Battle Dynamics
Battle of Aljubarrota

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In this paper, we present a model of the Battle of Aljubarrota simulated in Mathematica. Our model includes multiple layers of detail, from individual soldier movement to overall unit strategy. We incorporate factors such as the effect of a soldier's skill on combat outcomes, terrain limitations on troop movement, and the impact of the battle's course of action on troops' morale.

Historical Background: The timeline of the battle, along with the historical context provide a reference for our model.

Mathematical Model: The way we improved and developed our model throughout its lifespan. Explanation of the numbers behind each parameter.

I. HISTORICAL BACKGROUND

A. Setting the Stage

The Battle of Aljubarrota was a significant conflict that took place on August 14, 1385, between the Kingdoms of Portugal and Castile, during the 1383-1385 Portuguese Succession Crisis. The crisis arose when the untimely death of King D. Fernando of Portugal in 1383 left his only daughter Leonor as the queen regent until she bore a son. She was married to Juan I, the king of Castile, which meant that their son would inherit both thrones, potentially leading to a union between Portugal and Castile. However, the Portuguese were opposed to this idea, fearing that it would result in their subjugation to Castile.

At the time, Juan I of Castile did not focus on Portugal, as he was preoccupied with other matters, assuming that no resistance would arise. This created an opportunity for João, the head of the Aviz dynasty and D. Fernando's illegitimate half-brother, to unite the Portuguese people and lead them in a series of battles, culminating in the decisive Battle of Aljubarrota, which confirmed the independence of Portugal from Castile.

B. The Battle Unfolds

The Portuguese were led by King João I, who had come to power as a result of the crisis, and the Castilian forces were led by King Juan I of Castile. The Portuguese army was outnumbered and outmatched in terms of military equipment and training. However, they had a strategic advantage in the form of their position on the battlefield, which was on the top of a hill, situated between two rivers. The battle began with a Castilian assault on the Portuguese position, through a cavalry charge. The Portuguese were able to repel the attack. The Castilian forces then attempted to flank the Portuguese, but the Portuguese were able to hold their ground and eventually launched a counter-attack.

The Portuguese victory at the Battle of Aljubarrota is often attributed to the leadership of Nuno Álvares Pereira, a nobleman who commanded the Portuguese army. He is said to have come up with a clever tactic to use the long-range weapons to keep the Castilian cavalry at bay, while the Portuguese infantry advanced to engage the Castilian foot soldiers.

The Battle of Aljubarrota is considered one of the most important battles in Portuguese history, as it solidified the independence of the Kingdom of Portugal and ended the threat of Castilian invasion. The battle also had significant political and social implications, as it helped to establish the legitimacy of the new Portuguese royal dynasty and strengthened the power of the nobility.

Thus, the Battle of Aljubarrota marked a critical moment in Portuguese history, as it established their sovereignty and independence, while also consolidating the power of the Aviz dynasty and legitimizing the Portuguese royal lineage.

II. MATHEMATICAL MODEL

A. Existing Theory

Lanchester's Laws are a set of mathematical equations that describe the behaviour of military forces in combat. They were developed by Frederick W. Lanchester, a British engineer and mathematician, in the early 20th century.

Lanchester's Laws come in two forms: the Square Law and the Linear Law. The Square Law describes the relationship between the number of forces and their effectiveness in combat, while the Linear Law describes the relationship between the firepower of the forces and their effectiveness.

The Square Law states that the combat effectiveness of a force is proportional to the square of the number of units in the force. This means that if one force has twice as many units as another force, it will be four times as effective in combat.

The Linear Law states that the combat effectiveness of a force is proportional to the firepower of the force. This means that if one force has weapons that are twice as powerful as the weapons of another force, it will be twice as effective in combat.

Lanchester's Laws are widely used in military simulations and operations research, and have also found applications in other fields such as economics and business. However, it should be noted that Lanchester's Laws are based on several simplifying assumptions and may not always accurately reflect the complex dynamics of real-world combat situations. Our model is separate from these laws, allowing for greater user input when defining the parameters.

B. The Progress

In order to achieve the current model, we started off by creating a series of simpler more rudimentary models. Initially, we defined the starting positions of 2 armies, composed of 1000 "soldiers", each of them being allocated a position with coordinates (x, y), where x, y are random real numbers within a specified interval in a 2-dimensional plane. We found the closest "opponent" (element of the opposite army) for each element of each army, making them move towards that target using a recursive function.

