a block of size 8x8

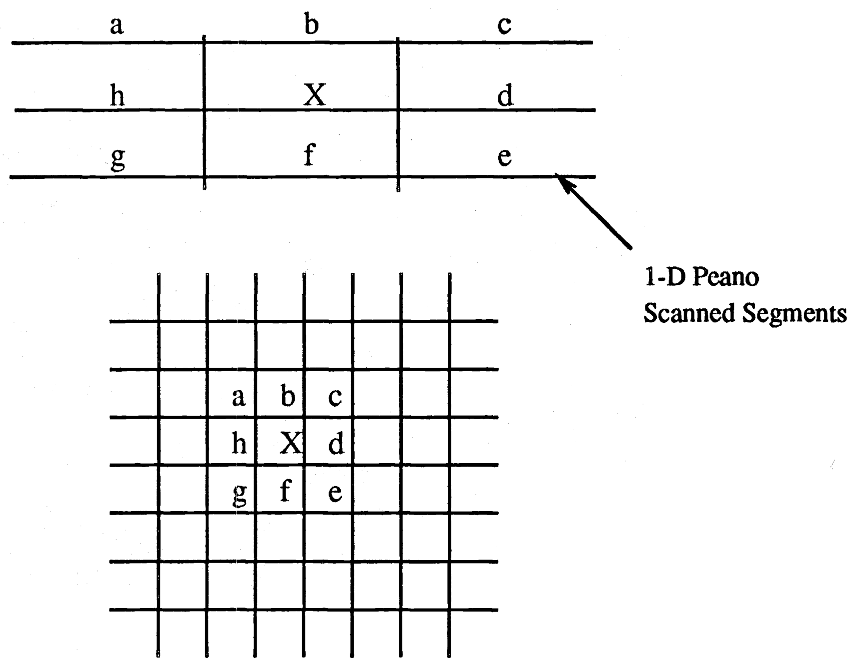

Fig. 2: Ordering of sub-blocks of size 8x8 pixels

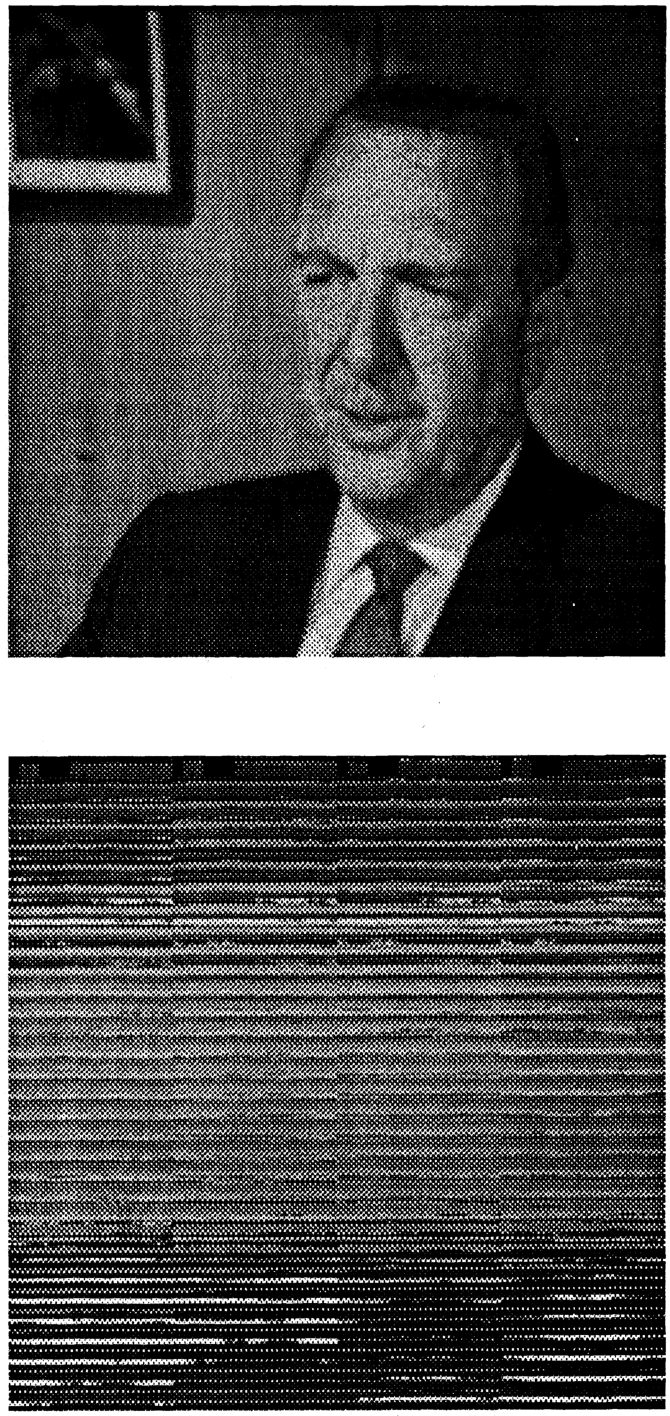

Fig. 3: (a) Original Raster scanned image (b) Clustered bands of correlated data in a Peano scanned image

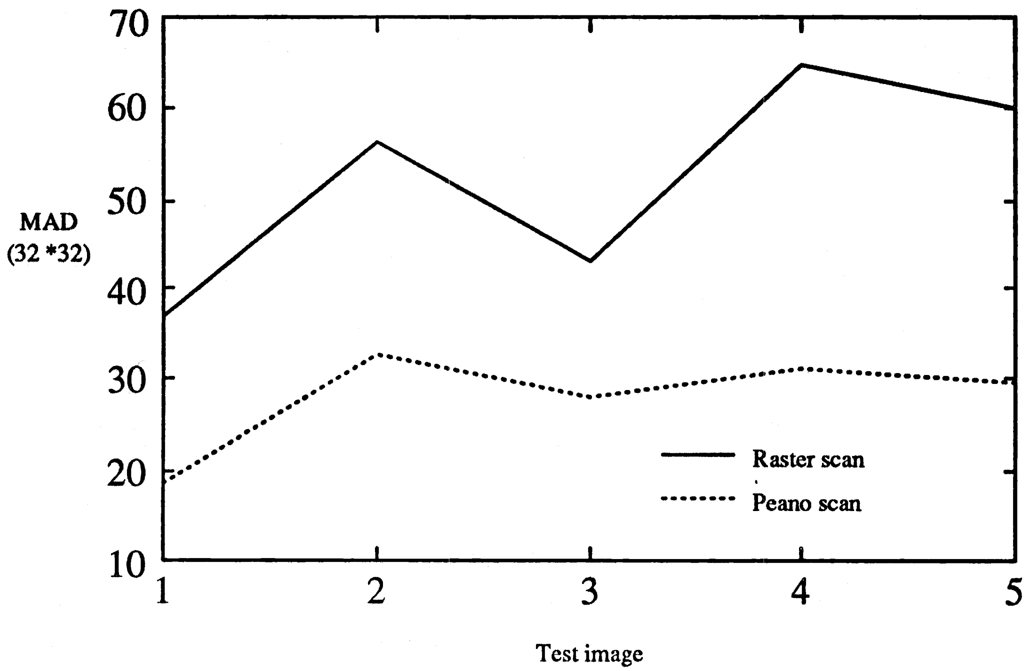

Fig. 4: Mean absolute difference (MAD) between successive blocks of 32x32 by Raster and Peano scans for different test images

4. THE COMBINED APPROACH

The motivation behind combining clustering algorithms with VQ encoding is to tailor the image data to improve the performance of the compression algorithm. The gains could be in terms of speed and more importantly, the amount of information to be transmitted. It should be noted that the cost in doing so must be measured in terms of reduction in the fidelity of the reconstructed information. For image compression, the goal would then be good quality image reproduction at reduced bit-rate. A description of some of the experiments performed and results obtained using the combined approach as well as plain VQ follows.

