Predator-Prey Models with Diffusion

Predator-Prey models are often presented without regard to the spatial distribution of the species. These models result in ordinary differential equations. A more realistic set of models includes diffusion terms in which the species can migrate to different spatial locations

Physics and Math Background

In this project you will consider a predator-prey system in dimensionless form that has the form [1],

$$\begin{aligned}
\frac{\partial u}{\partial t} &= u(1-u) - \frac{uv}{u+\alpha} + \nabla^2 u \\
\frac{\partial v}{\partial t} &= \beta\frac{uv}{u+\alpha} - \gamma v + \delta\nabla^2 u
\end{aligned} \tag{1}$$

Notice that both terms now include a spatial dependence term, ∇^2. The terms u, v are the populations of the predator (v) and the prey,(u). The terms $\alpha, \beta, \gamma, \delta$ are positive parameters.

The Project

You are to describe what each term in the equations represents in a *real system*. Then you are to solve Eqs (1) numerically using finite differencing methods. Use a grid size of 400×400, a grid step size of $h = 1$, an initial time step $= 1/3$ and run the system for about $T = 150$. Use the following initial conditions,

$$\begin{aligned}
u^0_{i,j} &= 6/35 - 2\times 10^{-7}\,(x_i - 0.1y_j - 180)\cdot(x_i - 0.1y_j - 800) \\
v^0_{i,j} &= 116/245 - 3\times 10^{-5}\,(x_i - 400) - 1.2\times 10^{-4}\,(y_i - 150)
\end{aligned}$$

Take the constants to be

$$\alpha = 0.4, \beta = 2.0, \gamma = 0.6, \delta = 1. \tag{2}$$

These will not change. However, repeat using the smaller time steps, $\delta t = 1/24$ then $\delta t = 1/384$. You should plot the prey densities on a spatial grid as a function of time. You might think about animating this plot using the MatLab command `pause`.

References:

[1] R. Marcus Garvie, *Finite difference schemes for reaction-diffusion equations modeling predator-prey interactions in MATLAB*, Bulletin of Mathematical Biology, 69:931–956, 2007.