After we had found the closest path, we defined a proximity function and gave each soldier 0.5 probability of "killing" (making it disappear) its opponent on contact. This was later refined when different unit types were created. Depending on the unit types fighting each other, the probabilities of hitting were different. This was done using a function that determined these probabilities once the fight started by checking the unit type of the opponent.

The next additions were the parameters of skill, morale, and health, which we continuously refined as the project progressed. With the implementation of soldier health, there would have to be several successful "hits" by an opponent to kill one another, each reducing health, rather than killing in a single hit. The skill parameter would affect the probability of hitting.

For simplicity, the morale parameter started off being a constant value. As we continued developing the model, we made it dynamic so that it reflected as faithfully as possible the way soldiers may feel after seeing other units die or flee.

Initially, we also imported information from the map of the battle's area, adding a height combat advantage to the units. This, however, didn't work well for a few reasons: the terrain where the battle took place has changed dramatically, with previously existing rivers no longer there; and the spikes in height located in the area made it so certain areas would be tremendously steep whilst others would not have a noticeable height difference. We then tried to implement a simple height model by imposing a speed and attack advantage to units approaching from the left-hand side (lower x-value) and the opposite to the others (higher x-value), but the way we had written the code, made it impossible to implement without recreating everything from scratch.

We also defined an area where units would be slowed down, meant to represent the defences deployed by the Portuguese. This is especially noticeable during the first phase of the battle where the Castilian cavalry is charging.

Another parameter we fine-tuned was the "reach range", determining when a unit would start its engagement with the opponent. It is important to note this is a one-way relation, where one unit could begin engaging its opponent, whilst the opponent might not be hitting them yet. In our model, the archers have the longest reach range, meaning that we consider this reach range to start a fight between two soldiers. Then we check the unit types of the soldiers and based on that we let them hit the opponent or not depending on whether they are in their reach range.

After doing this, we then worked on applying a model for routed units, that is, the troops that fled, another important aspect of our model. As in the historical battle, at certain points, part of the Castilian army fled. This happened in three waves: when half of the cavalry units died, when the first 5000 men from the entire army were lost and when the next 5000 men died. This has been considered in our model by removing the troops from the army units who are able to fight as if they were dead units, but still storing their positions and not subtracting them from the current unit number count. We then make these units run away from the battlefield by finding the closest enemy unit and making them move in the opposite direction. In certain simulations this allows us to have units that were surrounded by enemies and therefore unable to run away. In order to simulate the panic that the units abandoning the battlefield might be experiencing, we add some randomness to the path they were following.

C. Final Model

The battle is split into different phases, much like the original battle. The first phase consists of a cavalry charge by Castile, where their knights charged into the Portuguese, being slowed at a certain area right in front of the Portuguese vanguard (representing the Portuguese entrenchments). At this stage, since the Portuguese lances have a longer reach range than the Castilian cavalry (their lances are longer, hence the longer reach), they are able to strike at the cavalry for a long time before they are able to strike back. Concurrently, the Portuguese archers are firing at the cavalry meaning this first phase is a complete disaster for Castile. Around half of the cavalry flees the battle, entering a rout mode (visualised by frantic movement towards the edge of the map), which is intended to represent a mix of captured surrendering units, fleeing units who escaped, and fleeing units who were killed.

The second stage of the battle consists of a forward march by Castile's army. Once they reach the Portuguese, a complete engagement ensues, marking the start of the main battle. Castile's rapid loss of units, triggers a mass rout after 16% losses (corresponding to 5000 men in the actual battle) meaning that around half of their remaining forces flee the battle. The remaining forces continue fighting but after another 5000 men die, everyone routs.

D. The Numbers & Equations

Transforming real battle parameters into numbers proved a challenge, seeing as there is no direct relation between them. Looking at the Lanchester's Laws, we notice that they merely take into account the number of units fighting for each side and their "kill rate", which represents the number of enemy units each side can kill per time step. The choice of kill rate is arbitrary in a way, where if you know the outcome of the battle you can change it accordingly, whereas if you don't, it becomes harder to select. Within our model, through the addition of several different parameters, we can fine-tune each numerical input based on logical research-based decisions.