4.1 Codebook generation

Five images of 256*256 were chosen for generating training vectors. These images were Peano scanned using blocks of 8*8 as described earlier. The clustered images of 32*2048 were then divided into blocks of 4*4 pixels. The sixteen pixels corresponding to one block constitute a vector. A data file comprising of vectors from these images was generated and used as training data for the LBG algorithm. Three codebooks of sizes 128, 512 and 1024 codevectors were obtained in this manner.

4.2 Encoding-decoding without the Peano scan

Each digital image of size 256*256 is subdivided into blocks of 4*4 pixels without Peano scanning. The sixteen pixel values form the elements of one vector. For each vector the entire codebook is scanned and the one that yields the minimum mean absolute difference is chosen to encode the block. Thus each block is represented by the number or index of the codevector in the codebook used to encode it. The bit-rate, thus becomes the number of bits required to represent the index divided by the number of elements in a vector. For example, the bit-rate for the codebook of size 512 is 9/16=0.5625 bits/pixel. In this manner all the blocks in the image are are encoded in terms of the "best" codevector from the codebook. The decoder needs to have the same codebook with the same indexing of codevectors. A measure of distortion between the reconstructed image and the original is the SNR which in turn is a function of the mean square error (MSE) between the images. It is given by

$$SNR = 10.0 \log(\frac{255 * 255}{MSE})\ dB \tag{1}$$

The above procedure was implemented for six images; five of which were used as training vectors for codebook generation and one outside. It must be noted that plain VQ does not exploit any feature of a particular image and its performance in terms of bit-rate is entirely dependent on the size of the codebook used and the number of elements in each codevector. Therefore the bitrate for a selected codebook is the same for all images. This is an inefficient technique because, in practice, most images have significant redundancy that can be exploited by proper scanning and reordering.

4.3 Peano scan and VQ with coding of repeat pattern

A digital image of size 256*256 pixels is subdivided and transformed into clustered bands of 32*2048 pixels using the Peano scan as before. These bands of clustered data are then subdivided into blocks of 4*4 and encoded using VQ as described earlier. Since neighbouring blocks are highly correlated (due to Peano scanning), frequently, a series of successive blocks are encoded using the same codevector. Transmission of the same codevector index for each of the blocks is inefficient. In this method, additional bits are assigned to indicate the number of blocks for which the codevector index is to be used. It was found that the number of repeats vary from 2-10 for images with detail and as high as 30-70 for images with high spatial redundancy. At the decoder, the codevector index and the repeat code are interpreted and the image is reconstructed using the same codebook and an inverse Peano scan for declustering.

This technique performs well for most images considered. For each of the codebooks, substantial lowering of bit-rate is achieved, without change in SNR. For images with less detail, this technique performs very well whereas, for images with significant detail, the gains are lower. A key issue to address here is the choice of number of additional bits that would be optimal. If the number of additional bits assigned per block is less, bit-rate reduction is not significant. On the other hand if the number of additional bits is high, one may exceed the earlier bit-rate. Results were obtained for 2,3,4 and 5 repeat bits for each image. These are listed in table 1 corresponding to the codebook of 512 codevectors. It was observed that assigning more than 5 bits to encode the repeat pattern increased the bit-rate for all images.(In some cases, even using four bits increased the bit-rate). It must be noted that bit-rate reduction is achieved without any degradation in SNR from the plain VQ case.

Image	No rep. (bpp)	2 bit rep. (bpp)	3 bit rep. (bpp)	4 bit rep. (bpp)	5 bit rep. (bpp)
Camera Man	0.5625	0.3962	0.3895	0.4045	0.4287
Lena	0.5625	0.3051	0.3391	0.3729	0.4069
Hat	0.5625	0.4831	0.5134	0.5528	0.5952
Peppers	0.5625	0.4116	0.4168	0.4426	0.4736
Cronk	0.5625	0.3333	0.3100	0.3164	0.3362

Table 1: Bit-rates using fixed number of bits to encode repeat pattern of codevector indices. Codebook size is 512

