IMAGE DATA ORDERING AND COMPRESSION USING PEANO SCAN AND LOT

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Abstract

An efficient method for visual data compression is presented which combines generalized Peano Scan and a Lapped Orthogonal Transform (LOT). The Peano Scan, an application of the Peano curve to the scanning of images, is used locally in order to cluster highly correlated pixels. The new set of data is compressed using a LOT. The LOT is a tool for block transform coding with basis functions that overlap adjacent blocks [1]. The blocking effect, which constitutes a major problem at very low bit rates, is reduced significantly by applying LOT. An adaptive procedure is developed to encode image blocks at low bit rates without any visible blocking effects.

1 Introduction

A major challenge in the design of an image encoding system is to reduce the memory required for storage and the transmission bandwidth subject to some image quality constraints. This can be accomplished through reduction of the spatial and temporal redundancy of the image source. An image source is normally highly correlated both spatially and temporally and there is a strong correlation among the values of neighboring pixels. There are a variety of techniques for image compression, an excellent review of these methods can be found in [2].

Incorporating an efficient scan strategy into an encoding scheme results in clustering highly correlated areas of the image. By applying an adaptive compression technique to each cluster, higher compression performance can be achieved. Space-filling curves or Peano curves were studied for the first time in 1890 and 1891 by G. Peano [3] and D. Hilbert [4] , and are the theoretical basis for mapping a multidimensional space to a one dimensional space. The Peano scan is a discrete series of points obtained from the finite process of generating Hilbert's spaces filling curves. The Peano scan may be applied in cases where the scanning order is important for an image transformation, such as data compression or halftoning, because transformation of a pixel affects the adjacent pixels. The Peano scan or the Hilbert scan is applied to target detection [5] , image representations and manipulations [6], fractal based image coding [7], and one-dimensional mapping for vector quantization of images [8].

Space filling curves are self-similar and continuous, and also can be used in real time for on-line visual communication systems. As the scan trajectory determines the sequences of data fed into a compression algorithm, Sorek and Zeevi [9] showed that for on-line visual data compression along a one-dimensional scan, the Hilbert or Peano Scan is close to the optimum scanning trajectory.

A major problem in image compression at low bit rates is the blocking effect. The blocking effect is a natural consequence of the independent processing of each block of pixels. In images the blocking effect is perceived as visible discontinuities along the inter-block boundaries. In the case of speech coding which is the auditory analogy to image coding, blocking effects create extraneous tones and in image coding, blocking effects create extraneous edges. Methods for reducing the blocking effects have been proposed [10, 11]. These methods can be divided into two classes: *Overlapping*, and *Filtering*. In the overlapping method, the blocks overlap slightly so that redundant information is transmitted for the samples in the block boundaries. The disadvantage of this approach is the increase in the total number of samples to be processed, and thus

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an increase in the bit rate. In the filtering method, disjoint blocks are encoded, transmitted, and at the receiver a low-pass filter is applied only on the boundary pixels. Although this method does not increase the bit rate, it blurs the the signal across the inter-block boundaries.

The LOT provides similar benefit in reducing the blocking effects as compared to the overlapping method but it does not increase the bit rate. Cassereau [12] introduced a new class of optimal unitary transforms and LOT for image processing and Malvar [1] improved upon them and developed fast algorithms for the computation of LOT. The fast LOT needs approximately 20 percent more computations than an equal size DCT. In this paper an adaptive approach for image compression using a LOT on a Peano scan ordered image is presented. In the next section we describe the Peano scan and a new approach in which the Peano scan is used to cluster highly correlated pixels. In section 3, the improved inter-block correlation image data ordering is presented. The LOT and its application to image compression are presented in section 4.Experimental results follow in section 5.