The amount of units in our model maintains the ratio of the original battle, with each unit on our plane representing 100 soldiers. The morale of the Portuguese side is higher than the Castilian for several reasons: Portugal was fighting to maintain its sovereignty; Portugal was on the defending side of the battle; The Castilian army had recently failed to siege Lisbon; The Castilian army had suffered from the plague.

Each unit has a base health of 1, with each successful hit reducing it by 0.05. These numbers are arbitrarily chosen, as "damage" is not modelled. Instead, the chance of hitting will determine how fast a unit can defeat another, modelling how well it would perform in battle, having a similar effect to damage increases and decreases (where a damage increase would mean reducing more opponent health on hit and damage decrease a reduction). This also adds a level of randomness to the battle which would have also been a factor in real life.

The probability of hitting an opponent is determined by skill, morale and unit type. The equation was made up by us and looks as follows:

$$P_{hit} = skill^2 \cdot morale \cdot k$$

All the parameters have values between 0 and 1, which assures that the probability is also $\in [0, 1]$. The k parameter is a special number, whose value depends on which unit type attacks which, due to the fact that some units are stronger or weaker against others. The dependence is as follows:

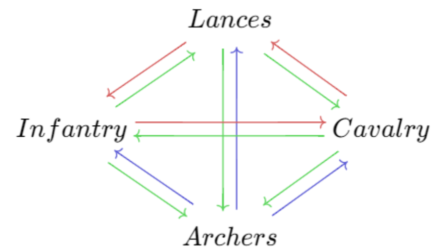

FIG. 1. Dependence between units: green arrow means that the unit is stronger against the given type and red that they're weaker. The blue arrow means normal strength.

Skill is meant to represent the individual abilities/training of each soldier. In an army, each individual's aptitude for battle would vary, taking into account many factors, such as training, personal interest, quality of weapons, and armour. We expected the biggest number of soldiers to be similarly trained and hence on a similar level, but also a few that are exceptionally good or bad. Due to this, we wanted normally distributed values between 0 and 1. However, since the Gaussian distribution could give us values outside this range, we applied the von Mises distribution, a continuous probability distribution function that gives values between 0 to 2π. In this case, we took a mean of π with a concentration of 0.8, and then divided all the numbers by 2π.

Morale is a very important parameter, especially in our battle. The equation we created for it is the following:

$$morale = \begin{cases} \left(40 - N_{1\ dead} \cdot \frac{40}{N_{1\ total}} \cdot 0.5\right)/40 & \text{– Portugal} \\ \left(30 - N_{2\ dead} \cdot \frac{30}{N_{2\ total}}\right)/40 & \text{– Castile} \end{cases}$$

Initially, both armies (each soldier) have a certain value set as morale – 40 for Portugal and 30 for Castile (both are normalised, so divided by 40). It decreases as soldiers die, which represents the remaining soldiers losing faith in the battle as their peers die – this is dynamic and updated at each time step. This decreasing factor is multiplied by a half for Portugal, so the Portuguese soldiers would never lose all of their morale, contrary to Castile.

E. Relevance and Usage of the Model

Firstly, our model can help scholars gain a better understanding of military conflicts and how factors like unit types, initial distribution, tactics, soldiers' morale and other parameters can affect the outcome of a battle. This can be particularly beneficial for those studying military history or those analysing current conflicts from a political perspective.

Secondly, the model can assist historians and researchers in reconstructing past battles with greater detail. By running different simulations and testing hypotheses, researchers can develop a more comprehensive understanding of what might have happened under different circumstances. This can be especially helpful for battles where the historical record is incomplete or where there are varying theories about what occurred.

Lastly, when comparing it to the existing theory, namely Lanchester's laws, it provides a more detailed and in-depth list of editable parameters.

