**The Lorentz System**

In 1963, Edward Lorenz, with the help of Ellen Fetter who was responsible for the numerical simulations and figures,[1] and Margaret Hamilton who helped in the initial, numerical computations leading up to the findings of the Lorenz model,[2] developed a simplified mathematical model for atmospheric convection. [1] The model is a system of three ordinary differential equations now known as the Lorenz equations.

## Physics and Math Background

The Lorenz equation relate the properties of a two-dimensional fluid layer uniformly warmed from below and cooled from above. In particular, the equations describe the rate of change of three quantities with respect to time: $x$ is proportional to the rate of convection, $y$ to the horizontal temperature variation, and $z$ to the vertical temperature variation. The equations are

$$\begin{aligned}\frac{\mathrm{d}x}{dt} &= \sigma(y-x)\\ \frac{\mathrm{d}y}{dt} &= x(\rho-z)-y\\ \frac{\mathrm{d}z}{dt} &= xy-\beta z\end{aligned} \tag{1}$$

where the constants $\sigma, \rho$, and $\beta$ are the system parameters proportional to the Prandtl number, Rayleigh number, and certan physical dimensions of the layer itself [3]. These constants are normally assumed to be positive.

## The Project

In this project you will plot a typical orbit (the solutions to $x(t).y(t), z(t)$) for various initial conditions. The ODEs are to be solved using an adaptive step-size Runge-Kutta algorithm. You are to use the following values for the parameters: $\sigma = 10, \beta = 8/3, \rho = 28$. You are to perform the following analysis:

(1) For the initial condition $(x_1, y_1, z_1) = (1, 1, 1)$ solve the Lorenz system for times greater than $t = 40$.

(2) Plot $(x(t), y(t)$ and $(x_1(t), y_1(t), z_1(t))$ and try to interpret the results. Note, in this plot there is no time axis, you are plotting one variable versus the other(s). Then plot just $x(t)$ vs $t$ and again interpret the results.

(3) Change the initial conditions to $(x_2, y_2, z_2) = (1, 1, 1.00001)$. Before solving the equations, predict whether the solutions will be much different for these initial conditions. Then plot the $(x_2 - x_1)$ versus $t$ and discuss the results.

(4) Make other small changes in the initial conditions $(x_1, y_1, z_1)$ and do similar analysis

**References:**

[1] Lorenz, Edward Norton (1963). "Deterministic nonperiodic flow". Journal of the Atmospheric Sciences. 20 (2): 130–141.

[2] Lorenz, Edward N. (1960). "The statistical prediction of solutions of dynamic equations". Symposium on Numerical Weather Prediction in Tokyo.

[3] Sparrow, Colin (1982). The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors. Springer.