2 Peano Scan

In a Peano scan, a rectangular block is divided into sub-blocks in a recursive manner and an ordering procedure is applied to each sub-block. Let $B(M,N)$ be a rectangular block with M columns and N rows, the following cases are considered:

(a) if $M \geq N$ and $M \geq 3$, then $B(M,N)$ is divided into three vertical sub-blocks, $B(M_1,N)$, $B(M_2,N)$, $B(M_3,N)$ respectively, M_1, M_2, and M_3 are computed in the following manner:

$$M_1 = INT(\tfrac{M}{3})$$

$$M_2 = \begin{cases} 1 + INT(\frac{M}{3}) & REM[INT(\frac{M}{3})] \geq 1 \\ INT(\frac{M}{3}) & Otherwise \end{cases}$$

$$M_3 = \begin{cases} INT(\frac{M}{3}) & REM[INT(\frac{M}{3})] = 0 \\ 1 + INT(\frac{M}{3}) & Otherwise \end{cases} \quad (1)$$

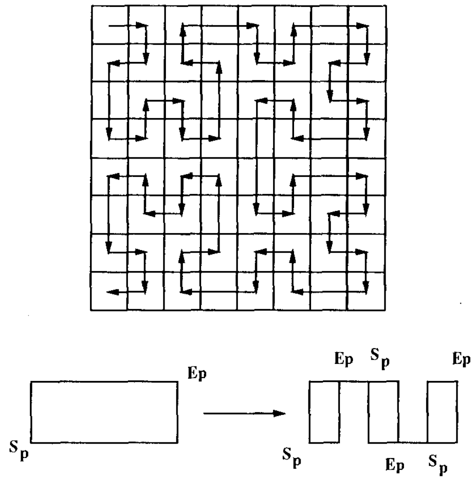


Figure 1: Peano scan of a block of size 8x8

where $INT(\frac{p}{q})$ is the integer value of the division of p by q, and $REM[INT(\frac{p}{q})]$ corresponds to the remainder of the division operation.

(b) if $N > M$ and $N \geq 3$ then $B(M,N)$ is divided into three horizontal sub-blocks, $B(M,N_1)$, $B(M,N_2)$, $B(M,N_3)$. N_1, N_2, and N_3 are computed by applying the same technique used in the computation of M_1, M_2, and M_3

(c) For other possible cases, $(M,N) \leq (2,2)$ sub-blocks are not divided.

The above procedure is applied until all sub-blocks cannot be divided. The next step is to reorder subblocks on a rectangular array. A starting point S_p and an ending point E_p for each sub-block is chosen. S_p and E_p are located on a diagonal line joining the corners of the sub-blocks. Figure 1 shows a Peano scan for a rectangular block of size 8x8.

3 Image Data Ordering

Image data ordering or clustering highly correlated areas of an image is a crucial factor in very low bit rate image compression because clusters of uniform textures can be encoded at very low bit rates with less distortion [13]. In our approach the Peano scan is used locally on the two-dimensional image data in order to transform the original data into