Another technique that can be used to encode the repeat pattern is to use variable number of bits. If the repeat pattern can be characterized on the basis of frequency or probability of occurence, an efficient scheme such as Huffman coding can be employed. On the basis of large number of experiments, frequency/probability of a particular number of repeats can be obtained. It can be expected that the frequency/probability decreases exponentially as the number of repeats increases. This is evident from Fig. 5. On the y-axis is the frequency of occurence and on the x-axis the number of repeats. For example, from the figure we can infer that out of 4096 blocks the number of times a codevector remains the same over three successive blocks is greater than 100. By limiting the number of repeats allowed to 16 (others can be expressed as combinations of 1-16), a Huffman code can be systematically obtained. (Fig. 6). This is more efficient than assigning four additional bits per block. This technique brings out the essence of the combined approach. It exploits the correlation between blocks in a Peano scanned-reordered image.

A few issues still need to be investigated in this regard. One is the problem of synchronizing the transmitter and receiver especially if a Huffman coding set up is used. Since the Huffman code is derived with the "prefix" condition satisfied, in the absence of channel errors, perfect decoding can be attained. Since there is less signaling involved between transmitter and receiver, a signaling tone can be used for a very short period to signify the end of transmission of a packet.

Table 2 gives a list of images and the bit-rates obtained by plain VQ, Peano scan and VQ with four bits for encoding repeat pattern, and bit-rate obtained using Huffman coding for the repeat pattern. It can be seen that substantial gains are achieved in encoding the repeat pattern. For example, for the image "Cronk", the bit-rate is reduced from 0.625 to 0.273, at the same SNR.

Image	Plain VQ (bpp)	Fixed 4 bits (bpp)	Huffman coded, 16 level(bpp)
Camera Man	0.6250	0.4010	0.3858
Hat	0.6250	0.4960	0.4680
Lena	0.6250	0.3921	0.3810
Peppers	0.6250	0.4210	0.3850
Cronk	0.6250	0.2800	0.2730

Table 2: Bit-rates obtained by ordinary VQ, fixed bit coding of repeat pattern and Huffman coding of repeat pattern. Codebook size is 1024