III. FIGURES

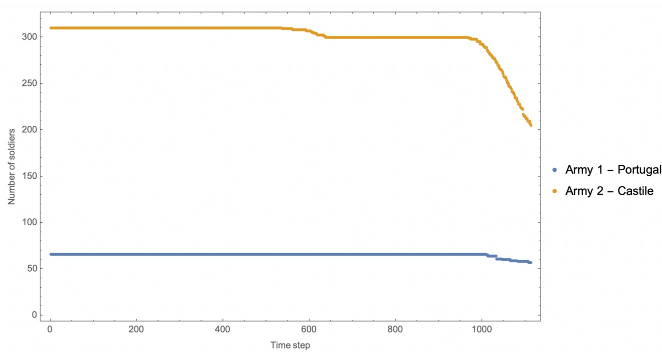

FIG. 2. Number of alive soldiers throughout the simulation

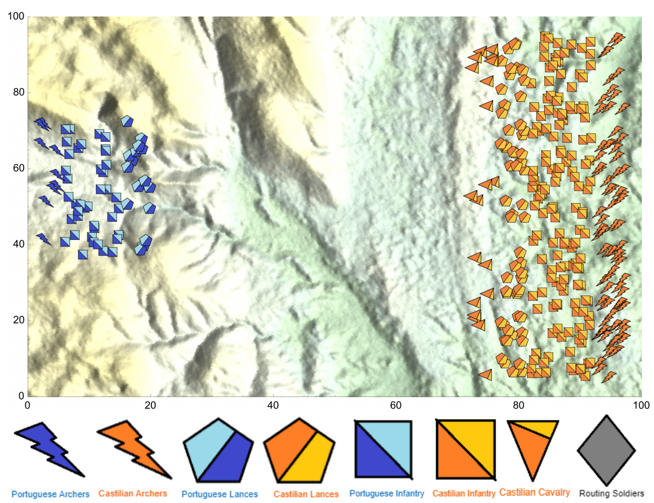

FIG. 3. Initial distribution of forces - Timestep 1 - and Troop Legend

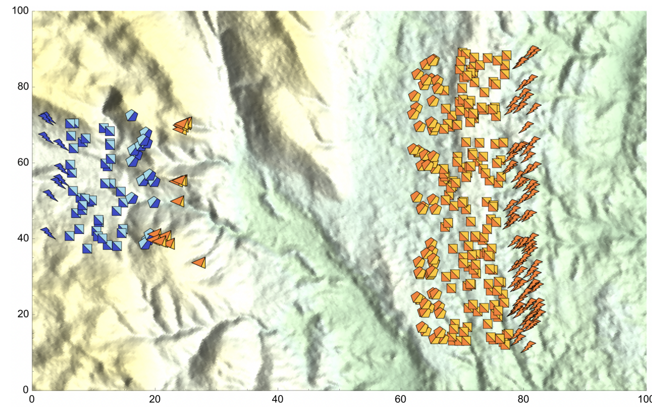

FIG. 4. Phase 1 - Castilian cavalry charge - Timestep 552

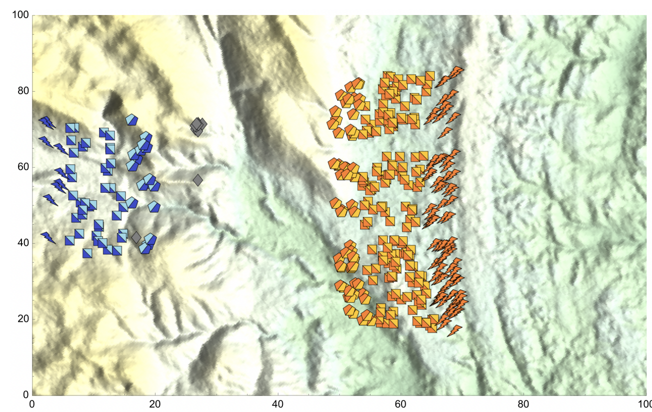

FIG. 5. Phase 2 - Castilian army charge - Timestep 684

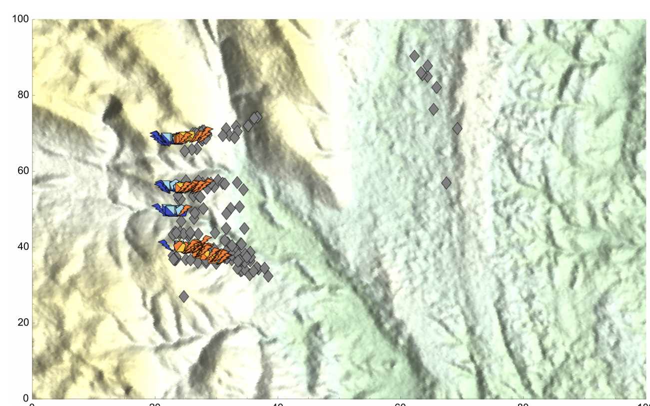

FIG. 6. Start of Castilian mass rout - Timestep 1091

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