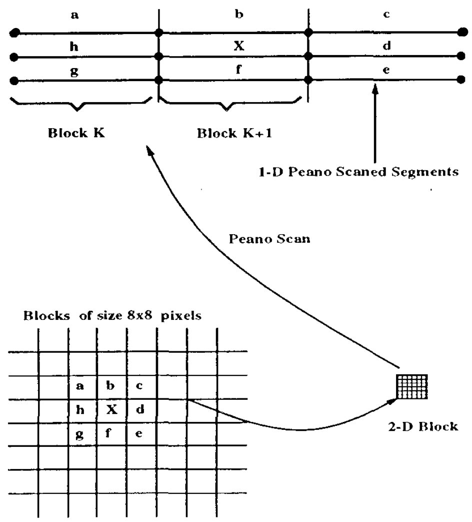


Figure 2: Ordering of sub-blocks of size 8x8 pixels

another set of two dimensional data in which correlated pixels are clustered. Let $I(R, C)$ be an image with R rows and C columns. A collection of $(\frac{R}{8} * \frac{C}{8})$ sub-blocks are obtained by dividing the image $I(R, C)$ into small rectangles of size (8x8) pixels. The Peano scan is applied on each sub-block and the one-dimensional Peano scanned lines of 64 pixels corresponding to the original sub-blocks are ordered with respect to the interblock correlation existing in the original image (Figure 2). The image $I(R, C)$ will be transformed into an image $I'(R', C')$, where $R' = \frac{R}{8}$ and $C' = \frac{C}{8} * 64$. For an image with $R = C = 256$, the output image will be $I'(32, 2048)$. Figure 3-a shows an original image (256x256), and the Peano scanned image (32,2048) in a (256x256) format is shown in Figure 3-b. On this new image, the bands with similar structures which represents clusters of correlated pixels are highly visible. The Mean Absolute Difference (MAD) is chosen as a measure to determine the degree of correlation between consecutive blocks of size 32x32 pixels. The MAD is defined as follows

$$AD(k) = \frac{1}{1024} \sum_{i=0}^{31} \sum_{j=0}^{31} |I_k(i,j) - I_{k+1}(i,j)|$$

$$MAD = \frac{1}{t} \sum_{k=0}^{t} AD(k) \tag{2}$$

#	Test Images	MAD Raster Scan	MAD Peano Scan
1	Cronk	37.03	18.52
2	Lena	56.33	32.85
3	Baboon	43.18	28.08
4	Hat	64.61	31.34
5	Pepper	60.04	29.64

Table 1: Mean Absolute Difference between consecutive sub-blocks (32x32)

where $AD(k)$ represents the Absolute Difference between two consecutive blocks B_k and B_{k+1} of size 32x32 pixels, the MAD is the average value of $AD(k)$ over the entire image, and t represents the total number of blocks for which the AD is computed.

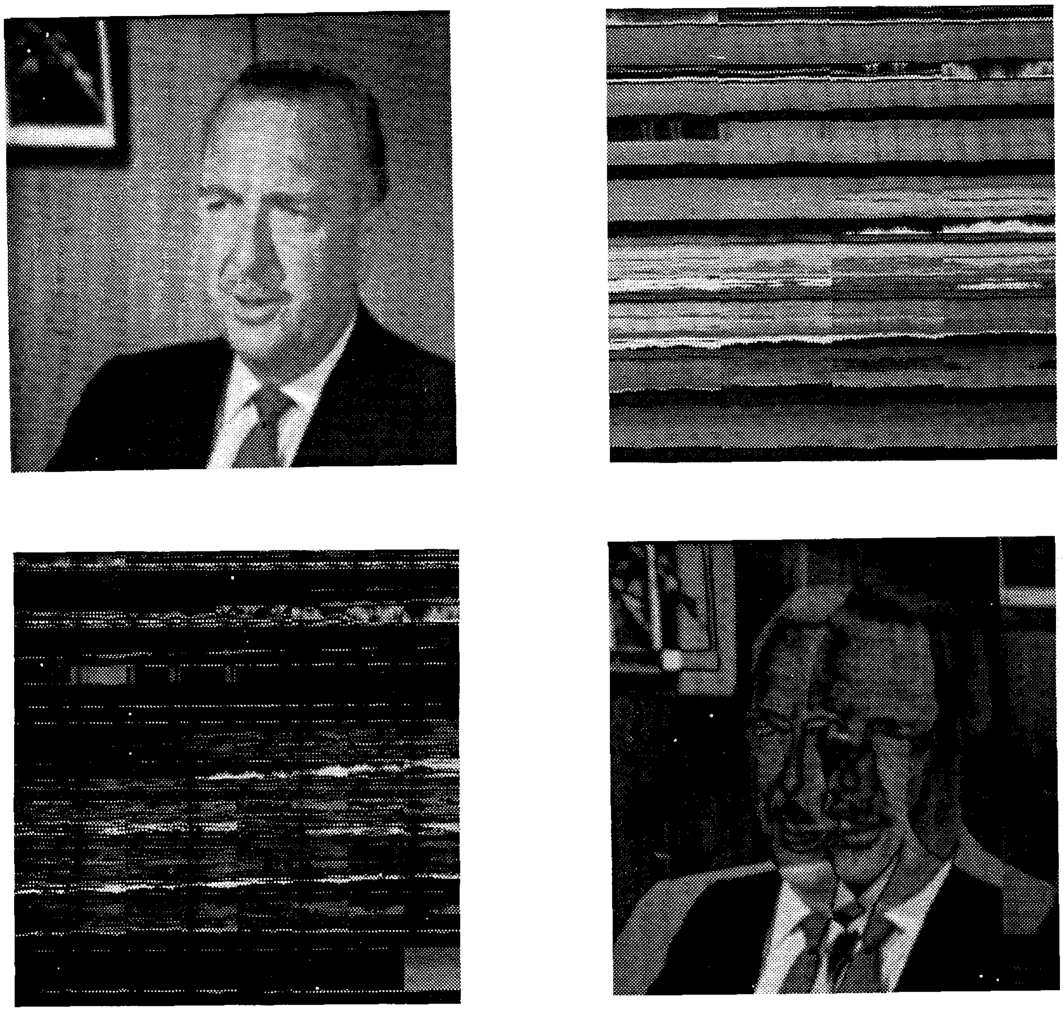
Figure 3-c corresponds to the interblock Mean Absolute Difference (MAD) which is given by equation (2) for the Peano scanned image. In this image the effects of Peano scanning are illustrated by large bands with very low luminance (black areas). The interblock MAD of the Raster image is shown in Figure 3-d. This image shows a significantly lager difference between successive blocks as is illustrated by areas of higher luminance.