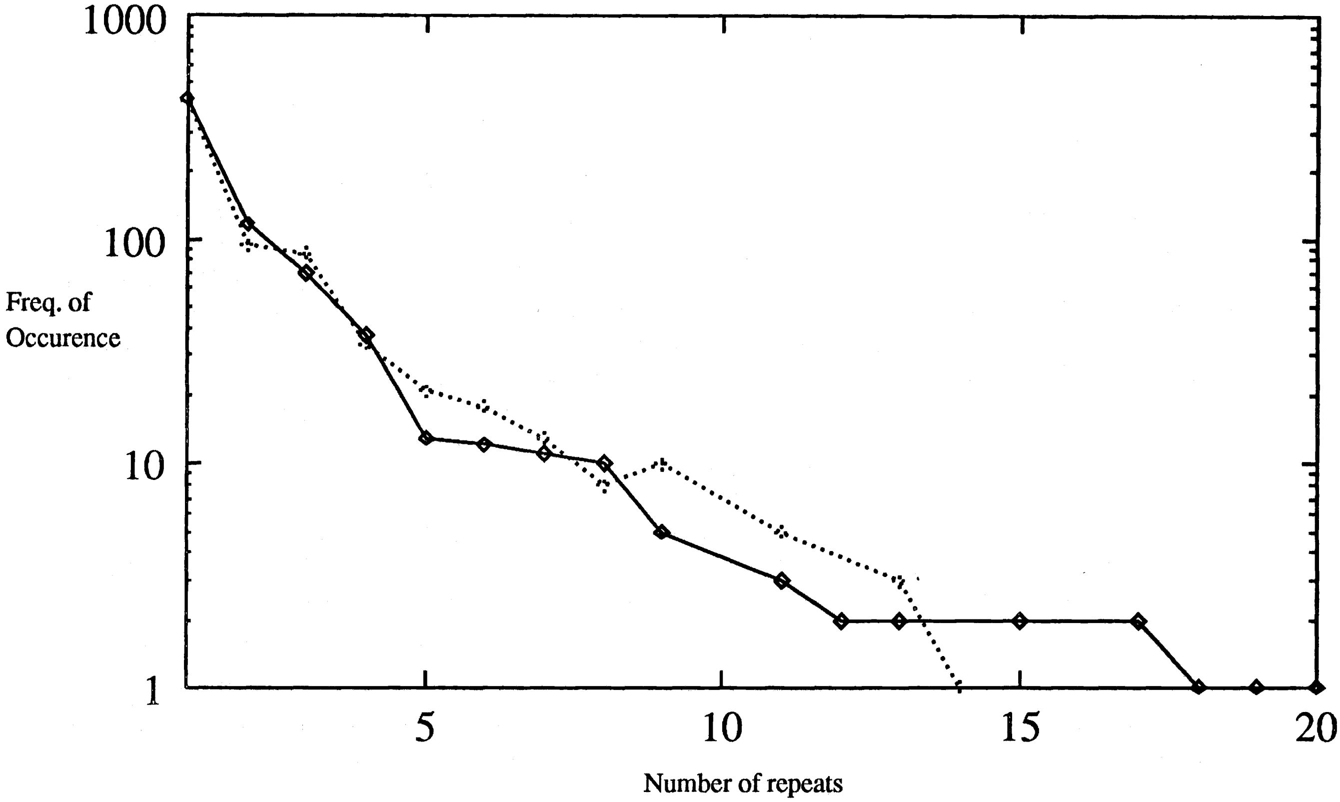

Fig. 5: Frequency of occurence of different repeats for two images. Total number of blocks in each image is 4096.

