Homework 1 Report

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Exercise 1

A Scatter plot of the Collatz's function's stopping time as a funcion of starting integer n is shown in figure 1.

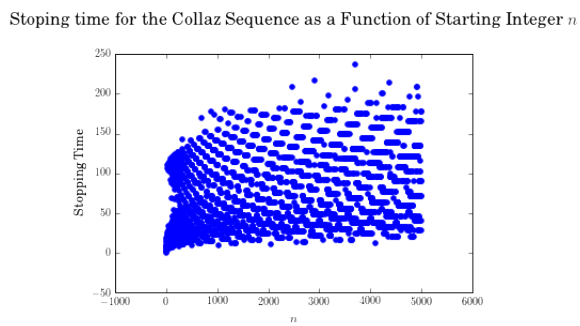

Figure 1: Collatz's function's stopping time as a funcion of starting integer n: (1,5000)

The Collatz conjecture states that no matter which n $\in \mathbb{N}$ *you begin with the sequence of integers nk will be finite and end with the number one.*

From figure 1 we can see for integers 1 to 5000 inclusive the conjecture holds, as they all have finite stopping time. However, we can see a general trend of increasing stopping time for increasing n. Considering this its seems less like the conjecture would hold for increasing n. We inestigate this further by plotting the stopping time as a function of n for integers 5000 to 10000 inclusive, shown in figure 2.

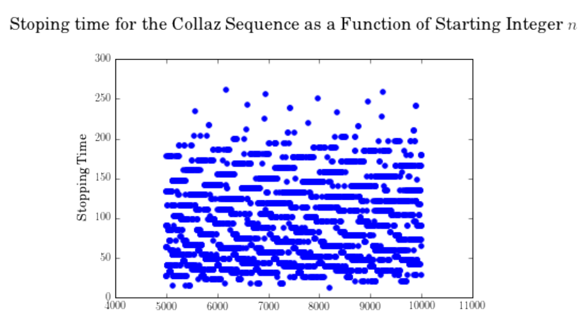

Figure 2: Collatz's function's stopping time as a funcion of starting integer n: (5000,10000)

We can see from figure 2 that for n between 5000 and 10000 inclusive that the stopping time does level off as a function of n. This suggests that the conjecture may be true. However, to be sure we would have to check all integers, which is impossible. Hence from these plots we cannot say definitively if the conjecture is true.

Exercise 2

We plot iterations of the gradient descent method for the function Cos(x) to find the one of its local minima. We start with and initial guess of $x = 0.1$ and compute the gradient descent method with $\sigma = 0.6$. This is shown in figure 3.

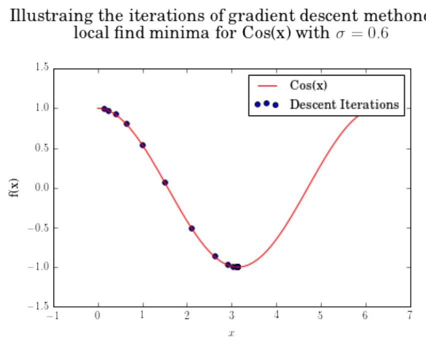

Figure 3: Iterations of the gradient descent method for Cos(x) with inital guess of $x = 0.1$ and $\sigma = 0.6$ to find a local mininma.

If $\sigma > 1$ then the gradient descent method would take larger steps in each iteration. This may result is the method overshooting the local minima, and oscillating about it before settling down, which increases the run time of the method. Also the method could completely miss the local minima all together.

Exercise 3

We plot the residuals r_k for the Jacobi and Gauss Seidel iterative solvers as a function of iteration number k. We have the convergence radius ϵ set at 1×10^{-8}. This plot is shown in figure 4.

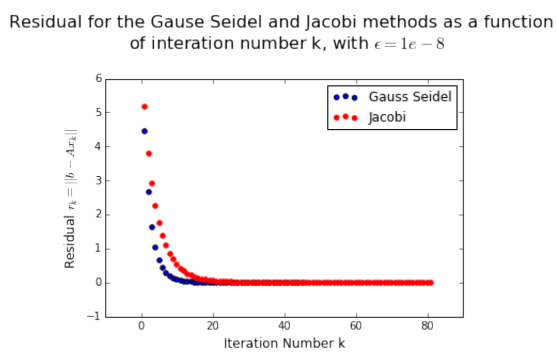

Figure 4: residuals r_k for the Jacobi and Gauss Seidel iterative solvers as a function of iteration number k. $\epsilon = 1 \times 10^{-8}$.

We can see from figure 4 that the gauss Seidel method converges in less iterations. This is because it flattens off before the Jacobi method.

Now we plot the same data as figure 4 but we decrease ϵ to 1×10^{-20}. This is shown in figure 5.

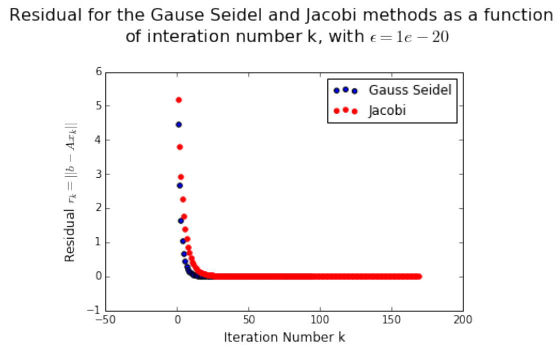

Figure 5: residuals r_k for the Jacobi and Gauss Seidel iterative solvers as a function of iteration number k. $\epsilon = 1 \times 10^{-20}$.

From figure 5 it still appears that gauss seidel converges faster than the Jacobi, and there is no real difference from figure 4. It is clear that decreasing ϵ increases the numeber of iterations for both the Jacobi and Gauss Seidel methods.