In order to confirm the effect of clustering correlated pixels by using a Peano scan, experiments were performed on different test images. The MAD between successive blocks is computed for both Raster and Peano scan. Successive blocks of size (32x32 pixels) are more correlated in a Peano scanned image than in Raster scanned image. This is illustrated in Table 1 and in Figure 4, where it is shown that the MAD for Peano scanned images is almost 50 percent less than for Raster scanned images. In the next section we describe the LOT and a new adaptive method for image compression.

4 LOT in Data Compression

In this section, the properties of the LOT [1] are briefly reviewed and the new adaptive image compression algorithm is presented. For ease of comprehension it is assumed that signals to be processed are unidimensional: the extension to two-dimension is achieved by defining separable transforms based on the one-dimensional profile.

Consider a discrete signal containing $m * n$ samples, where n is the block size. In a traditional transform encoding technique m blocks of length n would be independently transformed and coded.



a	b
c	d

Figure 1: (a) Original image, (b) Peano scanned image, (c) Interblock Mean Absolu te Difference of the Peano scanned image, (d) Interblock Mean Absolute Difference of the Raster s canned image

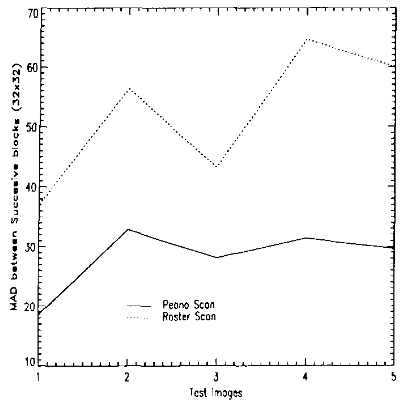


Figure 3: Mean Absolute Difference between sub-blocks of size (32x32) using Raster and Peano Scan. Successive blocks are more correlated in a Peano scanned image than in a Raster scanned image.