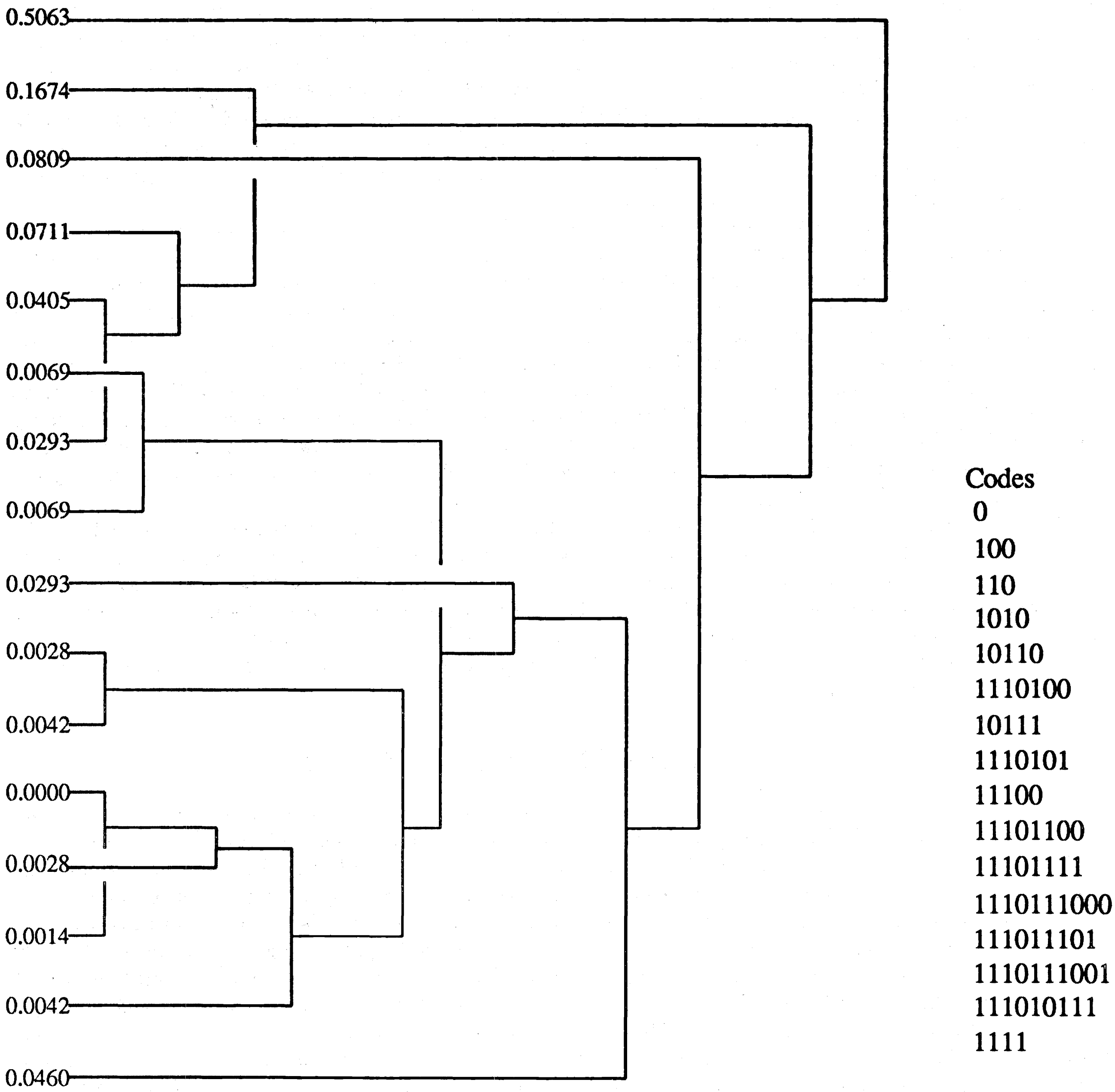

Fig. 6. Systematic 16 level Huffman code for encoding the repeat pattern

4.4 Peano scan and VQ with dynamic codebook partitioning

This technique employs a dynamic partitioning of the codebook. It can reduce the bit-rate and the time required for encoding-decoding. As before digital images are Peano scanned and then subdivided into blocks of 4*4. The first block is encoded, as was done earlier, by scanning the whole codebook and choosing the codevector that yields the minimum mean absolute difference. Additionally, a set of the next best vectors that could have been used to encode the block are identified (on the basis of minimum mean absolute difference). These vectors now constitute the active part of the codebook. The next block is then compared to the previous block and the mean square error is computed. This error is a measure of the correlation between the blocks. If the blocks are highly correlated the MSE is low. If the MSE is below a predetermined threshold, only the vectors in the active part of the codebook are scanned to encode the block. If the MSE is higher, then as for the first block, the entire codebook is scanned and an active part is created as before. This MSE comparison to threshold and active part formation goes on until the entire image is encoded. Interblock correlation is exploited in this manner.

There are two distinct advantages of this approach. One is that if the image contains blocks that are highly correlated, large amount of the encoding is done using the active part of the codebook. If we assume that a similar partitioning is done at the decoder, then for those blocks, only bits required to represent the correct index in the smaller, active part of the codebook, need to be transmitted. For example, experiments were performed for codebooks of sizes 128, 512, 1024 codevectors, with dynamic partitioning into an active part of four codevectors. Therefore for blocks encoded using the active part of the codebook only two bits are required. Fig. 7. shows the bit-rates obtained for five test images for two values of the threshold, 50 and 200 (MSE).There was no lowering of SNR for the threshold of 50 and for 200 it was marginal. A fairly large fraction of the blocks is encoded using the active part. Therefore, a lower bit-rate than plain VQ is obtained. Another advantage is that the time required for encoding is less since for several blocks only the smaller, active part needs to be searched.

There is a trade off involved for the choice of the threshold. If it is too low the entire codebook will be searched for most blocks and reduction in bit-rate will be small. The time for encoding will also be high. On the other hand if the threshold is high, the bit-rate will be low, but there will be loss in visual quality. For a specific application, the optimal threshold will have to be adaptively learnt. As in the case of coding repeat pattern, the problem of synchronization betwen encoder and decoder can be solved by transmitting a brief tone, when new partitioning is required. This partitioning technique is analogous to the address-VQ, suggested by Nasrabadi and Feng[5].