With the LOT each block has l samples, with $l > n$ so that neighboring blocks overlap by $l-n$ samples. In that way LOT maps the l samples of each block into n transform coefficients. In matrix notation, if X is the original input vector of length $m*n$, then Y, the vector representing the transform coefficient of all blocks, is given by

$$Y = A^{\tau} X \tag{3}$$

where A^{τ} is the transpose of an $(mn * mn)$ block diagonal matrix which has the following form

$$A = \begin{pmatrix} L_1 & 0 & \cdots & 0 & 0 \\ 0 & L_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & L_0 & 0 \\ 0 & 0 & \cdots & 0 & L_2 \end{pmatrix}.$$

L_0 is an $l*n$ matrix that contains the LOT basis functions for each block. The L_1 and L_2 matrices are introduced because the first and the last blocks of a segment have only one neighboring block and then the LOT of the first and last blocks must be defined in a slightly different way. It is clear that the LOT of a single block L_0 is not invertible, since L_0 is not square. Nevertheless, in terms of reconstructing the whole segment X, all that is needed for A to be invertible. In order for A to be invertible, the columns of L_0 must be orthogonal,

$$L_0{}^{\tau} L_0 = I, \tag{4}$$

where τ denotes matrix transpose and I is an $n*n$ identity matrix, and the overlapping functions of neighboring blocks must also be orthogonal,

$$L_0{}^{\tau} W L_0 = L_0{}^{\tau} W^{\tau} L_0, \tag{5}$$

where the shift operator W is an $l*l$ matrix defined as

$$W = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}.$$

The identity matrix above is of the order $l-n$, where $l \leq 2n$. A LOT matrix L_0 is feasible if it satisfies (4) and (5). An optimal LOT should minimize the required bit rate for a given distortion level. This is equivalent to maximizing the 'energy compaction' measure as defined in [14]. The LOT is also feasible for any orthogonal matrix Z,

$$\begin{gathered} L_0 = LZ \\ L_0{}^{\tau} L_0 = Z^{\tau} L^{\tau} L Z = Z^{\tau} Z = I. \end{gathered} \tag{6}$$

A feasible LOT matrix can be defined from the known block transform basis functions by

$$\mathrm{L} = \begin{pmatrix} D_e - D_o & D_e - D_o \\ J(D_e - D_o) & -J(D_e - D_o) \end{pmatrix}$$

where D_e and D_o are the $n*n/2$ matrices containing the even and odd basis functions of an orthogonal block transform, respectively, and J is the counteridentity matrix. This particular choice leads to a fast computation of the LOT. It is shown [15] that for fast computation and maximum energy compaction, chosen DCT basis functions in the computation of LOT, leads to higher performance. The resulting L_0 for a general case can be written as

$$L_0 = \frac{1}{2} \delta \cdot I_1 \cdot I_2 \cdot \zeta \tag{7}$$

where

$$\delta = \begin{pmatrix} D_e & D_o & 0 & 0 \\ 0 & 0 & D_e & D_o \end{pmatrix},$$

$$I_1 = \begin{pmatrix} I & I & 0 & 0 \\ I & -I & 0 & 0 \\ 0 & 0 & I & I \\ 0 & 0 & I & -I \end{pmatrix},$$

$$I_2 = \begin{pmatrix} 0 & 0 \\ I & I \\ I & -I \\ 0 & 0 \end{pmatrix},$$

$$\zeta = \begin{pmatrix} I & I \\ 0 & Z^r \end{pmatrix},$$

where D_e and D_o are the $n*n/2$ matrices containing the even and odd DCT functions respectively. The matrix I represents the identity matrix, and Z^r is an orthogonal matrix obtained by

$$Z^r = T_1 T_2 \cdots T_{\frac{n}{2}-1} \tag{8}$$

and T_i is represented as

$$T_i = \begin{pmatrix} I & 0 & 0 \\ 0 & \xi(\theta_i) & 0 \\ 0 & 0 & I \end{pmatrix}$$

$$\xi(\theta_i) = \begin{pmatrix} \cos(\theta_i) & \sin(\theta_i) \\ -\sin(\theta_i) & \cos(\theta_i) \end{pmatrix}.$$

The matrix $\xi(\theta_i)$ is a 2x2 butterfly, where θ_i is the rotation angle.

4.1 Adaptive LOT Coding

In an adaptive transform technique, image blocks are classified into different categories according to the spatial frequency information which characterizes each image block [16]. A category index is associated with each transform block and an encoder modifies its parameters to fit the statistical parameters of the image block. A major difficulty in this technique is to define an optimal number of categories to represent adequately different types of block structure. This is achievable using Vector Quantization (VQ) [17]. However, the computational complexity and memory requirements for VQ increases as the number of vectors used and their dimensions increase. Figure 5 shows a block diagram of the compression/decompression stages of a new and efficient procedure which is developed to obtain high quality images at low bit rates and to reduce the computational complexity. During compression 'significant' LOT coefficients of image blocks of size (8x8) are selected using a threshold sampling technique; coefficients above a certain threshold T are selected and coefficients below the threshold T are replaced by zero. To indicate the location of significant LOT coefficients after normalization , a 'clipping' procedure is applied on each normalized LOT block to form a binary pattern. This 'clipping' procedure is given by

$$BLOT(i,j) = \begin{cases} 1 & LOT(i,j) \geq T \\ 0 & LOT(i,j) < T \end{cases} \tag{9}$$

where $BLOT(i,j)$ is the binary representation of the selected $LOT(i,j)$ after clipping. An index is assigned for each binary pattern by matching its coefficients to those of a small sized binary codebook (16 entries) using a Mean Absolute Difference (MAD) criterion. To generate the codebook, training data consisting of 7168 binary patterns (from 7 different images) are computed using the above procedure (i.e. LOT coefficients selection and binary clipping). The LBG algorithm [17] is applied to cluster the training data into 16 different regions (Figure 6). These regions represent location maps of the image blocks with different textures. Upon selecting the codebook entry 'closest' to the input binary pattern, the selected LOT coefficients are uniformly quantized by rounding to the nearest integer value. Quantized values are classified into 8 planes according to their magnitude, and coded by generating a Variable Word Length Code (VWLC) using the Huffman coding technique.

To decompress an image block the binary code index is used to retrieve the codebook entry which indicates the location of significant LOT coefficients (Figure 5-b). The quantized values of these coefficients are obtained by table lookup (according to the Huffman code), and an inverse LOT is applied to reconstruct the image block.

5 Results

The new compression method which is presented in the previous section is tested on several different images. During simulation each Raster scanned image is transformed into another image using the Peano clustering and reordering procedure. Each original image is divided into small blocks of 8x8 pixels, each block is Peano Scanned, and the 1-D Peano segment is ordered with respect to its spatial location in the original image. The transformed