5. CONCLUDING REMARKS

In this paper, a new combined approach of Peano scanning and reordering followed by VQ has been described. Several possibilities for exploiting interblock correlation have been explored. One approach is to assign a fixed number of bits to encode the repeat pattern of codevector indices. This results in a significant lowering of bit-rate. Another technique proposed is to use Huffman coding to encode the repeat pattern. It was also suggested that a signaling tone be used to indicate end of transmission of a packet, so that the transmitter and receiver are always well synchronized. The second approach suggested is to dynamically partition a large codebook into a smaller active part and a large passive part. This is done on the basis of comparing interblock correlation with a predetermined threshold. Due to the clustering achieved by Peano scanning and reordering, a large number of blocks are encoded using the active part, resulting in lower bit-rates. The technique for determining the threshold needs further investigation. This involves the tradeoff between visual quality and very low bit-rate, and is application dependent. Visual quality can be further improved if a set of codebooks with different type of edge patterns[6] is used along with the above techniques.

6. REFERENCES

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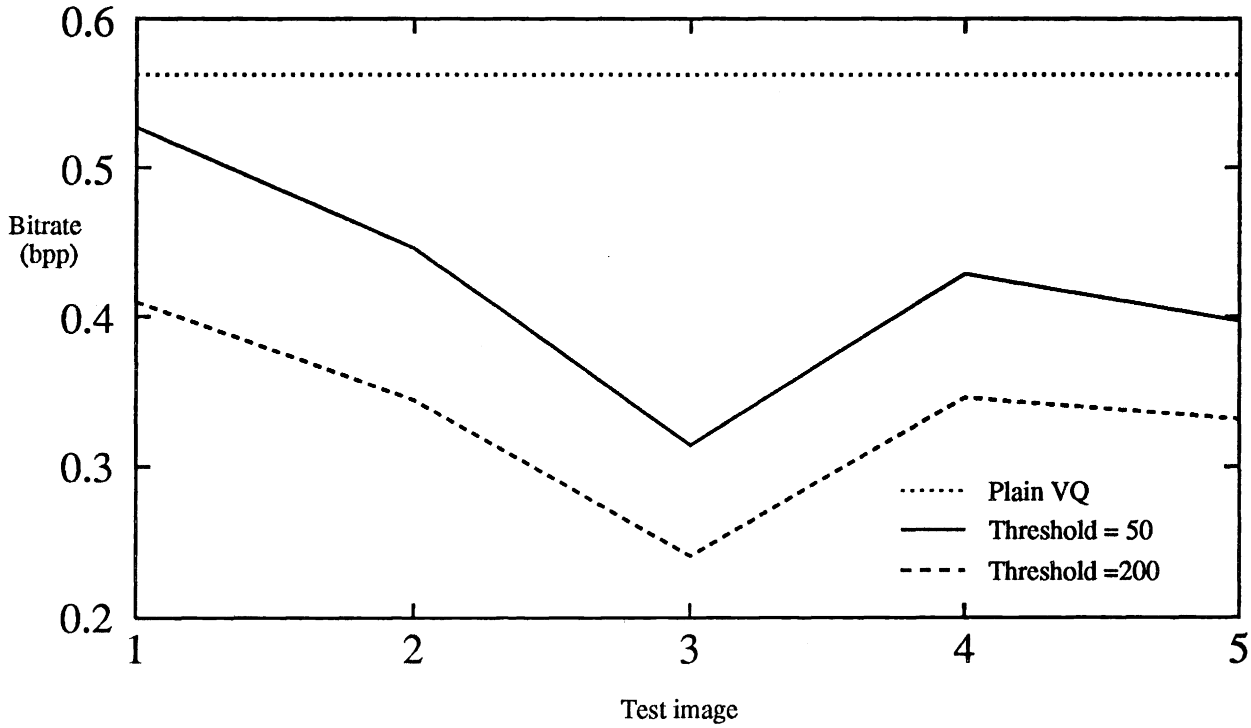

Fig. 7: Bit-rates obtained by plain VQ and dynamic partitioning using thresholds of 50 and 200 (MSE), for different test images