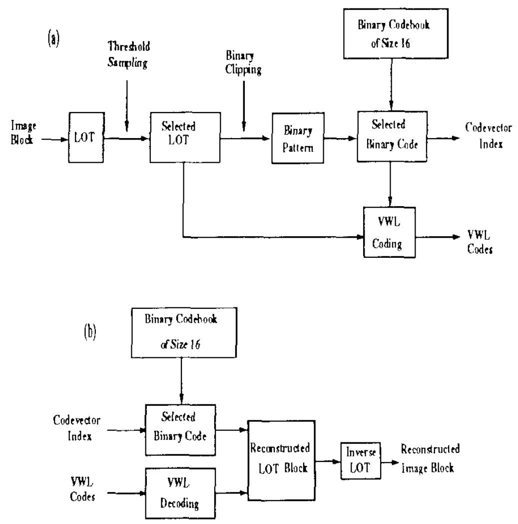


Figure 4: Block Diagram of the (a) Compression, (b) Decompression

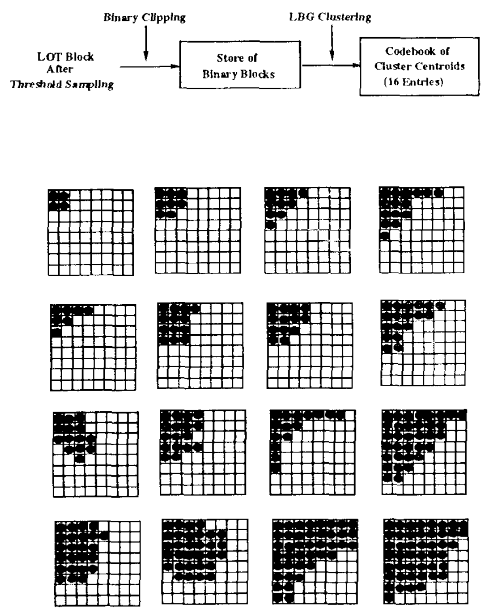


Figure 5: Binary pattern generation

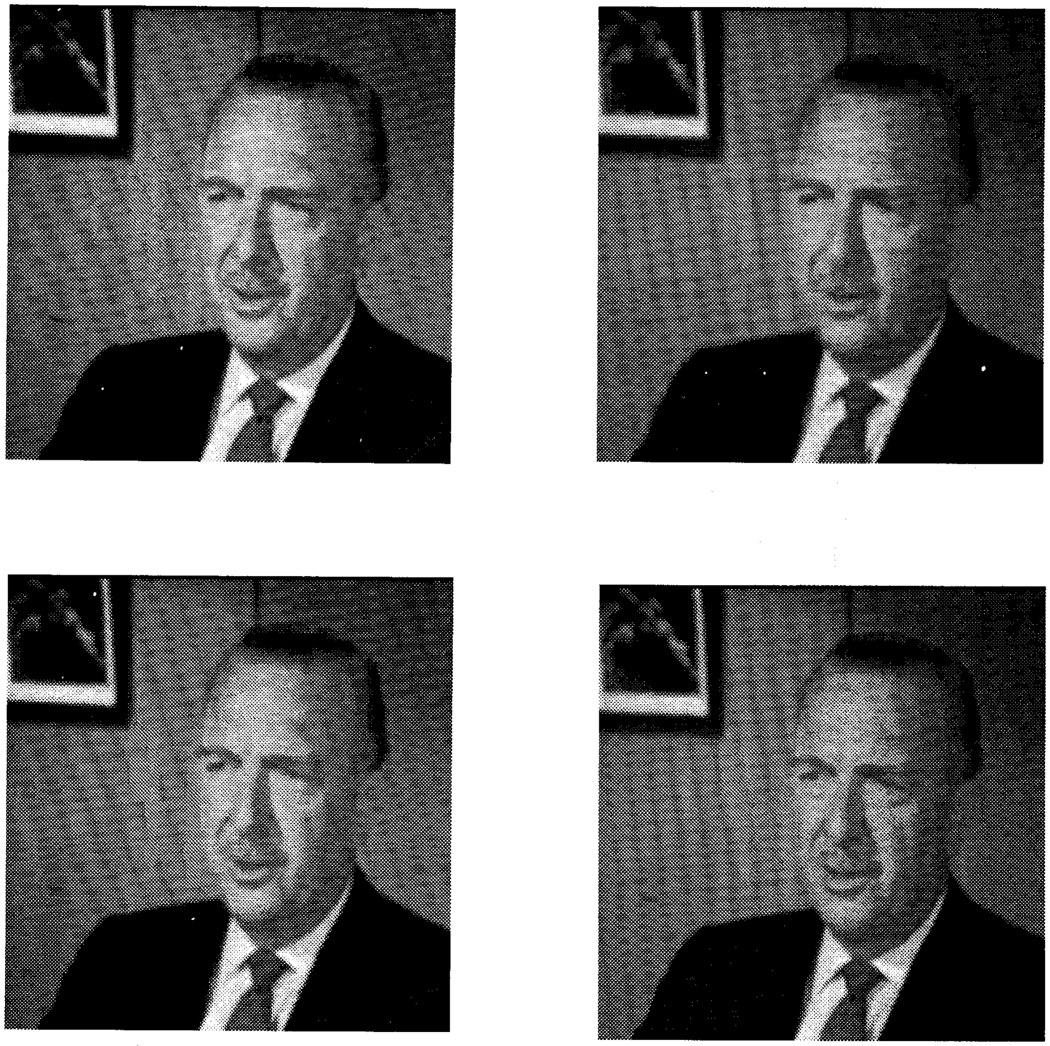
set of data is then compressed by the new adaptive LOT technique. During decoding, inverse LOT is used to reconstruct the Peano scanned image and by inverse Peano scanning the original Raster scanned image is obtained (see Figure 7). A comparative study between DCT and LOT is performed by compressing each image using the LOT and the DCT at the same bit rates. The effect of clustering highly correlated pixels using the Peano scan on the compression ratio and on the quality of the compressed image is also studied in the simulations, Raster scanned images and their corresponding Peano scanned images are compressed using the adaptive LOT technique at very low bit rates and the quality of the compressed images are judged on the basis of subjective visual quality and signal-to-noise ratio (SNR) which is defined by the following equation:

$$SNR = 10 * \log[\frac{(255 * 255)}{MSE}]. \qquad (10)$$

In Figure 8-a, a compressed image at 0.28 bit per pixel, with a SNR of 34.3dB is shown using the new LOT method. Figure 8-b corresponds to the same image compressed at the same bit rate using DCT, with an SNR of 34.1dB. The blocking effect for the LOT compressed image is almost invisible and the SNR is slightly higher than the DCT compressed image where important blocking effect can be noticed. In both cases, the images have been reordered using the new clustering procedure prior to their encoding. In Figure 8-c a compressed Raster scanned image is shown with a bit rate of 0.35 bit per pixel and a SNR of 34.8. The corresponding Peano scanned image is compressed at 0.29 bit per pixel with an SNR of 34.7. Both images have been judged subjectively to have the same visual quality. These experiments show that higher compression ratios can be obtained from a Peano scanned image. The above results have been confirmed on different type of test images.

6 Summary and Conclusions

In this paper a new method for image data ordering has been introduced. The Peano scan which maps a multidimensional space into one-dimensional space is used efficiently in order to cluster highly correlated pixels in an image. The new adaptive LOT



a	b
c	d

Figure 2: (a) LOT compressed image at 0.28 bpp, SNR 34.3dB, (b) DCT compressed image at 0.28, SNR 34.1, (c) Raster scanned image compressed at 0.35 bpp, SNR 34.8dB, (d) Peano scanned image compressed at 0.29, SNR 34.7dB.

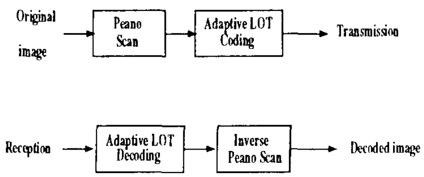


Figure 6: Block diagram of the adaptive coding and decoding scheme

method performs efficiently and reduces the blocking effect significantly. Simulation results showed that for the same bit rate, LOT compressed images have better visual quality than the DCT compressed images. The computational complexity is higher in the case of LOT, but it still can be implemented in real time with current technology. It is also shown that lower bit rate can be achieved in a reordered Peano image than in a Raster scanned image. The reordering procedure is computationally inexpensive and can be integrated into an encoding scheme. For example, in vector quantization (VQ), a major challenge is to obtain highly compressed images with a minimum distortion. By taking advantage of the interblock correlation, which is much stronger for a Peano scanned image than for a Raster scanned image, a strategy for codebook design and search can be defined. For instance, in a large codebook a small portion of it can be defined as active and be updated using the interblock correlation. This approach can be generalized to other block encoding techniques. In a wavelet decomposition for encoding images, adaptive sub-band compression techniques can exploit the Peano scan; as the high frequency sub-bands contain vertical and horizontal edge lines of the image, by using the Peano reordering method these two dimensional high frequency sub-bands can transformed into one dimensional signal and compressed efficiently using Run Length encoding technique.

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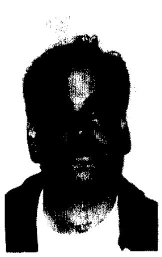

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